# Chapter 14
# Designing Instructional Materials to Help Students Make Connections: A Case of a Singapore Secondary School Mathematics Teacher's Practice

**Wei Yeng Karen Toh, Yew Hoong Leong, and Lu Pien Cheng**

**Abstract** It is widely acknowledged that making connections is an important part of learning mathematics—instead of seeing mathematics as comprising merely isolated procedures to follow, it is desirable that students learn the distinctiveness of mathematics as being a tightly connected subject. In fact, the Singapore mathematics curriculum framework listed "connections" as part of mathematical processes—one of the five areas of major foci. In the study reported here, we look specifically at how an experienced and competent secondary mathematics teacher listed "making connections" as one of her ostensible principles in the design of the instructional materials for her lessons on quadratic equations. The method used can be summarised as one of progressive widening of the analytical lens: we started by conducting an in-depth examination of one unit of her instructional material to uncover the connecting strategies she built into it. Based on the strategies we uncovered, we widened the analysis to include its adjoining unit. From here, not only did we test the applicability of these strategies on the next unit, we also explored her design principles on how she connected between units. Finally, we further widened the lens of focus to the whole set of instructional material to study other connecting strategies she used across all the units in the material. The four design principles she used are: connections across multiple modes of representation, conceptual connections, temporal connections, and connections across different solution strategies. This teacher's design principles with respect to making connections challenge conventional stereotypes of how Singapore mathematics teachers carry out instruction—it is not merely repeated practice of unrelated procedures; rather, it is a careful structuring of instruction such that the underlying mathematical connections are made explicit. Not only so, the deliberate design was not just carried out in-class; it was, as reflected in the

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careful crafting of the instructional material, an intentional plan prior to the teaching of the unit. The principles used by the teacher hold useful lessons for mathematics teachers, especially within the context of teacher professional development. These are discussed towards the end of the chapter.

**Keywords** Making connections · Secondary mathematics · Instructional materials

## 14.1 Introduction

This chapter reports a case of how an experienced and competent secondary mathematics teacher, Teacher 8, designed her instructional materials to "link everything together" (Post-Lesson Interview after Lesson 5, 00:18:20). It is this remark, coupled with ten other phrases with equivalent meanings that she made throughout her interviews, that intrigues us and motivated us to embark on our inquiry. We are curious to investigate the way she designed her instructional materials to achieve this goal. When we examined her instructional materials, we found that she had designed some of her tasks for the purpose of explicating certain intended *connections*. Hence, we surmise that her conscious intents to create links within the topic is an undergirding design principle that she applied in crafting the instructional materials. During our inquiry, we also found other teachers in the project, though not as ubiquitous as compared to Teacher 8, that consciously included making connections in the design of instructional materials. This is shown in Table 11.6 of Chapter 11.

A quick scan of the literature shows that there are researchers who had examined how mathematics teachers can teach in an interconnected manner (e.g. English & Halford, 1995; Hill, Ball, & Schilling, 2008; Ma, 1999; Pepin & Haggarty, 2007; Sun, 2019). Pepin and Haggarty (2007) for instance, reported on English, French, and German lower secondary textbooks containing tasks which provide opportunities for students to learn mathematics through making connections. They asserted that if we assume that "learning with understanding is enhanced by making connections, then mathematical tasks should reflect this" (p. 1). And in Sun's (2019) study, she clarified how Chinese textbooks make connections between whole numbers and fractions. However, little is reported about how mathematics teachers design instructional materials with a deliberate goal of helping students see the mathematics they learn in a connected way. Therefore, we are motivated to uncover how Teacher 8 incorporated *connections* in the design of her instructional materials. We begin by reviewing some literature on the *connectionist perspective* and how mathematics teachers *make connections* before describing the details of the case study.

## 14.2 Teaching with a Connectionist Perspective

In their study on the standard algorithms for the four basic arithmetic operations, Raveh, Koichu, Peled, and Zaslavsky (2016) implemented a framework with a *connectionist perspective*. This perspective is built on the recommendations of mathematics education researchers who highlight the importance of teachers' competency in perceiving different interconnections among the mathematics topics they teach (e.g. English & Halford, 1995; Hill et al., 2008; Ma, 1999). Ma (1999) underscored the need for mathematics teachers to have a "thorough understanding" of mathematics. She stated that it is best for teachers to be able to make connections within mathematics with both "depth" and "breadth"—that is, to make connections within and across topics. Likewise, English and Halford (1995) emphasised the importance for mathematics teachers to know the connections within the curriculum so as to provide sufficient connections between mathematical procedural skills and conceptual knowledge in their lessons for students. This is so that students will be less prone to develop difficulties in their learning.

The connectionist perspective is traced back to Askew, Brown, Rhodes, Wiliam, and Johnson (1997) who wrote about three *orientations* that mathematics teachers generally possess: transmission; discovery; or connectionist. A teacher with a transmission orientation views mathematics as a series of facts and algorithms that must be imparted to students and he/she teaches in a didactic manner with an emphasis for students to attain procedural fluency in computational skills. A teacher with a discovery orientation views mathematics as pieces of constructed knowledge and he/she facilitates students' learning by encouraging them to explore solutions on their own. And a teacher with a connectionist orientation views mathematics as a linkage of concepts that he/she constructs collaboratively with students through discussions. These three orientations are "ideal" types and a typical teacher may possess a mixture of orientations.

A connectionist orientation is aligned to a commitment to both "efficiency" and "effectiveness" in mathematics—that is, that students become "numerate". A numerate student has the "awareness of different methods of calculation" and the "ability to choose an appropriate method" when he/she solves a problem. With this belief, a connectionist orientated teacher "emphasise[s] the *links* [emphasis added]" (p. 31) between various aspects of the mathematics curriculum so that students can acquire mathematical concepts that are related in tandem. Askew et al. (1997) described how a mathematics teacher taught a class of Year 6 students fractions, decimal fractions, percentages, and ratios in an integrated manner, rather than as separate topics. The students were given one value and they worked among the different forms of representations. As an evaluation, the teacher and students discussed the appropriate contexts in which each form of representation could be used.

