Chapter 11
Singapore Secondary School Mathematics Teachers' Selection and Modification of Instructional Materials for Classroom Use

Lu Pien Cheng, Yew Hoong Leong, and Wei Yeng Karen Toh

Abstract This chapter examines how experienced and competent Singapore secondary school mathematics teachers select and modify materials for instructional practice. For the empirical section, we begin by analysing survey responses of 677 participants across a wide range of secondary schools to determine the extent of modification among teachers before identifying which instructional materials were used as reference materials in their modification. The findings showed that the teachers relied heavily on their school-based materials as reference materials. We next analyse the instructional materials of 30 experienced and competent teachers which reveal that the teachers' selection and modification of instructional materials were carried out in such a way as to integrate into their own instructional conceptions. The characteristics of the instructional materials that help teachers enact worthy instructional goals of teaching mathematics, such as making connections, reasoning, and challenge, were distilled from the 30 experienced and competent teachers' interview transcripts and their instructional materials.

Keywords Instructional materials · Task design · Task modification · Reference materials · Instructional design moves

L. P. Cheng (✉) · Y. H. Leong · W. Y. K. Toh
National Institute of Education, Nanyang Technological University, Singapore, Singapore
e-mail: lupien.cheng@nie.edu.sg

Y. H. Leong
e-mail: yewhoong.leong@nie.edu.sg

W. Y. K. Toh
e-mail: karen.toh@nie.edu.sg

B. Kaur et al. (eds.), *Mathematics Instructional Practices in Singapore Secondary Schools*, Mathematics Education – An Asian Perspective,
https://doi.org/10.1007/978-981-15-8956-0_11

11.1 Introduction

In this chapter and the next three chapters we turn to another aspect of the project (see Chapter 2 for details) that focus on Singapore mathematics teachers' use of instructional materials. In an earlier paper, we reported a teacher's use of instructional materials that he crafted to realise his goal of "making things explicit" (Leong, Cheng, Toh, Kaur, & Toh, 2019a). The paper also illuminated how the teacher selected and modified his instructional materials. In this chapter, we broaden our investigation to more Singapore secondary school mathematics teachers to: (i) gain deeper insight on the selection or modification of materials for instructional practice, and (ii) examine the characteristics of the instructional materials that help teachers enact worthy instructional goals of teaching mathematics.

11.2 Instructional Materials

Teachers are key to effective curriculum delivery. "The effectiveness of their curriculum delivery is connected to the quality of instructional materials (Ko & Sammons, 2014)" (as cited in Lashley, 2019, p. 2). Indeed, instructional materials are important mediators to connect teaching and learning. Not only are instructional materials resources designed to support or supplement instruction (Remillard & Heck, 2014), they are "one which is classroom-ready and that carries the teachers' actual instructional goals" (Leong, Cheng, & Toh et al., 2019a, p. 50). Instructional materials include textbooks, curriculum guides, descriptions of mathematical tasks, and instructional software (Remillard & Heck, 2014). It is important that instructional materials maintain high standards because "the standards of the instructional materials in the classroom for curriculum delivery directly impact the quality of the learning experiences" (Lashley, 2019, p. 3). However, designing high-quality instructional materials requires considerable thought in order to achieve the needed impact (Lashley, 2019).

11.2.1 Selection and Modification of Instructional Materials

"Textbooks and curriculum guides are the most common form of instructional materials used throughout the world and continue to play a critical role in national education systems" (Remillard & Heck, 2014, p. 707). Teachers also frequently develop their own materials (Steiner, 2018) and they seek after instructional materials that would address their students' learning needs. Their selection of instructional materials may be based on, for example, professional judgement and experience in selection of instructional materials (Bugler et al., 2017). They also added that criteria such as accuracy and visual appeal, alignment to standards and depth of knowledge, ease of use and support, and engagement and ability to meet student needs are also used for the selection of instructional materials. In order to improve the effectiveness of the selected and produced instructional materials, learners' interest and diversity should be considered (Lashley, 2019). Research on learning styles, and the design of instructional materials for flexibility, diversity, and balance can also be taken into consideration (Rowntree, 1992). In the selection of mathematical tasks, it is critical that the tasks selected to match the instructional objectives and that teachers recognise the nature of tasks in order to maximise learning opportunities afforded through different tasks (Lee, Lee, & Park, 2019).

"Teachers can use textbooks in any number of different ways, adapting and adding to them – or omitting some or all of any given activity (e.g. Grammatosi & Harwood, 2014; Gray, 2010; Menkabu & Harwood, 2014; Shawer, 2010)" (Harwood, 2017, p. 264). It is sometimes necessary for teachers to modify textbook tasks to respond to new curriculum standards or educational aspirations. For example, nurturing creativity is one of the essential twenty-first-century skills (Coil, 2013, 2014; Piirto, 2011) and creative thinking can be fostered through tasks designed for higher-order thinking (Kaur & Yeap, 2009). According to Lee et al. (2019), although creativity is explicitly addressed in the Republic of Korea's mathematics curricula, secondary school mathematics teachers did not feel the need for task modification (Kim & Kim, 2014) as many tasks in the middle and high school mathematics textbooks still require students to obtain correct answers by using procedures or algorithms (Kim & Kim, 2013). Lee (2017) reported that "few mathematics teachers design new tasks or adapt the tasks from textbooks to be appropriate for a high-level cognitive approach (Remillard 1999; Smith 2000; Stein, Grover, & Henningsen, 1996; Stigler & Hiebert 2004)" (p. 997).

Teachers also draw from a variety of resources or references to design their instructional materials. These reference materials, also known as base materials (Leong, Cheng, & Toh et al., 2019a), undergo some modification and selection process before the teachers morphed them into a form that is considered suitable for use in classroom work to advance their instructional goal. The teacher's modification of the textbook (reference materials) for his instructional materials could be to make things more explicit for his students (Leong, Cheng, & Toh, et al., 2019a). Three strategies were detailed in their report: (i) "Explicit-from" reference materials to fill gaps in the textbook content such as critical ideas and links between representations, (ii) "explicit-within" for students to revisit similar tasks in sequential units for skill consolidation and concept linkage, and (iii) "explicit-to" in order to direct students from the questions in the instructional materials to planned classroom enactments. Several other strategies for task modification have been reported in the literature. For example, Zaslavsky (1995), showed how to modify standard tasks that have only one correct answer into open-ended tasks that allow learners to explore more solutions to the tasks, to pose more questions, and to try various strategies. In the same vein, Yeo (2018) provided examples of modification of a textbook problem into a more open-ended task "that make[s] assumptions on the missing information" (p. 200). Lee, Lee, and Park (2016) reported three types of task modification strategies by pre-service teachers:

(i) *context modification* refers to modification by changing the context of tasks, making them student-friendly or diverse
(ii) *condition modification* refers to modification by adding, deleting, or transforming the conditions in tasks (Prestage & Perks, 2007). This can also be characterised by adding questions to remind students what they have learned, "changed the condition of the task to step questions to facilitate students with constructing mathematical concepts" (Lee et al., 2019, p. 979) and when students' cognitive level was considered, the condition of the task was modified to "provide the opportunity of inductive reasoning or informal justification" (p. 980).
(iii) *question modification* refers to modification by changing what students are required to answer. This can also be characterised by the opportunity "to facilitate students' reflective thinking" (Lee et al., 2019, p. 980). For example, including questions that require students to reflect whether their solutions have any meaning in real life. It included also questions that required learners to provide explanation about their solutions.

