Chapter 10
The Enacted Curriculum—Students' Perspectives of Good Mathematics Lessons in Singapore Secondary Schools

Ngan Hoe Lee, Berinderjeet Kaur, and Liyana Safii

Abstract This chapter presents the characteristics of good mathematics lessons from the lens of typical secondary school students in Singapore. This chapter begins by examining the student perception in relation to the five inter-related problem-solving components embodied in the Singapore School Mathematics Curriculum Framework (SSMCF): concepts, skills, processes, metacognition and attitudes. Data from post-lesson student interviews which were stimulated by videos of the lesson revealed that the development of proficiencies in mathematics skills was most commonly emphasised in the "highs" of mathematics lessons while emphasis on metacognitive strategies was the least emphasised. This was true for all four courses of study (i.e. Integrated Programme, Express, Normal (Academic) and Normal (Technical)). The chapter further categorises the student data into teacher approaches and class activities that have been perceived by the students as the highs of mathematics lessons. While the perceived value for teacher approaches differ across all four courses of study, class practice and peer discussion were the most commonly cited class activities for all courses of study. Findings from the study provide important implications on the way to better engage students in the teaching and learning of mathematics.

Keywords Students' lens · Good mathematics lesson · Singapore · Secondary mathematics

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10.1 The Student Perspective

Classroom instructions are no longer teacher-centred. Students are increasingly playing an active role in classroom learning. The shift towards a collaborative participation of teachers and students suggests that the mechanisms underlying teaching and learning in the classroom cannot be construed only by examining the processes that encapsulates the teacher's participation in the classroom. In other words, "as learning is dependent upon the situations and circumstances in which it is engendered and the feelings these situations provoke in students, any attempt to improve mathematics teaching must take into account both teacher practice, student practice and their responses to each other's practice" (Kaur, 2008, p. 951). In relation to the learning of mathematics, this could mean that teaching and learning is perceived as "the product of interactions among the teacher, the students and the mathematics" (Kilpatrick, Swafford, & Findell, 2001). This implies that the students' perception and participation in the classroom should be emphasised along with the teachers' perception and participation in the classroom (Clarke, Keitel, & Shimizu, 2006).

Research on classroom instructions through the teacher's lens (e.g. teacher beliefs and perceptions) have been widely explored and detailed in the literature. However, the understanding of the teaching processes in the classrooms as experienced by the learners could also provide valuable insights on how teachers deliver their lessons. Ahmad and Aziz (2009) highlight that student perception plays an important role in research on classroom instructions as their perception is "coloured by challenging and interesting experiences that allow them to observe the learning and teaching behaviours more intimately than the teacher" (p. 19). This suggests that students' perceptions not only promote heightened awareness of their own classroom learning experiences and their teachers' classroom instructions, but also forms part of a feedback channel for teachers to reflect and improve on their classroom instructions (Ahmad & Aziz, 2009). The study of student perception thus can provide valuable contributions in the improvement of teaching and learning in the classroom.

Prior research have explored mathematics teaching through the learner's perspective, providing insights into what students consider valuable for their classroom learning. These studies are varied and include student perception on what constitutes good teaching, effective teaching or a good teacher (e.g. Attard, 2011; Kaur, 2009; Martinez-Sierra, 2014; Murray, 2011; Shimizu, 2009; Wang & Hsieh, 2017). Student perception gathered from these studies, however, have been mixed, possibly attributed by various social and cultural norms that underlie the educational system in different countries. Pang (2009) highlights that existing classroom instructions need to be studied in relation to these norms in order to understand the beliefs and values on which these practices are based upon. This is also emphasised in The Learner's Perspective Study (LPS), a large international comparative study on mathematics education which takes into account students' perceptions in the study of mathematics classrooms, learning and student outcomes around the world (Clarke, Keitel, & Shimizu, 2006). The researchers of the LPS note that the findings from the study showed how "culturally-situated are the practices of classrooms around the world and the extent to which students are collaborators with the teacher, complicit in the development and enactment of patterns of participation that reflect individual, societal and cultural priorities and associated value systems" (Clarke, Emanuelsson, Jablonka, & Mok, 2006, p. 1).

