ANN-Based Random First-Ply Failure Analyses of Laminated Composite Plates

Subrata Kushari, A. Chakraborty, T. Mukhopadhyay, S. R. Maity, and S. Dey

Abstract This paper presents the random first-ply failure analyses of laminated composite plates by using an artificial neural network (ANN)-based surrogate model. In general, materials and geometric uncertainties are unavoidable in such structures due to their inherent anisotropy and randomness in system configuration. To map such variabilities, stochastic analysis corroborates the fact of inevitable edge towards the quantification of uncertainties. In the present study, the finite element formulation is derived based on the consideration of eight-noded elements wherein each node consists of five degrees of freedom (DOF). The five failure criteria namely, maximum stress theory, maximum strain theory, Tsai-Hill (energy-based criterion) theory, Tsai-Wu (interaction tensor polynomial) theory and Tsai-Hill's Hoffman failure criteria are considered in the present study. The input parameters include the ply orientation angle, assembly of ply, number of layers, ply thickness and degree of orthotropy, while the first-ply failure loads for five criteria representing output quantity of interest. The deterministic results are validated with past experimental results. The results obtained from the ANN-based surrogate model are observed to attain fitment with the results obtained by Monte Carlo Simulation (MCS). The statistical results are presented for both deterministic, as well as stochastic domain.

Keywords First-ply failure · Laminated composite: Monte Carlo Simulation (MCS) · Artificial Neural Network (ANN) · Uncertainty quantification

S. Kushari (✉) · S. R. Maity · S. Dey
National Institute of Technology, Silchar, India
e-mail: subrata734@gmail.com

A. Chakraborty
Indian Institute of Technology, Guwahati, India

T. Mukhopadhyay
University of Oxford, Oxford, UK

S. K. Saha and M. Mukherjee (eds.), *Recent Advances in Computational Mechanics and Simulations*, Lecture Notes in Civil Engineering 103,
https://doi.org/10.1007/978-981-15-8138-0_11

1 Introduction

The probability of failure in a laminated composite can be quantified by the amount of load it can withstand until its fracture point. Due to the inheritance of randomness and uncertainty in material and geometrical properties, stochastic regime seems to be an efficient way of modelling the failure analysis as compared to the deterministic way. An alternative approach for the analyses necessitating monotonous model evaluation is the utilisation of approximation models, also referred to as metamodels [1]. A result found by Onkar et al. [2], where first-ply failure load is analysed to evaluate the mean and variance of the failure statistics showed that the stochastic finite element has high accuracy. An experimental investigation designed by Reddy and Pandey [3], who are considered as the pioneer of failure analysis of composites conducted computational, as well as numerical investigation on laminated composite, with different ply orientation, ply angle with in-plane and out-of-plane failure load.

Laminated composite plates failure can be seen by debonding or delamination, fibre pull-out, fibre breakage and matrix cracking. These modes of failure are major limitations of laminated composite plate. Delamination is the most common mode of failure among all other kinds of failure. These modes of failure are major limitations of laminated composite. Many researchers investigated on deterministic failure analysis of laminated composite plate. ANN incorporated in analysing the probability of delamination of laminated composite by Chakraborty [4], highlighted that the trained network can predict the delamination for different shape, size and location with less computational time. ANN is applied to predict the kinetic parameters like high-velocity impact on a carbon reinforced fibre composites (CRFC) [5]. In a particular case where split growth in the notched composite is analysed under constant amplitude fatigue by using ANN and power law, ANN proved to be a better predictive tool than power law [6]. End milling process of a Glass fibre reinforced plastic (GFRP) composites ANN provided significant performance increase in analysing the damage factor by developing five learning algorithms and training them which contributed in the reduction of cost as well as time in conducting experiment [7]. Prediction of impact location using ANN during damage based on the kinetic energy of an impact by incorporating the limited strain signatures as inputs provided a warning system in damage initiation [8]. In another instance, prediction for impact resistance of aluminium–epoxy-laminated composites were analysed and it was found that ANN can be used as a substitutive approach to evaluate the effect of bonding strength of laminated composites [9]. ANN also proved to a better option in analysing the failure of a cross-ply composite tube under torsion, as well as axial tension/compression compared to Tsai-Wu theory and tensor polynomial theory [10]. Some of the important works done on the first-ply failure of laminated composites and ANN are hereby mentioned [11–16]. The present study deals with uncertainty quantification for first-ply failure of laminated composite plates by incorporating ANN as surrogate model to reduce computational time.

