Chapter 13
Production of Superposition of Coherent States Using Kerr Non Linearity

Shivani A. Kumar, H. Prakash, N. Chandra, and R. Prakash

Abstract The coherent states are often called as Glauber coherent states and were named after the American Scientist Glauber who was first to realize the extraordinary usefulness of these coherent states for explanation and analysis of many optical phenomena. These states were first introduced by Sudarshan also and are now been extensively studied and applied to quantum-optical problems. The most explicit form of these states are expressed as, $|\,\alpha\,\rangle = \sum_{n=0}^{\infty} e^{-\frac{1}{2}\,|\,\alpha\,|^2} \frac{\alpha^n}{\sqrt{n!}}\,|\,n\,\rangle$ where, the Fock states $|\,n\,\rangle$ is the eigen state of the number operator $N = a^{\dagger}a$, i.e., $N\,|\,n\,\rangle = n\,|\,n\,\rangle$ and $\alpha = \alpha_r + i\alpha_i$ is a complex number. These Glauber coherent states are the eigen states of annihilation operator and are well known. They play a very important role in many applications of quantum information processing including quantum teleportation. But it has been a long dream for physicists to generate these superposed coherent states in the most general desired form $|\,\psi\,\rangle = N\left(\cos\frac{\theta}{2}\,|\,\alpha\,\rangle \pm \sin\frac{\theta}{2}\,e^{i\,\varphi}\,|{-\alpha}\,\rangle\right)$ where, N is the normalization factor. In this paper, we propose a scheme to generate any such general superposition of coherent states $|\,\alpha\,\rangle$ and $|{-\alpha}\,\rangle$ using Kerr effect, two beams in coherent states, a single photon beam and optical devices like polarization beam splitter and mirrors. In the output, if a single photon is detected in a polarization state defined by angle θ and φ, the desired superposition of coherent states $|\,\alpha\,\rangle$ and $|{-\alpha}\,\rangle$ results. If the photon is detected in an orthogonal polarization state (the state in which the electric field strength at a given point in space is normal to the direction of propagation), a superposition state different from the desired one results.

13.1 Introduction

The coherent states, often called as Glauber states [1–3] after the American Scientist who was first to realize their extraordinary usefulness for the description of optical

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phenomena. These states were introduced by Sudarshan [4] also and are now been extensively studied and applied to quantum-optical problems. The explicit form of these states are expressed as,

$$|\alpha\rangle = \sum_{n=0}^{\infty} \mathrm{e}^{-\frac{1}{2}|\alpha|^2} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \tag{1.1}$$

where, the Fock states [5] $|n\rangle$ is the eigen state of the number operator $N = a^\dagger a$, i.e.

$$N|n\rangle = n|n\rangle \tag{1.2}$$

and $\alpha = \alpha_r + i\alpha_i$ is a complex number. These state are the eigen states of annihilation operator and are well known. Their coordinate representation is the minimum-uncertainty packet of harmonic oscillators [6]. Although these states are non-orthogonal, they do form a complete set of states, i.e., they obey a completeness relation and hence form a good set of basis states. The overcompleteness of coherent states allows the expansion of many important field operators as a single integral over projectors on these states.

It has been a dream for many physicists to generate superposition of coherent states $|\pm\alpha\rangle$ which are out of phase with each other by a phase difference of 180°

$$|\psi\rangle_{\pm} = N_{\pm}(\,|\alpha\rangle + |-\alpha\rangle) \tag{1.3}$$

where $N_{\pm}$ is the normalization factor. But it has now been shown [7–10] that a coherent state propagating through an optical Kerr medium may evolve into a superposition of a set of coherent states differing in phases by multiple of some constant. This superposition of macroscopically distinguishable quantum states is popularly known as Schroedinger cat states [9, 11]. In practice, the superposition states can be generated in various non-linear process [7, 8, 12–19], back action evading measurements [20–22], Jaynes-Cummings Model [23–27], quantum non-demolition measurements [28], and resonant cavity [29–33].

