Mathematical Modeling of Elbow-Actuated Wearable Robotic Arm for Muscular Disorders

V. M. Karthicraja, M. Mythily, and D. Manamalli

Abstract Musculoskeletal disorders are major concern globally not just for the pain and suffering by the individual but also due to its adverse impact on the economy of the individual and the society. This results in productivity loss in both manufacturing and service sector, which has an adverse impact over the economy. The main aim of this work is to design and test an upper body exoskeleton arm that will be used for empowering the able-bodied, i.e., healthy users. The exoskeleton arm is intended to power or amplify the ability of the human elbow. Inverse dynamic model of the system is simulated through kinematic analysis and workspace analysis. The mathematical model developed by MATLAB is validated using ADAMS simulation tool. The obtained model is used to analyze the assistive torque requirements and position analysis of each task defined in the problem statement. Finally, the forward kinematic model, the inverse kinematic model, and the workspace analysis of the system are done which is universally flexible with respect to the dynamic parameters of the material used.

Keywords Musculoskeletal disorders · Upper body exoskeleton · Inverse dynamic model · Kinematic analysis · Workspace analysis

1 Introduction

Musculoskeletal disorders (MSDs) is considered as the most common work-oriented health problem around the world affecting tens of millions of workers. A statistical study by European Union reports 25% of workers suffer from back ache and 23% complain muscular pain, which are resulted mainly by heavy physical work, repetive movements, manual handling, and even improper posture. The above-mentioned strenuous functions results in MSDs and can provoke intense pain to the point of creating it tough or not possible to carryout even day to day tasks [1]. Thus, based on MSDs impact on individual, organization, and the whole economy, there is a need

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to give attention to the ways in which these problems can be solved. In twenty-first century, these work-related MSDs are tackled in many ways, through the enormous development in the field of the automation and robotics [2–6].

One such solution is a wearable robot which may also save the workers from the curse of automation by aiding the worker to carry out a particular strenuous task without replacing the workers. Analysis and modeling of human movement have become a topic of greater importance as it helps to understand the complex systems by efficient initialization of mathematics, mechanics, and concepts of physiology. Most of the key features are extracted and are used to create a simplified representation of the system.

This work deals with the design, modeling, and simulation of a wearable robot. This proposed system can be considered as a biomechtronic system which will aid the wearer by amplifying his force and strength, by contributing most of the force required to do the job. The designed system mainly concentrates on transport and material handling. It has the ability that allows direct transfer of mechanical power. The wearer will be the control order generator and the biomechatronic systems will cooperate with the wearer and his commands. Such a model allows one to closely observe the insights of the systems and to make predictions regarding the performance under bounded input conditions and various system parameters. The data from real physical models could be used to validate derived models for increasing reliability.

2 Anatomy of Human Arm Model

To model any system, the fundamental concepts of the system is required. Hence, the human arm model's anatomy gives us the needed data regarding the functionality of the upper arm. Different range of motions of human arm is given in Fig. 1. The theoretical concepts of the upper limb reveal three important articulations, namely the shoulder, elbow, and wrist. The proposed model is being designed only for the elbow actuation with a single degree of freedom [2–6].

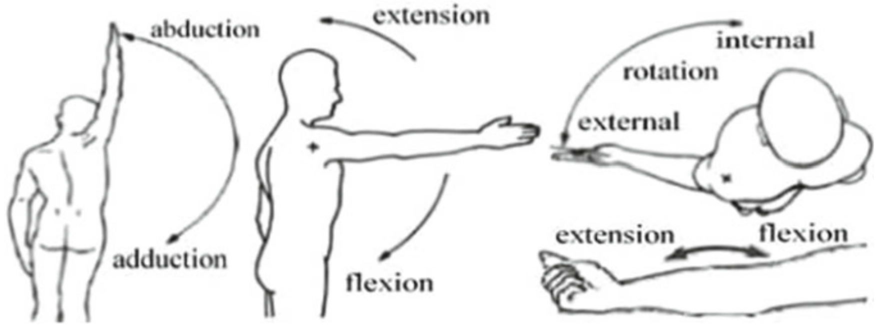


Fig. 1 Different range of motions of human arm

Table 1 Range of motions shoulder and elbow joint

	Shoulder abduction	Shoulder flexion	Shoulder rotation	Elbow flexion
Range of motion	0°–180°	−50°–180°	−80°–100°	0°–145°

The range of motion of all the articulation is given in the Table 1.

The values in Table 1 represent the average range of joint motion and our interest is to select the range of motion of human elbow while doing strenuous tasks. This has been finalized based on the joint range of motion of elbow as 0°–145°. This will be a parameter that will be considered as a boundary for the modeling and designing of the prototype.

3 Modeling by Reverse Dynamic Analysis

Dynamic analysis is generally performed by reverse and forward dynamics analysis. The latter uses forces to predict motion and the former uses system's motions to predict the forces required to produce the motion. In this work, reverse dynamic analysis is implemented which will help us for actuator sizing and testing control strategies.