Interestingly, from the transcriptions of the post-lesson interview after Teacher 8's fifth lesson, we find evidence that suggests that she is inclined to a connectionist orientation:

> [T]he big idea I was trying to drive at in this lesson was really this part: helping my students *link* the completing the square method and the [quadratic] formula because I think this [quadratic] formula is often taught as the teacher telling the students … (Post-Lesson 5 Interview, emphasis added, 00:02:59)

This motivated our study of Teacher 8 as a case of using instructional materials to support her connectionist agenda. However, we do not claim that she was aware of the connectionist theory. It is plausible that she had designed her instructional materials primarily to help her students make the connections within the topic better. In the next section, we list some specific strategies for making connections explicated in the literature. Some of these strategies were also employed by Teacher 8, and will be elaborated in the Findings section.

## 14.3 Making Connections

Mathematics teachers worldwide are encouraged to incorporate connections to deepen students' understanding of concepts (Fyfe, Alibali, & Nathan, 2017; Ma, 1999; Turner, 2015). For instance, the National Council of Teachers of Mathematics (NCTM, 2000) encourage students from Grades 9 through 12—between the ages of 14 and 19—to "develop an increased capacity to *link* mathematical ideas" (p. 354). Likewise, Singapore's mathematics curriculum framework advocates *connections* as one of the processes for proficient problem solving; and one of the aims of the secondary mathematics syllabus is to enable students to "*connect* ideas within mathematics …" (Ministry of Education, 2012, p. 8). This emphasis of making connections in mathematics is important because "mathematical meanings are developed by forging connections between different ways of experiencing and expressing the same mathematical ideas" (Healy & Hoyles, 1999, p. 60).

The specific strategies of making connections that we discuss in this chapter as described from the literature are: (1) connections across multiple modes of representations; (2) conceptual connections; (3) temporal connections; and (4) connections across different strategies to solve problems.

### *14.3.1 Connections Across Multiple Modes of Representations*

Mathematical concepts are naturally abstract (De Bock, van Dooren, & Verschaffel, 2015). Thus, representations are made to communicate their meanings. However, according to Duval (2006), no single representation can entirely elucidate a mathematical concept so multiple modes of representations are required to help facilitate students' understanding. When multiple modes of representations are used, students are able to harness the different advantages each representation offers. Thus, many different modes of representations which complement each other are typically required for the development of an idea (e.g. Ainsworth, 2006; Elia, Panaoura, Eracleous, & Gagatsis, 2007; Tall, 1988). Studies have also shown that when teachers make connections across multiple modes of representations, they can facilitate greater understanding for students because they emphasise the conceptual connections (more in the next section) among the representations (Crooks & Alibali, 2014; Rittle-Johnson & Alibali, 1999). As an example, Dreher, Kuntze, and Lerman (2016) described vividly the use of multiple modes of representations for "$\frac{3}{4}$" such that students can have a comprehensive conceptual understanding of this fraction.

### *14.3.2 Conceptual Connections*

Teachers can also help students learn mathematics in a connected way by helping them make conceptual connections. Leong (2012) described how connections can be made when a teacher extends the ideas students had learnt in a prior topic to a current one. He suggested that the ideas in the topic of "Special Quadrilaterals" which students learn in Year 7 in Singapore can be extended to the ideas in the topic of "Cyclic Quadrilaterals" which they will learn in Year 9. Similarly, teachers can lead students in Years 9 and 10 respectively to realise that the algorithm for computing the length of a line segment and the magnitude of a vector, respectively, is actually based on Pythagoras' Theorem which they would have learnt in Year 8. As such, it connects the concept of length of line segment on the Cartesian plane to the concept of Pythagoras' Theorem.

### *14.3.3 Temporal Connections*

Even though a single lesson is frequently regarded as a unit for teaching and planning, teachers tend to take into consideration the planning for a topic as a module over a series of lessons. As stated by Leong (2012), "teachers think about the content suitable for a lesson in terms of what goes *before* and what is to come *after*" (p. 244). In the language of "connections", the components in a lesson will not only connect with one another within itself, but they will also be linked to what precedes and follows in prior and subsequent lessons. He described how a Year 9 topic in Singapore on "angle properties in a circle" is usually taught in an interconnected manner such that students could see the connections among the four theorems—(i) angle at the circumference is twice angle at the centre; (ii) angle at semicircle; (iii) angles in the same segment; and (iv) opposite angles in a cyclic quadrilateral—taught over a few lessons. From this example, he also highlighted that the underlying instrumental link for temporal connections is time—in the chronological sense of it. It is over time that the connections across the four theorems in the topic are made consistently.

Conceptual connections and temporal connections appear similar as both involve connecting prior knowledge to new knowledge. However, there is a difference. When teachers make conceptual connections of an idea, it need not be developed over a series of lessons bounded by a specific period, but when teachers make temporal connections of an idea, this idea is being morphed chronologically over several lessons within an extended duration of time.

### *14.3.4 Connections Across Different Methods to Solve Problems*

Students can learn to make connections within mathematics by solving problems using different methods (Fennema & Romberg, 1999; Leikin & Levav-Waynberg, 2007; Toh, 2012). During this process, mathematical knowledge is constructed when students shift between representations, comparing methods, and connecting different concepts and ideas (Fennema & Romberg, 1999). Toh (2012) suggested that this could be achieved by teaching students to use different methods to solve the same problem. He illustrated his point by describing how the solutions to a rich problem can be used as a summary to link several topics together. He urged teachers to adopt this strategy so that students who perceive mathematics as a fragmented subject can learn to appreciate its connectedness.