Several other possibilities also exist to generate the non-classical state of electromagnetic field with the help of superposition of two or more states [34–50]. One of the earlier methods for generating these superposition states was proposed by Yurke and Stoler who proposed to generate a coherent superposition of the form

$$|\psi\rangle = \frac{1}{\sqrt{2}}\big[|\alpha\rangle + \mathrm{e}^{\mathrm{i}\pi/2}|-\alpha\rangle\big], \tag{1.4}$$

using a Kerr nonlinearity.

A scheme of back action evading coupling to correlate the signal and readout modes was proposed by Song et al. [21]. The calculations done by the authors show that a superposition of quantum states is generated in the signal mode as evidence by interference fringes in a homodyne measurement of the quadrature component orthogonal to the axis of maximum separation between the superposed states. Yurke et al. [28] interchanged the parametric amplifier and the back action evader in their calculations and showed that both devices generated the same Schroedinger cat wave function provided a suitable choice of gain parameter is made.

In 1993, Tara et al. [51] showed production of Schrodinger macroscopic quantum superposition states in a Kerr medium. They also proposed a scheme for production of superposition of squeezed coherent states. Gerry [31] proposed a method for generation of Schrodinger cat states and entangled coherent states [41, 51] in the motion of a trapped ion by a dispersive interaction. They showed that these entangled coherent states may be generated that are a particular form of Schrodinger-cat state showing strong correlations between the modes.

In 1999, Gerry [52] proposed a method to generate Schroedinger cat states for optical fields. Their method involved two modes of the field interacting in a Kerr medium. In 2005, Kim and Paternostro [10] also proposed a scheme to generate a superposition of coherent states using small Kerr effect and a single photon or two entangled twin photons. However, the authors of [52] and [53] used cross Kerr nonlinearity whose evolution operator is given by $\hat{U}_{CK} = \exp(-\mathrm{i}\,\chi\,\hat{n}_a\hat{n}_b)$ which affects the phase of the system depending on the photon numbers of the two modes a and b ($\hat{n}_b$ is the photon number operator for mode b). If mode a is in a coherent state of its amplitude α and mode b is in single photon state, then by the action of cross Kerr nonlinearity,

$$\hat{U}_{CK}|\alpha\rangle_a|n\rangle_b = \left|\alpha e^{\mathrm{i}\,n_b\chi\,t}\right\rangle_a|n\rangle_b \tag{1.5}$$

i.e. the Fock state $|n\rangle_b$ remains unaffected by the interaction but the coherent state $|\alpha\rangle_a$ picks up a phase shift directly proportional to the number of photons n_b in the $|\,n\,\rangle_b$ state.

These coherent states have several attractive non-classical properties. [7, 8, 10, 53, 54]. These states lead to squeezing [55, 56], normal second order squeezing, higher order squeezing [116, 122] and sub-poissonian statistics [55, 57]. These superposed coherent states also show maximum simultaneous squeezing, maximum antibunching, and maximum higher order squeezing, higher order sub poissonian statistics [57, 58].

In this chapter, we propose a scheme to generate any desired superposition of coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ using simple Kerr effect and linear optical devices like polarization beam splitters and mirrors. We show that in the output, if a single photon is detected in a definite polarization state defined by angle θ and φ, the desired superposition of coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ results. If the photon is detected in an orthogonal polarization state, a superposition different from the desired one results.

It may be noted that other authors who studied generation of superposition of coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ have taken $|\in_H| = |\in_V|$ and have not considered the general case. Also, those who considered use of cross Kerr effect have not taken the Kerr effect into account. As is obvious, the cross Kerr effect is a dimension of the Kerr effect only.

13.2 Generation of Superposed Coherent State Using Kerr Non-linearity

Figure 13.1 depicts our scheme for generation of superposed coherent states. Each number, 1–15 represents a set of two modes having the same direction of propagation but one having a horizontal linear polarization and the other having a vertical linear polarization. We refer to the two modes for the number n by nH and nV.

