

Fault Estimation and Compensation for Fuzzy Systems with Sensor Faults in Low-Frequency Domain

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Abstract. A new fault estimation and compensation scheme of fuzzy systems with sensor faults is addressed in low-frequency domain. A descriptor observer is proposed to ensure dynamic error's stability and H_{∞} performance for low-frequency range. The faults estimation is obtained via the observer above. By considering estimation of faults, a H_{∞} output feedback controller is shown such that controlled model with sensor faults considered has certain fault-tolerant function. A simulation proves this results' effectiveness.

Keywords: Compensation \cdot Observer design \cdot Fault estimation \cdot Finite low-frequency domain \cdot T-S fuzzy model

1 Introduction

Fuzzy logic has been used to describe complicated nonlinear system, which is effective. By applying the existing fuzzy approaches, the fuzzy IF-THEN rule has been firstly used to describe this kind of model [\[1](#page-7-0)], namely T-S fuzzy model. This method can simplify analysis of nonlinear model. By applying T-S fuzzy methodology, different linear models are described by local dynamics in different state-space regions. Therefore, membership functions smoothly blend these local models together so that overall fuzzy model is obtained. The issues [\[2](#page-7-1)[–6](#page-7-2)] on fuzzy systems attracted attention.

Recently, [\[7\]](#page-7-3) has introduced generalized Kalman–Yakubovich–Popov (GKYP) lemma which is a very significant development. Frequency domain property is converted into a LMI for a finite-frequency range. GKYP lemma

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has many practical applications. T-S fuzzy models' H_{∞} control [\[8\]](#page-7-4) was solved. Fault detection problem [\[9](#page-7-5)] was proposed in T-S fuzzy networked models. Fuzzy filter problem for nonlinear systems [\[10](#page-7-6)] was solved. T-S fuzzy fault detection method was designed in [\[11\]](#page-7-7).

In past decades, when demands for industrial manufacture are growing, more and more people pay attention to fault-tolerant control (FTC). For obtaining the same control objective, different design methodologies are used in two approaches: passive FTC (PFTC) and active FTC (AFTC) in accordance with how redundancy is used. The system performance lies on the availability of redundancy and FTC design method. Each method can produce some unique properties based on the distinctive design approaches used. Passive and active FTC were considered simultaneously in [\[12](#page-7-8)[,13](#page-7-9)]. PFTCs were provided for affine nonlinear models $[14]$ and stochastic systems $[15,16]$ $[15,16]$ $[15,16]$, respectively. At the same time, there were also many results on AFTC. Fault estimation and AFTC of discrete systems [\[17\]](#page-7-13) are considered by finite-frequency method. Nonlinear stochastic AFTC system [\[18\]](#page-7-14) was analyzed by applying fuzzy interpolation approach. For stochastic systems [\[19](#page-8-0)], the descriptor observer was given to solve fault estimation and FTC problem by the sliding mode method. FTC [\[20](#page-8-1)] for fuzzy delta operator models was proposed.

However, there are few papers on fault estimation and compensation for T-S fuzzy models with sensor faults in low-frequency domain. Therefore, this paper solves the problem above, whose contributions are that: For this models considered, in terms of the descriptor system approach, a fuzzy observer is designed so as to make the stability of dynamic error in low-frequency domain be ensured. State and fault's estimations are showed by the observer above, then a H_{∞} output feedback controller is proposed so as to make controlled systems with sensor faults have certain fault-tolerant function. A numerical simulation shows the effectiveness of this designed scheme.

2 Problem Formulation

T-S fuzzy model is shown: Plant Rule *i*: If θ_{1k} is ϕ_1^i , θ_{2k} is ϕ_2^i , ..., θ_{mk} is ϕ_m^i , then

$$
x_{k+1} = A_{1i}x_k + B_{1i}u_k + D_{1i}d_k,
$$

\n
$$
y_k = A_2x_k + F_2f_k,
$$
\n(1)

where $i = 1, ..., M, M$ is IF-THEN rules's number; $\theta_{1k}, \theta_{2k}, ..., \theta_{mk}$ are premise variables; $\phi_1^i, \phi_2^i, \ldots, \phi_s^i$ are fuzzy set; $x_k \in \mathcal{R}^g$ is state, $y_k \in \mathcal{R}^{gy}$ is output. $d_k \in \mathcal{R}^{gd}$, $u_k \in \mathcal{R}^{gu}$ and $f_k \in \mathcal{R}^{gf}$ are disturbance, input and sensor fault, respectively, which belong to $\mathcal{L}_2[0,\infty)$. A_{1i} , B_{1i} , D_{1i} , A_2 , and F_2 are matrixes. Hypothesis that (A_{1i}, A_2) is observable and F_2 has full rank.

