

Analysis of Anterior Cruciate Ligament of the Human Knee Using a Mathematical Model



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Abstract Anterior cruciate ligament (ACL) of the knee is often injured particularly in sports and other strenuous activities. Approaches for replacement of the ligament surround controversies due to complexities and varied outcome. In the present study, mechanical behavior of the ligament is analyzed during knee motion considering changing geometric arrangements of the ligament fibers and their material properties during both unloaded and loaded states. An analytical model with sagittal plane representation of the knee was employed. The two cruciate ligaments were modeled as bundles of fibers that were nonlinearly elastic. Flexion motion in the unloaded state was guided by selected fibers. Effects of external loads were then superimposed to simulate activity. In a simulated Drawer test with 130 N external force applied during 0–120° flexion, the ACL stretched increasingly in early to mid-flexion. From mid to high flexion range, the stretching effect decreased continuously. Further, anterior bundle of fibers in the ligament resisted the external load at each flexion angle. The posterior bundle of fibers contributed to resistance in low and high flexion only. The model calculations compared reasonably with in vitro experimental measurements on human knees that are available in the literature. In conclusion, the analysis indicates that surgical replacement of the ACL requires careful attention to ligament insertion positions on bones. Also, rehabilitation of ACL-injured or replaced knee requires carefully designed exercises to be safe and effective. Future work will also focus on parametric analysis.

Keywords Knee ligaments · Anterior cruciate ligament replacement · ACL injuries · Rehabilitation of ACL · ACL properties

1 Introduction

Cruciate ligaments of the knee connect one bone to another and perform crucial role in providing a delicate balance between stability and mobility during a variety of

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activities that may range from daily routine for the common population to strenuous activities for the sports persons [1–4]. The ligaments stretch while restricting the distraction of the connected bones. Certain postures or positions of the joints during specific activities can predispose involved ligaments to injuries [3, 4]. Anterior cruciate ligament or ACL at the knee is often damaged resulting in partial or complete tear of the constituent fibers, thus restricting activity and causing pain. ACL is more prone to damage during activities near extension or low flexion angles and with tibial rotation [1, 3]. As healing characteristics of the ligament are poor or nearly absent [5], restoration of the ligament function is achieved through surgical repair or replacement of the damaged structure with single or double bundle approach [6, 7]. However, such approaches for treatment vary in procedures and patient outcomes including restricted level of activity [5–7]. Also, understanding of the ligament behavior through experimentation is prone to uncertainties and errors as making reproducible experimental setup is many times difficult [8, 9]. Moreover, possible changes in the ligament behavior due to experimental configuration can also contribute to difficulties in interpretation of outcome as well as in comparison with other similar studies [8, 9].

Realistic computer-based models, therefore, can be useful tools to analyze the ligament behavior. This requires appropriate mathematical representation of the ligament-intact knee with anatomical parameters and physiological features that can guide simulation of the joint function. The present study is an attempt in this direction.

In this work, an analytical model of the human knee is used to study mechanical behavior of the ACL in different joint positions. Effects of external loads are superimposed in each joint position.

2 Methods

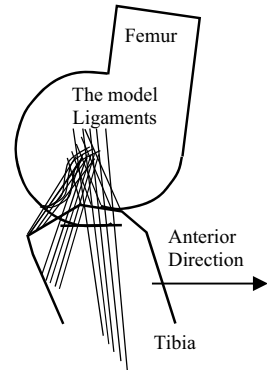
A mathematical model of the knee with intact cruciate ligaments was used in the sagittal plane similar to that reported in [3, 10, 11]. A brief description follows here.

The model knee joint is formed with two bones, namely femur and tibia, connected together with ligaments that comprised of bundles of fibers. Anterior cruciate ligament and posterior cruciate ligament (PCL) as well as collateral ligaments were modeled as bundles of nonlinear elastic fibers [3, 4, 9–12].

Figure 1 gives a graphical representation of the model knee at full flexion showing the bones connected through the cruciate and collateral ligaments. Effects of the collateral ligaments in the sagittal plane are shown to be relatively insignificant [9]. Accordingly, the model collateral ligaments had insignificant effect in the present analysis.

The model allows analysis of contributions of separate bundles of ligament fibers such as those located in the anterior part of the ACL that are reported in the literature as antero-medial bundle [3, 4, 9–11]. Similarly, fibers in the posterior part of the ACL were modeled that are reported in anatomical studies as postero-lateral bundle of the

Fig. 1 Graphical representation showing outlines of the model knee and the ligaments represented as straight lines when taut or as curves when slack like those used in references [10, 16]



ligament [3, 4, 9–11]. The model articular surfaces were frictionless and impenetrable resisting compressive forces.

The knee motion was defined during 0–120° flexion such that in the absence of any external load or muscle action, one specific fiber in each cruciate ligament maintained constant length or isometricity throughout the range of motion. However, the isometric fibers changed their respective position and orientation relative to the bones [9]. The isometric fibers were located most anteriorly in the ACL and somewhat in the middle of the PCL [9]. The flexion motion of the joint guided by these isometric fibers defined what is considered as passive motion of the knee [10].

