Driving Condition Recognition and Optimisation-Based Energy Management Strategy for Power-Split Hybrid Electric Vehicles

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Abstract Power-split hybrid electric vehicles (PSHEVs) have the advantages of low fuel consumption, low emissions, and no mileage limitations, and their performance is largely determined by the control strategy. The purpose of the study was to solve the problem of driving condition recognition and energy management strategy (EMS) of PSHEVs. For this purpose, the parametric description method for the driving cycle conditions, the driving condition recognition method based on learning vector quantisation (LVQ) neural network (NN), and the energy management optimisation strategy for hybrid power systems based on predictive information were studied. Energy management optimisation under certain conditions was carried out by using Pontryagin's minimum principle. A test bench platform for hybrid power systems was built to verify the effectiveness of the energy management and control strategy for HEVs based on condition recognition. Computer simulation and experimental results show that the presented EMS can effectively control hybrid power systems and significantly improve fuel economy compared with other control strategies.

Keywords PSHEVs · Condition recognition · Neural network · Pontryagin's minimum principle · Energy management optimisation

1 Introduction

The performance of hybrid electric vehicles (HEVs) is closely related to choice of control strategy. The practical operational condition of such vehicles is a random and uncertain process. The optimisation effect of energy management strategy (EMS) under certain driving cycle conditions shows a certain limitation in adaptability

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to practical working conditions. HEVs are sensitive to changes in working conditions. Under some specific working conditions, clarifying future working conditions contributes much to the reduction of energy consumption [\[1,](#page-13-0) [2\]](#page-13-1).

Given a driving cycle condition, the globally optimal solution can be attained by utilising an EMS based on the dynamic programming (DP) algorithm, however, owing to driving cycle conditions being unforeseeable and the DP algorithm suffering from the drawback of an onerous computational burden, the EMS based on DP algorithm is not used on-line. DP-based EMS [\[3–](#page-13-2)[5\]](#page-14-0) is generally regarded as a benchmark for evaluating the control effect of other EMSs. To improve the fuel economy of power-split plug-in hybrid electric vehicles (PHEVs), Chunting Chris Mi proposed an EMS based on DP algorithm [\[6\]](#page-14-1): two neural network (NN) modules are trained according to optimised results from use of the DP algorithm, while length of stroke (LS), and its duration are incorporated into the controller. The simulation result showed that whether the LS and duration are known or not, the EMS can improve fuel economy. On the basis of estimating driving cycle conditions, Yang Chao proposed a globally optimal EMS based on a modified DP algorithm aiming at hybrid electric buses to optimise fuel economy and drivability [\[7\]](#page-14-2). Chen Bo-Chiuan extracted a rule-based EMS from the DP algorithm, which is used for real-time control of stroke-lengthened electric vehicles [\[8\]](#page-14-3). Zhang Hui proposed an improved DP algorithm [\[3\]](#page-13-2) which presents finite computational complexity and can reduce emissions by optimising the size of ground vehicles.

Aiming at the problem that EMSs based on optimisation algorithms depend on driving cycle conditions, scholars have explored methods for predicting and identifying driving cycle conditions based on historical information about vehicles or traffic information fusion, thus implementing EMSs based on condition prediction. By taking people–vehicle–road as a unified whole, research into EMSs for HEVs is conducted, which is the third stage in the development of EMSs. By comparing and analysing the predictive effects of three methods, that is, exponential condition predictors, Markov condition predictors, and NN condition predictors, Sun Chao proposed an EMS for power-split hybrid electric vehicles (PSHEVs) based on model predictive control (MPC) [\[9\]](#page-14-4). According to predicted working conditions, optimisation is conducted within the time domain of prediction. Li Liang et al. proposed an EMS based on condition prediction with a Markov chain and random driving behaviour perception [\[10\]](#page-14-5). By utilising the *K*-means algorithm, the random driving behaviours between two PHEB (parallel hybrid electric bus) stations are classified. Aiming at vehicles with dual-motor-propulsion systems, Zhang Shuo proposed a condition predictor based on NN, thus proposing a predictive power management system based on condition predictors [\[11\]](#page-14-6).

The purpose of the study is to solve the problem of condition recognition and the EMS for PSHEVs. To that end, the research explored the parametric description method for driving cycle conditions, the condition recognition method based on LVQ (learning vector quantisation) NN, and the optimisation-based EMS for HEVs based on predictive information. Furthermore, a test bench for hybrid power systems was established to conduct experimental verification of the proposed control strategy.