A fuzzy inference and weighted center average defuzzifier is given by considering a singleton fuzzifier, the form of [\(1\)](#page-1-0) is

$$
x_{k+1} = \sum_{i=1}^{M} \eta_{i\theta_k} [A_{1i} x_k + B_{1i} u_k + D_{1i} d_k],
$$

$$
y_k = A_2 x_+ F_2 f_k,
$$
 (2)

where $\eta_i \theta_k = \frac{\prod_{i=1}^m \phi_{i\theta_{lk}}^i}{\sum_{i=1}^{\mathcal{M}} \prod_{i=1}^n \phi_{i\theta_{lk}}^i}, \sum_{i=1}^{\mathcal{M}} \eta_i = 1.$ Define $\left[x_k^T f_k^T\right]^T = \bar{x}_k$, then the dynamic global model is that

$$
\begin{aligned} \bar{E}\bar{x}_{k+1} &= \sum_{i=1}^{\mathcal{M}} \eta_{i\theta_k} [\bar{A}_{1i}\bar{x}_k + \bar{B}_{1i}u_k + \bar{D}_{1i}d_k], \\ \bar{y}_k &= \bar{A}_2\bar{x}_k, \end{aligned} \tag{3}
$$

where $\bar{E} = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$, $\bar{A}_{1i} = \begin{bmatrix} A_{1i} & 0 \\ 0 & 0 \end{bmatrix}$, $\bar{B}_{1i} = \begin{bmatrix} B_{1i} \\ 0 \end{bmatrix}$ θ $\overline{D}_{1i} = \begin{bmatrix} D_{1i} \\ 0 \end{bmatrix}$ θ $\Big\vert \, , \, \bar{A}_2 = \big[\, A_2 \; F_2 \, \big].$ For the same IF-Then rule, design the fuzzy obser

$$
\begin{aligned}\n\hbar_{k+1} &= (\bar{A}_{1i} - L_{1i}\bar{A}_2)\hat{x}_k + \bar{B}_{1i}u_k + L_{1i}\bar{y}_k, \\
\hat{x}_k &= (\bar{E} + L_2\bar{A}_2)^{-1}(\hbar_k + L_2\bar{y}_k), \\
\hat{y}_k &= \bar{A}_2\hat{x}_k,\n\end{aligned} \tag{4}
$$

and the output feedback controller

$$
u_k = K_v y_k = K_v \bar{y}_k,\tag{5}
$$

with \bar{x}_k 's estimate is that \hat{x}_k , x_k 's estimate is that $\hat{x}_k = \begin{bmatrix} I & 0 \end{bmatrix} \hat{\bar{x}}_k$, f_k 's estimate is $\hat{f}_k = [0 I] \hat{x}_k$, and y_k 's estimate is \hat{y} . L_{1i} , L_2 , K_v are gains to be determined.

Hence, dynamic global model is obtained as follows:

$$
\begin{aligned}\n\hbar_{k+1} &= \sum_{i=1}^{M} \eta_{i\theta_k} \left[(\bar{A}_{1i} - L_{1i} \bar{A}_2) \hat{\bar{x}}_k + \bar{B}_{1i} u_k + L_{1i} y_k \right], \\
\hat{\bar{x}}_k &= (\bar{E} + L_2 \bar{A}_2)^{-1} (\hbar_k + L_2 \bar{y}_k), \\
\hat{\bar{y}}_k &= \bar{A}_2 \hat{\bar{x}}_k,\n\end{aligned} \tag{6}
$$

and

$$
u_k = \sum_{v=1}^{\mathcal{M}} \eta_{v\theta_k} K_v y_k = \sum_{v=1}^{\mathcal{M}} \eta_{v\theta_k} K_v \bar{y}_k.
$$
 (7)