11.2.2 Characteristics of Instructional Materials

Many instructional materials have been published to respond to "new" curriculum standards over the years, "with the explicit intent of helping teachers and students enact reform-oriented subject matter and pedagogical goals" (Lloyd & Behm, 2005, p. 48). According to González, Estrada, and González (2017), *The Guide for Evaluating Teaching Materials and Development* reported in Travé, Pozuelos, Cañal, and Rodríguez (2016) is a tool that can be used to evaluate instructional materials in terms of six aspects:

(1) epistemological aspects of teaching material, e.g., material identifies school knowledge results from the "interaction between scientific and everyday knowledge" (p. 976)
(2) axiological aspects, e.g., "inclusion of cultural elements and promotion of respect for the environment" (p. 976)
(3) psychological aspects, e.g., takes into account the kind of learning promoted by the material and the role of previous knowledge
(4) pedagogical elements, e.g., considers:
    - key competences
    - objectives
    - contents (e.g., organisation and connection with environment)
    - methodology approach (e.g., non-directive, inquiry-based)
    - activities (e.g., sequencing according to some structure; explanatory orientation that is applicable to textbook; theoretical–practical design that requires description, explanation, argument and requires diversity of sources of information and materials)

- assessment where evaluation is viewed "as a process of understanding and reflection for improving learning and teaching" (p. 977)

(5) teaching design, e.g., "based around the textbook with other complementary material" or "based around self-produced materials complemented by various other resources" (p. 978)
(6) professional development, e.g., "material promoted the design, development and evaluation of the syllabus from an enquiry-based perspective" (p. 982).

As seen from the literature review, much has been reported about the quality of instructional materials, but relatively less is reported about how teachers design these materials. If we assume that teachers do not usually create their instructional materials from scratch, it necessarily implies that they select and modify from reference materials. It will thus be interesting to examine these processes and mechanisms teachers engage in when they select and modify reference materials for their instructional materials. This is the focus for the rest of this chapter as we proceed to the empirical section.

11.3 Teachers' Reference Materials

We first report findings from four survey items completed by 677 experienced and competent Singapore secondary school mathematics teachers. The results of the survey items inform us of the most useful reference materials for the teachers, the reference materials that they based their modification upon, and the extent of modification among teachers.

11.3.1 Item 1

Item 1 of the survey requires the teacher respondents to rank the materials (e.g., main textbook, school-based material, etc.) in order of usefulness given a list of reference materials.

> Item 1: *The following is a list of reference materials. Rank the materials in order of usefulness, 9 being the one most useful to you.*

Our analysis of this survey item revealed that the main reference material which had the most influence on teachers was the main textbook, followed by school-based materials. As shown in Table 11.1, out of 677 respondents to the survey, 432 chose main textbook as what they consider as the most useful reference materials, followed by 143 (21%) who chose school-based materials. The main workbook supplements

Table 11.1 Most useful reference materials chosen by 677 survey participants

Reference materials	9	8	7	6	5	4	3	2	1	Total
Main textbook	432	117	55	18	7	7	7	11	23	677
Supplementary textbook(s)	6	93	73	91	112	113	104	65	20	677
Main workbook	16	143	95	97	102	80	66	66	12	677
Supplementary workbook(s)	3	10	27	80	90	137	144	122	64	677
School-based resource(s)	143	125	135	77	81	50	31	21	14	677
Commercial materials	5	33	82	81	98	90	115	133	40	677
Online resources	28	119	149	136	66	71	73	27	8	677
MOE-produced resources	8	29	51	79	100	94	92	190	34	677
Others	36	9	11	18	20	35	45	42	461	677

the main textbook and allows for more practice, assessment and development of problem-solving and thinking skills.

While we expect textbooks to be the main reference materials for teachers, because in Singapore, mathematics textbooks are "part of the official curriculum to the extent that they are incorporated into the designated curriculum through authorised selection or adoption processes" (Remillard & Heck, 2014, p. 710), we were surprised at how highly the teachers valued school-based materials as references. This appears to be a "new" finding as prior studies of this scale within Singapore had not revealed a similar significant preference for school-based materials. If so, this may indicate a quite recent phenomenon where school-based materials are gaining more influence on secondary mathematics teachers. While we did not anticipate how highly the teachers valued school-based materials (as shown in the finding from Item 1), we were aware—through noticing that many of the teachers we studied in Phase 1 of the research relied on school-based materials—that they were also used in a number of secondary schools. As such, we wanted to survey the type of school-based materials referred to across a broad range of schools. This is the purpose of Item 2.

11.3.2 Item 2

Item 2: *Do you use any school-based resource(s) as your reference materials? If yes, please specify.*

We counted and sorted teachers' responses to their use of school-based resources into three categories (Fig. 11.1), namely, (i) past year papers (ii) other school papers and (iii) within-school materials such as notes and teaching packages, lessons developed by the mathematics department. More than half of the respondents (410 out of 677) indicated the use of within-school materials. This seems to signal a shift

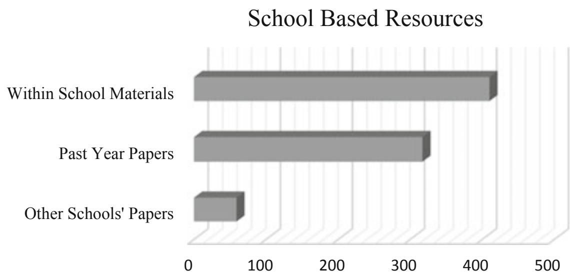


Fig. 11.1 Categories of school-based resources used by teachers as reference materials

towards supplementing externally designed materials (such as textbooks) with internally designed school-based materials. As schools rely more on their "in-house" expertise for instructional materials, what are some implications for teaching and research? For one, since the quality of instruction is largely influenced by the quality of the materials referred to, a study into the kind of actual within-school materials used by schools would be a productive inquiry. However, to date, there have been scarce research in this area within Singapore.

We then examined the reference materials that the teachers used for modifications and selection to design their own instructional materials by analysing Item 3.

11.3.3 Item 3

Item 3: *What were used or modified from the reference materials for the design of your instructional materials (you may select more than one item).*

The results showed a variety of tasks (e.g., practice items, challenging items, diagrams, activity, worked examples, organisation of content(s)) that were being modified from the reference materials for teachers' design of instructional materials (Fig. 11.2). Practice examples were found to be the most frequently modified. In separate studies, we zoomed-into the design principles used by some of these teachers in crafting sequences of practice examples (Leong, Cheng, Toh, Kaur, & Toh, 2019b, in press).

Lastly, we examined Item 4 to determine the relationship between Secondary mathematics teachers' reference materials and instructional materials. "By instructional materials (IM), we mean materials that teachers bring into the classroom for instructional purposes, and in a form that is classroom-ready for students' access in the learning of mathematics" (Leong, Cheng, & Toh, 2019a, p. 90).

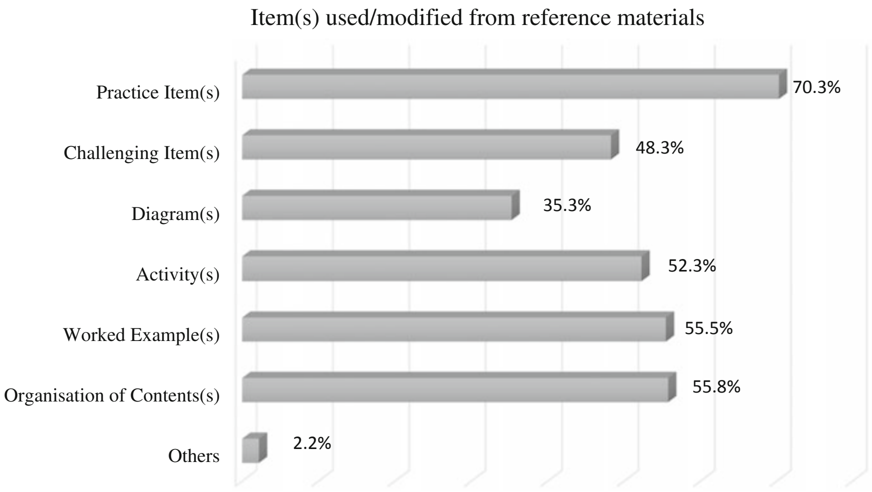


Fig. 11.2 Items used for modification and selection of instructional materials

11.3.4 Item 4

> Item 4: *What is the relationship between their reference materials and their instructional materials?*

Responses to this item were coded (see Table 11.2) as: (1) *Exactly the same* as reference materials, (2) *Sometimes* adaptation and modification were made, (3) *Frequently* adaptation and modification were made, and (4) *Mostly/Always* modifications were made. The results revealed that only 3.5% of the respondents did not make any adaptation and modifications from their reference materials, that is, 96.5% of the respondents made adaptations and modifications from their reference materials (see Fig. 11.3 for the graphical representation of this result). This means that a vast majority of Singapore secondary mathematics teachers do not view their duty as merely "lifting" items from reference materials to give to their students; rather, they see their role as necessarily one of mediation between the reference materials and