Singapore was also part of the LPS. It was discovered that Grade 8 (Year 2 in Singapore secondary school) students in Singapore perceived a good mathematics lesson as one where their teachers adopted some of the following classroom instructions (Kaur, 2009, p. 343):

1. Explaining mathematical concepts and demonstrating steps of procedures clearly
2. Showing demonstrations, or using manipulatives or real-life examples to make it easier for complex ideas to be understood
3. Reviewing previously taught knowledge
4. Introducing new knowledge
5. Giving individual or whole-class feedback using student individual work or group presentations
6. Giving clear instructions for activities that are expected to be completed during or after class
7. Providing students with opportunities to work on interesting activities individually or collaboratively in small groups
8. Allocating sufficient practices as part of exam preparation.

Drawing upon the same motivations that underlie the LPS, the current study examines students' learning experiences through their perspectives. We first detail the Singapore School Mathematics Curriculum Framework (SSMCF) to understand the context of mathematics teaching and learning in Singapore. We proceed to discuss the data and findings from one part of the project (detailed in Chapter 2) which examines Singapore secondary school students' perceptions of good mathematics lessons. These perceptions would be presented in the form of characteristics of good mathematics lessons, also referred to as the *highs* of the lessons. The highs of the lessons include moments of the lessons that the students feel would constitute part of a good lesson. These lesson characteristics would be analysed in relation to the five problem-solving components in the SSMCF (i.e. concepts, skills, processes, metacognition and attitudes). We also draw upon the data of students from four courses of study (i.e. Integrated Programme (IP), Express, Normal (Academic) (N(A)) and Normal (Technical) (N(T))) to help us understand the perceptions of students with diverse student learning profile. Details of the four courses of study are provided in Chapter 1, Sect. 1.2. The student data was examined from two perspectives—the teacher approach and class activity.

10.2 Mathematics Instruction in Singapore

10.2.1 Singapore School Mathematics Curriculum Framework

As briefly introduced in Chapter 1, mathematics instruction in Singapore is guided by a robust problem-solving framework for the teaching, learning and assessment of mathematics in the classroom. Known as the Singapore School Mathematics Curriculum Framework (SSMCF), the framework was developed in 1990, and has since undergone several changes and been an integral part of mathematics curriculum enactment in Singapore (Ministry of Education [MOE], 2012). The framework was constructed with the intention of providing teachers with directions to create a "more engaging, student-centred, and technology-enabled learning environment" as well as to "promote greater diversity and creativity in learning" (MOE, 2012, p. 17). The SSMCF (see Chapter 1, Fig. 1.2) draws upon five inter-related competencies that focus on mathematical problem solving: conceptual understanding, skills proficiency, mathematical processes, metacognition and attitudes, to develop students' ability in solving a wide range of problems including straightforward and routine tasks to complex and non-routine ones (MOE, 2018b). This is in line with Singapore Ministry of Education's (MOE) intention to equip students with twenty-first century competencies to prepare them for challenges brought about by the fast-changing world attributed by globalisation, shift in demographics and advancement in technology (MOE, 2018a). These twenty-first century competencies include skills such as critical and inventive thinking, and communication, collaboration and information skills. In the next section, we discuss the five components of SSMCF in further detail.

Introduce/construct mathematical concepts. Mathematical concepts in numbers, algebra, geometry, probability and statistics, and calculus are "connected and interrelated" (MOE, 2018b, p. 10). These concepts can be represented through numerical/tabular, pictorial, graphical, verbal, symbolic (equations or expressions) and physical/concrete (Cleaves, 2008). Goldin and Kaput (1996) postulate that students' comprehension of mathematical ideas is influenced by the mathematical representations that teachers use. In particular, conceptual understanding can be fostered through the use of multiple representations (Donovan & Bransford, 2005). Students who grasp a coherent understanding of mathematical concepts are able to "recognise the idea embedded in a variety of qualitatively different representational systems, flexibly manipulate the idea within given representational systems and accurately translate the idea from one system to another" (Lesh, Post, & Behr, 1987, p. 36). As such, teachers are encouraged to adopt a wide range of learning experiences that involve "hands on activities and the use of technological aids to help students relate abstract mathematical concepts with concrete experiences" (MOE, 2012, p. 15).