The whole scheme is based mainly on two components, viz, Kerr-cells and polarizing beam splitters. The Kerr-cells have a non-linearity expressed by the Hamiltonian

$$H_I = gN^2 = g(N_H + N_V)^2. \tag{2.1}$$

This gives the time evolution operator

$$U_I = \mathrm{e}^{-\mathrm{i}H_I t} = \mathrm{e}^{-\mathrm{i}\chi N^2}, \quad \chi = gt \tag{2.2}$$

Constant χ represents the interaction time and is proportional to the third order non-linear susceptibility and the length of the Kerr-cell. If a light beam, initially in modes IH and IV passes through the Kerr-cell, and emerges in the modes FH and FH, the initial and final mode operators are related to each other by,

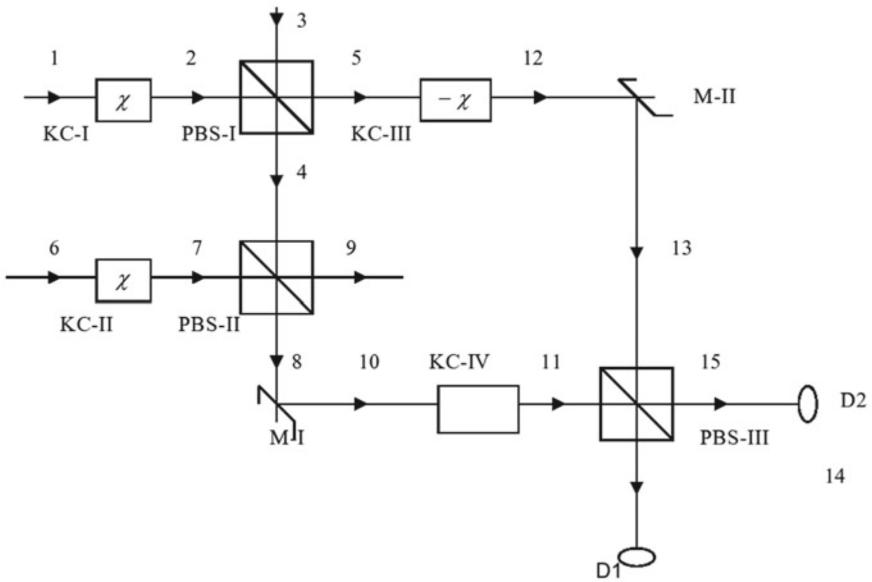

Fig. 13.1 Schematic diagram showing coherent superposition state generated by an entangled pair of photons

$$a_{IH,IV} = U^{\dagger} a_{FH,FV}\, U = \mathrm{e}^{\mathrm{i}\chi (N_{FH}+N_{FV})^2} a_{FH,FV} \mathrm{e}^{-\mathrm{i}\chi (N_{FH}+N_{FV})^2} \tag{2.3}$$

since, $N_{IH} = N_{FH}$ and $N_{IV} = N_{FV}$.

For the other element, the polarizing beam splitter connection between the four input modes and four output modes is obvious if we take into account the fact that the horizontal polarization is transmitted through the beam splitter but the vertical polarization is reflected.

Consider the input having a single photon in modes $3H$ and $3V$ and two coherent beams in the state $|\beta\rangle$ in modes $1H$ and $6V$ (see Fig. 13.1). The input state can be written as,

$$|\Psi\rangle_{1,3,6} = \exp[\beta a_{1H}^{\dagger} - \beta^* a_{1H}]\Big(\in_H\, a_{3H}^{\dagger} + \in_V\, a_{3V}^{\dagger}\Big) \exp[\beta a_{6V}^{\dagger} - \beta^* a_{6V}]|vac\rangle \tag{2.4}$$

Here, a and $a^{\dagger}$ are annihilation and creator operators for the modes denoted by the subscripts.