## 14.4 Method

Teacher 8 was one of 30 experienced and competent teachers who participated in the first phase of the project detailed in Chapter 2. As mentioned briefly at the start of this chapter, the choice of Teacher 8 as a case study of making connections was predominantly because she articulated that one of her teaching goals was to "link everything together". In addition, other factors about Teacher 8's practices lends itself to a rich unpacking of her work—a characteristic feature of case study: (1) During interviews, she expressed comprehensively her objectives for many tasks. This allowed us to uncover her intentions behind the activities we recorded in her classroom; (2) she crafted a full set of handouts for students' use in class (hereafter referred to as "Notes") before the start of the module. In other words, her work generated a rich set of instructional materials on which to ground our study; (3) she constantly made references among her objectives, her actual activities in class, and her use of instructional materials. This allowed us to study the interactions among these major pieces of her instructional processes.

The class that Teacher 8 taught as resident teacher was a Year 9 Express class. It comprised 39 students whose age range was 14–16 years old. The module that she taught was "Quadratic Equations". The contents—as stipulated by the Ministry of

Education (2012)—that she had to cover were: (i) solving quadratic equations in one variable by (a) the use of formula, (b) completing the square for $y = x^2 + px + q$, and (c) the graphical method; (ii) solving fractional equations that can be reduced to quadratic equations; and (iii) formulating a quadratic equation in one variable to solve problems.

### *14.4.1 Data*

Under instructional materials, Teacher 8 used mostly the set of Notes she designed for her students. This forms the first primary source of data. The next source of data is the set of transcripts of interviews we conducted with Teacher 8. We conducted one pre-module interview before she conducted her suite of lessons and three post-lesson interviews after Lessons 2, 5, and 8, based on her selection. All interviews were video recorded and transcribed verbatim. We designed an interview protocol with two sets of questions and prompts respectively for the pre-module interview and post-lesson interviews.

The pre-module interview was conducted to find out what Teacher 8's instructional goals were and how she had designed and planned to utilise her instructional materials to fulfil her goals. Some prompts in the pre-module interview were:

- Please share with me what mathematical goals you intend to achieve for this set of materials that you will be using.
- How different is this set of materials that you developed compared to those in the textbook?
- Are there any other specific instructional materials that you are going to prepare for this module?

The post-lesson interviews were conducted to find out if she had met her instructional objectives with the instructional materials she had designed and planned to use. Some of the questions were:

- Did you use all the materials that had you intended to use for the lesson?
- How did the materials help you achieve your goals for this lesson?

The third source of data is Teacher 8's enactment of her lessons in the module. We adopted non-participant observer roles during the course of our study. That is, one researcher sat at the back of the class to observe Teacher 8's lessons. This was so that the researcher would be able to make relevant and precise references to her teaching moves when pursuing some threads during the post-lesson interviews. A video camera was also placed at the back of the class to record Teacher 8's actions. We recorded a total of eight lessons. Three were 60-minute lessons while rest were 90-minute lessons.

### 14.4.2 Analysis of Data

We proceeded with the analysis along these stages:
*Stage 1: Identification of units of analysis of the Notes*
Each unit is a section in the set of Notes prepared by Teacher 8 (e.g. “Factorisation Method”, “Graphical Method”, “Completing the Square Method”, “Quadratic Formula Method”, “Thinking Activity”, etc.). We coded the units according to the mathematical contents targeted in each unit. We matched the comments in Teacher 8’s pre-module interview according to the references she made to these units. Together with the coded content, we were better able to verify the instructional objectives intended for each unit.
*Stage 2: Composition of chronological narratives*
For some of these selected units with rich related data on Teacher 8’s enactment and interview comments, we crafted chronological narratives for each of them. These are narratives that coherently bring together related data sources for that particular unit of analysis. In each chronological narrative, we integrated several data sources—pre-module interview transcriptions, post-lesson interview transcriptions, tasks in her Notes, and her classroom vignettes. The chronological narrative for “Completing the Square Method”, for instance, was composed by first examining the text in the pre-module interview. As we found her commenting at length about how she planned to develop the concept of “completing the square” with her Notes, we validated her intentions for designing the mathematical tasks and questions by examining the unit on “Completing the Square Method” in her Notes. After which, we proceeded to search the video recordings of the related lessons she conducted for evidence to corroborate her use of the instructional materials. We consolidated the evidence and organised them in a table. Table 14.1 presents the evidential ingredients for building the chronological narrative for “Completing the Square Method”.
*Stage 3: Strategies related to making connections*
We begin specifically to look for the strategies that Teacher 8 used to make connections by closely examining the chronological narrative on “Completing the Square

**Table 14.1** Main evidence leading to the building of the Chronological Narrative of the Unit on “Completing the Square Method”

| Chronological sequence | Main data source | Description |
|---|---|---|
| 1 | Pre-Module Interview | • Explained rationale for the way she designed the tasks in the unit on “Completing the Square Method”<br>• Explained how she planned to help her students connect their prior knowledge on perfect squares in lower secondary to the new knowledge on completing the square |

(continued)

**Table 14.1** (continued)

| Chronological sequence | Main data source | Description |
|---|---|---|
| 2 | Lesson 3 Video Recording and Notes | • Elicited students' prior knowledge on perfect squares<br>• Elicited students to illustrate pictorially the squares of 7, $(x + 1)$, and $(x - 2)$ on their Notes (as shown in Fig. 14.3)<br>• Emphasised that students have to make sense of "perfect squares" algebraically and pictorially<br>• Assigned students to express the squares of 7, $(x + 1)$, and $(x - 2)$ in words (as shown in Fig. 14.4)<br>• Assigned students to work in groups to give examples of "perfect squares"<br>• Conducted class discussion on the examples given by each group<br>• Explained geometrically the meaning of $(a + b)^2$ and $(a - b)^2$<br>• Assigned students to work in pairs on Task A2 (as shown in Fig. 14.5)<br>• Contrasted 120 to 121 by illustrating 120 as an "incomplete square" of side 11 on white board (as shown in Fig. 14.6)<br>• Utilised table (as shown in Fig. 14.7) in Notes to help students connect the concept of completing the square algebraically and geometrically<br>• Explained the first row of entry in the table for $x^2 + 2x$ by redrawing the diagram and relating to the algebraic expressions<br>• Assigned students to complete the table as homework |
| 3 | Lesson 4<br>Video Recording and Notes | • Conducted class discussion for the homework assigned at the end of Lesson 3<br>• Stressed that students have to connect the geometrical representations to the algebraic expressions so as to gain conceptual understanding of the "completing the square method" |