Develop proficiencies in mathematical skills. Mathematical skills include "carrying out the mathematical operations and algorithms and in visualising space, handling data and using mathematical tools" (MOE, 2018b, p. 10). Mathematical skills also comprise students' ability to use software in the learning and application of mathematics, especially in today's classroom settings where ICT tools are increasingly being incorporated into classroom learning. Bloom (1968) posits that for students to develop these mathematical skills, teachers should establish clear learning goals and complement student learning with formative assessments that serve as a medium for determining students' level of mastery. It is, however, important that mathematical skills are "taught with an understanding of the underlying mathematical principles and not merely as procedures" (MOE, 2012, p. 15). This means that the acquisition of both instrumental and relational understanding should be involved in the development of procedural fluency (Skemp, 1987). In other words, the acquisition of procedural skills should not just focus on the "how" but should also focus on the "why".

Emphasise on mathematical processes. Mathematical processes that are involved in the acquisition and application of mathematical knowledge require students to use certain skills. As identified in the SSMCF (MOE, 2018b, p. 11), these include:

1. *Abstracting and reasoning*—While abstraction is what makes mathematics powerful and applicable, justifying a result, deriving new results and generalising patterns involve reasoning;
2. *Representing and communicating*—Expressing one's ideas, solutions and arguments to different audiences involves representing and communicating and the use of notations in the mathematics language.
3. *Applying and modelling*—Applying mathematics to solve real-world problems often involves modelling, where reasonable assumptions and simplifications are made so that problems can be formulated mathematically, and where mathematical solutions are interpreted and evaluated in the context of the real-world problems.

These skills reflect the critical and inventive thinking competencies in the twenty-first Century Competencies Framework (MOE, 2018a). In particular, the skills required for mathematical problem solving could foster students' ability to "think critically" and "think out of the box" (MOE, 2018a). For students to develop proficiencies in such mathematical processes, teachers are encouraged to provide sufficient opportunities for students to engage in problem solving that involves complex and non-routine tasks (MOE, 2018b, p. 10).

Emphasise on metacognitive strategies. Metacognition, as defined by Flavell (1976), refers to "one's knowledge concerning one's own cognitive processes and products or anything related to them… Metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects or data on which they bear, usually in the service of some concrete goal or objective" (p. 232). Simply put, metacognition involves one's "awareness of, and the ability to control one's thinking processes, in particular the selection and use of problem-solving strategies" (MOE, 2018b, p. 12). These processes also involve students' ability to monitor and regulate their own thinking and learning. Metacognition, particularly in the learning of mathematics, in essence involves three facets—awareness, monitoring and regulating (Lee, Ng, & Yeo, 2019). To promote development of strategies that support metacognition, the SSMCF has advocated that teachers provide students with opportunities to "solve non-routine or open-ended problems" to provide opportunities for students to discuss their solutions, think aloud and reflect on what they are doing, keep track of how things are going and make changes when necessary (MOE, 2018b, p. 12).

Imbue desired learning attitudes. Attitudes towards mathematics learning reflect the affective facet of learning that includes one's "belief and appreciation of the value of mathematics, one's confidence and motivation in using mathematics, and one's interests and perseverance to solve problems using mathematics" (MOE, 2018b, p. 12). In line with Singapore's move to achieve balance between academic rigour and joy of learning, the Singapore MOE (2017) advocates that learning should promote students' discovery of their interests and passions, and love in the things that they do. In other words, learning should go beyond external motivations and achieving good grades. Teachers are recommended to incorporate fun learning experiences in the acquisition of knowledge and skills to instil the joy of learning among students. In particular, teachers are encouraged to use a wide range of resources to cater to varied student interest (variety), use these resources sufficiently (opportunity) and make connections between these resources and mathematics learning (linkage) (Yeo, 2018). These types of instructions are aimed at building students' desired attitudes towards the learning of mathematics.

10.3 Singapore Secondary School Students' Perspectives of Good Mathematics Lessons

To document students' perspectives of good mathematics lessons, data was collected through post-lesson video stimulated interviews that were conducted with 447 focus students. These focus students were students of the 30 experienced and competent teachers who were involved in the first phase of the project—the video segment—where their lessons were recorded (detailed in Chapter 2).