Equation [\(6\)](#page-2-0) becomes

$$
(\bar{E} + L_2 \bar{A}_2) \hat{\bar{x}}_{k+1} = \sum_{i=1}^{M} \eta_{i\theta_k} [(A_{1i} - L_{1i} \bar{A}_2) \hat{\bar{x}}_k + \bar{B}_{1i} u_k + L_{1i} \bar{y}_k + L_2 \bar{y}_{k+1}].
$$
 (8)

Meanwhile, adding L_2y_{k+1} on the two sides of [\(3\)](#page-2-1), then

$$
(\bar{E} + L_2 \bar{A}_2) \bar{x}_{k+1} = \sum_{i=1}^{M} \eta_{i\theta_k} [(A_{1i} - L_{1i} \bar{A}_2) \hat{\bar{x}}_k + \bar{B}_{1i} u_k + L_{1i} \bar{y}_k + \bar{D}_{1i} d_k]. (9)
$$

Define $\tilde{\bar{x}}_k = \bar{x}_k - \hat{\bar{x}}_k$, $r_k = \bar{y}_k - \hat{\bar{y}}_k$, then

$$
\tilde{\tilde{x}}_{k+1} = \sum_{i=1}^{M} \eta_{i\theta_k} [(\bar{E} + L_2 \bar{A}_2)^{-1} (A_{1i} - L_{1i} \bar{A}_2) \tilde{\tilde{x}}_k + \bar{D}_{1i} d_k],
$$

\n
$$
r_k = \bar{A}_2 \tilde{\tilde{x}}_k.
$$
\n(10)

3 Main Results

Theorem 1. *Considering constant* $\gamma > 0$ *, the gain of* [\(4\)](#page-2-2) *is addressed so as to make [\(10\)](#page-3-0)* with H_{∞} *level* γ *asymptotic stability in* $|\vartheta| \leq \vartheta_1$ *if there is symmetric matrixes* $P_i > 0$, $P_l > 0$, $Q > 0$ *and matrixes* X_i *for all* $i, l \in \{1, ..., M\}$ *so that [\(11\)](#page-3-1) and [\(12\)](#page-3-1) hold:*

$$
\begin{bmatrix}\nP_l - g_2 X_i - g_2 X_i^T & g_1 X_i + g_2 \xi_1 \\
\ast & -P_i - g_1 \xi_1 - g_1 \xi_1^T\n\end{bmatrix} < 0,\tag{11}
$$

$$
\begin{bmatrix}\n-P_l & Q + X_i & 0 & 0 \\
\ast & \varphi_1 & -X_i^T \bar{D}_{1i} & \bar{A}_2^T \\
\ast & \ast & -\gamma^2 I & 0 \\
\ast & \ast & \ast & -I\n\end{bmatrix} < 0,\n\tag{12}
$$

 $where \xi_1 = X_i^T (\bar{E} + L_2 \bar{A}_2)^{-1} \bar{A}_{1i} - Y_i \bar{A}_2, \ \varphi_1 = P_i - 2 \cos \vartheta_1 Q - \xi_1^T - \xi_1, \ L_2 =$ $\left[0 I\right]^T$, and g_1 , g_2 are arbitrary fixed scalars satisfying $g_1^2 \sigma_{\max}(P_i) < g_2^2 \sigma_{\min}(P_i)$. *Then gains of [\(4\)](#page-2-2)* are that $L_{1i} = (\bar{E} + L_2 \bar{A}_2) X_i^{-T} Y_i$.