2 Mathematical Formulation

In order to determine the first-ply failure of the laminated composite five failure criteria are taken into consideration as mentioned earlier. The finite element (FE) model is designed based on the failure criteria followed by this the surrogate model is implemented using ANN.

2.1 Failure Criteria for Laminated Composite

In the present work, a three-layered laminated composite as shown in Fig. 1, is considered to study the failure analysis. The orientation of the laminate is [45°, − 45°, 45°]. The five failure criteria are employed to analyse the first-ply failure load of the laminate and design the finite element model for the same.

Maximum Stress Theory. This theory involves two forms of stress (normal stress and shear stress) theories. It specifies that, when a material has exceeded its maximum stress enduring capacity in any of its axes, it fails. The mathematical formulation [17], can be expressed as

$$(\sigma_1^c)_u < (\sigma_1) < (\sigma_1^T)_u \tag{1}$$

$$(\sigma_2^c)_u < (\sigma_2) < (\sigma_2^T)_u \tag{2}$$

$$(\tau_{12})_u < (\tau_{12}) < (\tau_{12})_u \tag{3}$$

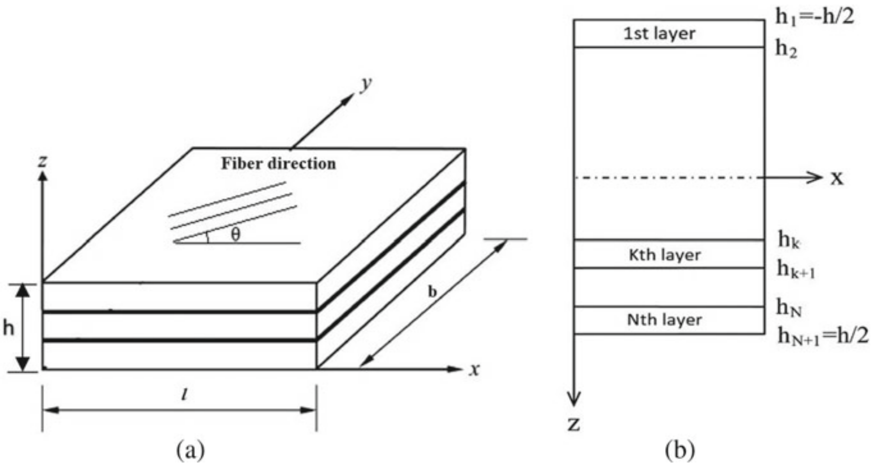

Fig. 1 **a** Isometric view of the laminate. **b** Layer thickness representation of the laminated plate

where σ_1, σ_2 represent the normal stresses in x-axis and y-axis, respectively. While τ_{12} represents the shear stress. σ^c and σ^T represent the compressive stress and tensile stress, respectively, through the laminate. Here the suffix 'u' is used to signify the ultimate stress point.

Maximum Strain Theory. This theory is based on the maximum normal strain theory of St. Venant and Tresca's strain equivalent of maximum stress theory for isotropic materials. According to this theory, when the shear and principal strain exceeds the ultimate strain the material tends to rupture or fail. The mathematical deduction [18], for the same can be expressed as

$$(\varepsilon_1^c)_u < (\varepsilon_1) < (\varepsilon_1^T)_u \tag{4}$$

$$(\varepsilon_2^c)_u < (\varepsilon_2) < (\varepsilon_2^T)_u \tag{5}$$

$$\gamma_{12} < \Gamma_{12} \tag{6}$$

where, ε_1 and ε_2 represent the normal strains in x-axis and y-axis represents while γ_{12} represents the shear strain. ε^c and ε^T represent the compressive strain and tensive strain, respectively through the laminate and Γ_{12} represents the ultimate shear strain.