Effect of external load (or muscle action) during activity is to cause relative movement between the bones resulting in either stretching or slackening of the involved ligament fibers compared to their neutral lengths. To simulate this effect, the model required that as the distance between bony attachments of a ligament increased, its fibers were selectively stretched. Conversely, as the distance between the bony attachments decreased, the ligament fibers selectively became slack and buckled. The material properties of the ligaments and other model parameters were taken from anatomical studies conducted on human cadaver joints available in the literature [9–16].

To simulate application of external load, like in a Lachman test or Drawer test to assess joint instability in the sagittal plane, 130 N anterior directed load was applied on the lower bone, or tibia, along with an external flexing moment that provided a balancing effect and thus held the joint at a fixed flexion angle. This resulted in anterior translation of the tibia (AT) and stretched fibers of the ACL. The simulation was repeated at intervals of 30° throughout the defined flexion range. Results from the model calculations were compared with similar experimental observations reported in the literature from independent work conducted on cadaver knees [14]. This supported validation of the model calculations. Further, contributions from each of the anterior and posterior bundles of the ACL were calculated using the model and analyzed.

3 Results and Analysis

Figure 2 gives a graphical representation of the model knee during flexion shown for 0° and 60° positions of the joint. During simulated passive flexion, the femoral bone translated in the posterior direction on tibial bone as depicted by respective contact positions in (a) and (b) represented by a+ sign where the circular femur contacted flat tibial surface. This observation agrees well with previously reported posterior movement of femoral condyles on tibial surface available in the literature [15, 16].

- (a) The model knee is shown at 0° flexion with fibers of the ACL just taut.
- (b) The model knee at 60° angle compared with that at 0° flexion. Notice that the bones rotate and slide during flexion and the ligament fibers slacken progressively with increasing joint angle.

In Table 1, model calculations for 130 N simulated test are presented with experimental measurements by Lo et al. [14]. The model calculations and experimental observations are similar. Tibial translation first increased from 0–30° and then decreased in high flexion.

Table 2 gives a comparison of contributions from each of the two bundles of model ACL over 120° flexion in steps of 30°. In the simulation, these fibers resisted 130 N force that was applied on the model tibia. At each flexion position, the anterior fibers stretched and resisted the external force. In comparison, the posterior bundle

Fig. 2 Model knee at 0° and 60° flexion showing only ACL fibers. Other ligaments were removed from the graphics for clarity

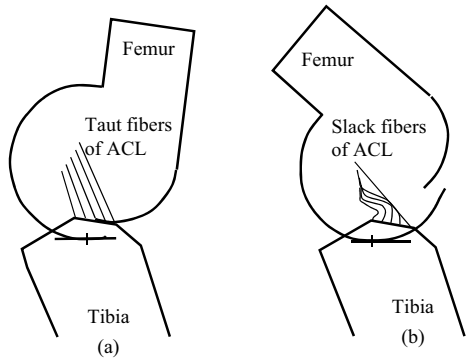


Table 1 AT corresponding to 130 N simulated test—comparing the model calculations with in vitro measurements from Lo et al. [14]

Flexion angle	Model results AT (mm)	In vitro measurements [14] mean (Std. Dev.) (mm)
0	3.7	4.1 (0.6)
30°	6.4	7.5 (1.8)
60°	6.3	7.4 (2.2)
90°	5.8	6.2 (1.9)
120°	4.5	–

Table 2 Force in each of the anterior and posterior bundle of the ACL with respect to the total force developed in the ligament

Flexion Angle	% Posterior force in anterior bundle of the ACL	% Posterior force in posterior bundle of the ACL
0	45.4	54.6
30°	96.3	3.7
60°	100	0
90°	100	0
120°	88.6	11.4

developed resistance only in low and high flexion positions of the joint. In the mid-flexion range, anterior bundle fully resisted the external force.

Observations from the two tables suggest that as the external load is applied at different flexion positions, the tibia is pulled anterior to the femur stretching the anterior fibers much more than the posterior fibers of ACL. In fact, the posterior fibers did not stretch at 60 or 90° flexion. Since most activities involving the knee take place in early to mid-flexion range, these observations suggest possibly more critical role for the anterior part of the ACL during activity. Consistent with differential contributions of the ligament fiber bundles, several practitioners prefer double bundle replacement of the ACL against a single bundle approach. However, differences exist relating positioning and orientation of the bundles.

For the same reason that early to mid-flexion resulted in more tibial translation and involved the anterior fibers much more than the posterior fibers, rehabilitation exercises for ACL after reconstruction or injury must consider sufficient protection in these joint positions in order to avoid harm.

4 Conclusion and Clinical Relevance

Model calculations during a simulated 130 N anterior laxity test at the knee joint suggest that anterior fibers of the ACL stretch in all knee positions while the posterior fibers stretch mainly in extreme positions of the joint motion. This and more observations from the model agree with those from experiments in the literature. The current analysis shows that mathematical modeling of the human knee joint with appropriate representation for its structures can be useful for gaining understanding of the ligament characteristics. On the other hand, analysis of the ACL function with experimental techniques involves several limitations which make comparison between different studies difficult. Though the present study used a planar model, the results corroborated experimental observations and helped in gaining insight into the behavior of the knee ligaments. Future development of the model involves a parametric analysis of the ACL fibers to account for person-to-person variations in the ligament anatomy and material properties as well as effects of surgical positioning of

the ligament fiber bundles during reconstruction. This work has relevance to ligament reconstruction or rehabilitation.

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