Support Vector Model Based Thermal Uncertainty on Stochastic Natural Frequency of Functionally Graded Cylindrical Shells

Vaishali and S. Dey

Abstract This paper presents the effect of temperature on stochastic natural frequencies of cylindrical shells, composed of functionally graded materials (FGM) by using machine learning quadratic Support Vector Machine (SVM). An eight noded isoperimetric quadratic element is considered for the finite element formulation. The power law is employed to construct the material modelling of FGM cylindrical shells. Monte Carlo Simulation (MCS) is carried out in conjunction with stochastic eigenvalue solution. In the present study, zirconia (ceramic) and aluminium (metal) are considered to compose the FGM. The machine learning SVM model is constructed to reduce the computational iteration time and cost and validated with the traditional MCS model. The statistical analyses are conducted to portray the first three modes of frequencies. The results show that due to the increase of the temperature, the values of both deterministic as well as the stochastic mean of the first three natural frequencies decreases along with the decrease in sparsity. Sensitivity analysis is also carried out to enumerate the significant important input parameters contributing to influence the output quantity of interest (QoI). The statistical results obtained are the first known results.

Keywords Functionally graded materials · Cylindrical shells · Support vector machine · Monte Carlo simulation and sensitivity

1 Introduction

Functionally graded materials (FGM) are the advanced materials which have gained immense popularity because of its exclusive properties like heat and corrosion resistance in addition to high stiffness and strength. They are extensively used in aerospace, marine, civil construction and mechanical industries. These materials are the example of nonhomogeneous materials which is graded at intervals. Therefore

Vaishali (✉) · S. Dey
National Institute of Technology Silchar, Assam 788010, India
e-mail: vaishali765@gmail.com

S. K. Saha and M. Mukherjee (eds.), *Recent Advances in Computational Mechanics and Simulations*, Lecture Notes in Civil Engineering 103,
https://doi.org/10.1007/978-981-15-8138-0_50

at different sections, unique properties are present which doesn't resemble the properties of parent materials. By using power law, the gradations of material properties are varied across the thickness.

In general, engineering structures are susceptible to vibration, so dynamic analysis plays an important role. Using a deterministic approach, various authors [1–4] have carried out a vibration analysis of FGM structures. Some researchers worked on the analytical solution [5, 6] of functionally graded plates. In various dimensions, many researchers worked on functionally graded (FG) structures [7–11]. But stochastic responses of FGM shells are yet not intrusively addressed. In the present study, it is aimed to assess the stochastic first three natural frequencies of functionally graded cylindrical shells considering the effect of variation of temperature. In this paper, various sections are presented after Sect. 1 is an introduction, such as Sect. 2: which illustrates the theoretical formulation of FGM cylindrical shells, Sect. 3: depicts the stochastic results obtained followed by discussion while Sect. 4: portrays the concluding remarks.

2 Theoretical Formulation

In FGM, the variability in material properties [12] vary with change in temperature which can be shown by given formulation,

$$Q = Q_0 + Q_{-1}T^{-1} + 1 + Q_1T + Q_2T^2 + Q_3T^3 \tag{1}$$

where the temperature coefficients are represented by Q_0, Q_{-1}, Q_1, Q_2, Q_3 while T represents temperature (in Kelvin). The material properties variation across the depth can be expressed by various laws such as sigmoid law, exponential law and power law. In the present study, power law [13] is considered which can be expressed as

$$R(\hat{w}) = R_m(\hat{w}) + \left[R_c(\hat{w}) - R_m(\hat{w})\right]\left[{}^{w}/_{t(\hat{w})} + 0.5\right]^P \tag{2}$$

whereR_{m} and R_{c} represent properties of metal and ceramic respectively. Here, $\hat{w}$ represents the degree of stochasticity. The geometry of the cylindrical shell is shown in Fig. 1. Finally, the equation of motion for free vibration is obtained in the global form [7]

$$\left[M(\hat{w})\right]\left\{\ddot{\delta}_p\right\} + \left(\left[K(\hat{w})\right] + \left[K_\sigma(\hat{w})\right]\right)\left\{\delta_p\right\} = 0 \tag{3}$$

where $\{\delta_p\}$ is the displacement vector, $[M(\hat{w})]$ is the mass matrix while $[K(\hat{w})]$ and $[K_\sigma(\hat{w})]$ are the random elastic stiffness matrix and the random geometric stiffness matrix, respectively. Considering eigenvalue problem [14], the stochastic natural frequencies $(\omega(\hat{w}))$ can be obtained from Eq. (3) as