(continued)

**Table 14.1** (continued)

| Chronological sequence | Main data source | Description |
|---|---|---|
| 4 | Lesson 5<br>Video Recording and Notes | • Conducted class discussion to help students recall the algorithm for the "completing the square method" and generalise the theorem<br>• Conducted class discussion on the practice items on p. 5 of the Notes to help students learn to apply the method<br>• Assigned students to complete practice items on p. 6 to p. 8 |

Method". This chronological narrative was chosen as a first-entry study because it is one where Teacher 8 articulated that she "actually took great trouble to prepare [the] worksheets" (Pre-Module Interview, 00:03:17). This chronological narrative became an intensive source of analysis for emerging themes related to her strategies in making connections. We underwent many rounds of discussions, conjecturing, refuting, and re-conjecturing until we reached stability in agreement among the members of the research team (authors of this chapter)—where the purported strategies could be substantiated from all the data sources. Figure 14.1 shows the various units of analysis. It also highlights that the chronological narrative on "Completing the Square Method" is the first in the process of analysis. The report of this analysis is given in Sect. 14.5.1.

*Stage 4: Confirmation and expansion of strategies*

In the final stage of analysis, we examined the preliminary strategies we conjectured in Stage 3 to check it against two other chronological narratives following this process: we repeated the process of the analysis as in Stage 3 on "Quadratic Formula Method"; we then drew connections between these two adjacent units of analysis (this stage of analysis is presented in Sect. 14.5.2); finally these conjectures were further refined as we examined across a number of units of analysis (the next stage is presented in Sect. 14.5.3). These sequential phases of analysis are also presented diagrammatically in Fig. 14.2.

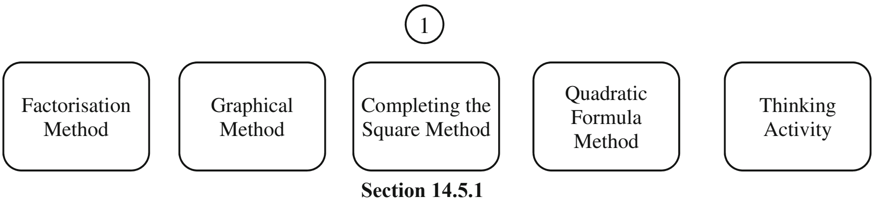



**Fig. 14.1** Diagram showing different units of analysis and the first unit of analysis

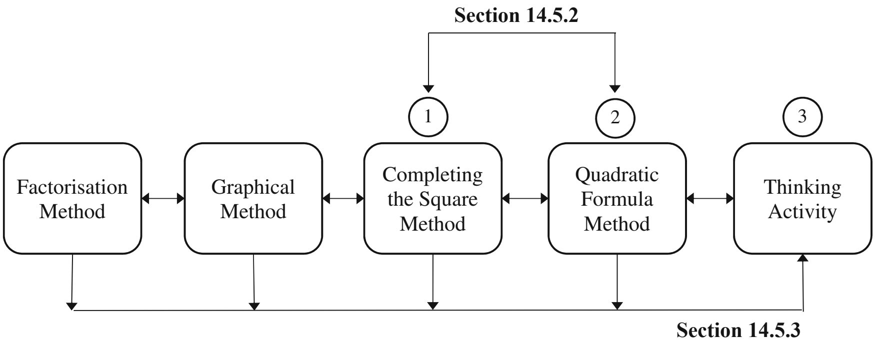



**Fig. 14.2** Diagram illustrating analyses across different units of analysis

## 14.5 Findings

### *14.5.1 Making Connections Within a Unit*

In the Notes that Teacher 8 prepared for the unit on "Completing the Square Method", she designed three sections which we labelled: A, B, and C. We focus our report on certain tasks in Sections A and B whereby she had applied one or more strategies to make connections to develop the completing the square method. Our analysis will exclude Section C as it comprises mainly of practice items.

When we first looked at Task A1, (as shown in Fig. 14.3), we were curious as to why it was designed in this manner. We noticed that in the first column, the top row had "$5^2$" written in it, and in the bottom row, there was a diagram of a square with sides of 5 units. We also noticed that throughout the four columns, the top row was presenting a kind of symbolic representation; and students were expected to produce a geometrical representation. The diagram in the first column had been provided to them as an example. It appeared that Teacher 8 designed in it such a way that students could revise the meaning of squares of numbers and then connect them to the geometrical meaning of squares with areas of given sides. We noticed her

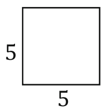
| A1. Draw a geometric representation of each of the following. The first one has been done for you. | | | |
|---|---|---|---|
| $5^2$ | $7^2$ | $(x+1)^2$ | $(x-2)^2$ |
|  | | | |

**Fig. 14.3** Task A1 in Section A of Notes on "Completing the Square"

deliberate design for students to relate symbolic terms to geometric figures. Also, this activity extends to squares with sides that involve algebraic expressions. At this point, we saw explicitly what she meant by "to link everything together"—numeric to geometric; algebraic to geometric; and from numeric numbers (left) to symbolic algebra (right)—and found obvious pieces of evidence for her use of **the strategy to connect across multiple modes of representations**. In addition, from her pre-module interview, we verified her intention when she expressed that her insertions of diagrams were "so that they *have the algebraic* procedure and they also *have the pictorial* representation of what they are doing algebraically" (Pre-Module Interview, emphases added, 00:02:23).