Proof. Define $V_k = \tilde{x}_k^T \left[\sum_{i=1}^M \eta_{i\theta_k} P_i\right] \tilde{x}_k$, differences of V_k along [\(10\)](#page-3-0) is that $\Delta V_k = (\sum_{i=1}^{M} \eta_{i\theta_k})^2 \sum_{l=1}^{M} \eta_{l\theta_{k+1}} \tilde{x}_{k}^T [\varrho^T P_l \varrho - P_i] \tilde{x}_k$, where $\varrho = (\bar{E} + L_2 \bar{A}_2)^{-1} (A_{1i} L_{1i}\bar{A}_2$). Notice that $\overline{\varrho}^T P_l \varrho - P_i < 0$ so $\Delta V_k < 0$. Easily, it is obtained that

$$
\begin{bmatrix} \varrho \\ I \end{bmatrix}^T \begin{bmatrix} P_l & 0 \\ 0 & -P_i \end{bmatrix} \begin{bmatrix} \varrho \\ I \end{bmatrix} < 0. \tag{13}
$$

There exist g_1 and g_2 such that $g_1^2 \sigma_{\max}(P_i) < g_2^2 \sigma_{\min}(P_i)$, then

$$
\begin{bmatrix} g_1 I \\ g_2 I \end{bmatrix}^T \begin{bmatrix} P_l & 0 \\ 0 & -P_i \end{bmatrix} \begin{bmatrix} g_1 I \\ g_2 I \end{bmatrix} = g_1^2 P_l - g_2^2 P_i < 0.
$$
 (14)

Since that $\left[g_2I - g_1I\right]^{T\perp} = \left[g_1I g_2I\right]$ and $\left[\varrho^T I\right]$ belongs to the null subspace of $\left[-I\varrho\right]^T$, it has

$$
\begin{bmatrix} -I \\ \varrho^T \end{bmatrix} X_i \begin{bmatrix} g_2 I \\ -g_1 I \end{bmatrix}^T + \begin{bmatrix} g_2 I \\ -g_1 I \end{bmatrix} X_i^T \begin{bmatrix} -I \\ \varrho^T \end{bmatrix}^T + \begin{bmatrix} P_l & 0 \\ 0 & -P_i \end{bmatrix} < 0.
$$
 (15)

Set $Y_i = X_i^T (\bar{E} + L_2 \bar{A}_2)^{-1} L_{1i}$, [\(15\)](#page-3-2) holds after some matrix manipulations. According to GKYP Lemma [\[7](#page-7-3)], then

$$
\begin{bmatrix} \bar{A}_{1\eta} \ \bar{D}_{\eta} \\ I \end{bmatrix}^T \begin{bmatrix} -P_{\eta+} & Q \\ Q & P_{\eta} - 2\cos\vartheta_1 Q \end{bmatrix} \begin{bmatrix} \bar{A}_{1\eta} \ \bar{D}_{\eta} \\ I \end{bmatrix} + \begin{bmatrix} \bar{A}_2 \ 0 \\ I \end{bmatrix}^T \begin{bmatrix} I & 0 \\ 0 & -\gamma^2 I \end{bmatrix} \begin{bmatrix} \bar{A}_2 \ 0 \\ 0 \end{bmatrix} < 0.
$$

Define $P_{\eta+} = \sum_{l=1}^{N} \eta_{l\theta(k)} P_l$, $P_{\eta} = \sum_{i=1}^{N} \eta_{i\theta(k)} P_i$, then

$$
\begin{bmatrix} \bar{A}_{1i} & \bar{D}_i \\ I & 0 \end{bmatrix}^T \Theta_1 \begin{bmatrix} \bar{A}_{1i} & \bar{D}_i \\ I & 0 \end{bmatrix} + \begin{bmatrix} \bar{A}_2 & 0 \\ 0 & I \end{bmatrix}^T \Theta_2 \begin{bmatrix} \bar{A}_2 & 0 \\ 0 & I \end{bmatrix} < 0,\tag{16}
$$

where $\Theta_1 = \begin{vmatrix} -P_l & Q \\ Q & R_l - 2\rho q \end{vmatrix}$ $Q \quad P_i - 2\cos\vartheta_1 Q$ $\theta_2 = \begin{bmatrix} I & 0 \\ 0 & -\infty \end{bmatrix}$ $0 - \gamma^2 I$, then (16) 's form is that $\Upsilon^{\perp T} \left(\varrho_1^T \Theta_1 \varrho_1 + \varrho_2^T \Theta_2 \varrho_2 \right) \Upsilon^{\perp} < 0,$ (17)

where $\Upsilon^{\perp} = \begin{bmatrix} \bar{A}_{1i}^{T} I & 0 \\ \bar{D}_{1i}^{T} & 0 & I \end{bmatrix}$ $\bar{D}_i^{\bar{T}}$ 0 1 $\int_{\Omega} \rho_1 = \left[\begin{array}{cc} I & 0 & 0 \\ 0 & I & 0 \end{array} \right]$ 0 I 0 $\left[\begin{matrix} T \\ 0 & A_2 \end{matrix} \right]$ 0 0 I .