Two parts of the post-lesson student interviews which were stimulated by videos of the lesson were analysed when identifying characteristics of good mathematics lessons. These parts involved portions of the interview where the focus students were asked to identify the highs of a particular mathematics lesson in which they were the focus students. The highs of mathematics lessons were referred to as moments of the lessons that the students felt would constitute part of a good lesson. In particular, students were asked, "Can you share with me what the highs of this lesson were?" and were provided with the recorded video of that particular lesson. The recorded video served to help students in recalling how the lesson was taught. The students were instructed to fast forward the recorded lesson video to the parts of the lesson that they perceive to be the highs of the lesson.

A total of 636 responses were collected from this part of the interview (i.e. 108 from IP, 196 from Express, 194 from N(A) and 138 from N(T)). Most of the students shared at least one high moment of the lesson that they sat for. These responses were categorised into two perspectives: teacher approach and class activity. The findings will be presented in two parts. In the first part, we outline the teacher approaches and class activities in relation to the five problem-solving competencies as embodied in the SSMCF (i.e. concepts, skills, processes, metacognition and attitudes). In the second part, we delve into the types of teacher approaches and class activity that were valued by the students. The student interview data is also compared across the four courses of study.

10.3.1 Problem-Solving Competencies in the SSMCF

Analysis of the post-lesson student interviews revealed that the focus students perceived a variety of teacher approaches and class activities as the highs of mathematics lessons. Table 10.1 shows how commonly cited the teacher approaches and class activities were in relation to the five problem-solving competencies embodied in the SSMCF. The interview data revealed that the development of proficiencies in mathematics skills (42%) was most commonly emphasised in the highs of mathematics lessons, followed by emphasis on mathematical processes (27%), imbuement of desired learning attitudes (15%) and introduction of concepts to students or engagement of students in constructing concepts (12%). The emphasis on metacognitive strategies was the least emphasised (4%) in the highs of mathematics lessons.

The data also revealed that, generally, students across all four courses of study placed similar emphasis on these competencies. Regardless of the courses of study they were in, it appeared that students emphasise most on the development of proficiencies in mathematics skills (at least 40% for all courses of study) and least on metacognitive strategies (at most 5% for all courses of study) in the highs of mathematics lesson. As compared to students in other courses of study, students in the Express course (5%) appeared to place lesser emphasis on the introduction of

Table 10.1 Student perception of good mathematics lessons in relation to problem-solving competencies

Problem-solving competencies	Percentage of responses				
	IP (n = 108)	Express (n = 196)	N(A) (n = 194)	N(T) (n = 138)	Total (N = 636)
Skills	41	43	43	40	42
Processes	35	25	28	21	27
Attitudes	5	22	13	17	15
Concepts	14	5	11	22	12
Metacognition	5	5	5	0	4

concepts or engagement of students in constructing concepts in the highs of mathematics lessons. On the other hand, students in the IP course (5%) seemed to emphasise the imbuement of desired learning attitudes comparatively lesser than students in the other courses of study.

10.3.2 Teacher Approach and Class Activity

The teacher approaches and class activities were further examined to understand the nature of the highs of mathematics lessons, as identified by the students, as well as the reasons underlying their choices. Table 10.2 shows the percentages of responses for the different types of teacher approach and class activity that had been cited by the students. For the purpose of discussion, only the teacher approaches and class activities that recorded a frequency of at least 10 student responses, i.e. at least more than 1% of the total responses, would be discussed.

A comparison of the teacher approaches and class activities across all four courses of study revealed some similarities and differences. Class practice and peer discussion were commonly cited by students in all four courses of study as the highs of mathematics lessons. Apart from class practice and peer discussion, students in the IP course deemed the parts of the lessons where their teachers reviewed student work (12%) as the highs of mathematics lessons. On the other hand, students in the Express course tended to value teachers' attempt to make jokes (6%) and share alternative ways of solving problems (5%) during lessons. For students in the N(A) course, the teachers' attempt to review student work (9%) and explain how to solve a worked example (8%) were some of the high moments of the lessons. Six percents of students in the N(A) course also identified assessment for learning, such as use of the entry and exit cards, as the highs of the mathematics lessons. In addition, students in the N(T) course appeared to appreciate teachers' use of manipulatives when concepts were demonstrated in the lesson (7%).