Subsequently, we noticed that she intended to connect numeric, algebraic, and geometric representations in Task A1 to *words* in Task A2, as shown in Fig. 14.4. Her instructions clearly stated: "Explain in *words* what each of the following represents with reference to its *geometric* representation"; and the first column of the table are the same numeric and algebraic expressions as those in Task A1. The sample statement in the first row of the table also exemplifies how she expected her students to explain in terms of "area of a square".

We surmise that she had purposefully designed Task A2 such that students could learn to use words to connect to numbers; and algebraic expressions to their geometrical representations so that students can make connections across multiple modes of representations. We validated her intention to connect across multiple modes of representations from her pre-module interview transcript:

> [F]or the worksheets right … first I [will] elicit prior knowledge: "What does it mean when you square a *number*? What does it mean [when] you square the *algebraic expression*?" … [T]hen after that I [will] try to get them to write in *words so* they [can] get used to the math language. … [A]fter that, I [will] show them the *pictorial representation* … (Pre-Module Interview, 00:14:52, emphases added)

The second task which caught our attention was Task A6. From the way the task was designed, we infer that it was a continuation to help students make connections across

| A2. Explain in words what each of the following represents with reference to its geometric representation. The first one has been done for you. | |
|---|---|
| $5^2$ | $5^2$ represents the area of a square with sides of 5 units in length |
| $7^2$ | |
| $(x+1)^2$ | |
| $(x-2)^2$ | |

**Fig. 14.4** Task A2 in Section A of Notes on "Completing the Square"

multiple representations. The table that Teacher 8 had drawn up as shown in Fig. 14.5 was to get her students to discern if the numeric and algebraic expressions in the first column (on the left) were perfect squares. She expected them to write in words in the third column the reasons for their conclusion. She had provided sample statements for the first and second numbers—81 and 120. She had planned for students to state whether the given expressions in the first column could be "expressed as $k^2$".

However, upon closer analysis, we notice another strategy being used in this task—Teacher 8 intended to **help her students make conceptual connections**. She designed this task to help her students make sense of the concept of "incomplete squares" to the concept of "perfect squares"—which they had learnt previously in Years 6 or 7. She developed the concept of an "incomplete square" from the associated concept of "perfect squares" in her lesson by discussing the number "120" and highlighting the difference between "121" and "120". She elicited from students that "121" was a perfect square—that is "$11^2$"—but "120" was not. To make a geometric connection to this number, she illustrated "120" as a square with sides 11, but was one that was "short of that little bit" (Lesson 3, 01:19:13). The diagram she drew on the board is shown in Fig. 14.6. She went on to ask her students: "If I want to make 120 into a perfect square, what shall I do?" (Lesson 3, 01:19:40). Her point was to show students that the concept of completing the square was to complete an 'incomplete' square by adding on a small square with a specific side. So for the case of "120", she explained that she would have to add a small square of sides 1 to make "120" a complete square with sides 11; and then she said: "So this is the idea behind completing the square" (Lesson 3, 01:19:54).

Teacher 8's attempts to make conceptual connections can also be seen from the entries in the fourth and fifth row in the first column. She selected "$x^2 + 2x + 1$" as an introductory example because it is the expanded quadratic expression of the most basic polynomial in the $(ax + b)^2$ form. And this expanded quadratic expression for $(x + 1)^2$ can be expressed in terms of a perfect square, "$k^2$". The difference between "$x^2 + 2x + 1$" (in the fourth entry) and "$x^2 + 2x$" (in the fifth entry) is 1—just

| Expression | Perfect Square Expression? (Yes/No) | Reason |
|---|---|---|
| 81 | Yes | Can be expressed as $9^2$ |
| 120 | No | Cannot be expressed as $k^2$, where $k$ is an integer |
| 100 | | |
| $x^2 + 2x + 1$ | | |
| $x^2 + 2x$ | | |

**Fig. 14.5** The first five rows of Task A6 in Section A of Notes

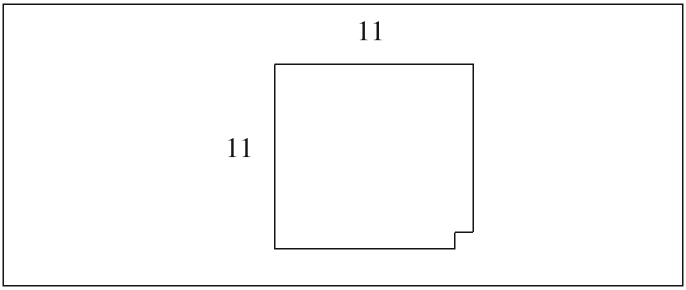



**Fig. 14.6** Diagram which Teacher 8 drew on the whiteboard to illustrate 120

like the difference between "121" and "120". In other words, "$x^2 + 2x + 1$" is a 'perfect square' like "121" but "$x^2 + 2x$" is an 'incomplete square' that is short of a square of side 1, just like "120". She made this careful selection so as to help her students "construct the *new* knowledge [of an 'incomplete' square] by *connecting* to *prior* knowledge [of a perfect square]" (Pre-Module Interview, emphases added, 00:15:17).

Teacher 8's use of both strategies to make connections across multiple modes of representations and to make conceptual connections continues for the third time in Section B of her Notes. For this section, she created a table. As shown in Fig. 14.7, the table had four columns. We name them as Columns B1, B2, B3 and B4 (from left to right) for easy reference. Teacher 8 designed the table such that Column B1 is

When we write $x^2 + bx + c$ in the form $(x - h)^2 - k^2$ where $b$, $c$, $h$ and $k$ are real numbers, we are **completing the square**. Study the examples shown and complete the table below.

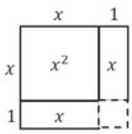
| Expression | Geometric Representation | Term to be Added | Algebraic Representation |
|---|---|---|---|
| $x^2 + 2x$ |  | $1^2 = 1$ | $x^2 + 2x$<br>$= x^2 + 2x + 1^2 - 1^2$<br>$= (x + 1)^2 - 1$ |
| $x^2 + 4x$ | | | |

**Fig. 14.7** Table in Section B of Notes showing first two rows

for algebraic expressions; B2 for geometric representations; B3 for students to write down a "term to be added"; and B4 for algebraic representation.