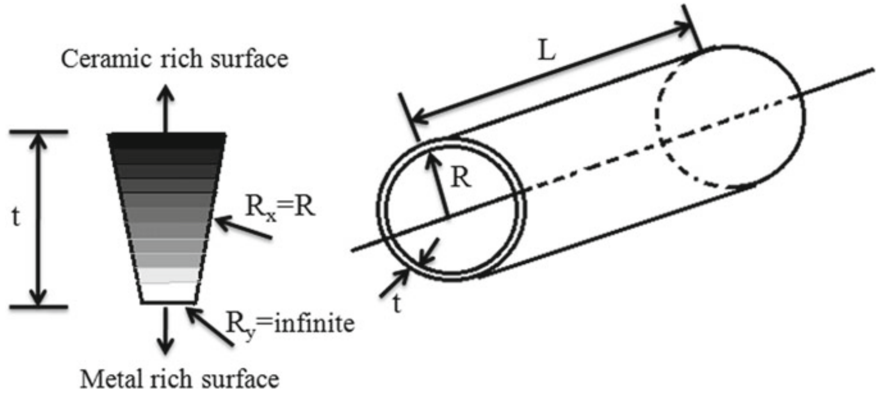

Fig. 1 Functionally graded cylindrical shell geometry

$$[A(\hat{w})]\{\phi\} = \lambda(\hat{w})\{\phi\} \tag{4}$$

where

$$\left[A(\hat{w})\right] = \left(\left[K(\hat{w})\right] + \left[K_\sigma(\hat{w})\right]\right)^{-1}\left[M(\hat{w})\right] \\ \text{and}\,\lambda(\hat{w}) = 1/\left[\omega(\hat{w})\right]^2 \tag{5}$$

The summary of the entire procedure is represented in the form of flow diagram as shown in Fig. 2.

3 Results and Discussion

In the present study, Monte Carlo simulation is employed for stochastic natural frequency analysis of functionally graded cylindrical shells having Rx = 0.1e20, Ry = 2. The composition of the materials considered for the present analysis is, namely, zirconium (ceramic) and aluminium (metal), whose properties are furnished in Table 1. In the present FE formulation, an eight noded isoperimetric quadratic element is considered. As a surrogate, machine learning quadratic Support Vector Machine (SVM) is used. From the errors (see Table 2) obtained for sample 256, 512 and 1024, the root mean square error (RSME) for sample size 1024 is minimum and the scatter plot (see Fig. 3) for same sample size 1024 shows the least deviation from the perfect prediction, so for further study sample size 1024 is considered. The first three natural frequencies are determined by varying the temperature of functionally graded cylindrical shells i.e. 300, 600, 900 and 1200 K. The present study is validated with the previous research results obtained in past literature [11] as furnished in Table 3. The results show (see Fig. 4) that due to the increase of the temperature, the values of both deterministic, as well as the stochastic mean of the first three natural frequencies