Tsai-Hill (Energy-Based Criterion) Theory. Tsai-Hill theory for the failure of laminate is a combination of two energy principle, the first one is the distortion energy (which is responsible for change of the shape) and the second one is dilation energy (which causes volumetric changes in the material). The failure of the material takes place when the following equation [19], holds true.

$$f(\sigma_{ij}) = F(\sigma_2 - \sigma_3)^2 + G(\sigma_3 - \sigma_1)^2 + H(\sigma_1 - \sigma_2)^2 + 2L\sigma_4^2 + 2M\sigma_5^2 + 2N\sigma_6^2 = 1 \tag{7}$$

where, F, G, H, L, M and N signify strength parameters of the material and σ_4, σ_5, σ_6 are the shear stress components.

Tsai-Wu (Interaction Tensor Polynomial) Theory. The Tsai-Wu failure criterion is a special case of the general quadratic failure criteria developed by Gol'denblat and Kopnov. It can be written in a scalar form as [20]

$$F_i\sigma_i + F_{ij}\sigma_i\sigma_j \geq 1 \tag{8}$$

where, F_i and F_{ij} are the first order and fourth order strength tensors of the material. Here σ_i denotes the difference between compressive and tensile induced stress. The term $\sigma_i\sigma_j$ defines an ellipsoid along with the stress space.

Tsai-Hill's Hoffman Failure Criteria. The Tsai-Hill's Hoffman criterion is a special condition of Tsai-Hill failure criteria. In Hoffman's failure criteria the difference between the strength of tension and compression is considered which is ignored in the case of Tsai-Hill failure criteria which is significant if brittle materials are considered. The modified criteria are established by adding the odd functions of the principal stress components (σ_1, σ_2 and σ_3) in the actual expression of Tsai-Hill criteria [21]. Thus

$$C_1(\sigma_2 - \sigma_3)^2 + C_2(\sigma_3 - \sigma_1)^2 + C_3(\sigma_1 - \sigma_2)^2 + C_4\sigma_1 + C_5\sigma_2 + C_6\sigma_3 + C_7\sigma_4^2 + C_8\sigma_5^2 + C_9\sigma_6^2 = 1 \quad (9)$$

Here C_1–C_9 denote the material parameters.

2.2 ANN-Based Surrogate Model

Artificial neural network (ANN) is analogous to the working phenomenon of a human brain based on which the present computational model is being prepared. The algorithm in ANN acquires its working procedure based on the input and internal hidden neuron configuration also known as weights after which the output of the data is compared with known correct values. The modelling of intricate association between input and output data involves non-linear statistical data modelling tool, which is further incorporated and executed through ANN. The input and output data are channelised through a training process which continues until a significant reduction in error is achieved. The input data moves forward on a layer basis, training the data in hidden networks, and simultaneously it is supervised to reduce the error through the back-propagation algorithm. The hidden layers can be more than one according to the design required. The principal objective of using ANN is its ability to compensate the computational time by developing an efficient model similar to the finite element model. The structure of input and output in an ANN is shown in Fig. 2.