Table 11.2 Relationship between reference materials and instructional materials

	Code(s)	Frequency	Percentage (%)
Exactly the same	1	24	3.5
Sometimes	2	253	37.4
Frequently	3	262	38.7
Almost always	4	138	20.4
Total		677	

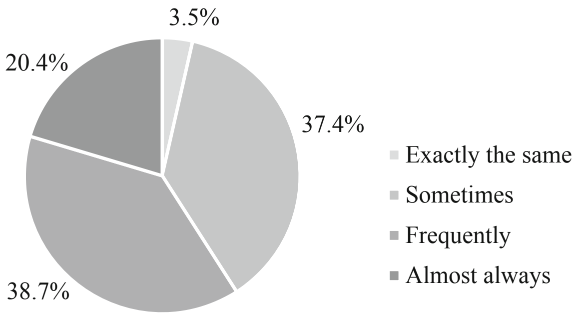


Fig. 11.3 Relationship between reference materials and instructional materials

student learning: they are required to value-add by modifying them. This leads to a natural question: how do teachers select and modify materials? This is the substance of the next section of this chapter.

11.4 Teachers' Strategies in Selection and Modification for Their Instructional Materials

We inquire into this aspect of the investigation through two research questions.

11.4.1 Research Question 1: How Do Singapore Secondary School Mathematics Teachers Select or Modify Materials for Instructional Practice?

11.4.1.1 Method

The 30 experienced and competent Singapore secondary school mathematics teachers who participated in the first phase of the project submitted their instructional materials they planned for the mathematics topics before they were interviewed (pre-module interview) and before any observations on their mathematics lessons using the planned instructional materials were made. From the instructional materials that they initially submitted, we were able to trace several examples of modification and selection from their reference materials, which was chiefly the main textbook (see Sect. 11.3.1, Item 1). This suggests that modification and selection of materials was done before enactment of the mathematics lesson (Stage 1 in Fig. 11.4).

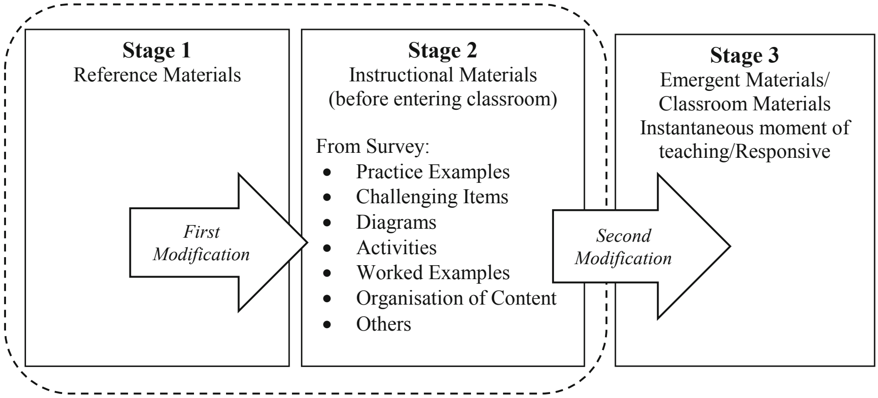


Fig. 11.4 Modification and selection before and during enactment of lesson

When asked whether the teachers have any special features that they have put in place, through their instructional materials, that will help them assess whether their students have attained the mathematical goals of the lesson, Teacher 1 said,

> It would be through certain questions. These are the questions that, if they are able to answer, that means, they will have learned what they are supposed to learn, that means, that *sub, that small content goals, smaller sub goals*. So they'll be like, so called particular questions, that, by doing, by going through these questions, if they are able to answer, that means they know, and then we can move on. Because if not, we probably have to *go back and think of other examples* [modification during enactment]. Either other examples, other ways of showing them, or they just need more practice questions. It really depends.

Teacher 9 said,

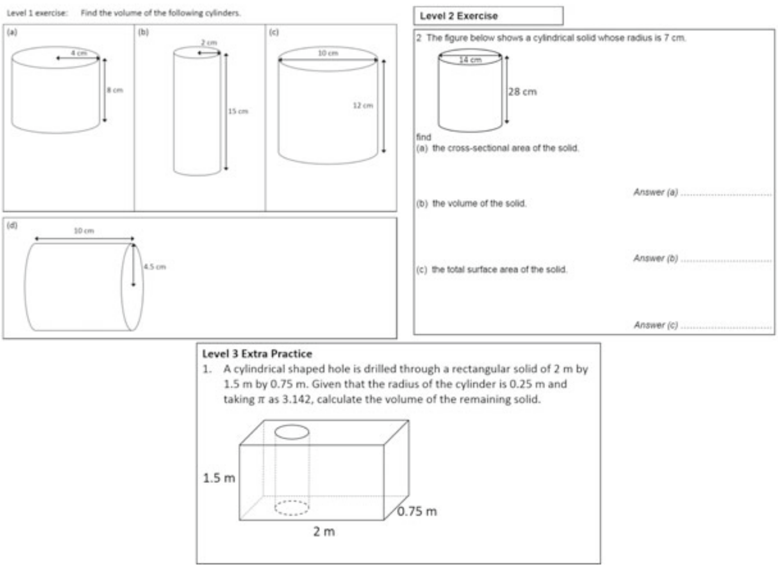
> So, besides the notes, and the worksheets, I give them quizzes, and if I find that the classes, not able to handle certain things like yesterday … *I may need - will recap*… So I *will prepare something* [modification during enactment] to, a very short recap to go through the… Not misconceptions, *the gaps that they still have*.

This suggests that modification and selection of materials were also made during the enactment of the planned lessons, at different junctions of the topic (Stage 2 in Fig. 11.4). During the enactment of the lesson, the teachers sometimes came up with examples on the spot to respond to the students' learning needs and we refer this as *emergent material* to meet the instantaneous moment of teaching (Stage 3 Fig. 11.4).

In this chapter, we analysed only modification of instructional materials before the enactment (First modification from Stage 1 to 2 in Fig. 11.4).

Phase 1: We begin by examining in detail the interview transcripts of two randomly selected exemplary teachers (Teacher 1 and 3) and looked for instances when they explicitly communicated their instructional design moves before triangulating with their instructional materials and reference materials. Here, we used reference materials as textbooks (mainly the teachers' school main textbooks) approved by Ministry of Education (MOE) in Singapore. The template in Table 11.3 serves as a way to

Table 11.3 Example of analysis to determine participants' modification and selection moves from reference materials (Textbook)

Interview Transcript
Teacher 14: *I'll say that my questions that I gave them it's actually from level 1, level 2,…* basically for *level 1 um it's really more on the, the simple one* like, even like identifying which one is a prism,… *another level 1 question is also to be able to do the direct questions* as well. *So level 2 will be a little bit more wordy … So I slowly build up…* I foresee that they might not be able to see, so that's why I give them the different orientation for them to get used to it… my building up to the volume of cylinder they will need to find the area of the, the 2D figure. So for area of 2D figure, I ask them to actually memorise like, to find the area of the circle is actually πr^2. So that's why … link it to the *r*. So the students will be more, I'll say, … they will relate better when it comes to radius … instead of diameter. So I give them the radius first, then after that the diameter … some of *the higher ability one, the HA students right they can finish this exercise pretty fast, so that's why my level 3 question is to actually stretch them.*
Reference Materials
We are unable to reproduce here the diagrams from page 254 of the source due to copyright reasons. Source: Toh, T. L. (2014). *Maths 360 Normal (Technical)* 2. Singapore: Marshall Cavendish Education.
Instructional Materials (Modified Tasks)


collate and organise the relevant data for the first two teachers. Based on the template, we derive categories of how these two teachers selected and modified reference materials for their instructional materials.