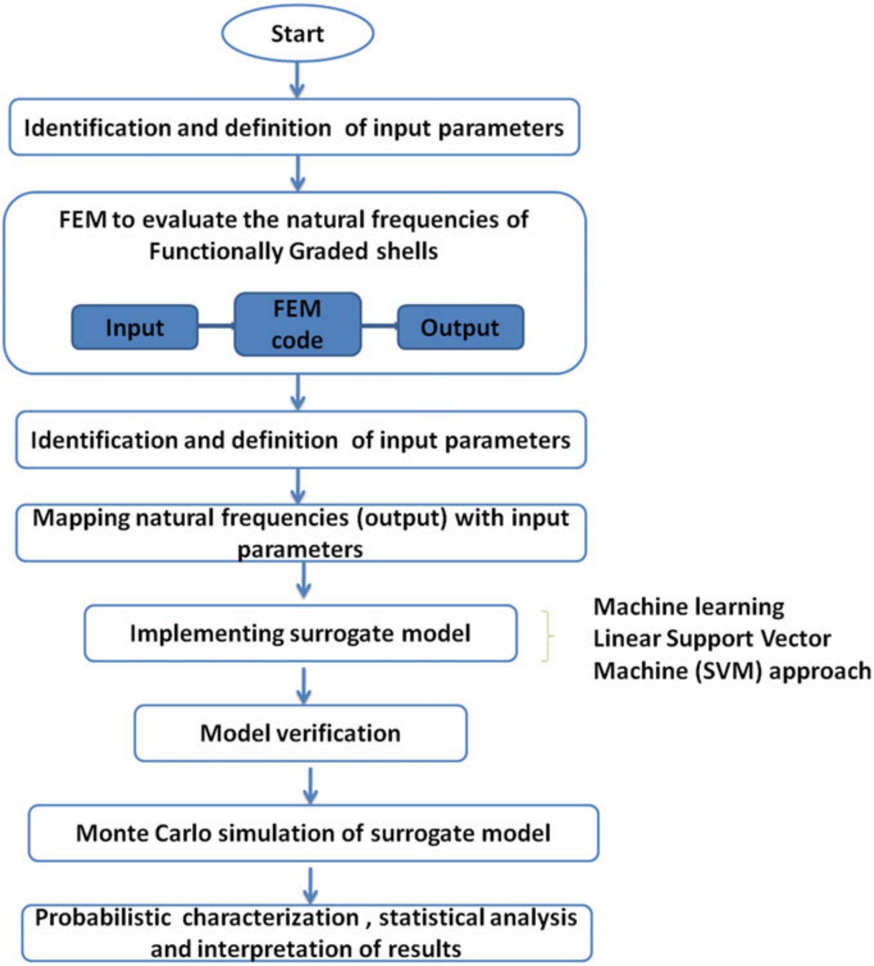

Fig. 2 Flowchart for natural frequency (N_f) analysis using a linear support vector machine (SVM) surrogate model

Table 1 Material properties [10]

	E [GPa]	υ	ρ [kg/m^3]
Metal	700	0.3	2707
Ceramic	168	0.3	5700

decreases along with the decrease in sparsity. Further sensitivity analysis (see Fig. 5) is also carried out which shows the influence of individual material properties. It is observed that Young's Modulus has a maximum effect while the effect of shear modulus and mass density is moderate and Poisson's ratio has the least influence.

Table 2 Root Mean Square Error (RMSE) for first, second and third natural frequency (rad/sec) while using quadratic SVM surrogate on functionally graded cylindrical shells with a sample size of 256, 512 and 1024

	Fundamental natural frequency (rad/s)	Second natural frequency (rad/s)	Third natural frequency (rad/s)
256	0.838	0.2487	0.79457
512	0.14649	0.15137	0.43908
1024	0.088871	0.14839	0.26091