ANN procures their result by following the patterns and relationships in data and learn through experience, instead of following a fixed set of programmable arrangement. The working process of an ANN is followed by a series of steps [22], as specified here. At first, the input data (x_n) is fed to the neurons in the input unit followed by calculating the output (y_n) from the hidden layer of neurons utilising the transfer function

$$y_n = \sum W_{nm} x_n + \psi \quad (10)$$

$$H_m = \frac{1}{1 = \exp(-\alpha y_n)} \quad (11)$$

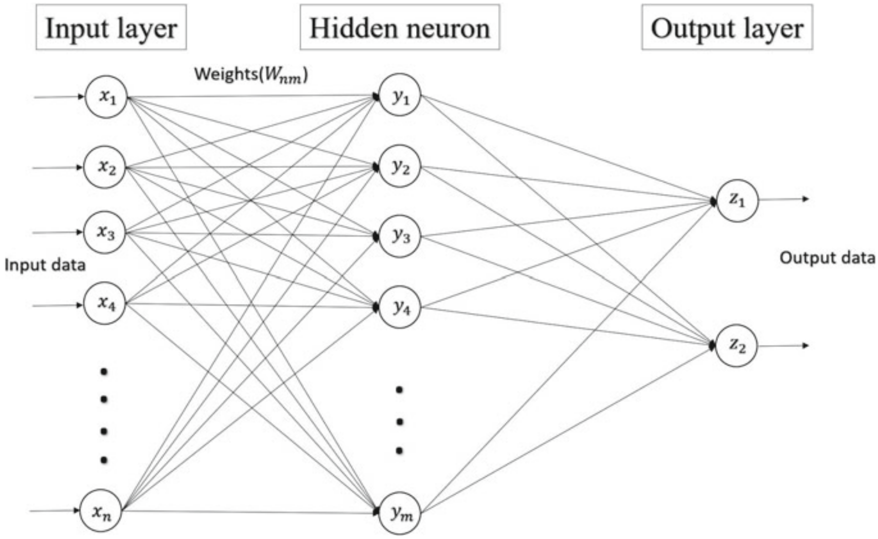

Fig. 2 Structure of an ANN

where W_{nm} denotes the connection weight for the neurons n and m, Ψ is the bias or threshold value for neuron m that can be observed as the non-zero offset in the data, H_m is the output of neuron m, and α signifies the non-linear parameter for the neuron's operation. Following this stage, compute the parameter to be studied (P_k) for the output neuron in a similar manner as mentioned in Eqs. (10) and (11). Since the algorithm will face an error at this stage, the error for each weight in the output value P_k and target output t_k is analysed by Eq. (13). This process is known as backpropagation.

$$\delta_k = (t_k - P_k)P_k(1 - P_k) \tag{12}$$

where k is the output neuron. According to the error correction factor, the change in weights and bias are updated in this stage

$$W_{mk}^{\text{new}} = W_{mk}^{\text{old}} + \Delta W_{mk}(p) \tag{13}$$

$$\Delta W_{mk}(p) = \eta \delta_k H_m + \mu \Delta W_{mk}(p - 1) \tag{14}$$

where W_{mk} signifies the adjusted value of the weight between output neuron k and hidden layer neuron m, p and $p-1$ refer to the present and previous cycles of correction, respectively. Also, η denotes the learning rate and μ *signifies momentum.*

Now the error for the hidden layer (δ_m) is calculated as Eq. (15) and an updated weight (W_{nm}) is formulated due to the hidden unit based on Eqs. (16) and (17).

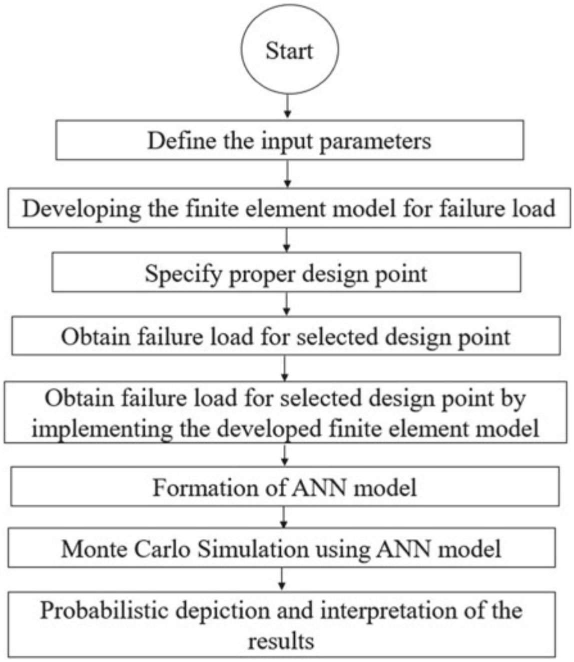

Fig. 3 Flowchart of the first-ply failure analysis incorporating ANN as the surrogate model