Fig. 1 Schematic diagram of the transmission mechanism of a hybrid power system

2 Methods for Condition Recognition Based on LVQ NN

2.1 Introduction of a Power-Split Hybrid Power System

The schematic diagram of transmission mechanism of a power-split hybrid power system is shown in Fig. [1:](#page-2-0) this mainly consists of an engine, a front gear box, motors A and B, clutches C0 and C1, brakes Z1 and Z2, and a coupling mechanism (four planet rows). The power from the engine is transmitted to the coupling mechanism through the clutch C0, the front gear box, and the planet carrier of the planet row k2, and power is decomposed and synthesised through the coupling mechanism; motor A is connected to the gear ring of planet row k1 while motor B is linked with the sun gears of planet rows $k1$, $k2$, and $k3$; the planet carrier of planet row $k1$ is solidly fixed with the gear ring of planet row k2 while the gear ring of planet row k3 is solidly fixed to the sun gear of planet row k4; the planet carrier of planet row k3 and the gear ring of planet row k4 are both connected to the output shaft; the planet carrier of planet row k1 is joined to the output shaft through clutch C1, and their working conditions are controlled by engagement and disengagement thereof.

2.2 Analysis of Characteristic Parameters for Working Conditions

The common characteristic parameters for operation condition of HEVSs include the maximum speed, maximum acceleration, maximum deceleration, average speed, average acceleration, average deceleration, the ratios of travel time at a high speed $(v > 80 \text{ km/h})$, intermediate speed (45 km/h $\lt v \lt 80 \text{ km/h}$), and low speed (*v* \lt

45 km/h), the ratio of drive time when idling (the proportion of drive time at idle of the total time under working load), and standard deviations of speed and acceleration. By utilising the aforementioned characteristic parameters we form a characteristic vector for condition recognition, the methods for condition recognition are thereby explored.

The typical standard working conditions (including ① HWFET, ② NREL2VAL, ➂ INDIA_URBAN_SAMPLE, ➃ CSHCR, ➄ NYCC, and ➅ MANHATTAN) for extracting characteristic parameters are selected. Through analysis based on data, it is preliminarily supposed that $\mathcal D$ and $\mathcal Q$ refer to working conditions in highways; $\mathcal Q$ and \circled{a} represent working conditions in suburbs; \circled{a} and \circled{b} denote working conditions in urban districts.

2.3 Condition Recognition Algorithm Based on NN

LVQ NN combines supervised learning with supervisory signals with competitive learning in self-organising maps without supervisory signals. By taking strictly classified information (supervisory signals) as input information, the samples are classified and various network parameters are determined [\[12,](#page-14-7) [13\]](#page-14-8).

Vector quantisation is the extension of scalar quantisation and is applicable to highdimensional data. The principle of vector quantisation is summarised as follows: a high-dimensional input space is divided into several different zones. A central vector is separately determined as the clustering centre in each zone which can represent the input vector located in the same zone. In this way, a point set, in which various central vectors are considered as clustering centres, is formed.

By using the list of typical characteristic parameters for working conditions, the input samples for training LVQ NN are established:

$$
P = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]
$$
 (1)

where, x_k = $\left[\text{vmax } a_{\text{max}} \ d_{\text{max}} \ \bar{v} \ \bar{a} \ \bar{d} \ i_{high} \ i_{mid} \ i_{low} \ i_{idle} \ v_{std} \ a_{std} \right], k$ = $1, 2, \cdots, 6$

The working conditions in the last section are classified by applying clustering analysis: $\mathcal D$ and $\mathcal Q$ denote working conditions in highways; $\mathcal Q$ and $\mathcal Q$ refer to working conditions in suburbs; \circled{a} and \circled{b} represent the working conditions in cities. The driving cycle conditions are divided into three types. In NN, the supervisory signal is set to $C = [90, 90, 60, 60, 30, 30]$, in which 90, 60, and 30 denote working conditions in highways, suburbs, and cities, respectively. The output value of NN is only one of 30, 60, or 90, that is, the recognition result by using NN only corresponds to one of the three types. In MATLAB[™], the LVQ NN is trained, and then the classification result is as shown in Fig. [2.](#page-4-0)

In the figure, different types of working conditions are represented by different colours, in which blue, red, and green represent working conditions in highways,

suburbs, and cities, respectively. The eigenvector of typical working conditions appears as 12 dimensions while the typical working conditions can only be symbolically expressed by using 2-d coordinates in the figure. After NN training, verification will be further conducted aiming at random working conditions.