Phase 2: We broaden the analysis to five other teachers (randomly selected from the different tracks) using the categories by looking at their instructional materials to trace modification and selection from their reference materials. We continued our analysis of the instructional materials using Table 11.3. In summary, in this phase, (i) three of the instructional materials are from the Express, Teacher 1, 3, and 8; (ii) two from Normal (Technical), Teacher 9 and 14; (iii) one Normal (Academic), Teacher 11, and (iv) one from the Integrated Programme, Teacher 13.

Phase 3: In this phase of the analysis, we scanned through the rest of the teachers' instructional materials to confirm and refine the categories. We examined the instructional materials of 30 teachers.

11.4.1.2 Findings

Our analysis resulted in three categories that illuminate the modification and selection design moves made by the teachers: (i) *modified*, (ii) *new*, and (iii) *smoothened*. We elaborate and provide examples of the three modification and selection design moves below.

(i) Modified

Teachers modified and selected their materials from the textbooks for varied purposes. For example, the tasks in Fig. 11.5 were for Secondary 4 Express students on the topic of Vectors. Textbook item (iii) was modified to item (c) in the teacher's instructional material. The modified item (c) required more thinking on the part of the student as compared to item (iii) and thus the modification increased the cognitive demand of the tasks.

(ii) New

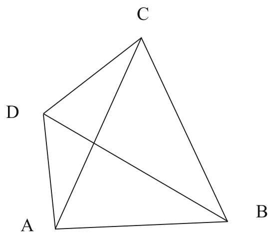
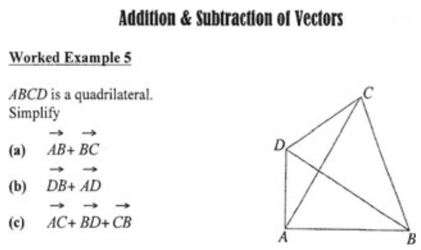
Reference Materials	Instructional Materials (Modified Tasks)
The diagram shows a quadrilateral ABCD. Simplify (i) $\overrightarrow{AB} + \overrightarrow{BC}$, (ii) $\overrightarrow{DB} + \overrightarrow{AD}$, (iii) $\overrightarrow{AC} + \overrightarrow{CB} + \overrightarrow{BD}$.  [Reproduced from Yeo, J., Teh, K. S., Loh, C. Y., & Chow, I. (2016). *New syllabus mathematics 4* (7th ed.). Singapore. With permissions from Shinglee Publishers Pte Ltd]	 **Addition & Subtraction of Vectors** Worked Example 5 *ABCD* is a quadrilateral. Simplify **(a)** $\overrightarrow{AB}+\overrightarrow{BC}$ **(b)** $\overrightarrow{DB}+\overrightarrow{AD}$ **(c)** $\overrightarrow{AC}+\overrightarrow{BD}+\overrightarrow{CB}$

Fig. 11.5 Modification from textbook for Secondary 4 Express Vectors (Teacher 1)

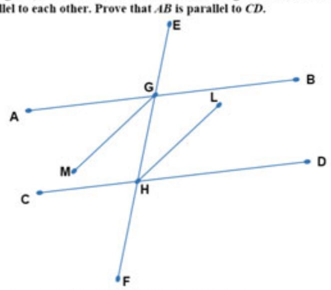
Interview Transcript	Instructional Materials (New)
Interviewer: … what is one special feature that you put in place, based on the package, special features? Teacher 3: I think it would be the *step by step approach,* sequential [manner]. Because I think textbook they only teach you the concept. Based on the concept, they do not tell you where to begin with. So we tell them … So what are some of the key steps that you need to do, in writing out what are some of the theorems, before you even get started? So that, *it sort of eases the students into the question itself.*	**Chapter 10: Proofs in Plane Geometry** Name:________________( ) Date:________ Class:________ **Method 1 – Denoting the unknown (Basic Proofs in Plane Geometry)** **Eg 1 In the diagram, bisector *GM* and *HL* of alternate angles *AGH* and *DHG* respectively are parallel to each other. Prove that *AB* is parallel to *CD*.**  Step 1: Mark all given information on diagram and deduce. Step 2: State what you want to prove and outline your approach. Step 3: Write out your proof step by step, giving reasons where necessary. Let $\angle AGM = x$. $\angle MGH =$ ________ $\angle MGH = \angle GHL =$ ________ ( ) $\angle GHL = \angle LHD =$ ________ $\angle AGH = \angle GHD =$ ________ $\therefore AB \parallel CD$ ( )

Fig. 11.6 New items for Secondary 4 Express geometrical proofs (Teacher 3)

Teachers also created new instructional materials which were clearly not from the teachers' school main textbook and the innovations were for varied purposes. An example is provided in Fig. 11.6.

It is clear from the teacher interview that the new materials are created to gradate the level of difficulty. This gradation also reflects the teacher's sensitivity and response to students' responses to pre-existing or available materials before class instruction. The teacher created new materials at specific junctures of the topic to fill learning gaps anticipated by the teacher for the group of students that he will be carrying out the instructions.

The next example (Fig. 11.7) illustrates new material created in order to facilitate the connections of mathematical concepts. Notice that the subheadings "Eliciting Prior Knowledge" in the new instructional material explicitly highlighted students' attention to recall area of a square and then connected this geometric representation to the perfect square expressions.

The purpose of this new material is to connect to completing the square method from geometric to algebraic representation.

(iii) Smoothened

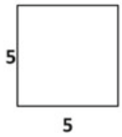
While the teachers we examined do indeed modify items and add new items—as described in the earlier sections—our further analysis shows that these two moves alone do not adequately explain the teachers' strategy of instructional design. When both actions are visible to us within the same tasks, there is smoothening of the

Instructional Materials (New)

2.3 Completing the Square Method

Eliciting Prior Knowledge

1. Draw a geometric representation of each of the following. The first one has been done for you.

5^2	7^2	$(x+1)^2$	$(x-2)^2$
			

2. Explain in words what each of the following represents with reference to its geometric representation. The first one has been done for you.

5^2	5^2 represents the area of a square with sides of 5 units in length.
7^2	
$(x+1)^2$	
$(x-2)^2$	

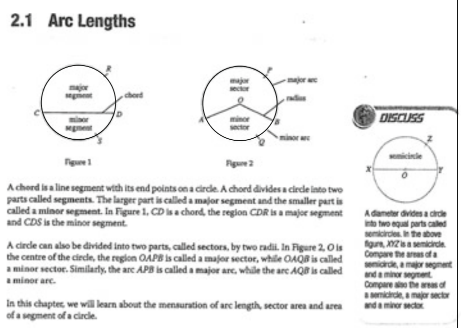
Fig. 11.7 New items for Secondary 3 Express quadratic equations (Teacher 8)

instructional materials. We define tasks as a series of work students are required to do organised around an ostensible goal, e.g., identify four types of angles. Figure 11.8 illustrates smoothening of instructional materials.

The teacher combined two figures in Sect. 2.1 of the textbook into one. This modification of diagrams (collapsing the number of diagrams) summarises several key terms for this topic and draws out key differences between the terms such as chord and radius. A table was created (new material) below the modified diagram in the teacher's instructional material to repeat some of the key terms such as radius, diameter, chord, arc, and sector. Not only that, the students are required to describe those terms in the space provided in the table. The students also have to draw the radius in the circle provided in the table, diameter in the circle provided in the next row of the table etc. The modifications made and the new material added appears as an "entity" rather than separate activities. In other words, the teacher also *smoothens* these components to provide continuity and connection in the students' learning. The reference materials are insufficient to help the targeted students learning and the teacher modifies, adds, and smoothens the learning materials to facilitate this learning transition. The teacher, Teacher 11, said during the interview for the topic arc length,

Reference Materials	Instructional Materials (Smoothened)

Reference Materials



2.1 Arc Lengths

A **chord** is a line segment with its end points on a circle. A chord divides a circle into two parts called **segments**. The larger part is called a **major segment** and the smaller part is called a **minor segment**. In Figure 1, *CD* is a chord, the region *CDR* is a major segment and *CDS* is the minor segment.

A circle can also be divided into two parts, called **sectors**, by two radii. In Figure 2, *O* is the centre of the circle, the region *OAPB* is called a **major sector**, while *OAQB* is called a **minor sector**. Similarly, the arc *APB* is called a **major arc**, while the arc *AQB* is called a **minor arc**.