$$\delta_m = H_m\left(1 - H_m \sum \delta_k W_{mk}\right) \tag{15}$$

$$W_{mk}^{\text{new}} = W_{mk}^{\text{old}} + \Delta W_{nm}(p) \tag{16}$$

$$\Delta W_{mk}(p) = \eta \delta_m H_n + \mu \Delta W_{nm}(p-1) \tag{17}$$

At the end terminating condition is checked after every new sample is calculated and finally a significant reduction in error is achieved.

The working module for the ANN-based random first-ply failure analysis can be depicted by the flowchart in Fig. 3.

3 Result and Discussion

In this section, the result obtained for the different failure criteria as mentioned above are discussed in brief. Firstly, a deterministic study is carried out for the analysis of first-ply failure loads with respect to the mentioned failure criteria and then the stochastic analysis is designed based on ANN as the surrogate model.

Table 1 Material properties of T300/5208 graphite-epoxy laminate [23] with ply orientation of [45°, −45°, 45°]

E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	G_{13} (GPa)	G_{23} (GPa)	μ	ρ (kg/m^3)
132.37	10.7	5.65	5.65	3.37	0.3	3202

The laminated composite considered is a three-layered T300/5208 graphite-epoxy laminate with ply orientation of [45°, −45°, 45°]. The mean value of the material properties for the specified material is specified in Table 1.

The plate has a dimension of 0.22 m length, 0.127 m breadth and 3×10^{-4} m as thickness. The plate is subjected to uniformly distributed load on the top surface in the z-direction. Three mesh size is considered for the verification of the deterministic model of sizes (2 × 2), (4 × 4) and (8 × 8) and they are simultaneously validated with the finite element model of Reddy and Pandey [3]. The mesh plane area is considered of (8 × 8) configuration comprising of 64 elements and 225 total number of nodes. The deterministic validation of the five different failure criteria from Reddy and Pandey [3], is shown in Table 2. The present deterministic validation of the failure modes is also validated by Karsh et al. [16], for the spatial vulnerability study for the first-ply failure for laminated composite.

Subsequently, the deterministic model is validated in Table 2, the ANN model is designed based on the parent MCS model. For the ANN model, the main MCS model which is of 10,000 samples sized is compared with the ANN-based MCS for three different sample size which is 64,128 and 256 as depicted in Fig. 3.

As it can be perceived from Fig. 4, that out of the three sample size considered, 256 samples sized data converges with the parent MCS to a great extent for the five different failure criteria considered. For the lesser sample size data, the PDF tends to deviate from the parent MCS model. Thus, it can be concluded that as the sample size increases the ANN model tends to improve its accuracy comparatively.

Table 2 Validation of the present finite element model with experimental results [3] for in-plane loading of different failure criteria for the laminate with [45°, −45°, 45°] ply orientation

Failure theory	Failure load					
	(2 × 2)		(4 × 4)		(8 × 8)	
	Reddy et al. [3]	Present FE model	Reddy et al. [3]	Present FE model	Reddy et al. [3]	Present FE model
Max. stress	2854.40	3408.70	2164.32	2486.50	1908.16	1962.50
Max. strain	2947.68	3273.20	2268.60	2421.70	1940.48	1994.75
Tsai-Hill	2788.80	3091.40	1803.84	1897.91	1530.40	1563.70
Tsai-Wu	2886.72	3337.70	2218.88	2432.73	1917.76	1957.32
Hoffman	2850.24	3224.50	2156.80	2269.53	1905.76	1962.10