In inverse dynamics methodology, the motion is given and the force-angular momentum method of Newton–Euler is used to resolve for the joint torques and the joint reaction force at the joints. The human arm was idealized as two-dimensional two-segment-coupled pendulum systems and the branching pattern of those segments is shown in Fig. 2 with two mass points one on each segment. The connected pattern corresponds to an acyclic graph and that one segment is anchored to the origin of the coordinate system point (*o*) [1, 7–10].

Figure 2 shows the schematic diagram of two degree of freedom (DOF) of the robot arm with the robot arm link 1 and link 2. *Z*, *L*, *m* and θ represents joint displacement, length, mass of each link and torque for the link 1 and 2, respectively.

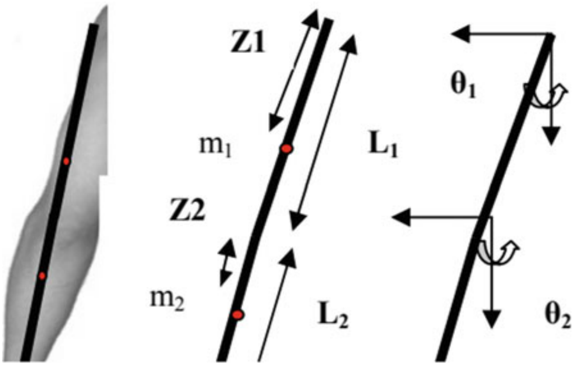


Fig. 2 Idealization of human hand

The energy (work) based approach of Lagrange is used to formulate the equations of motion of the suggested mechanical system. Since the independent dynamic variable is chosen as position, the equations of motion are formulated as second-order equation. The Lagrange method is used since it is usually more direct than either Newton's or D'Alembert's methods for arriving at the correct set of independent motion equations. The energy (Work) based approach of Lagrange is based on the difference between the kinetic and potential energy of the system. More precisely, the Lagrangian function is defined by Eq. 1 [11–14].

$$L = K - P \tag{1}$$

where K and P represents kinetic and potential energy, respectively.

In the model, the following assumptions are made:

i. The actuators dynamics (motor and gear boxes) is not taken into account.
ii. The effect of friction forces is assumed to be negligible.
iii. The mass of each link is assumed to be concentrated at the end of each link.

The ith equation of motion corresponding to the ith degree of freedom in general is given in Eq. (2).

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{\theta}_i}\right] - \frac{\partial L}{\partial \theta_i} = 0 \tag{2}$$

For the two-segmented model proposed ($i = 1{,}2$), θ_i represents the segment angle with the vertical axis, viewing the segments from the lateral view. The differential model for the two torque equations is given by Eqs. 3 and 4.

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial L}{\partial \dot{\theta}_1}\right] - \frac{\partial L}{\partial \theta_i} = 0 \tag{3}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial L}{\partial \dot{\theta}_2}\right] - \frac{\partial L}{\partial \theta_i} = 0 \tag{4}$$

The kinetic and potential energy are found and are represented in Eqs. 5 and 6 respectively.

$$K = \frac{1}{2}\sum_{i-1}^{3} m_i\left(\dot{x}_i^2 + x\dot{y}_i^2\right) \tag{5}$$

$$P = \sum_{i=1}^{3} g m_i y_i \tag{6}$$

To formulate the system equation of motion, both kinetic and potential energy's are found and are given in Eqs. 7, 8, 9 and 10.

$$X = [Z_1 \sin\theta_1 L_1 \sin\theta_1 + Z_2 \cos\theta_2]^{\mathrm{T}} \tag{7}$$

$$Y = [Z_1 \cos\theta_1 L_1 \cos\theta_1 + Z_2 \sin\theta_2]^{\mathrm{T}} \tag{8}$$

$$X' = \left[Z_1\theta_1^{'} \cos\theta_1 \quad L_1\theta_1^{'} \cos\theta_1 + Z_2\theta_2^{'} \cos\theta_2 \right] \tag{9}$$

$$Y' = \left[Z_1\theta_1^{'} \sin\theta_1 \quad L_1\theta_1^{'} \sin\theta_1 + Z_2\theta_2^{'} \sin\theta_2 \right] \tag{10}$$

Thus *K* and *P* is represented in Eqs. 11 and 12 respectively.

$$\begin{aligned} K = {} & \frac{1}{2}(Z_1 + Z_2)L_1^2\theta_1'^2 + \frac{1}{2}\left(Z_2L_2^2\theta_1'^2 + Z_2L_2^2\theta_1'\theta_2'\right) \\ & + \frac{1}{2}\left(Z_2L_2^2\theta_{21}'^2\right) + Z_2L_1L_{12}\cos\theta_2\left(\theta_1'\theta_2' + \theta_1'^2\right) \end{aligned} \tag{11}$$

$$P = Z_1gL_1\cos\theta_1 + Z_2g(L_1\cos\theta_1 + L_2\cos(\theta_1 + \theta_2)) \tag{12}$$