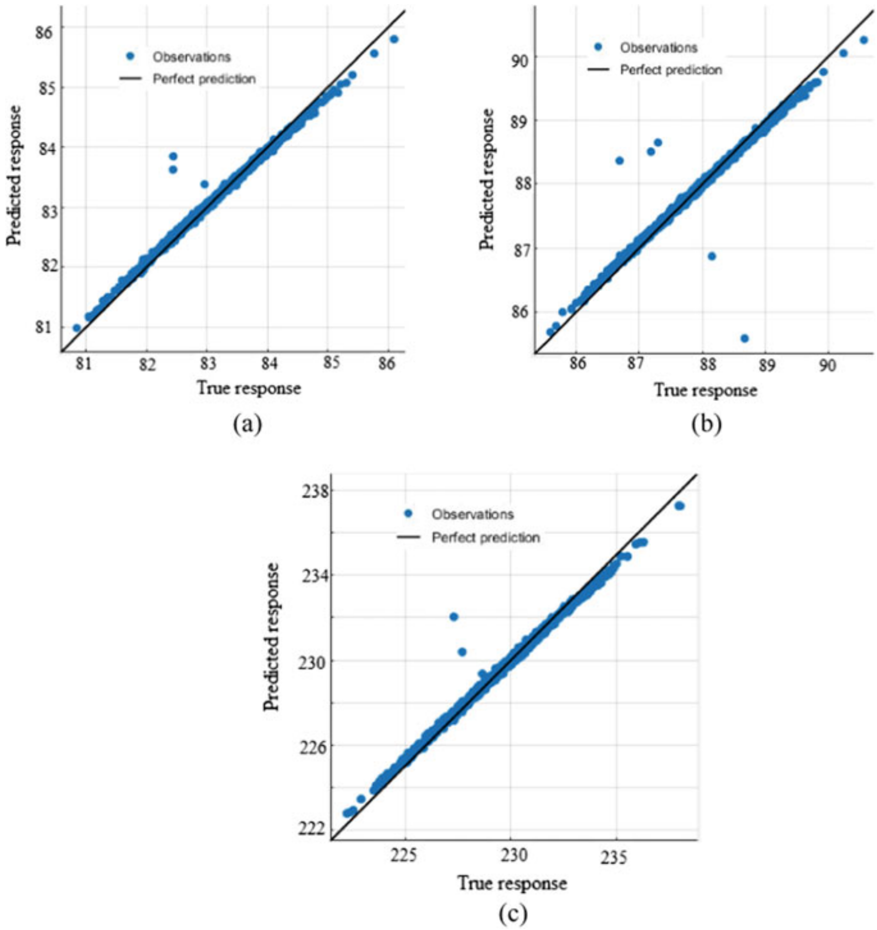

Fig. 3 Scatter plot due to combined variation of random input parameters considering quadratic SVM surrogate model with a sample size of 1024

Table 3 The fundamental natural frequency of FG square plate for symmetric boundary conditions

P	t/L	Baferani et al. [11]	Present Study
1	0.1	0.0891	0.0883
2	0.1	0.0819	0.0797

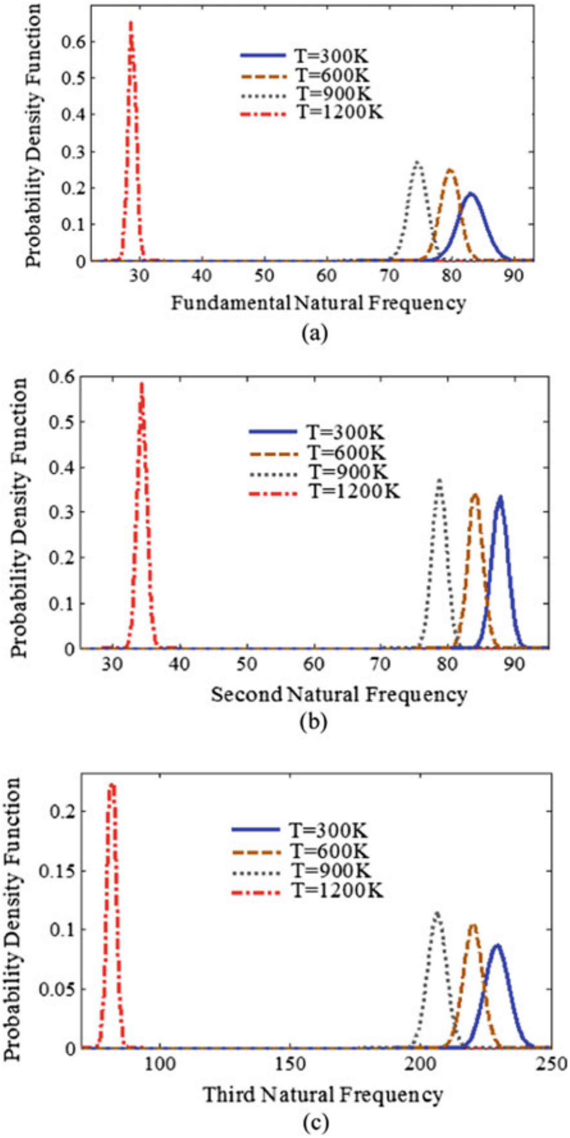

Fig. 4 Stochastic natural frequency (rad/s) of FGM cylindrical shells (**a**) Fundamental (**b**) Second and (**c**) Third modes considering temperature(T) = 300 K, 600 K, 900 K, 1200 K