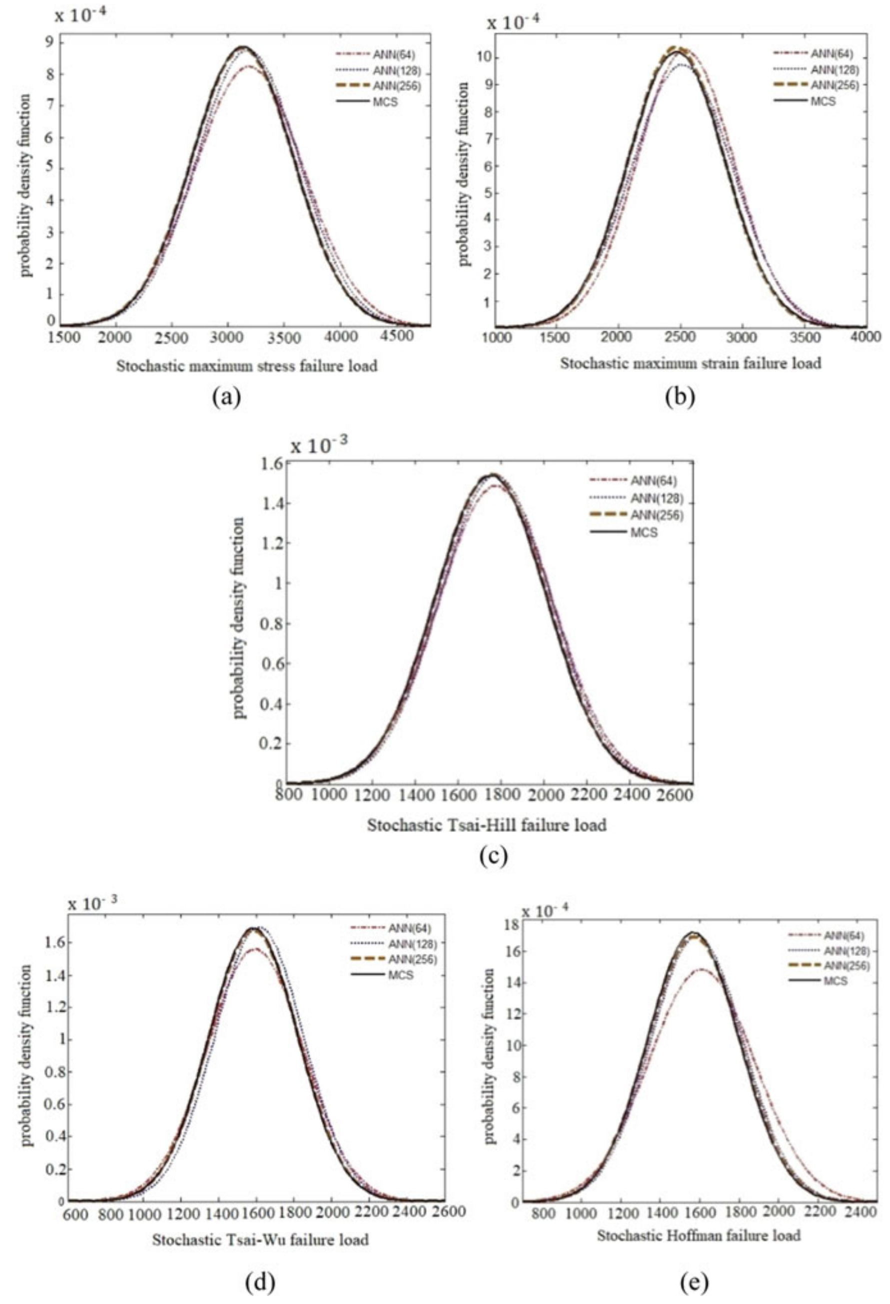

Fig. 4 Probability density functions (PDF) of parent MCS and ANN model considering first-ply failure for **a** Maximum stress. **b** Maximum strain. **c** Tsail-Hill. **d** Tsai-Wu and **e** Hoffman failure criteria