The effect of Kerr-cell KC-I on mode $1H$ would be given by,

$$a_{1H} = \mathrm{e}^{\mathrm{i}\chi N_{2H}^2} a_{2H} \mathrm{e}^{-\mathrm{i}\chi N_{2H}^2}. \tag{2.5}$$

Modes $2H$, $3H$ and $3V$ pass through the polarizing beam splitter PBS-I for which we have,

$$a_{2H} = a_{5H}, \quad a_{2V} = a_{4V}, \quad a_{3H} = a_{4H}, \quad a_{3V} = a_{5V} \tag{2.6}$$

Using (1.5) and (2.1), we can express the state of light written in earlier (1.4) in the form,

$$|\Psi\rangle_{5,4,6} = \mathrm{e}^{\mathrm{i}\chi N_{5H}^2} \exp\Big[\beta\, a_{5H}^{\dagger} - \beta^* a_{5H}\Big]\Big(\in_H\, a_{4H}^{\dagger} + \in_V\, a_{5V}^{\dagger}\Big) \\ \times \exp\Big[\beta a_{6V}^{\dagger} - \beta^* a_{6V}\Big] \mathrm{e}^{-\mathrm{i}\chi N_{5H}^2} |vac\rangle \tag{2.7}$$

For light mode in $6V$ passing through the Kerr-cell KC-II, we can write

$$a_{6V} = \mathrm{e}^{\mathrm{i}\chi N_{7V}^2} a_7 \mathrm{e}^{-\mathrm{i}\chi N_{7V}^2} \tag{2.8}$$

Effect of Polarizing beam splitter PBS-II is given by,

$$a_{4H} = a_{8H}, \quad a_{4V} = a_{9V}, \quad a_{7H} = a_{9H}, \quad a_{7V} = a_{8V} \tag{2.9}$$

Also, if the mirror M-I gives

$$a_{8H} = a_{10H}, \quad a_{8V} = a_{10V}$$

(2.2) takes the form,

$$|\Psi\rangle_{5,10,9} = \mathrm{e}^{\mathrm{i}\chi\left(N_{5H}^2+N_{10V}^2\right)}\ \exp\left[\beta a_{5H}^{\dagger} - \beta^{*} a_{5H}\right]\left(\in_H a_{10H}^{\dagger} + \in_V a_{5V}^{\dagger}\right) \times \exp\left[\beta a_{10V}^{\dagger} - \beta^{*} a_{10V}\right]\mathrm{e}^{-\mathrm{i}\chi\left(N_{5H}^2+N_{10V}^2\right)}|vac\rangle \tag{2.10}$$

There are no photons in the modes $9H$ and $9V$ and it is therefore left out of consideration and the subscript 9 on $|\Psi\rangle$ is dropped.

Light beams in modes $5H$ and $5V$ and $10H$ and $10V$ are now passed through Kerr-cells KC-III and KC-IV, each of which has interaction time $-\chi$. This can be obtained by having a nonlinear material in the cells which has a different sign of the third order nonlinearity.

For these Kerr cells we have,

$$a_{5H,5V} = \mathrm{e}^{-\mathrm{i}\chi(N_{12H}+N_{12V})^2} a_{12H,12V}\, \mathrm{e}^{\mathrm{i}\chi(N_{12H}+N_{12V})^2} \tag{2.11}$$

and

$$a_{10H,10V} = \mathrm{e}^{-\mathrm{i}\chi(N_{11H}+N_{11V})^2} a_{11H,11V}\, \mathrm{e}^{\mathrm{i}\chi(N_{11H}+N_{11V})^2} \tag{2.12}$$