With reference to the video recorded lessons, Table 10.3 details the reasons underlying the students' perceived value on the various teacher approaches and class activities.

As seen in our inferences made in Table 10.3, the students' reasons on their choice of teacher approaches and class activities add pedagogical value to mathematics lessons. It appears that the characteristics of mathematics lessons that students thought as important also reflect pedagogically sound practices. In other words, the students seemed to value the importance of pedagogically sound practices in the choice of teacher approaches and class activities that they had identified.

Table 10.2 Teacher approaches and class activities of good mathematics lessons as perceived by students in the different courses of study

Problem-solving competencies	Characteristics	Percentage of responses			
		IP ($n = 108$)	Express ($n = 196$)	N(A) ($n = 194$)	N(T) ($n = 138$)
Teacher approach					
Skills	Explaining how to solve a worked example • Convey expectations explicitly to students when explaining how to solve a worked example by showing the process of arriving at the solution and demonstrating the proper steps in the workings	–	–	8	–
Skills	Reviewing student work • Provide timely feedback • Provide opportunities for students to learn from strengths and weaknesses of their peers' work	12	–	9	–
Attitudes	Making jokes • Inject jokes into the lessons, making classroom learning experiences more enjoyable than otherwise	–	6	–	–
Concepts	Demonstrating a concept using manipulatives • Use manipulatives to enhance students' understanding of concepts, especially abstract concepts, through demonstration or visualisation of such concepts	–	–	–	7
Metacognition	Sharing alternative ways of solving problems • Encourage students to compare and contrast different problem-solving approaches, fostering their metacognitive strategies	–	5	–	–

(continued)

Table 10.2 (continued)

Problem-solving competencies	Characteristics	Percentage of responses			
		IP ($n = 108$)	Express ($n = 196$)	N(A) ($n = 194$)	N(T) ($n = 138$)
Class activity					
Skills	Assessment for learning • Opportunities provided for students to check their understanding of acquired knowledge and skills • Opportunities provided for students to monitor their progress in learning	–	–	6	–
Skills	Class practice • Opportunities provided for students to apply the knowledge and skills that they have learned through a variety of mathematical problems	21	29	16	23
Processes	Peer discussion • Opportunities provided for students to express their ideas/solutions and their underlying reasons or arguments that could sharpen their mathematical thinking through collaborative learning	11	7	8	8

Note The table only includes teacher approaches and class activities that recorded a frequency of at least 10 student responses

Table 10.3 Reasons underlying the students' perceived value on the various teacher approaches and class activities

Characteristic	Lesson context	Student interview response	Inferences
Explaining how to solve a worked example	Topic: Vectors Teacher 2 (T2) (N(A)) in Lesson 7 explained a worked example that involved the expression of the relationship between two vectors (i.e. $\overrightarrow{BC}$ and $\overrightarrow{DA}$) as an equation. The conditions of the vectors are such that they share the same length but have opposite directions. In his explanation, the teacher emphasised why $\left\|\overrightarrow{BC}\right\| = \left\|\overrightarrow{DA}\right\|$ cannot be a possible answer as this equation expresses a weak relationship. He added that the relationship between the two vectors should be expressed as $\overrightarrow{BC} = -\overrightarrow{DA}$ as this equation reflects the given conditions of the vectors (i.e. same length and opposite directions)	**Student (P13)**: So first I thought you can put like that. The magnitude, everything, but I learned that it's, you cannot put that. Because it's that, that, the two lengths was opposite direction, so you cannot put the magnitudes, so it (this part of the lesson) was high. I always put the line (the symbol for absolute value) one **Interviewer (I)**: You always put the magnitude **P13**: So I learned that you cannot put that. **Because the length was opposite, the arrow was opposite, one is showing below, one is showing on top, so you cannot put the magnitude** **I**: The magnitude not equal **P13**: Ya **I**: If they are pointing different directions **P13**: This was, wait ah. Ya, it's the length that is the same. But, the relation was opposite, the strong relation was opposite direction **I**: Ok. So by doing that, it doesn't tell the opposite direction **P13**: Ya **it is only telling that the length is the same. It doesn't tell that it is opposite. You need to put the negative sign** **I**: So at that point, you suddenly realise, it cannot… **P13**: Ya I suddenly realise that you cannot put that	The explanation of a worked example helps the student to understand how certain mathematical expressions can/cannot be expressed in mathematical problems