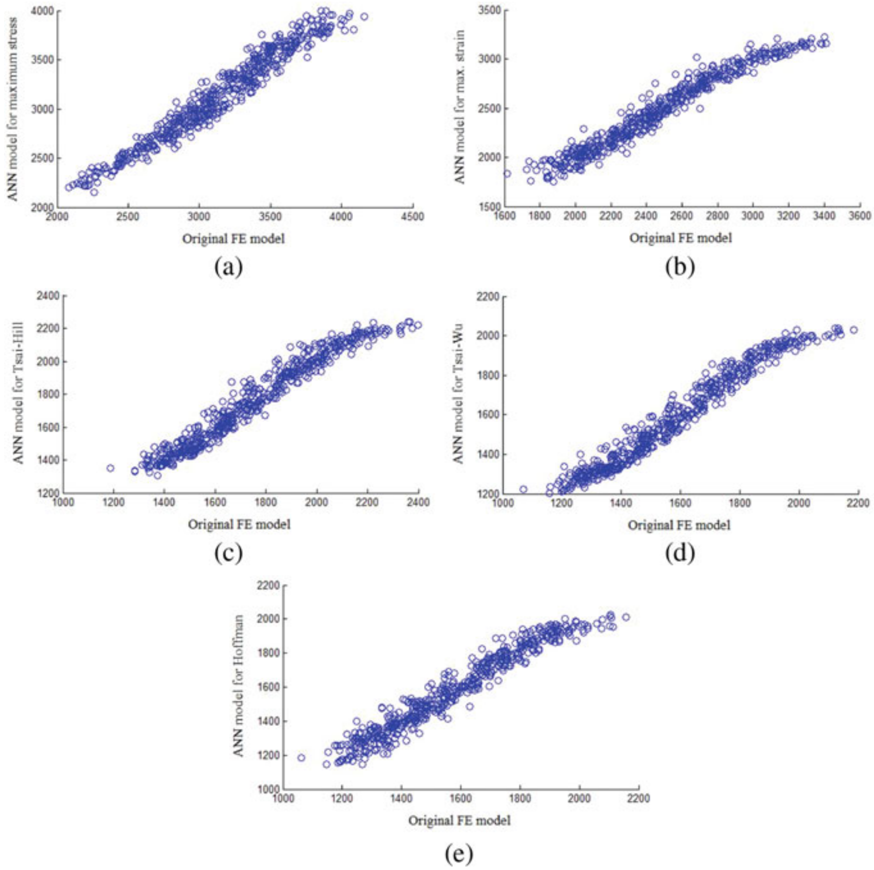

Fig. 5 Scatter plot of the 256 samples sized ANN model with respect to the original finite element (FE) model for **a** max. stress **b** max. strain. **c** Tsai-Hill. **d** Tsai-Wu and **e** Hoffman failure criteria

Since the 256 samples sized data of the ANN-based model almost converges with the parent MCS, the scatter plot shown in Fig. 5, shows that the present ANN-based model can substitute the time-consuming MCS model with the 256 samples sized ANN model.

4 Conclusion

The novelty of the present work is that ANN is incorporated along with stochastic finite element modelling for first-ply failure analysis of a three-layered laminated composite. It is concluded that as the sample size increases for the ANN-based model, the accuracy level of the model with respect to the parent MCS also increases. The scatter plot for the efficiently matched ANN model is depicted to conclude that the present parent MCS can be replaced with the efficient ANN-based model.

Acknowledgments The authors would like to acknowledge the Aeronautics Research and Development Board (AR&DB), Government of India (Project Sanction no.: ARDB/01/105885/M/I), for the financial support for the present research work.