Mirror M-II gives,

$$a_{12H,V} = a_{13H,V} \tag{2.13}$$

The state of light then can be written as,

$$\begin{aligned}
|\Psi\rangle_{13,11} &= \exp\left[-\mathrm{i}\chi\{N_{13H}+N_{13V}\}^2 + \{N_{11H}+N_{11V}\}^2\right]\mathrm{e}^{\mathrm{i}\chi\left(N_{13H}^2+N_{13V}^2\right)} \\
&\exp\left[\beta a_{13H}^{\dagger} - \beta^{*} a_{13H}\right]\left(\in_H a_{11H}^{\dagger} + \in_V a_{13V}^{\dagger}\right)\exp\left[\beta a_{11V}^{\dagger} - \beta^{*} a_{11V}\right] \\
&\mathrm{e}^{-\mathrm{i}\chi\left(N_{13H}^2+N_{13V}^2\right)}\exp\left[\mathrm{i}\chi\{N_{13H}+N_{13V}\}^2 + \{N_{11H}+N_{11V}\}^2\right]|vac\rangle \\
&= \exp\left[-i\chi\left(N_{13V}^2 + 2N_{13H}N_{13V} + N_{111H}^2 + 2N_{11H}N_{11V}\right)\right] \\
&\quad \times \exp\left[\beta a_{13H}^{\dagger} - \beta^{*} a_{13H}\right]\left(\in_H a_{11H}^{\dagger} + \in_V a_{13V}^{\dagger}\right)\exp\left[\beta a_{11V}^{\dagger} - \beta^{*} a_{11V}\right]|vac\rangle
\end{aligned} \tag{2.14}$$

For terms with $\in_H$, $N_{11H} = 1$ but $N_{13V} = 0$ and for terms with $\in_V$, $N_{11H} = 0$ but $N_{13V} = 1$. Hence, we can write (2.14) in the form

$$\begin{aligned}
|\Psi\rangle_{13,11} = \mathrm{e}^{-i\chi}\Big\{&\in_H a_{11H}^{\dagger}\exp\left[\beta a_{13H}^{\dagger} - \beta^{*} a_{13H}\right]\mathrm{e}^{-2\mathrm{i}\chi N_{11V}}\exp[\beta a_{11V}^{\dagger} - \beta^{*} a_{11V} \\
&\times \mathrm{e}^{2\mathrm{i}\chi N_{11V}} + \in_V a_{13V}^{\dagger}\mathrm{e}^{-2i\chi N_{13H}}\exp\left[\beta a_{13H}^{\dagger} - \beta^{*} a_{13H}\right]\mathrm{e}^{2\mathrm{i}\chi N_{13H}} \\
&\times \exp\left[\beta a_{11V}^{\dagger} - \beta^{*} a_{11V}\right]\Big\}|vac\rangle
\end{aligned} \tag{2.15}$$

$$= \mathrm{e}^{-\mathrm{i}\chi}\Big\{\in_H\, a^{\dagger}_{11H}\, \exp\Big[\beta a^{\dagger}_{13H} - \beta^* a_{\,13H}\Big]\exp\Big[\beta a^{\dagger}_{11V} - \beta^* a_{11V}\Big] + \in_V\, a^{\dagger}_{13V}\exp\Big[\beta a^{\dagger}_{13H} - \beta^* a_{13H}\Big]\exp\Big[\beta a^{\dagger}_{11V} - \beta^* a_{11V}\Big]\Big\}|vac\rangle \tag{2.16}$$

$$= \mathrm{e}^{-\mathrm{i}\chi}\Big[\in_H\, a^{\dagger}_{11H}|\beta\rangle_{13H}\big|\beta'\big\rangle_{11V} + \in_V\, a^{\dagger}_{13V}\big|\beta'\big\rangle_{13H}|\beta\rangle_{11V}\Big] \tag{2.17}$$

where

$$\big|\beta'\big\rangle = \big|\beta \mathrm{e}^{-2\mathrm{i}\chi}\big\rangle. \tag{2.18}$$

The polarizing beam splitter PBS-IV gives

$$a_{11H} = a_{15H},\quad a_{11V} = a_{14V},\quad a_{13H} = a_{14H},\quad a_{13V} = a_{15V} \tag{2.19}$$

and therefore light is in the state

$$|\Psi\rangle_{14,15} = \mathrm{e}^{-\mathrm{i}\chi}\Big[\in_H\, a^{\dagger}_{15H}\big|\beta, \beta'\big\rangle_{14H,14V} + \in_V\, a^{\dagger}_{15V}\big|\beta', \beta\big\rangle_{14H,14V}\Big] \tag{2.20}$$