In Fig. [4,](#page-4-1) the recognition effect of LVQ NN for working conditions is tested by using UDDS typical working conditions. In the experiment, an LVQ NN with five neural units in implicit strata is established, in which "trainr" and "mse" are separately employed as training and performance functions. After undergoing 59 training steps, the training error of LVQ NN reaches design requirements (Fig. [3\)](#page-4-2). Through comparison and analysis, the sampling time is determined to be 50 s.

The classification and recognition results of typical working conditions by using the proposed LVQ NN (a means of condition recognition) are compared with those obtained by applying commonly used clustering analysis (a means of condition recognition), as shown in Fig. [5.](#page-5-0)

By comparing and analysing the test results, it can be seen that two recognition algorithms employ different numbers of characteristic parameters. In addition, due to contrasting mechanisms, the two algorithms also exhibit a certain difference in

Fig. 5 A comparison of recognition results attained by utilising two recognition algorithms

their recognition results despite use of the same characteristic parameters. Compared with clustering analysis algorithms, more eigenvalues are applied in the LVQ NN algorithm for condition recognition. Thus, the LVQ NN algorithm is influenced by many types of characteristic parameters during classification. When a certain eigenvalue exhibits a strong enough influence, the recognition result obtained by using LVQ NN differs from that attained with the clustering analysis algorithm.

3 Optimisation-Based EMS Based Condition Recognition

Current control strategies are mainly divided into two types: rule-based and optimisation-based control strategies. On the basis of rule-based control strategy, the optimal control strategy based on condition recognition is established. The basic idea of optimal control strategy based on condition recognition is summarised as follows: at first, under off-line conditions, different types of typical working conditions are optimally solved. After the optimised result is subjected to data processing, the optimal control parameters of EMSs corresponding to each working condition are obtained; thereafter, when driving, the control parameters corresponding to different working conditions are selected according to the result of condition recognition to control various energy sources powering such vehicles.

3.1 Off-Line Optimisation of Standard Working Conditions

Practical working conditions are partitioned into three types: working conditions in cities, suburbs, and highways. Before optimising different types of representative working conditions under off-line conditions, it is necessary to determine the typical

working conditions able to represent the three aforementioned types. In simulation, an existing working condition is divided into three parts, each of which represents working conditions in highways, suburbs, and cities, respectively (Fig. [6\)](#page-6-0).

The characteristic parameters of the three working conditions are separately extracted and there are 12 types of characteristic parameters determined (Sect. [2.2\)](#page-2-1), which include the maximum speed v_{max} (km/h), the maximum acceleration a_{max} (m/s²), the maximum deceleration d_{max} (m/s²), average speed \bar{v} (km/h), average acceleration \bar{a} (m/s²), average deceleration \bar{d} (m/s²), ratios of travel time at a high speed (i_{high}) , intermediate speed (i_{mid}) , and low speed (i_{low}) , ratio (i_{idle}) of travel time when idling, and the standard deviations of speed (v_{std}) and acceleration (a_{std}) . The characteristic parameters under each working condition are used to form corresponding eigenvectors; afterwards, the eigenvectors are taken as clustering centres, providing a classification basis for the condition recognition system.

As shown in Fig. [6,](#page-6-0) three working conditions are subjected to off-line optimisation. The differences between these optimisations lie in that: under working conditions in cities, the speed of vehicles is low and there is a small power demand, so the preset terminal value of SOC (state of charge) is slightly larger than its initial value, so that vehicles driving in cities tend to charge the battery pack; under working condition in suburbs, the initial value of SOC is equivalent to its terminal value; under working condition in highways, the terminal value of SOC is slightly lower than its initial value, so that vehicles tend to compensate for the demand for driving power by utilising the motor, allowing the engine to operate steadily and efficiently.

3.2 Energy Management Optimisation Algorithm Based on Condition Recognition and Prediction

By utilising an optimisation algorithm based on Pontryagin's minimum principle in optimal control theory, a performance function for equivalent fuel consumption is established. By optimising the predicted working conditions within the short-term future, a series of continuous optimal operating points for both engine and motors are found. Afterwards, only the first optimal control parameter in the optimised result is taken as the control command of various energy sources. As prediction and optimisation continue, the control command is constantly updated, which allows predictive control of full working conditions.