Based on surface analysis of the table, we note from the entries in the first row how she had intended to let her students see that the algebraic expression $x^2 + 2x$ in B1 could be represented geometrically with a diagram as shown in B2. The diagram of an incomplete square in B2 was intended to help students visualise that if $x^2 + 2x$ were to be represented as a square with side "$x + 1$", there would be a missing corner. And this corner is actually a small square with side of 1 unit—that is, $1^2$—to sensitise students to the need to add this $1^2$ to "complete the square". This information is contained in B3. The algebraic working in B4 is the algebraic documentation of what goes on in B2 and B3.

Upon careful inspection of the entries, we realise that Teacher 8 designed the table to harness the connections she had made in the earlier tasks. There were links across multiple modes of representations—from algebraic to geometric (from B1 to B2), geometric to numeric (B2 to B3), and geometric plus numeric to algebraic (B2 + B3 to B4)—just like those in Tasks A1 and A2.

In addition, she carefully linked Section B to A by deliberately repeating the choice of $x^2 + 2x$ as the first entry. This was the same entry in the fifth row of Task A6. And the geometric representation of this expression—an "incomplete square" of sides $x + 1$—corresponded to the perfect square "$(x + 1)^2$" which is the third entry in both Tasks A1 and A2. Figure 14.8 explains how the algebraic working in Column B4 connects to the other tasks.

When we link up all the details in our analysis in this unit, we uncover aspects of how Teacher 8 planned to "link everything together" using her instructional materials. She crafted her tasks within this unit such that her students could make connections by progressing from stage to stage until they arrived at the concept for the completing the square method. The tasks in Section A set the background for what was to come

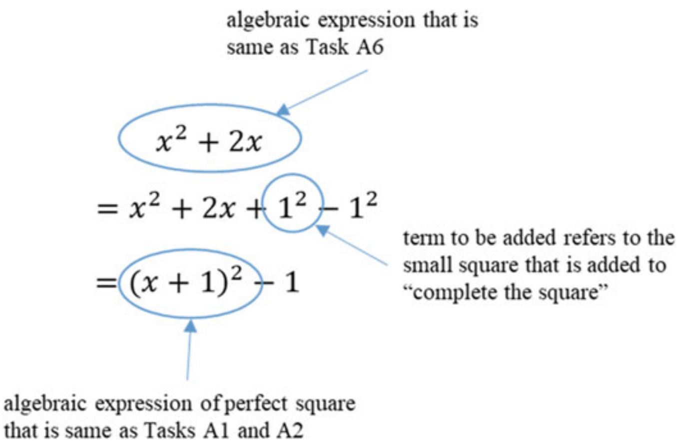



**Fig. 14.8** Breakdown of the algebraic working

in the table in Section B. She designed it such that when her students completed the tasks in Section A, they would be prepared to make the conceptual connections to the tasks that progress across the columns in the table.

### 14.5.2 Making Connections Between Adjacent Units

We proceed to analyse the next unit on “Quadratic Formula Method”. As details of the analysis process of a unit were given in the previous section, we will be brief in this section. The first page of this unit is shown in Fig. 14.9. The formula is in a text box, placed at the top of the page—occupying one-third of it—while two-thirds of the page is left blank. We did not fully understand how she intended to let her students “derive this formula by applying the completing the square method” until we uncover them from the transcriptions of her pre-module interview, post-module interview after Lesson 5 and that of Lesson 5.

From the videos recordings, we observed that Teacher 8 only started teaching the unit on Quadratic Formula in Lesson 5 after explaining three practice items from the earlier unit of Completing the Square: Solve (i) $x^2 - 16x - 4 = 0$, (ii) $x^2 + 5x + 4 = 0$, and (iii) $2x^2 + 15x + 10 = 0$. She had presented her solutions (with students’ participation) in three separate columns on the whiteboard sequentially from left to right. After which, she erased only her written solutions for items (i) and (ii), and left the solution of (iii) on the extreme right column on the whiteboard. We reproduce her actual working steps on the whiteboard for Item (iii) in Fig. 14.10.

**Quadratic Formula**

The roots of the general quadratic equation $ax^2 + bx + c = 0$ can be obtained by the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Let’s try to derive this formula by applying the completing the square method.

**Fig. 14.9** Task on first page of the unit on Quadratic Formula

$$2x^2 + 15x + 10 = 0$$

$$x^2 + \frac{15}{2}x + 5 = 0$$

$$x^2 + \frac{15}{2}x = -5$$

$$x^2 + \frac{15}{2}x + \left(\frac{15}{4}\right)^2 = -5 + \left(\frac{15}{4}\right)^2$$

$$\left(x + \frac{15}{4}\right)^2 = \frac{145}{16}$$

$$\frac{15}{4x +} = \pm\sqrt{\frac{145}{16}}$$

$$x = \pm\sqrt{\frac{145}{16}} - \frac{15}{4}$$

$$x = -0.740 \; or \; x = -6.76$$

**Fig. 14.10** Actual working steps for item (iii)

Upon analysis of Lesson 5, we discovered that she had planned to make use of the numeric workings of item (iii) from the unit on completing the square to help her students cope with the abstract and complex algebraic manipulations they had to handle when they derive the quadratic formula from the general quadratic equation $ax^2 + bx + c = 0$. We found her telling her students: "I want to show you what exactly I am doing here [Item (iii)] with number coefficients [as it] is exactly the same way as what you are doing here [with the general quadratic equation] with algebraic coefficients" (Lesson 5, 00:42:50). And in the post-module interview after Lesson 5, we found her explanation for leaving the working steps of item (iii) by the right-hand side of the white board—she articulated that she had placed it "side by side" (Post-Lesson Interview after Lesson 5, 00:05:02) to the derivation steps so that students "can see the parallel" (Post- Lesson Interview after Lesson 5, 00:05:02). From this vignette, we observe once again how Teacher 8 helped her students learn a concept **by making connections across multiple modes of representations**.