If we consider linear polarizations in directions dividing the horizontal and vertical directions and denote these modes by + and −, defined by,

$$a_H = \frac{1}{\sqrt{2}}(a_+ + a_-), a_V = \frac{1}{\sqrt{2}}(a_+ - a_-), \tag{2.21}$$

we have,

$$\big|\beta,\ \beta'\big\rangle_{13H,13V} = \left|\frac{1}{\sqrt{2}}\big(\beta + \beta'\big),\ \frac{1}{\sqrt{2}}\big(\beta - \beta'\big)\right\rangle_{13+,13-} \tag{2.22}$$

and a similar relation for $\big|\beta', \beta\big\rangle$.

Since $\beta' = \beta \mathrm{e}^{-2\mathrm{i}\chi}$,

$$\frac{1}{\sqrt{2}}\big(\beta + \beta'\big) = \sqrt{2}\beta \mathrm{e}^{-\mathrm{i}\chi}\cos\chi$$
$$\frac{1}{\sqrt{2}}\big(\beta - \beta'\big) = \sqrt{2}i\beta \mathrm{e}^{-\mathrm{i}\chi}\sin\chi, \tag{2.23}$$

If we write

$$\alpha \equiv \sqrt{2}i\beta\, \mathrm{e}^{-i\chi}\sin\chi$$
$$\alpha_0 \equiv \sqrt{2}\beta\, \mathrm{e}^{-i\chi}\cos\chi = -i\alpha\cot\chi, \tag{2.24}$$

(2.16) takes the form,

$$|\Psi\rangle_{14,15} = \frac{1}{\sqrt{2}} e^{-i\chi} \Big[\in_H a^{\dagger}_{15H} |\alpha\rangle_{14-} + \in_V a^{\dagger}_{15V} |-\alpha\rangle_{14-} \Big] |\alpha_0\rangle_{14+} \tag{2.25}$$

If we now define modes 15± using (2.19), we get,

$$|\Psi\rangle_{14,15} = \frac{1}{\sqrt{2}} e^{i\chi} \Big[a^{\dagger}_{15+} \big(\in_H |\alpha\rangle_{14+} + \in_V |-\alpha\rangle_{14-} \big) + a^{\dagger}_{15-} \big(\in_H |\alpha\rangle_{14+} - \in_V |-\alpha\rangle_{14-} \big) \Big] |\alpha_0\rangle_{14+} \tag{2.26}$$

This makes it clear that if a photon is detected in mode 15+, radiation in mode 14+ will be in the superposed coherent state,

$$\in_H |\alpha\rangle_{14+} + \in_V |-\alpha\rangle_{14-}, \tag{2.27}$$

while if a photon is detected in 15−, the state

$$\in_H |\alpha\rangle_{14+} - \in_V |-\alpha\rangle_{14-} \tag{2.28}$$

will be generated.

If we write $\in_H / \in_V = \tan\big(\frac{1}{2}\theta\big)\, e^{i\delta}$, $0 \le \theta \le \pi,\ 0 \le \delta \le 2\pi$,

The probabilities of detection of a single photon in the modes 15± are

$$\frac{1}{2}\Big[1 + 2\mathrm{Re}\big(\in_H^* \in_V\big)\, e^{-2|\alpha|^2}\Big] = \frac{1}{2}\Big[1 \pm \sin\theta \cos\delta\, e^{-2|\alpha|^2}\Big]. \tag{2.29}$$

If $|\alpha|^2$ is appreciable, the probabilities are close to 1/2 and hence it may be expected that the desired superposition state should be produced within two attempts. It may also be noted, that for $0 < \delta < \pi$ if the probability of success in any one attempt is >1/2.