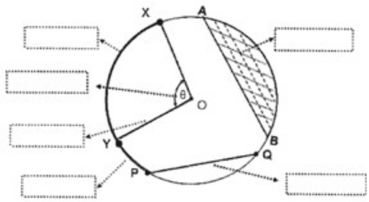
In this chapter, we will learn about the mensuration of arc length, sector area and area of a segment of a circle.

A diameter divides a circle into two equal parts called semicircles. In the above figure, *XYZ* is a semicircle. Compare the areas of a semicircle, a major segment and a minor segment. Compare also the areas of a semicircle, a major sector and a minor sector.

[Source: Chow, W. K. (2016). *Discovering mathematics Normal (Academic) 4A* (2nd ed.). Singapore: Star Publishing Pte Ltd]

Instructional Materials (Smoothened)

<u>Activity 1:</u>
Fill in the mathematical terms for the highlighted parts or shaded regions of the circle in the following diagram.



Parts of a circle	Description	Illustration
Radius		
Diameter		
Chord		
Arc		
Sector		

Fig. 11.8 Smoothened materials for Normal (Academic) Secondary 4 arc length (Teacher 11)

> From my years of experience … I find that students are not comfortable … in listing the radian … radian measure…my goal is really for them to be able to accept this radian measure and be able to use it … when it's required … they are not comfortable. So… that is something I would like to … make it easy for the students.

Figure 11.9 illustrates another example where smoothening of instructional materials can be observed from Teacher 11.

New rows for Fig. 4 and 5 in the teachers' instructional materials were added as compared to the textbook which provides rows for Fig. 1, 2, and 3. This provided more specific examples for students to observe patterns and relationships from the data generated for rows from Fig. 1 to 5 in the teacher's instructional materials to facilitate the generalisation process. The instructions in the teacher's material were added with each column labelled as (1), (2), and (3) to modify the instructions to the task into a form that was less wordy. Question (c) and (d) in the teachers' instructional material is a modification to Question 2 and 3 respectively in the textbook. Question (c) and (d) are less wordy and have a more direct approach towards the derivation of the formula for the length of arc in terms of r and θ as compared to Question 2. This suggests the modification and selection move to "remove unnecessary work". Once again, the modifications made and new material added appears as a coherent activity which clearly smoothens students' learning. Here, sensitivity towards students' responses to the existing materials before class and facilitating

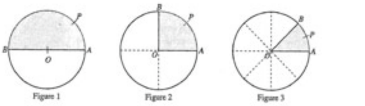
Reference Materials

CLASS ACTIVITY 1

Objective: To find the arc length in a circle by considering it as a fraction of the circumference of the circle.

Express your answers in this activity in terms of π where appropriate.

1.



(a) Cut a circle of radius 6 cm from a piece of paper and fold it into 2 halves, 4 quadrants and 8 equal sectors as shown in the above figures, where O is the centre of the circle.

(b) Work together with the classmate sitting next to you.

(i) Observe the size of $\angle AOB$ in each figure formed.

(ii) What is the ratio of the length of arc APB to the circumference in each figure?

[Source: Chow, W. K. (2016). Discovering mathematics 4A, 2nd Edition, p. 39. With permissions from Star Publishing Pte Ltd]

(c) Fill in the following columns, $\angle AOB$, $\frac{\angle AOB}{360^{\circ}}$ and $\frac{Length\ of\ arc\ APB}{Circumference}$.

	$\angle AOB$	$\frac{\angle AOB}{360^{\circ}}$	$\frac{Length\ of\ arc\ APB}{Circumference}$	Length of arc APB
Figure 1	180°	$\frac{180^{\circ}}{360^{\circ}}=\frac{1}{2}$	$\frac{1}{2}$	
Figure 2				
Figure 3				

(d) What can you say about the fractions $\frac{\angle AOB}{360^{\circ}}$ and $\frac{Length\ of\ arc\ APB}{Circumference}$?

(e) Given that the circumference of the circle = 12π cm, express the length of the arc APB in each figure in terms of π. Using your answer, complete the table in (c).

2. In Figure 4, a circle O and radius r cm is divided into 6 equal sectors.

(a) Write down the circumference of the circle in terms of r.

(b) Express $\angle AOB$ as a fraction of 360°.

(c) Express arc APB as a fraction of the circumference of the circle.

(d) Hence find the length of arc APB in terms of r.

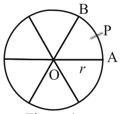


Figure 4

3. In Figure 5, AOB is a sector of the circle O and radius r cm, and $\angle AOB = x°$.

(a) Express $\angle AOB$ as a fraction of 360°in terms of x.

(b) Express arc APB as a fraction of the circumference of the circle.

(c) Hence derive a formula of expressing the length of arc APB in terms of r and x.

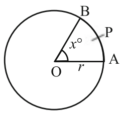
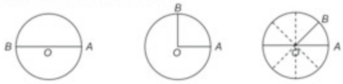


Figure 5

[Reproduced from Chow, W. K. (2016). Discovering mathematics 4A, 2nd Edition, p. 40. With permissions from Star Publishing Pte Ltd]

Instructional Materials (Modified Tasks)

Activity 2: To find the arc length in the circle



	(1) $\angle AOB$	(2) $\frac{\angle AOB}{360°}$	(3) $\frac{Length\ of\ arc\ APB}{Circumference}$	(4) Length of the arc APB
Figure 1				
Figure 2				
Figure 3				
Figure 4				
Figure 5				

a) Fill in the columns (1), (2) and (3).
What can you say about the columns (2) and (3)?

____.

b) If the circumference of the circle is $2\pi r$ and the radius is r cm, complete column (4)

c) If Figure 4 is a circle of centre O and radius r cm that is divided into 6 sectors, complete the columns (1) to (4).

d) If Figure 5 is a circle of centre O and radius r cm that is divided into sectors with an angle $x°$ each, complete the columns (1) to (4).

Hence , can you derive the formula for the length of arc in terms of r and x.

Fig. 11.9 Smoothened materials for Normal (Academic) Secondary 4 formula for arc length (Teacher 11)

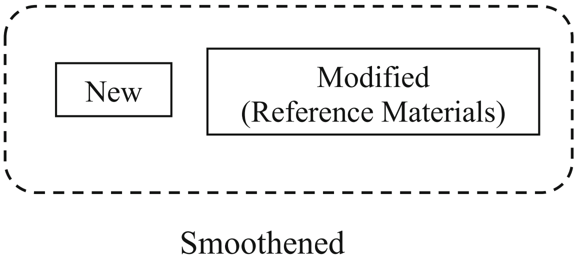


Fig. 11.10 Modification and selection instructional design moves

students' connections of mathematical ideas are evident from the modification and addition of new materials.

Figure 11.10 summarises the teachers' modification and selection of instructional design moves. One can modify without adding items in the instructional materials. Smoothening occurs when teachers modify and add new items within tasks in their instructional materials towards an ostensible goal. Our findings also reveal the teachers' selection and modification of instructional materials were carried out in a way to integrate with their conceptions of instruction, such as (i) increasing cognitive demand of task to raise students' level of thinking, (ii) gradating level of difficulty, (iii) being sensitive to students' responses to the existing materials before class, (iv) helping students to make connections, and (v) removing unnecessary work. Not only do teachers create their own instructional materials, they also modify from their reference materials and smoothen it to become a coherent unit for their students.

Out of the 30 experienced and competent teachers, 29 of them modified material from reference materials and 23 of them also inserted new materials. Out of this, 23 teachers modified, added, and smoothened the material.

11.4.2 Research Question 2: What Are the Characteristics of Instructional Materials That Will Help Teachers Enact Worthy Instructional Goals of Teaching Materials and Help Students Achieve Desirable Outcomes?

11.4.2.1 Phase 1

In order to investigate the characteristics of the instructional materials that help teachers enact worthy instructional goals of teaching mathematics and help students achieve desirable outcomes, we analysed interview transcripts of the 30 experienced and competent teachers to the interview questions, in particular, the goals articulated by the teachers for the following pre-module interview questions:

(i) Please share with us your goals for this series of lessons. You may include both content and non-content goals

(ii) Please share with us what mathematical goals you intend to achieve for each set of materials that you will be using to determine.