(continued)

Table 10.3 (continued)

Characteristic	Lesson context	Student interview response	Inferences
Reviewing student work	Topic: Logarithm Teacher 12 (T12) (IP) in Lesson 6 used students' work to explain the correct solutions to logarithmic equations which involved common and natural logarithms. While reviewing the students' solutions, the teacher highlighted common mistakes and discussed more efficient way(s) of solving such equations	**Student (P11)**: I would say the high point is when she starts pointing out some common mistakes that are made so that we can also **get a better understanding on the topic** because she starts going through the answers **then she wants to see whether we have different methods in solving the questions.** So she takes our workings and shows it on the projector, and if you make any mistakes, **she will tell us where we have gone wrong and how we should prevent (it)** **P11**: It's good to show us different ways of doing it because sometimes we may do much longer method when there's easier way to solve **Interviewer (I)**: So what did you learn from them? Why did you find (this moment) high? **P11**: Because if we know how to do the question in a faster method, then it will save more time. So have more time to work on other harder questions **I**: Then why do you say that it's a high? **P11**: Because it also help us get a good understanding on how to apply the different laws of logarithm. Then she uses our friend's workings then she writes it on the board to explain to us what we should not do 'cause my friend only derived at one solution but there were actually two because he made a wrong assumption on the question **I**: So when the teacher does this pointing out during your example, something wrong, and talking about it, this kind of discussion, how do you find it? **P11**: I find it very helpful because sometimes we also make this kind of mistakes in our test papers and we actually do not know where we went wrong. So if she goes through it like that we are able to know how to prevent ourselves from making these kind of mistakes. Maybe sometimes we make the wrong assumptions but if we understand the laws better then we will not make these kind of assumptions	The student finds value when the teacher uses their peers' work to explain how to obtain the correct solution to the problem; the student is then able to learn from mistakes made by the peers and also learn to use more efficient problem-solving approaches

(continued)

Table 10.3 (continued)

Characteristic	Lesson context	Student interview response	Inferences
Making jokes	Teacher 22 (T22) (Express) in Lesson 1 responded to student's mischievous comment by making a witty remark	**Student (P02)**: I like how she can give very witty remarks to those mischievous students. It kind of **makes it funny, interesting**, because like maths lesson can be a little boring and like, because it's the first lesson of the day and people are like "ha, so tired", but she can give witty remarks that kind of makes it like you kind of like, **that little laugh can give you a lot of adrenaline to enjoy the day** **Interviewer (I)**: Could you remember anything, any one of those witty remarks? **P02**: She gave it to many students, a few students so it's like, Student A. She likes to tease Student A because Student A is like very, like to give a lot of unnecessary comments which like make people like "omg", eye roll thing. So she gives a remark, he will keep quiet, it's kind of funny	Teacher's witty remarks make lessons enjoyable
Demonstrating a concept using manipulatives	<u>Topic: Mensuration</u> Teacher 9 (T9) (N(T)) in Lesson 6 had prepared a circle that was cut from a large piece of vanguard sheet for the lesson. While showing the students the circle she had prepared, she asked the students to think about whether a cone could be created from that particular circle. After eliciting responses from the students, the teacher demonstrated how a cone could be formed from a circle by cutting a minor sector from the circle. She showed the students that by cutting a minor sector from the circle, she was also able to form a major sector. Throughout the demonstration, the teacher emphasised on the part of the circle that she has cut as a minor sector, and the remaining part of the circle as a major sector. She concluded the demonstration by highlighting that cones (without a base) are created from sectors	**Student (P11)**: It's a high point because I can understand it. I understand when she told me that, she explained which one is minor and which one is major **Interviewer (I)**: So when you saw this do you find that it's a very good way to express it? **P11**: Yes **I**: Do you like it when teacher bring in manipulatives? **P11**: Yeah **I**: How come? **P11**: **So that I'm able to know more, better** **I**: Know more better, what do you mean by know more? **P11**: If she write it on whiteboard, I don't know which one is, like draw it out on whiteboard, I don't know what is minor, what is major. **When she cut down, I do understand that the small one is minor, and the big one is major. It's clearer**	The use of manipulatives helps the student in visualising concepts