For Υ^{\perp} , then $\Upsilon = \begin{bmatrix} -I & \bar{A}_{1i} & \bar{D}_i \end{bmatrix}$. According to Projection Lemma, then $\varrho_1^T \Theta_1 \varrho_1 + \varrho_2^T \Theta_2 \varrho_2 < \Upsilon X_i R^T + (\Upsilon X_i R^T)^T$. Let $R^T = [0 I 0]$, then [\(18\)](#page-4-1) holds:

$$
\begin{bmatrix} -P_l & Q & 0 \ * & P_i - 2\cos\vartheta_1 Q & 0 \ * & * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} 0 \\ \bar{A}_2^T \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ \bar{A}_2^T \\ 0 \end{bmatrix}^T - \begin{bmatrix} 0 - X_i & 0 \\ * & \nu & X_i^T \bar{D}_i \\ * & * & 0 \end{bmatrix} < 0. \tag{18}
$$

with $\nu = \bar{A}_{1i}^T X_i + X_i^T \bar{A}_{1i}$. By matrix manipulations, [\(18\)](#page-4-1) and [\(12\)](#page-3-1) are equivalent. Therefore, [\(12\)](#page-3-1) satisfies H_{∞} index γ in $|\vartheta| \leq \vartheta_1$ if [\(16\)](#page-4-0) holds. The proof is finished.

Secondly, we give the controller in low-frequency domain. This system may not work normally with sensor faults. This motivates us to consider the faulttolerant method, which is shown in the following.

Based on observer technique, f_k is designed as $\hat{f}_k = \begin{bmatrix} 0 & I \end{bmatrix} \hat{x}_k$. By subtracting \hat{f}_k from y_k , then $y_{ck} = y_k - F_2 \hat{f}_k = A_2 x_k + F_2 f_k - F_2 \hat{f}_k = A_2 x_k + \bar{F}_2 \tilde{x}_k$ with $\bar{F}_2 = F_2 [0 I].$

Consider controller [\(5\)](#page-2-3), and use y_{ck} to replace y_k , then u_{ck} = $\sum_{v=1}^{M} \eta_v(\theta_k) K_v y_{ck}$ so closed-loop models are that

$$
x(k+1) = \sum_{i=1}^{M} \eta_{i\theta_k} \sum_{v=1}^{M} \eta_{v\theta_k} [(A_{1i} + B_{1i}K_v A_2)x_k + B_{1i}K_v \bar{F}_2 \tilde{x}_k + D_{1i}d_k],
$$

$$
y_{ck} = A_2x_k + F_2 [0 \ I] \tilde{x}_k.
$$

Considering system (10) , a new system is obtained

$$
X_{ek} = \sum_{i=1}^{M} \eta_{i\theta_k} \sum_{v=1}^{M} \eta_{v\theta_k} [\bar{A}_{iv} X_k + \bar{D}_{1i} d_k],
$$

$$
y_{ck} = \bar{A}_2 X_k,
$$
 (19)

where $X_k = \begin{bmatrix} x(k) \\ \frac{z}{k} \end{bmatrix}$ $\tilde{\bar{x}}_k$ $\left| \right.,\, \bar{\bar{D}}_{1i} = \left| \frac{D_{1i}}{\bar{D}_{1i}} \right|$ \bar{D}_{1i} $\left. \begin{array}{c} \end{array} \right|, \, \bar{\bar{A}}_{iv} = \left[\begin{array}{cc} \Upsilon_{11} \; \Upsilon_{12} \ 0 \; \; \Upsilon_{22} \end{array} \right] \!, \, \bar{\bar{A}}_{2} = \left[\, A_{2} \; \bar{F}_{2} \, \right] \!, \, \Upsilon_{11} =$ $A_{1i} + B_{1i}K_{v}A_{2}, \Upsilon_{12} = B_{1i}K_{v}\bar{F}_{2}, \Upsilon_{22} = (\bar{E} + L_{2}\bar{A}_{2})^{-1}(\bar{A}_{1i} - L_{1i}\bar{A}_{2}).$

It is not difficult to show that equation [\(19\)](#page-5-0) for this controlled system adopting the static output feedback control strategy, by applying this proposed technology, it can reduce sensor fault's influence on controlled models, so whole closed-loop ones have a certain fault-tolerant function.