Besides this, Teacher 8 had another objective for leaving two-thirds of the page blank for her students to derive the quadratic formula by applying the completing the square method. She had planned this because she did not wish for them to "just memorise [the quadratic formula] blindly" (Pre-Module Interview, 00:03:24) and apply on practice items or problems. She wanted them to **make the conceptual connections** between the completing the square method and the quadratic formula. She stressed this in her pre-module interview when she said she "want[ed] them to listen to the conceptual development" (Pre-Module Interview, 00:18:51) before memorising and applying the quadratic formula. We verified her plan when we observed how she established the conceptual connections in Lesson 5. She had asked students to make close reference to item (iii) on the right-hand side of the board to help them derive the quadratic formula from the general quadratic equation.

Subsequently, we found that Teacher 8 used a third strategy in making connection between two adjacent units of analysis—she also incorporated **temporal connections** in her development of the quadratic formula. She had cautiously timed her lessons such that she would demonstrate the derivation of the quadratic formula from the general quadratic equation immediately after she completed item (iii). We found her explanation for this design principle during her pre-module interview that substantiates our conjecture:

> I actually took great trouble to prepare this worksheet … to help them appreciate this idea of completing the square. Then after that right, I will go on to the quadratic formula … and I want to show them how… [the] *formula is derived from completing the square*. That's why I *sequenced* the worksheets in this order …. (Pre-Module Interview, 00:03:15, emphases added)

### *14.5.3 Making Connections Across Units*

Teacher 8's goal of helping students make connections across units was to let them see the links across the whole topic of solving quadratic equation. This was observed in the third unit of analysis: "Thinking Activity". It was through this task sheet that we are able to observe how she "tie[d] everything together" (Pre-Module Interview, 00:02:28). She stressed in her pre-module interview that this task sheet was designed because her "ultimate goal [was] to help [students] appreciate the affordance and constraint of each method". And during the lesson when students were assigned to work on this task sheet, she explained to them that they were to "consolidate everything that [they] had learnt" (Lesson 7, 00:50:44) from the past few lessons.

There were altogether five tasks—Task 1 to Task 5—in this "Thinking Activity" task sheet. As we found Task 1 and Task 3 particularly interesting, we focused our analyses on them.

As shown in Fig. 14.11, Teacher 8 presented the solutions for the quadratic equation $x^2 - 4x - 5 = 0$ using four different methods. She articulated explicitly in her pre-module interview that she had purposefully displayed the solutions "side by side instead of sequentially" (Pre-Module Interview, 00:14:52) so that students could "make comparisons" (Pre-Module Interview, 00:08:35). She stated clearly in the instructions for Task 1: "discuss the pros and cons of the method, and give suggestions on *when* the method should be used".

We surmise that her intention of presenting the solutions using all the four methods was to demonstrate to students that the same problem could be solved by more than one strategy. In addition, she had probably wanted her students to learn to apply the most suitable strategy to solve a problem, depending on its context. It seems clear to us that her goal was to **make connections across different methods**. We validated our conjecture with the evidence we found in her pre-module interview and lesson transcript. In her pre-module interview, she emphasised her objective for designing this "Thinking Activity":

You can use one of the following four methods to solve the quadratic equation $x^2 - 4x - 5 = 0$.

**Method 1: By Factorisation**

$x^2 - 4x - 5 = 0$

$(x + 1)(x - 5) = 0$

$(x + 1) = 0 \ or \ (x - 5) = 0$

$x = -1 \ or \ x = 5$

**Method 2: By Completing the Square**

$x^2 - 4x - 5 = 0$

$(x - 2)^2 - 2^2 - 5 = 0$

$(x - 2)^2 - 9 = 0$

$(x - 2)^2 = 9$

$x - 2 = \pm 3$

$x = 3 + 2 \ or \ x = -3 + 2$

$x = 5 \ or \ x = -1$

**Method 3: By using the Formula** $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x^2 - 4x - 5 = 0$

$a = 1, b = -4, c = -5$

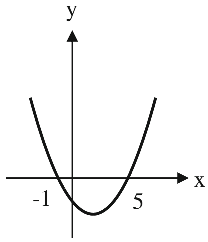
$x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-5)}}{2(1)}$

$x = \dfrac{4 \pm \sqrt{16 + 20}}{2}$

$x = \dfrac{4 \pm 6}{2}$

$x = 5 \ or \ x = -1$

**Method 4: By Graphical Method**




From the graph,
$x = 5 \ or \ x = -1$

<u>**Task 1**</u>: Working in pairs, take turns to discuss each solution with your partner. Each person is to talk about 2 solutions. You should describe the method used, discuss the pros and cons of the method, and give suggestions on when the method should be used.

**Fig. 14.11** Task 1 in "Thinking Activity" task sheet

> They are actually required to choose or identify key characteristics by themselves, they are supposed to *make comparisons*, they are supposed *to reason out why a method is more efficient than the other* … (Pre-Module Interview, 00:08:32.02, emphases added)

And during the lesson, she highlighted to students that they "must not rule out" (Lesson 7, 01:14:39) using another method even though they might prefer the factorisation method.