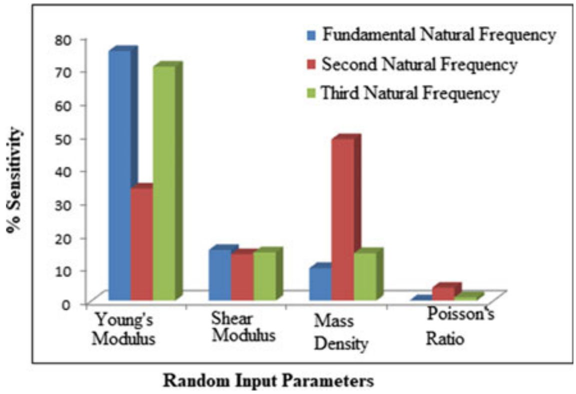

Fig. 5 Sensitivity analysis for various random input parameters for fundamental, second and third natural frequency

4 Conclusions

Based on machine learning quadratic Support Vector Machine (SVM) in combination with finite element formulation, the natural frequency of functionally graded (FG) cylindrical shell is studied. The surrogate used increases the computational efficiency along with the reduction in computational cost. The novelty of present work includes the effect of temperature on stochastic natural frequencies of FGM cylindrical shells. Due to unavoidable inherent randomness present in these structures, a band of deviation in the values of natural frequencies is observed compared to the deterministic mean value. Therefore, to map the degree of influence of sources causing uncertainties, sensitivity analysis is carried out in order to ensure the reliability and safety of such shell structures. Based on these observations, the present work can be further extended to more complex geometries and structures.

Acknowledgements The first author acknowledges Ministry of Human Resource and Development (MHRD) and Technical Education Quality Improvement Programme (TEQIP-III), Govt. of India, for the financial support provided during research work.

References

1. Reddy BS, Kumar JS, Reddy CE, Reddy KVK (2014) Free vibration behaviour of functionally graded plates using higher-order shear deformation theory. J Appl Sci Eng 17(3): 231–241
2. Wang YQ, Jean WZ (2017) Nonlinear dynamics of functionally graded material plates under dynamic liquid load and with longitudinal speed. Int J Appl Mech 9(4):1750054
3. Dey S, Sarkar S, Das A, Karmakar A, Adhikari S (2015) Effect of twist and rotation on the vibration of functionally graded conical shells. Int J Mech Mater Des 11(4):425–437
4. Sharma AK, Chauhan PS, Narwaria M, Pankaj R (2016) Vibration analysis of functionally graded plates with multiple circular cutouts. Int J Curr Eng Sci Res 2(3):15–21
5. Apuzzo A, Raffaele B, Raimondo L (2015) Some analytical solutions of functionally graded Kirchhoff plates. Compos B Eng 68:266–269

6. Barretta R, Raimondo L (2015) Analogies between Kirchhoff plates and functionally graded Saint-Venant beams under torsion. Continuum Mech Thermodyn 27(3):499–505
7. Karsh PK, Mukhopadhyay T, Dey S (2018) Stochastic dynamic analysis of twisted functionally graded plates. Compos B Eng 147:259–278
8. Dey S, Adhikari S, Karmakar A (2015) Impact response of functionally graded conical shells. Lat Am J Solids and Struct 12(1):133–152
9. KarshPK, Mukhopadhyay T, Dey S (2018) Stochastic investigation of natural Frequency for functionally graded plates. IOP Conf Series Mater Sci Eng 326(1)
10. Tornabene F, Erasmo V (2013) Static analysis of functionally graded doubly-curved shells and panels of revolution. Meccanica 48(4):901–930
11. Baferani AH, Saidi AR, Ehteshami H (2011) Accurate solution for free vibration analysis of functionally graded thick rectangular plates resting on elastic foundation. Compos Struct 93(7):1842–1853
12. Touloukian YS (1967) Thermophysical properties of high temperature solid materials. McMillan, New York
13. Loy CT, Lam KY, Reddy JN (1999) Vibration of functionally graded cylindrical shells. Int J Mech Sci 41(3):309–324
14. Bathe K-J, Edward LW (1973) Solution methods for eigenvalue problems in structural mechanics. Int J Numer Meth Eng 6(2):213–226