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Table 10.3 (continued)

Characteristic	Lesson context	Student interview response	Inferences
Sharing alternative ways of solving problems	Topic: Vectors Teacher 1 (T1) (Express) in Lesson 3 explained a worked example that involved the simplification of the expression, $\overrightarrow{PQ} - \overrightarrow{PS}$, based on a given diagram. The teacher elicited answers from the students but realised that they were facing difficulties in tackling the problem as $\overrightarrow{PS}$ was expressed in negative form. To help the students, the teacher taught them to change the expression to $\overrightarrow{PQ} + \overrightarrow{SP}$ to avoid dealing with negative vectors.	**Student (P06)**: Like when she goes through the examples then you have to think through and yah, and she finds like the shortest possible methods so it might not be something that I already know of or the method that I use. So it's experience, because if in the future there are similar examples, then I will be able to do it in the shortest method… 'Cause I feel like I benefitted quite a lot from it. **Interviewer (I)**: In what way? **P06**: In a way that I managed to learn to, in a way **see this from a different point of view**, like I managed to, instead of really going through the vectors one by one**, I managed to cut short quite a few steps**	Sharing an alternative way to approach a problem teaches the student to be more metacognitive when solving problem
Assessment for learning	Topic: Arc length, sector area and radian measures Teacher 11 (T11) (N(A)) in Lesson 5 made her students complete an exit card before the lesson ended. The exit card required students to express i. a given angle in radian, in degree, and ii. a given angle in degree, in radian	**Student (P11)**: I think the exit card is quite helpful because we, in the end of the day, we try not to get people to help us, so it's basically on what we have learned, at the end of the day what we've catch as much as possible from her. Then as she mark our answers, **she understands that his understanding is this type, his level of, which part of his workings are weak, then from there she will slowly try to help us in our weak areas** **Interviewer (I)**: So you like doing exit card activity? **P11**: Yeah. **It just shows like we understand that, understand her teaching**, at least we also don't have to worry about different types of question that might be coming out	The exit card serves not just as an activity for the student to check his/her own understanding but also for teachers to identify and address students' weaknesses
Class practice	Topic: Quadratic Equations Teacher 21 (T21) (IP) in Lesson 1 made her students solve several quadratic equations (using the factorisation method) independently in class	**Student (P01)**: Solving the questions, because I'm not sure why actually but I just like to solve the equation and if I get the answer correct right, I will feel quite satisfied because **it shows that I'm making progress** **Interviewer (I)**: So it makes you feel good because you are making progress? **P01**: Yeah	Independent class practice allows the student to check their progression in learning

(continued)

Table 10.3 (continued)

Characteristic	Lesson context	Student interview response	Inferences
Peer discussion	Topic: Cumulative Frequency In the class of Teacher 7 (T7) (N(T)) in Lesson 4 students held discussions with their peers while attempting class practice tasks that were assigned to them. The tasks involved the use of a cumulative frequency graph which represents the examination marks of a group of students. Students were required to identify the students' marks in relation to concepts such as the median, lower quartile, 20th percentile and the interquartile range using the graph	**Student (P07)**: And the highest is, I think working with my friends to find out the same answer, so we get the same marks[a] together **Interviewer (I)**: So when you were saying that you get the same marks together, work together, how did you feel, why is it that there is a high? **P07**: For **me it's motivation for me to work harder**. Instead of working by myself, and **sometimes I do not have anyone to ask**, the teacher may be busy at times ah **I**: So which of the words to describe you, excited, happy? Achieved, motivated? **P07**: Motivated… I think instead of the teacher trying to explain the whole thing at once, he check with us if we still understand. So, so after he go through this, I work with my friends ah to find out which one, to confirm that which one is the, which part we supposed to use ah. **Because we also have different thinking ah. So we all use the same method, then we agree on each other's answer**	The student enjoys collaborative learning

[a]*Note* Marks here refer to a variable in the class practice task

10.4 Conclusion

The present study has enriched our understanding of how secondary students in Singapore consider a mathematics lesson to be a good one. In particular, the purpose of this chapter was to explore students' perceptions of valued teaching and learning experiences in mathematics classrooms, especially in relation to the context of mathematics instructions in Singapore and for students with various learning profiles.