According to the theory governing vehicle dynamics, on condition of ignoring climb resistance, the driving force of vehicles can be calculated by using the following equation:

$$
F_d = F_f + F_w + F_a
$$

=
$$
mgf + \frac{1}{2}C_dA\rho v^2 + m\frac{dv}{dt}
$$
 (2)

where, F_f , F_w , F_a , m , g , and f refer to the rolling resistance of wheels, air resistance, inertial force of vehicles during acceleration (without considering the climbing resistance), mass of the vehicle, gravitational acceleration, and the coefficient of rolling resistance, respectively; C_d , A , ρ , and ν represent the coefficient of air resistance, the frontal area of the vehicle, air density, and the speed of the vehicle, respectively. According to predicted information about the future speed and acceleration of vehicles, the future demand for driving force of vehicles can be calculated, thus further attaining the output torque demand on the transmission system.

When predicting future output rotational speed and torque of the hybrid power system, according to the steady-state rotational speed–torque relationship of the hybrid power system, traversal optimisation is conducted on all feasible operating points of the engine. Through calculation and comparison, the performance function is minimised to determine the optimal operating points of the engine and motors.

The power balance equation for the system is expressed as follows:

$$
p_b = p_A(u)\eta_A^{-sign(P_A)} + p_B(u)\eta_B^{-sign(P_B)} + p_C
$$
 (3)

where, P_b refers to the charging and discharging powers of battery pack, in which a positive (negative) value means the discharging (charging); P_A and P_B separately denote the powers of motors A and B while η_A and η_B both stand for the efficiencies of motors; *sign*, *u*, and P_c represent a sign function, a vector $(T_e n_e)^T$ consisting of the torque and rotational speed of the engine and electrical power demand of the vehicle's auxiliary system, respectively.

The SOC is set as a state variable of the system while the torque and rotational speed $(u = (T_e, n_e)^T)$ of the engine are designed as the control variables, respectively. The state equation of the system is expressed as follows:

$$
S\dot{O}C = \dot{x} = \frac{\sqrt{E_b^2 - 4P_b(u)R_b} - E_b}{7200C_bR_b}
$$
(4)

where, E_b , C_b , and R_b refer to the electromotive force, capacity, and internal resistance of the battery pack, respectively.

According to the state equation of the system, the rate of change of SOC of batteries can be calculated. The rate of change of SOC is represented by using the control

command $(u = (T_e n_e)^T)$ of the engine; thereafter, the aforementioned parameters are substituted into the performance function, so the optimisation variable of the performance function only involves the control command (*u*) of the engine.

The performance function is expressed as follows:

$$
J = \Phi[SOC(N) - SOC_0] + \sum_{k=0}^{N-1} [\alpha|SOC(u(k)) - SOC_0] + \beta \dot{m}_{fuel}(u(k))]
$$
 (5)

where, $\Phi[SOC(N) - SOC_0]$ is used to emphasise the cost of the objective function when the terminal variable $SOC(N)$ does not approximate to the given value $SOC₀$; α and β refer to the control factors and it is feasible to adjust the trend in the power consumption of the hybrid power system by adjusting the values of α and β ; the larger the value of α relative to β , the more sensitive the system to the change of SOC and the greater the possibility of keeping the SOC stable, therefore, to satisfy the driving demand and reduce the power of the battery packs, the engine is more frequently invoked; \dot{m} _{*fuel*} denotes the rate of fuel consumption of the engine, which is directly determined by the control command *u* of the engine. The optimal control problem is to minimise the aforementioned objective function by determining and controlling the vector sequence $\{\mu(0), \mu(1), \cdots, \mu(N-1)\}.$

According to the physical properties of various parts, inequality constraints are added as follows:

$$
\begin{cases}\nSO C_{\min} \leq SO C \leq SO C_{\max} \\
P_{batt_min} \leq P_{batt} \leq P_{batt_max} \\
T_{e_min} \leq T_e \leq T_{e_max} \\
T_{mA_min} \leq T_{mA} \leq T_{mA_max} \\
T_{mB_min} \leq T_{mB} \leq T_{mB_max} \\
\omega_{e_min} \leq \omega_e \leq \omega_{e_max} \\
\omega_{mA_min} \leq \omega_{mA} \leq \omega_{mA_max} \\
\omega_{mB_min} \leq \omega_{mB} \leq \omega_{mB_max}\n\end{cases} \tag{6}
$$

where, $T_{e_{\text{min}}}$, $T_{mA_{\text{min}}}$, and $T_{mB_{\text{min}}}$ refer to the minimum torques of the engine, motors A and B; $T_{e_{\text{max}}}$, $T_{mA_{\text{max}}}$, and $T_{mB_{\text{max}}}$ denote the maximum torques of the engine, motors A and B; $\omega_{e_{\text{min}}}$, $\omega_{mA_{\text{min}}}$, and $\omega_{mB_{\text{min}}}$ represent the minimum rotational speeds of the engine, motors A and B, respectively; ω_e _{max}, ω_{mA} _{max}, and $\omega_{mB\text{max}}$ separately stand for the maximum rotational speeds of the engine, and motors A and B; *SOC*min and *SOC*max refer to the minimum and maximum number of charges in a system battery, respectively; P_{batt_min} and P_{batt_max} separately represent the minimum and maximum of instantaneous output power of the system battery.