The other task that illustrated how Teacher 8 helped students make connections across units is Task 3 as shown in Fig. 14.12. In this activity, students were asked to select the "most efficient" method for each "question". Notice that Question 1 can be solved by any of the four methods but it would be most efficient to use the factorisation method. Subsequently, it would be most efficient to solve Question 2 using the quadratic formula method though it could also be solved by the completing the square method; and lastly, it would be most efficient to solve Question 3 by first dividing the equation by 2, then taking the square root for the equation, though one could also expand the left-hand side of the equation and then solve it by any of the other three methods. Nevertheless, one may also apply the quadratic formula method for every question without thinking about its "affordance and constraint". Hence, we infer that Teacher 8 had crafted Task 3 so that students could learn to regulate their

**Task**: Examine each of the following questions. Discuss with your partner and decide which method ***you prefer*** to solve each of the following questions. Justify your choices.

| No. | Question | Your Preferred Method | Reason(s) for Your Choice |
|---|---|---|---|
| 1 | Find the roots of the equation $2x^2 - 5 = 9x$. | | |
| 2 | Solve $2x^2 + 9x - 15 = 0$, giving your answers correct to 2 decimal places where necessary. | | |
| 3 | Solve the equation $2(x-5)^2 = 100$. | | |

**Fig. 14.12** Task 3 in "Thinking Activity" task sheet

understanding on the four methods and apply the most appropriate one for each question.

We think this task indeed requires students to think about the suitability of each method and not merely apply one method throughout mechanically. Teacher 8's responses in her post-lesson interview support our inference:

> [For] these particular set of task sheets, the content goal is really to help students to understand *when* they should be using which method. They need to have this appreciation for each [of the] different types of questions [where] some methods are more efficient than others …. That was the idea behind this worksheet. … … [W]hen they came to Task 3, they were *forced to make a choice* on which was their most efficient method, and I can see from their many responses that *many of them chose different methods*. (Post-Lesson Interview after Lesson 8, 00:07:48, emphases added)

In short, Teacher 8 helped her students make connections across units by providing a platform for them to engage in problem-solving with different methods. Through this activity, they were given the opportunity to appreciate the connectedness of the four methods and the conditions in which each was more appropriate.

## 14.6 Discussion

As mentioned in the beginning of the chapter, a mathematics teacher who conducts their lessons to help students perceive the connections across mathematics concepts views teaching mathematics within a connectionist orientation (Askew et al., 1997; Raveh et al., 2016). Based on our analysis of the instructional materials she created, we argue that Teacher 8 is an illustration of one who subscribes to the connectionist perspective. Her commitment to tight connection in her instruction is not merely a cursory one; as described in the previous section, she deliberately worked in various strategies of connection in the way she planned and carried out the lessons. Her commitment is extended to the way she designed her instructional materials. From the way she embedded intermodal links in her instructional materials, it is clear to us that she wanted to use the materials as an instrument to help her enact her goal of "link everything together" in her series of lessons. Yet, Teacher 8's connectionism is not limited to only one level of analysis. Her version of connectionism goes beyond a particular level as mentioned in the findings—she views connections within a unit, between adjacent units, and across all the units within the topic.

Second, Teacher 8 sequenced her instructional materials in such a way that her students could make temporal connections throughout the topic. For instance, students could link the unit on the factorisation method to the graphical method for finding the roots of a quadratic curve; they were also led to draw temporal links between the unit on completing the square method to and the unit on the graphical method for finding the maximum or minimum point of a quadratic curve; the same was also evident in the link between the unit on quadratic formula to the unit on graphical method for finding the discriminant of a quadratic curve. This careful sequencing reflected her conscious planning—evidence of the hypothetical learning trajectory she had constructed for her students. Moreover, to be able to plan lessons such that the units were so tightly connected requires vision that spans beyond the temporal boundaries of one or two lessons. To enact temporal connections as indicated in the lessons, one needs to project one's temporal horizons and hence connections across the content development over the *whole* unit. This, to us, calls into question of whether there is sufficient professional development work for teachers to conceive of planning at this scale.

Third, we think that Teacher 8's conception of connections across methods has implications to the development of students' problem-solving abilities—in particular, this metacognitive awareness of multiple strategies (and their respective affordances); that is, the consciousness of looking across different solution methods requires an executive function at work psychologically, and this mechanism to take executive control is a component of metacognition (Holton & Clarke, 2006; Schoenfeld, 1992). This link between her move of making connections and her intentions to highlight metacognitive regulation as part of problem-solving is underreported in the literature, although it was mooted a long time ago: "Can you derive the result differently? … One of the first and foremost duties of the teacher is not to give students the impression that mathematical problems have little *connection* with each other, and no *connection* at all with anything else" (Pólya, 1945, p. 15, emphases added).

## 14.7 Conclusion

Mathematics is a subject that is interconnected. In order for students to master the concepts, teachers need to help students relate one mathematical idea to another. Nonetheless, though many studies have examined how teachers make connections during their lessons, little is known about how teachers design their own instructional materials to help students make these connections. This chapter exemplifies a mathematics teacher in Singapore who not only advocates teaching in an interconnected way, she deliberately integrates connections within a set of instructional materials she designed for a topic. The most interesting characteristic we discovered from her instructional materials is that she does not only incorporate connections within a sub-section in a mathematics topic; taking a "global" view of the topic, she was able to insert numerous places at using different strategies to help students make connections between adjacent sub-sections; and even across all sub-sections.

As this teacher's instructional goals are embodied in her instructional materials explicitly, we can present a rich case of a teacher who helps students make connections via her instructional materials. However, as there is currently limited literature on how teachers design connections with instructional materials, more research work can be concentrated in this area. We think this Singapore teacher presents an interesting portrait of how "making connections" can be an organising principle in teachers' design of instructional materials for teaching mathematics. It is unclear at this stage if this represents the "Singapore portrait", however, we propose that this in an area worthy of further pursuit—to broaden and test the extent of whether this teacher's portrait to other mathematics teachers in Singapore and perhaps, even beyond.

## Appendix

Section C on Page 5 of Teacher 8's Notes

**Applying New Knowledge**

C1. Solve the following equations

| | |
|---|---|
| (a) $x^2 = 4$ | (b) $(x + 1)^2 = 4$ |
| (c) $(x - 2)^2 = 25$ | (d) $(x - 4)^2 = 10$ |

C2. Given two equations $(x + 1)^2 = 4$ and $x^2 + 2x - 3 = 0$, how are they related? Which equation is easier to solve and why?

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