References

1. Dey S, Mukhopadhyay T, Adhikari S (2017) Metamodel based high-fidelity stochastic analysis of composite laminates: a concise review with critical comparative assessment. Compos Struct 171:227–250
2. Onkar AK, Upadhyay CS, Yadav D (2007) Probabilistic failure of laminated composite plates using the stochastic finite element method. Compos Struct 77(1):79–91
3. Reddy JN, Pandey AK (1987) A first-ply failure analysis of composite laminates. Comput Struct 25(3):371–393
4. Chakraborty D (2005) Artificial neural network-based delamination prediction in laminated composites. Mater Des 26(1):1–7
5. Fernández-Fdz D, López-Puente J, Zaera R (2008) Prediction of the behaviour of CFRPs against high-velocity impact of solids employing an artificial neural network methodology. Compos A Appl Sci Manuf 39(6):989–996
6. Choi SW, Song EJ, Hahn HT (2003) Prediction of fatigue damage growth in notched composite laminates using an artificial neural network. Compos Sci Technol 63(5):661–675
7. Erkan Ö, Işık B, Çiçek A, Kara F (2013) Prediction of damage factor in end milling of glass fibre reinforced plastic composites using artificial neural network. Appl Compos Mater 20(4):517–536
8. Watkins SE, Akhavan F, Dua R, Chandrashekhara K, Wunsch DC (2007) Impact-induced damage characterization of composite plates using neural networks. Smart Mater Struct 16(2):515
9. Nazari A, Sedghi A, Didehvar N (2012) Modeling impact resistance of aluminum–epoxy-laminated composites by artificial neural networks. J Compos Mater 46(13):1593–1605
10. Lee CS, Hwang W, Park HC, Han KS (1999) Failure of carbon/epoxy composite tubes under combined axial and torsional loading 1. Experimental results and prediction of biaxial strength by the use of neural networks. Compos Sci Technol 59(12):1779–1788
11. Chen G, Wang H, Bezold A, Broeckmann C, Weichert D, Zhang L (2019) Strengths prediction of particulate reinforced metal matrix composites (PRMMCs) using direct method and artificial neural network. Compos Struct 110951
12. Solati A, Hamedi M, Safarabadi M (2019) Combined GA-ANN approach for prediction of HAZ and bearing strength in laser drilling of GFRP composite. Opt Laser Technol 113:104–115
13. Balcıoğlu HE, Seçkin AÇ, Aktaş M (2016) Failure load prediction of adhesively bonded pultruded composites using artificial neural network. J Compos Mater 50(23):3267–3281
14. Reddy AR, Reddy BS, Reddy KVK (2011) Application of design of experiments and artificial neural networks for stacking sequence optimizations of laminated composite plates. Int J Eng Sci Technol 3(6):295–310
15. Kam TY, Lai FM (1999) Experimental and theoretical predictions of first-ply failure strength of laminated composite plates. Int J Solids Struct 36(16):2379–2395
16. Karsh PK, Mukhopadhyay T, Dey S (2018) Spatial vulnerability analysis for the first ply failure strength of composite laminates including effect of delamination. Compos Struct 184:554–567
17. Tresca HE (1864) Sur l’ecoulement des corps solides soumis a de fortes pressions. Imprimerie de Gauthier-Villars, successeur de Mallet-Bachelier, rue de Seine-Saint-Germain, 10, près l’Institut

18. Sokolnikoff IS (1956) Mathematical theory of elasticity. McGraw-Hill Book Company
19. Mises RV (1913) Mechanik der festen Körper im plastisch-deformablen Zustand. Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse, pp 582–592
20. Tsai SW, Wu EM (1971) A general theory of strength for anisotropic materials. J Compos Mater 5(1):58–80
21. Hoffman O (1967) The brittle strength of orthotropic materials. Compos Mater 1(2):200–206
22. Guo D, Wang Y, Xia J, Nan C, Li L (2002) Investigation of $BaTiO_3$ formulation: an artificial neural network (ANN) method. J Eur Ceram Soc 22(11):1867–1872
23. Köhler L, Spatz HC (2002) Micromechanics of plant tissues beyond the linear-elastic range. Planta 215(1):466