13.3 Conclusion

It may be mentioned that one could have chosen $\chi = \pi/2$ and $\beta = \frac{\alpha}{\sqrt{2}}$, necessitating the least possible value of $|\beta|$ for a given $|\alpha|$. The advantage of having a larger value of $|\beta|$ (note that $\beta = -i\alpha e^{i\chi}/\sqrt{2}\sin\chi$ or $|\beta| = |\alpha|/\sqrt{2}\ \sin\chi$) and a smaller value of χ is that only a smaller interaction time and therefore a smaller length of the Kerr-cell is required. In this case, however, the energy going in the mode 14+ is wasted in the signal in the state $|-i\alpha\cot\chi\rangle$.

Thus, one has to make a compromise between requirements of (i) larger length of Kerr-cell and (ii) wastage of the energy in the orthogonal mode 14+. It may be noted that the states $|\alpha\rangle_+|0\rangle_-$ and $|-\alpha\rangle_-|0\rangle_-$ can also be written as $\left|\frac{\alpha}{\sqrt{2}}, \frac{\alpha}{\sqrt{2}}\right\rangle_{H,V}$ and $\left|-\frac{\alpha}{\sqrt{2}}, -\frac{\alpha}{\sqrt{2}}\right\rangle_{H,V}$, respectively. Hence, the states $\in_H |\alpha\rangle_+ + \in_V |-\alpha\rangle_+$ is same as the state $\in_H |\gamma, \gamma\rangle_{H,V} + \in_V |-\gamma, -\gamma\rangle_{H,V}$. Note that such states are entangled in the modes H and V and have been used extensively in literatures.

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References

1. R.J. Glauber, Phys. Rev. **130**, 2529 (1963)
2. R.J. Glauber, Phys. Rev. **131**, 2766 (1963)
3. R.J. Glauber, In quantum optics and electronics. In: C. De Witt, A. Blandin, Cohen Tanaudiji C Gordon and Breach, New York (1965)
4. E.C.G. Sudarshan, Phys. Rev. Lett. **10**, 277 (1963)
5. V. Fock, Z. Phys. **49**, 339 (1928)
6. E. Schrodinger, Phys. Rev. **14**, 644 (1927)
7. B. Yurke, D. Stoler, Phy. Rev. Lett. **57**, 13 (1986)
8. G.J. Milburn, C.A. Holmes, Phys. Rev. Lett. **56**, 2237 (1986)
9. E. Schrodinger, Die gegenwartige Situation in der Quantenmechanik. *Naturewissenschaften* 23:807, 823, 844. An English translation of this can be found in Quantum Theory of Measurement, edited by J. Wheeler and W. H. Zurek (Princeton: Princeton University Press, 1983) (1935)
10. M.S. Kim, Paternostro M *arXiv quant-Ph*/0510057 v1
11. C.C. Gerry, Phys. Rev. A **55**, 2479 (1996)
12. C.M. Savage, W.A. Cheng, Opt. Comm. **70**, 439 (1989); N. Qureshi, H. Schmidt, A.R. Hawkins, Appl. Phys. Lett. **85**, 431 (2004); N. Qureshi, S.Q. Wang, M.A. Lowther, A.R. Hawkins, S. Kwon, A. Liddle, J. Bokor, Schmidt H. Nano Lett. **5**, 1413
13. P. Tombesi, A. Mecozzi, J. Opt. Soc. Am. B **4**, 1700 (1987); P.R. Cantwell, U.J. Gibson, D.A. Allwood, H.A.M. Macleod, J. Appl. Phys. **100**, 093910 (2006)
14. T. Ogawa, M. Uede, N. Imoto, Phys. Rev. A **43**, 6458 (1991)
15. J.J. Slosser, P. Meystre, E.M. Wright, Opt. Lett. **15**, 233 (1990)
16. J. Banacloche, Phys. Rev. A **44**, 5913 (1991)
17. J.J. Slosser, P. Meystre, Phys. Rev. A **41**, 3867 (1990)
18. P. Meystre, J.J. Slosser, M. Wilkens, Phys. Rev. A **43**, 4959 (1991)
19. M. Wilkens, P. Meystre, Phys. Rev. A **43**, 3832 (1991)
20. A. La Porta, R.E. Slusher, B. Yurke, Phys. Rev. Lett. **62**, 28 (1989)
21. S. Song, C.M. Caves, B. Yurke, Phys. Rev. A **41**, 5261 (1990)
22. B. Yurke, J. Opt. Soc. Am. B. **2**, 732 (1986)
23. E.T. Jaynes, E.W. Cummings, Proc. IEEE **51**, 89 (1963)
24. S.J.D. Phoenix, P.L. Knight, Ann. Phys. (New York) **186**, 381 (1988)
25. S.J.D. Phoenix, P.L. Knight, J. Opt. Soc. Am. B **7**, 116 (1990)
26. J. Gea-Banacloche, Phys. Rev. Lett. **65**, 3385 (1990)
27. S.J.D. Phoenix, P.L. Knight, Phys. Rev. Lett. **66**, 2833 (1991)
28. B. Yurke, W. Schleich, D.F. Walls, Phys. Rev. A **42**, 1703 (1990)
29. J. Eiselt, H. Risken, Opt. Comm. **72**, 351 (1989)
30. V. Buzek, H. Moya-Cessa, P.L. Knight, S.J.D. Phornix, Phys. Rev. A **45**, 8190 (1992)
31. A. Auffeves, Phys. Rev. Lett. **91**, 230405 (2003)