We also extracted instances when the teachers explicitly articulated their goals in the post-lessons interviews. We excluded affective goals in our analysis as this goes beyond the scope of the chapter. From the goals that the teachers articulated during their interviews, we locate examples of instructional materials that help teachers enact those goals and collated them in a table as shown in Table 11.4. From these two data sources, we elicited and coded the characteristics of the instructional materials. We collapsed our codes into categories as illustrated in Table 11.5.

We used the task analysis guide (lower-level demands, higher-level demands) by Stein, Smith, Henningsen, and Silver (2009) to determine whether the instructional materials were challenging. Tasks that were identified as having higher-level demands were those that required procedures with connections and doing mathematics.

Table 11.4 Example of analysis process for characteristics of instructional materials

Code(s)	Extracts from interview transcripts	Instructional materials	Researchers' notes
Relate from one thing to another; connection Category: Making Connections	Teacher: … So for the volume itself, volume itself I would like to link to understand prism. Because prism was covered in Sec 2, then if they—I want them to be able to relate from one thing to the other. Then the prism and the pyramid the volume is actually related so I want them to see the connection. Even though the… The cover page on the examination, the formulas are given, but I want them to understand how the formulas come about. It make meaning … otherwise it's just throw them the formula, it won't make meaning to what they are learning	**Activity:** • Find out what is the **relationship** between the **volume of prism** and **volume of pyramid** • Find out the **formula for Volume of Pyramid**	Teacher planned to help students make the conceptual connections between the formulae for the computation of the volume of a prism to that for a pyramid. e.g. connect volume of pyramid to volume of prism learnt in Sec 2

Table 11.5 Sample of codes and categories for characteristics of instructional materials

Codes from interview transcripts and instructional materials	Categories
Link to the various forms, building on past knowledge, build up, linkage, refer to, relate	Make connections
Infer, reason out, justify, explain why	Reasoning
Higher order thinking, higher level questions, stretch, challenging questions, advanced questions, complex	Challenge
Quizzes, exit pass, entrance pass, assessment, check students' understanding	Assessment
Step by step, systematically, structure	Template
Procedural, formula, practice examples, exercise	Deliberate sequencing of examples
Context, real-life, applications	Context
ICT, videos, on the portal, software	ICT-related materials include *space in instructional materials to record* e.g. ICT explorations, making conjectures

11.4.2.2 Findings

Table 11.4 illustrates an example where the teacher's goal is to make conceptual connections between the formulae for the computation of the volume of a prism to that for a pyramid, i.e., connect volume of pyramid to volume of prism learnt in Secondary 2. The teacher's instructional material clearly reflected this goal. Figures 11.11, 11.12, 11.13, 11.14, 11.15, 11.16, and 11.17 provide examples of ostensible goals of what the teachers made explicit in the design of their instructional materials: *reasoning, challenge, assessment, template, deliberate sequencing of examples, context* and *ICT* to help students achieve desirable outcomes. Next, we tabulated the number of teachers who made each of the above goals explicit in their instructional design moves during the interviews (Table 11.6, Column 2).

Interview Transcript	**Instructional Materials**
Teacher 2: Then the one that you are referring to about the non-content goals right, I mean that will be more the mathematics disposition itself. I want to ... you know, I want a student *to be able to do reasoning* and be able to apply also problem-solving skills … particularly in the last part when they are supposed to use vectors to solve geometrical problems itself. So that will be a built up to it. *Again, in the process of learning itself, there will be a lot of reasoning involved.*	Exercise: Which of the following pairs are parallel and why? (i) a – b and b – a (ii) $a + \frac{1}{2}b$ and $4a + 2b$ (iii) $\binom{2}{4}$ *and* $\binom{-1}{-2}$ (iv) $\binom{2}{1}$ *and* $\binom{1}{2}$

Fig. 11.11 Reasoning as a goal for instructional materials from a Normal (Academic) Secondary 5 class on vectors (Teacher 2)

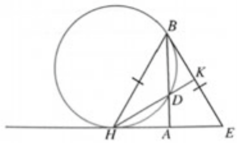
Interview Transcript	Instructional Materials
Teacher 3: I think whatever that I wanted to do during the lesson, more or less it is done. There were some additional practice. I know back in my mind, this topic is not very well received. But I have some motivated ones. So in my way, the motivated ones, they can do all the extra questions. But basically, everyone must attempt at least four similar questions. So that is the basic. But the better ones, *they are going to do the more advanced questions by attempting more.*	**9** In the figure, *HAE* is the tangent to the circle at *H*, *BH* = *BE* and *KH* is the angle bisector of ∠*BHE* and it cuts the circle at *D*. Given that *BD* produced meets *HE* at *A*, prove that **(i)** *HD* = *BD*, **(ii)** a circle can be drawn to pass through *A*, *D*, *K* and *E*.  [Source: Yeo, J., Teh, K. S., Loh, C. Y., & Chow, I. (2013). New syllabus additional mathematics 4 (9th ed.). Singapore. With permissions from Shinglee Publishers Pte Ltd]

Fig. 11.12 Challenge as a goal for instructional materials from an Express Secondary 4 class on geometrical proofs (Teacher 3)

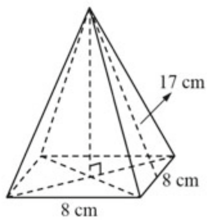
Interview Transcript	Instructional Materials (Quiz)
Teacher 9: … Besides the scaffolding and the examples …I also give them *quizzes.* Because quizzes they are actually quite motivated to have quizzes because when they have quizzes they like challenge each other, and they also have ... It's a form of *self-assessment for* them to say … *how did I do, am I better now or am I worse off.* So there's a bit of competition among the class itself. Then they also want to score well, because when they score well the self-esteem … So I like to give them quizzes, and the quizzes I give them is recurring, that means … *The topics I've covered many many weeks earlier I put them inside my quiz so it's like a recurring quiz* … So the questions keep repeating. And then once they … keep looking at the question they get accustomed to it.	10. (a) Draw the net of the square pyramid.  (b) Find the surface area of the pyramid.

Fig. 11.13 Assessment as a goal for instructional materials from a Normal (Technical) Secondary 4 class on volume and surface area- pyramid & cone (Teacher 9)

11.4.2.3 Phase 2

We also realised that there were many instances in the teachers' instructional materials that fit into some of the categories in Table 11.5, even though those goals were not articulated by the teachers during the interviews. For example, the categories *challenge, assessment, template, deliberate sequencing of examples, context,* and *ICT* are categories that are generally identifiable from the instructional materials. We apply the categories in Table 11.5 back to the teachers' instructional materials

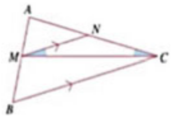
Interview Transcript	Instructional Materials (Quiz)
Teacher 3: … There are certain concepts. Ok. Concepts are actually stated in the textbook itself. Concepts for example similar triangles, midpoint theorem, as well as angles in alternate segment. These are the three major concepts that are being taught in this topic itself but because this is a topic that is not ah focusing on solving, it's focusing on proving on geometry, plane geometry. So the thing is that … I also added into the package itself what are some of the *step by step approaches* the students can start in identifying the question and then …I also shared with them some techniques we actually came up with ourselves so that is actually easier for students to actually do the proving.	Example 4  In the diagram, *MN* and *BC* are parallel lines, and $\angle NMC = \angle NCM$, prove that (a) ΔABC is similar to ΔAMN, (b) $NC \times AC = AN \times BC$. Step 1: Mark all given information on diagram and deduce. Step 2: State what you want to prove and outline your approach. Step 3: Write out your proof step by step, giving reasons where necessary. (a) $\angle BAC$ = __________ ( ) $\angle ABC$ = __________ ( ) $\angle ACB$ = __________ ( ) _________________________. (Proven) (b) (Hint: By similar triangle) _________________________. (Proven)

Fig. 11.14 Template as a goal for instructional materials from an Express Secondary 4 class on geometrical proofs (Teacher 3)