The findings revealed insights on students' perception of good mathematics lessons in relation to the five problem-solving components embodied in the SSMCF. Students across all four courses of study appeared to be fairly consistent in what they considered as valuable aspects of mathematics lessons. In particular, students across all courses of study gave most priority to the proficiencies in mathematics skills and least priority to the emphasis of metacognitive strategies when considering the characteristics of good mathematics lessons. The lack of priority given to the emphasis of metacognitive strategies could be explained by the possible lack of perceived value in metacognitive strategies or students' lack of vocabulary to articulate their perceived value in relation to metacognitive strategies. The findings also revealed that as compared to other courses of study, students in the IP course gave lower priority to the imbuement of desired learning attitudes in mathematics lessons. This observed lack of priority could be attributed by IP students' self-sufficiency in cultivating the desired learning attitudes in the learning of mathematics. Moreover, with a climate that is heavily dependent on national examinations and placement, IP students might consider themselves to be already academically successful, and so do not place as much emphasis on developing interest or appreciation for mathematics. Thus, they might perceive the imbuement of desired learning attitudes in mathematics lessons as less necessary than students in other courses. The findings also showed that the preference for the use of manipulatives to demonstrate a concept appears to be distinctive of students in the N(T) course. Manipulatives are often used as a pedagogical resource tool to guide students in understanding abstract mathematical ideas through concrete experiences, especially for weaker students. Thus, the lack of priority given by students in other courses on the use of manipulatives could be attributed to lesser use of manipulatives in mathematics lessons taught by teachers in the IP, Express and N(A) courses.

The findings also highlighted eight key characteristics of good mathematics lessons identified by the students. Despite the difference in student learning profiles, it was observed that students across all courses of study appeared to value individual mathematical task attempts allocated in class (class practice) and the exchange of ideas with their peers (peer discussion). This suggests that students place importance on opportunities for mathematical application and checking for mastery of learning and skills, as well as collaborative learning. It is also interesting to note that five of these lesson characteristics—demonstrating a concept using manipulatives, assessment for learning, class practice, explaining how to solve a worked example and reviewing student work—are similar to the characteristics of good mathematics teaching observed in Kaur's (2009) study. In a nutshell, the characteristics of good mathematics lessons as viewed from the students' perspectives generally seem to resonate well with the framework that supports mathematics instructions in Singapore (i.e. the SSMCF). The students' perspectives provided an enhanced understanding of teaching and learning processes that occur in mathematics lessons as experienced by learners, and provided directions in better engaging our students in the teaching and learning of mathematics.

While the students perspectives of good mathematics lessons generally reflect classroom instructions advocated in the SSMCF, students appeared to be lacking in the ability to articulate what they deemed as important in the teaching and learning of mathematics or have a superficial awareness of mathematical strategies. Our findings thus call on teachers to provide support in the development of students' vocabulary that will help them to express clearly their needs or what is important to them in the teaching and learning of mathematics. For instance, teachers could provide more student exposure to the idea of metacognition as well as the teaching and learning of metacognitive strategies. In other words, the address of metacognition in the mathematics classroom may require a more deliberate rationalisation and articulation.

The findings also suggest that there could be value in emphasising heterogeneous grouping in mathematics lesson, as reflected in the students' perceived value in peer discussion during lessons. Piaget (1932) postulated that peer interaction has its own advantages; peer interaction helps students to identify and correct their misconceptions, and develop high-level cognitive architecture. Research on groupings in general, have been inconclusive as it appears that none of the group composition (i.e. homogenous or heterogeneous grouping) is equally advantageous for high, average and low-achieving students (e.g. Huang, 2009; Kaya, 2015; Saleh, Lazonder, & de Jong, 2007). However, low-achieving students seem to benefit from heterogeneous composition through motivation and stimulation from high-achieving students (Chang, Singh, & Filer, 2009). In particular, low-achieving students can benefit from better performance and higher motivation (Saleh, Lazonder, & de Jong, 2005).

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