3.3 Simulation Result and Analysis

To verify the effect of the EMS based on condition prediction, analysis was conducted aiming at the UDDS working condition by taking the hybrid power system shown in Fig. [7](#page-9-0) as a research object. It is supposed that the initial SOC is 0.6 and the reference value of SOC is also 0.6. The established off-line calculation for the control strategy based on condition prediction was completed in MATLAB™.

It can be seen from the result of off-line calculation that the control strategy based on condition prediction established in this section can realise control of the hybrid power system in the case of satisfying various constraint conditions. By controlling motors A and B, not only is the driving demand satisfied but the engine can operate efficiently. In this way, the fuel economy is improved and the SOC fluctuates around the reference value of 0.6, which guarantees the longest service life of batteries. The rotational speed and torque of the engine did not fluctuate to any significant extent; the fluctuation of load on pavements is mainly offset through motors: motor A provides rotational speed compensation while motor B offers torque compensation, which ultimately sustains optimal engine operation. Only under special working condition such as rapid acceleration when the motors fail to satisfy the demand under driving cycle conditions did the engine adjusts the operating point at the cost of reducing fuel economy to satisfy driving demand.

Fig. 7 Off-line calculation output for the control strategy based on condition prediction

Fig. 8 Test platform for a hybrid power system

Table 1 Key parameters of the hybrid power system

4 Experimental Verification of the EMS

4.1 Test Bench Introduction

To verify the feasibility of the EMS based on condition prediction for HEVs, a test bench for the hybrid power system was established. The inertial system was added to replace the inertial mass of the vehicle. The test bed is shown in Fig. [8](#page-10-0) and key parameters are listed in Table [1.](#page-10-1)

4.2 Experimental Results and Analysis

Due to being restricted by the computing capability of control units on the test bench, it is inappropriate to set too large a time domain for condition prediction. During the experiment, the time domain for prediction is set to 1 s, that is, we predict working condition information for the next second. According to the predicted result, the future power demand for driving can be calculated. Afterwards, based on the power demand, optimisation is conducted to find the optimal control commands of the

(c) Changes in torque of various parts

Fig. 9 Experimental verification of energy management optimisation strategy based on condition recognition

Fig. 9 (continued)

engine and motors. In the experiment, the degree of opening of the accelerator pedal directly reflects the deviation between the actual speed and the expected speed of the vehicle. The output range of the accelerator pedal is $[-1, 1]$; when the output from the accelerator pedal is less than 0, the expected speed of vehicles is lower than the actual speed, so it is necessary to decelerate; on condition that the output is greater than 0, the expected speed of the vehicle exceeds its actual speed, so it needs to accelerate. By indirectly applying the output power of the accelerator pedal to the predicted power demand, the predicted power can be adjusted and the speed is rendered consistent with the expected value (Fig. [9\)](#page-11-0).

It can be seen from the experimental result that various parts can work in a coordinated manner to drive a vehicle; under the proposed control strategy, the SOC of battery pack can be maintained at, or around, the preset equilibrium point and the engine can operate sustainably within a zone of low fuel consumption. As shown in the figure, the EMS based on condition prediction is more likely to satisfy the power demand of the whole vehicle relying on the engine; the battery pack is used to compensate for the dynamic response characteristics of the engine and realise the recovery of braking energy; the two motors are mainly applied to coordinate the engine to adjust its operating point. By virtue of the power difference between the two motors, the power is supplied for electrical equipment and the battery pack can work in a sustainable manner.

5 Conclusions

Aiming at the demand for an optimised control strategy, predictions of information about future working conditions are provided by way of an NN. In the EMS based on condition recognition for HEVs, typical working conditions are classified. Then, when driving, the type of the current driving cycle condition was determined by using a recognition algorithm. According to the control strategy for this type of typical working condition, the vehicle was controlled, which can significantly increase the efficiency of utilisation of energy.

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