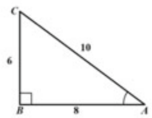
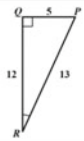
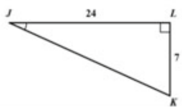
Interview Transcript	Instructional Materials (Quiz)
Teacher 19: ... I think it's more of uh, a lot of scaffolding, uh the scaffolding is there, and I think the questions is basically... progressive, difficulty level is progressive … it's not like I straightaway ask them to find uh the unknown sides and angles, but getting them at the first part, because, after 6.1 part (d), oh- I mean 6 point- before part (d), I'll get them to do the recognising of sides. So before they talk about finding the actual sine, cosine, tangent. Then after that from there, after they get more or less used to finding sine, cosine, tangent, the ratio for it, then they will move on to 6.2 which is finding the unknown sides …	**(D) Test Yourself!** 1. For the right-angled triangle shown, find a) sin *A* b) cos *A* c) tan *A*  2. For the right-angled triangle shown, find a) sin *P* b) cos *P* c) tan *R*  3. For the right-angled triangle shown, find a) sin *K* b) cos *J* c) tan *K* 

Fig. 11.15 Deliberate sequencing of examples as a goal for instructional materials from a Normal (Academic) Secondary 3 class on trigonometric ratio of acute angles (Teacher 19)

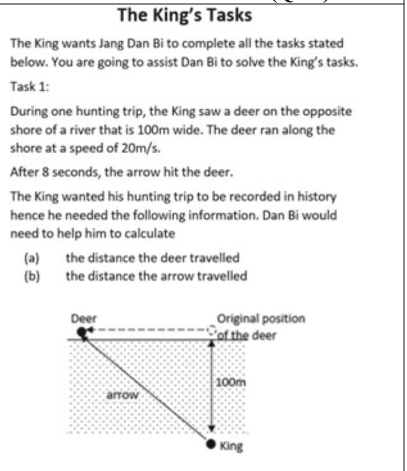
Interview Transcript	Instructional Materials (Quiz)
Teacher 6: …the goal of today's lesson is actually to er get them to apply the Pythagoras theorem and see actually how in real life context, how they can actually use Pythagoras theorem, ya. Interviewer: ... in your lesson, how did you go about achieving your goal? Teacher 6: So … what I did is actually edited the video, … to show them …a scene where the girl actually travel past time and how she actually uses Pythagoras theorem,… to actually help the king to solve the problem. And after that, they'll actually do some questions … to apply what they have learned in terms of the real life contexts.	 **The King's Tasks** The King wants Jang Dan Bi to complete all the tasks stated below. You are going to assist Dan Bi to solve the King's tasks. Task 1: During one hunting trip, the King saw a deer on the opposite shore of a river that is 100m wide. The deer ran along the shore at a speed of 20m/s. After 8 seconds, the arrow hit the deer. The King wanted his hunting trip to be recorded in history hence he needed the following information. Dan Bi would need to help him to calculate (a) the distance the deer travelled (b) the distance the arrow travelled

Fig. 11.16 Context as a goal for instructional materials from an Express Secondary 2 class on Pythagoras theorem (Teacher 6)

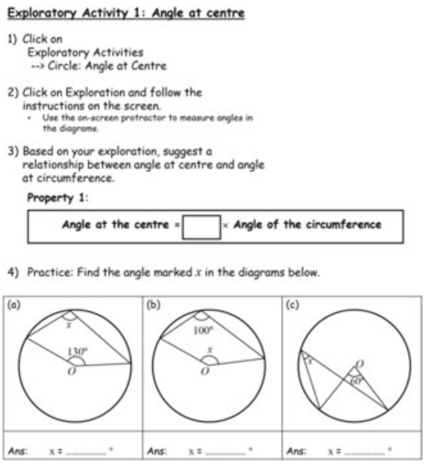
Interview Transcript	Instructional Materials (Quiz)
Teacher 5: … But …for this particular topic, I usually bring them to the computer lab to get them to explore first. So, my goal initially is to get them to go through the process of exploration. Self-discovery of the rules. So, it's more of a deductive approach. Because you don't see a few cases, in terms of how the properties play out. And then based on that, they are able to consolidate, which, so they're able to summarise, or conclude, that the relation between the angles is as what is being displayed.	Diagram is based on an activity that students have access to from a commercial online repository of materials.  Exploratory Activity 1: Angle at centre 1) Click on Exploratory Activities --> Circle: Angle at Centre 2) Click on Exploration and follow the instructions on the screen. • Use the on-screen protractor to measure angles in the diagrams. 3) Based on your exploration, suggest a relationship between angle at centre and angle at circumference. Property 1: Angle at the centre = ☐ × Angle of the circumference 4) Practice: Find the angle marked x in the diagrams below.

Fig. 11.17 ICT as a goal for instructional materials from an Express Secondary 3 class on angle properties of circles (Teacher 5)

Table 11.6 Characteristics of teachers' instructional materials

	Number of teachers (Interviews and instructional materials)	Number of teachers (Instructional materials only)
Challenge	21	27
Deliberate sequencing of examples	19	29
Making connections	18	21
Assessment	16	23
Support reasoning	12	19
Context	8	20
ICT	5	9
Template	4	13

to find their prevalence in the instructional materials. By doing so, we were trying to locate characteristics implicitly embedded into the design of the instructional materials. Table 11.6 (Column 1 and 3) summarises this result.

11.5 Discussion

In this chapter, we found that the textbook was the most useful reference material for teachers, followed by school-based materials. 96.5% of the respondents made adaptations and modifications from their reference materials. The teachers modified a variety of tasks, such as, practice items, challenging items, diagrams, activity, worked examples, organisation of content(s), from the reference materials when designing their instructional materials—with practice examples being the most frequently modified. From the finding above, it is clear to us that most of our teachers do not merely offload or adapt their reference materials into their instructional materials. Rather, they intentionally select materials that are suitable for their goals and make explicit efforts to coherently tie these in with new tasks that they construct for their students. In other words, the instructional materials were mediated through the goals of the teachers in a purposeful manner. This image of teachers' use of instructional materials was also depicted by Lee et al. (2019) as "active interpreter and user of textbook" (p. 966). The instructional design moves could be categorised into: (i) modified (ii) new, and (iii) smoothened. These instructional design moves are under-reported in the international literature and the examples that illuminate the design moves in this study can potentially provide a local-sensitive knowledge base for teacher professional development in designing quality instructional materials for effective teaching and learning.

González et al. (2017)—as reviewed earlier—provided us an overview of possible characteristics of instructional materials. Our findings are quite different as our study examined the actual moves that teachers pull together in their design of instructional materials. The teachers we studied reflected a number of the aspects reported in González et al. (2017)—e.g., psychological aspects in their deliberate sequencing of examples and pedagogical elements such as assessment—where all these are integrated together in their instructional design moves. "Challenge" is a characteristic in 27 out of the 29 instructional materials which suggests that most of the teachers made intentional effort to include challenging tasks in their instructional materials. In this same book, we have devoted Chapter 12 on challenging items where we elucidate all the connections between all these aspects. More than half of the instructional materials carry the characteristics of "support reasoning" and 'making connections'. This is not surprising as reasoning and making connections are two of the processes in the Singapore mathematics curriculum. Almost half of the instructional materials have templates and this interesting finding is reported in Leong, Cheng, and Toh (2019b).

Using the curriculum materials effectively includes not only being able to recognize and distinguish between high- and low-quality materials. Skilful selection and modification of instructional materials guided by clear goals of the teachers—in this study the characteristics inherent in the teachers' instructional materials—for classroom use are also critical. There has been a lot of interest recently in professional development research that draws upon task design and analysis, and instructional materials to develop teacher capacity (e.g. *Journal of Mathematics Teacher Education*, 2007, Volume 10). We see this study as a further contribution to this body of knowledge particularly suited for the local community.

References

Bugler, D., Marple, S., Burr, E., Chen-Gaddini, M., & Finkelstein, N. (2017). How teachers judge the quality of instructional materials: Selecting instructional materials, Brief 1—Quality. *WestEd*. Retrieved December 7, 2019, from https://www.wested.org/resources/selecting-instructional-materials-brief-1-quality/.