4 Numerical Simulations

A example can prove effectiveness of this proposed method. These systems have Rule 1 and 2, their membership functions are $\phi_{1x_{1k}} = \frac{1}{1+exp(-3x_{1k})}$, $\phi_{2x_{1k}} =$ $1 - \phi_{1x_{1k}}$, and consider x_{1k} is ϕ_1 and ϕ_2 , respectively, and

$$
A_{11} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.6 \end{bmatrix}, B_{11} = \begin{bmatrix} 0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, D_{11} = \begin{bmatrix} -0.2 \\ 0.5 \end{bmatrix}, A_{12} = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.5 \end{bmatrix},
$$

\n
$$
B_{12} = \begin{bmatrix} 0.4 & 0 \\ 0 & -0.4 \end{bmatrix}, D_{12} = \begin{bmatrix} 0.8 \\ 0.4 \end{bmatrix}, A_{2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, F_{2} = \begin{bmatrix} 1.0 & 0 \\ 0 & 1.0 \end{bmatrix},
$$

let $g_1 = 1, g_2 = 3$, in $|\vartheta_1| \leq 1.7$, for $\gamma = 0.15$, by [\(11\)](#page-3-1) and [\(12\)](#page-3-1), gains of [\(4\)](#page-2-2) are

$$
L_{11} = \begin{bmatrix} -2.2563 & 0.8171 \\ 1.1779 & -1.0420 \\ -0.0744 & -0.0817 \\ -0.1178 & -0.4958 \end{bmatrix}, L_{12} = \begin{bmatrix} -1.6636 & -1.0563 \\ -1.4917 & -0.3082 \\ -0.4336 & 0.1056 \\ 0.1492 & -0.4692 \end{bmatrix}, L_{2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.
$$

Assumed that $d_k = 0.1 \sin_k$ and $f_k = \begin{bmatrix} f_{1k} \\ f_{2k} \end{bmatrix}$ f_{2k} | with

 $f_{1k} = \begin{cases} 0, & 0 < k \le 30 \\ 0.3 \sin_k, & 70 \ge k \ge 300, \quad k > 70 \end{cases}, f_{2k} = \begin{cases} 0, & 0 < k \le 30 \\ 0.4 \sin_k, & 70 \ge k \ge 300, \quad k > 70 \end{cases}.$

Considering $x_0 = \left[x_{1k}^T x_{2k}^T\right]^T = \left[-1, 2\right]^T$ $x_0 = \left[x_{1k}^T x_{2k}^T\right]^T = \left[-1, 2\right]^T$ $x_0 = \left[x_{1k}^T x_{2k}^T\right]^T = \left[-1, 2\right]^T$, Fig. 1 depicts the estimation of fault f_k , where fo_i is the estimation of f_{ik} with $i = 1, 2$. \hat{y}_k 's estimations are shown in Fig. [2,](#page-6-1) where $y\overline{o_1}$ means \hat{y}_{1k} and $y\overline{o_2}$ means \hat{y}_{2k} .

Fig. 2 Estimate output \hat{y}_k

5 Conclusion

The fault estimation and compensation scheme of fuzzy T-S discrete models is addressed for low-frequency range. A fuzzy observer is given so as to ensure error model's stability with H_{∞} performance for low-frequency range. The fault estimations are obtained via the observer above, then a fuzzy H_{∞} output feedback controller is shown so as to ensure certain fault-tolerant function of controlled model with sensor fault considered. A numerical simulation proves the effectiveness of this method. The conclusion of this paper can also be expended into finite middle- and high-frequency domain.

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