From Eq. 1,

$$\begin{aligned} L = {} & \frac{1}{2}(Z_1 + Z_2)L_1^2\theta_1'^2 + \frac{1}{2}\left(Z_2L_2^2\theta_1'^2 + Z_2L_2^2\theta_1'\theta_2'\right) \\ & + \frac{1}{2}\left(Z_2L_2^2\theta_2'^2\right) + Z_2L_1L_2\cos\theta_2\left(\theta_1'\theta_2' + \theta_1'^2\right) \\ & - Z_1gL_1\cos\theta_1 + Z_2g(L_1\cos\theta_1 + L_2\cos(\theta_1 + \theta_2)) \end{aligned} \tag{13}$$

Now, thus, found,

$$\tau_1=[(Z_1+Z_2)L_1^{2}+Z_2L_2^{2}+2Z_2L_1L_2C_2]\Theta_1^{''}+[Z_2L_2^{2}+Z_2L_1L_2C_2]\Theta_2^{''}- 2Z_2L_1L_2S_2\Theta_1^{'}\Theta_2^{'}- Z_2L_1L_2S_2\Theta_2^{'2} + (Z_1+Z_2)L_1S_1g + Z_2gL_2S_{12.} \qquad (14)$$

(14)

$$\tau_2= [\ Z_2L_2^{2}+Z_2L_1L_2C_2]\ \Theta_1^{''} + Z_2L_2^{2}\ \Theta_2^{''} - Z_2L_1L_2S_2\Theta_1^{'2} + Z_2gL_2S_{12} \tag{15}$$

The derived dynamic equations represented in Eqs. 14 and 15 can be written in terms of inertial matrix, centrifugal force, and Coriolis force vector and gravitational force. The dynamic equation for this wearable robot is represented by the coupled nonliner differential equations which was derived from the lagrangian method

$$Z(\Theta)\ \Theta^{''}+C(\Theta,\Theta^{'})\ \Theta^{'} + g(\Theta) = \tau$$

4 Results and Discussions

The model was simulated and validated for different joint ranges, initial & final conditions. The values of torque required at each joint for performing various motions are plotted in Fig. 3. The deviation between the initial and final positions of the robotic arms tip is plotted in Fig. 4 [15–17].

From the figure, it is observed that θ_1 of joint 1 encounters high torque in relatively small time.

Fig. 3 Actual torques at joint 1 and joint 2

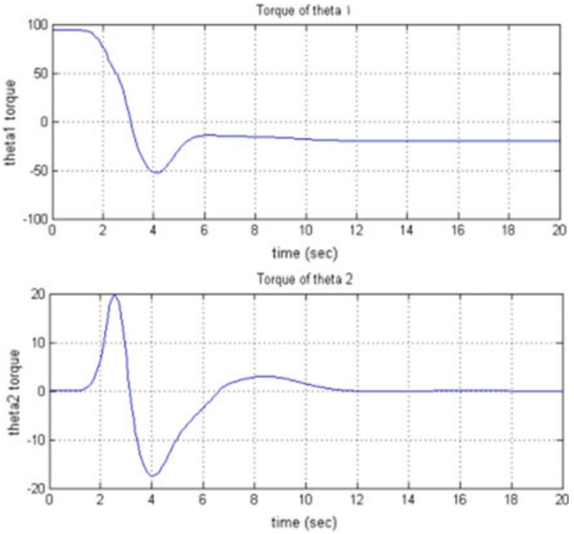
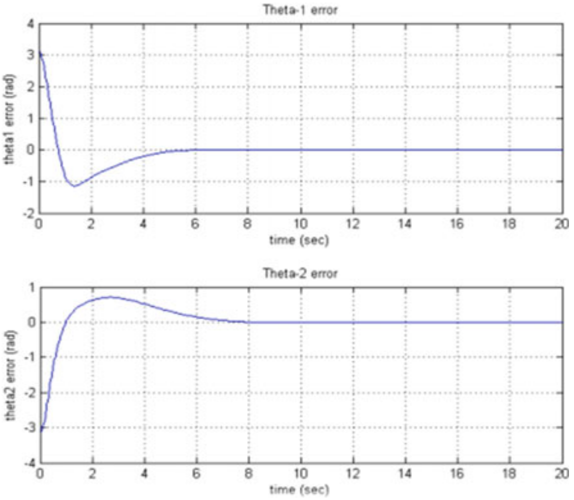


Fig. 4 Error waveforms of theta 1 and & theta 2



From the figure, it is observed that the response has acceptable overshoot, settling time, and linear behavior. Here, we have analyzed the torques resulted for the particular initial and final conditions. The control inputs here are the torques. However the actual joints torques are

$$\begin{bmatrix} f_{\theta_1} \\ f_{\theta_2} \end{bmatrix} = B(q)\begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

The mathematical model developed by MATLAB is validated using ADAMS simulation tool.

5 Conclusion

In this paper, wearable robot with single degree of freedom (DOF) is proposed. The robotic system was intended to power or amplify the ability of the human elbow. Kinematic analysis and dynamics analysis for the exoskeletal system were carried out which successfully led to the formulation of forward kinematic model, inverse kinematic model, and inverse dynamic model. From the forward kinematic model, the workspace analysis of the system was done successfully. Finally, the mathematical model was validated by position analysis and torque analysis and the results were presented.

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