Coil, C. (2013). The application of common core state standards to foster creativity and curiosity. In J. B. Jones & L. J. Flint (Eds.), *The creative imperative: School librarians and teachers cultivating curiosity together* (pp. 151–164). Santa Barbara, CA: ABC-CLIO.

Coil, C. (2014). Creativity in an assessment driven environment. *Knowledge Quest, 42*(5), 48–53.

González, G. T., Estrada, F. J. P., & González, G. T. (2017). How teachers design and implement instructional materials to improve classroom practice. *Intangible Capital, 13*(5), 967–1043.

Grammatosi, F., & Harwood, N. (2014). An experienced teacher's use of the textbook on an academic English course: A case study. In N. Harwood (Ed.), *English language teaching textbooks: Content, consumption, production* (pp. 178–204). Basingstoke: Palgrave Macmillan.

Gray, J. (2010). The ELT coursebook as cultural artefact: How teachers censor and adapt. *ELT Journal, 54*(3), 274–283.

Harwood, N. (2017). What can we learn from mainstream education textbook research? *RELC Journal, 48*(2), 264–277.
Kaur, B., & Yeap, B. H. (2009). *Pathways to reasoning and communication in the primary school mathematics classroom.* Singapore: National Institute of Education.
Kim, M., & Kim, G. Y. (2013). The analysis of mathematical tasks in the high school mathematics. *School Mathematics, 15*(1), 37–59.
Kim, D., & Kim, G. Y. (2014). Secondary mathematics teachers' understanding and modification of mathematical tasks in textbooks. *School Mathematics, 16*(3), 445–469.
Ko, J., & Sammons, P. (2014). *Effective teaching* (Reprint of 2014 Report 978-1-909437-75-3). University of Oxford: Education Development Trust.
Lashley, L. (2019). A Reflective analysis of the selection and production of instructional material for curriculum delivery at the primary level in postcolonial Guyana. *Sage Open, 9*(2), 1–15.
Lee, E.-J., Lee, K.-H., & Park, M.-S. (2019). Developing preservice teachers' abilities to modify mathematical tasks: Using noticing-oriented activities. *International Journal of Science and Mathematics Education, 17*(5), 965–985.
Lee, K.-W. (2017). Convergent and divergent thinking in task modification: A case of Korean perspective mathematics teachers' exploration. *ZDM—The International Journal on Mathematics Education, 49*(7), 995–1008.
Lee, K.-W., Lee, E.-J., & Park, M.-S. (2016). Task modification and knowledge utilization by Korean prospective mathematics teachers. *Pedagogical Research, 1*(2), 1–13.
Leong, Y. H., Cheng, L. P., & Toh, W. Y. K. (2019a). Chronologically-grounded survey. In G. Hine, S. Blackley, & A. Cooke (Eds.), *Mathematics Education Research Impacting Practice (Proceedings of the 42nd Annual Conference of the Mathematics Education Research Group of Australasia)* (pp. 90–93). Perth: MERGA.
Leong, Y. H., Cheng, L. P., & Toh, W. Y. K. (2019b). Formula: To derive or not to derive? *Maths Buzz, 19*(1), 2–5. Singapore: Association of Mathematics Educators.
Leong, Y. H., Cheng, L. P., Toh, W. Y. K., Kaur, B., & Toh, T. L. (2019a). Making things explicit using instructional materials: A case study of a Singapore teacher's practice. *Mathematics Education Research Journal, 31*(1), 47–66.
Leong, Y. H., Cheng, L. P., Toh, W. Y. K., Kaur, B., & Toh, T. L. (2019b). Example sequences to help students recognise and apply formula: The Singapore situation. *The Mathematics Educator, 18*(1&2), 23–46.
Leong, Y. H., Cheng, L. P., Toh, W. Y. K., Kaur, B., & Toh, T. L. (2019). Teaching students to apply formula using instructional materials: A case of a Singapore teacher's practice. *Mathematics Education Research Journal*. https://doi.org/10.1007/s13394-019-00290-1.
Lloyd, G. M., & Behm, S. L. (2005). Preservice elementary teachers' analysis of mathematics instructional materials. *Action in Teacher Education (Association of Teacher Education), 26*(4), 48–62.
Menkabu, A., & Harwood, N. (2014). Teachers' conceptualization and use of the textbook on a medical English course. In N. Harwood (Ed.), *English language teaching textbooks: Content, consumption, production* (pp. 145–177). Basingstoke: Palgrave Macmillan.
Piirto, J. (2011). *Creativity for 21st century skills.* Dordrecht: Springer Science and Business Media.
Prestage, S., & Perks, P. (2007). Developing teacher knowledge using a tool for creating tasks for the classroom. *Journal of Mathematics Teacher Education, 10*(4–6), 381–390.
Remillard, J. T. (1999). Curriculum materials in mathematics education reform: A framework for examining teachers' curriculum development. *Curriculum Inquiry, 29*(3), 315–342.
Remillard, J. T., & Heck, D. J. (2014). Conceptualizing the curriculum enactment process in mathematics education. *ZDM—The International Journal on Mathematics Education, 46*(5), 705–718.
Rowntree, D. (1992). *Exploring open and distance learning materials.* London: Kogan Page.
Shawer, S. F. (2010). Classroom-level curriculum development: EFL teachers as curriculum developers, curriculum-makers and curriculum-transmitters. *Teaching and Teacher Education, 26*(2), 173–184.

Smith, M. S. (2000). Balancing old and new: An experienced middle school teacher's learning in the context of mathematics instructional reform. *The Elementary School Journal, 100*(4), 351–375.
Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: *An analysis of mathematical tasks used in reform classrooms. American Educational Research Journal, 33*(2), 455–488.
Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2009). Implementing standards-based mathematics instruction: A casebook for professional development (2nd ed.). Reston, VA: New York: National Council of Teachers of Mathematics; Teachers College Press.
Steiner, D. (2018). Materials matter. *Learning Professional, 39*(6), 24–28.
Stigler, J. W., & Hiebert, J. (2004). Improving mathematics teaching. *Educational Leadership, 61*(5), 12–17.
Travé, G., Pozuelos, F. J., Cañal, P., & Rodríguez, F. P. (2016). Validación de un instrumento de análisis de materiales (AMADE). In P. Cañal, G. Travé, F. J. Pozuelos, et al. (Eds.), *La enseñanza sobre el Medio Natural y Social, Investigación y experiencias* (pp. 97–114). Sevilla: Díada.
Yeo, K. K. J. (2018). Extending textbook exercises into short open-ended tasks for primary mathematics classroom instruction. In P. C. Toh & B. L. Chua (Eds.), *Mathematics instruction: Goals, tasks and activities* (pp. 195–210). Singapore: World Scientific.
Zaslavsky, O. (1995). Open-ended tasks as a trigger for mathematics teachers' professional development. *For the Learning of Mathematics, 15*(3), 15–20.

Lu Pien Cheng is a Senior Lecturer in the Mathematics and Mathematics Education Academic Group at the National Institute of Education, Nanyang Technological University in Singapore. She received her PhD in Mathematics Education from the University of Georgia (U.S.) in 2006. She specialises in mathematics education courses for primary school teachers. Her research interests include the professional development of primary school mathematics teachers, task design in mathematics education and cultivating children's mathematical thinking in the mathematics classrooms.

Yew Hoong Leong is an Associate Professor at the National Institute of Education, Nanyang Technological University. He began his academic career in mathematics education with the motivation of improving teaching by grappling with the complexity of classroom instruction. Along the journey, his research has broadened to include mathematics problem solving and teacher professional development. Together with his project teammates, they developed "Realistic Ambitious Pedagogy" and its accompanying plan of action—the "Replacement Unit Strategy".

Wei Yeng Karen Toh worked as a research assistant at the National Institute of Education for seven years. She has a Bachelor of Science in Applied Mathematics from the National University of Singapore. She was awarded with a Distinction grade for the Post-Graduate Diploma in Education at the National Institute of Education. She taught in a few secondary schools as a mathematics and chemistry teacher for seven years. Her research interests include studying the history of mathematics, learning mathematical and pedagogical content knowledge, examining classroom practices and investigating teachers' designs of instructional materials. To advance her career in research, she pursued a Master of Education in Mathematics at the National Institute of Education.