32. M.S. Kim, G.S. Agarwal, J. Mod. Opt. **46**, 2111 (1999)
33. P.K. Pathak, G.S. Agarwal, Phys. Rev. A **71**, 043823 (2005)
34. N. Chandra, H. Prakash, Ind. J. Pure Appl. Phys. **9**, 688 (1971)
35. N. Chandra, H. Prakash, Indian J. Pure Appl. Phys. **2**, 767 (1971)
36. N. Chandra, H. Prakash, Lett Nuovo Cim **4**, 1196 (1970)
37. N. Chandra, H. Prakash, Phys. Rev. Lett. **22**, 1068 (1969)
38. D. Stoler, B.E.A. Saleh, M.C. Teich, Opt. Acta **32**, 345 (1985)
39. Y. Xia, G. Guo, Phys. Lett. A **136**, 281 (1989)
40. D.T. Pegg, S.M. Barnett, J. Mod. Opt. **46**, 1657 (1999)
41. B.C. Sanders, Phys. Rev. A **39**, 4284 (1989)
42. P.L. Knight, V. Buzek, Opt. Comm. **81**, 331 (1991)
43. C.L. Chai, Phys. Rev. A **46**, 7187 (1992)
44. B.C. Sanders, Phys. Rev. A **45**, 6811 (1992)
45. C.T. Lee, Phys. Rev. A **52**, 3374 (1995)
46. R. Ragi, B. Baseia, V.S. Bagnato, Int. J. Mod. Phys. B **12**, 1495 (1998)
47. K. Nemoto, B.C. Sanders, J. Phys. A **34**, 2051 (2000)
48. W. Schleich, M. Pernigo, F.L. Kein, Phys. Rev. A **44**, 2172 (1991)
49. V.V. Dodonov, S.Y. Kalmykov, V.I. Man'ko Phys. Lett. A **199**, 123 (1995)
50. K. Tara, G.S. Agarwal, S. Chaturvedi, Phy. Rev. A **47**, 5024 (1993)
51. H. Jeong, M.S. Kim, J. Lee, Phys. Rev. A **64**, 052308 (2001)
52. Christopher C. Gerry, Phy. Rev. A **59**, 4095 (1999)
53. M. Hillery, Phys. Rev. A **36**, 3796 (1987)
54. A. Mecozzi, P. Tombesi, Phys. Rev. Lett. **58**, 1055 (1987)
55. H. Prakash, N. Chandra, Phys. Rev. A **9**, 2167 (1974)
56. D.F. Walls, Nature **306**, 141 (1983)
57. R. Loudon, P.L. Knight, J. Mod. Opt. **34**, 709 (1987)
58. V.V. Dodonov, J. Opt. B **4**, R1 (2002)