Double Diffusion Due to Centrally Heated Strip in Porous Material

N. Ameer Ahamad, Azeem, Maughal Ahmed Ali Baig, and A. Praveen Kumar

Abstract This work describes the analysis being carried out to find out the diffusion of heat and mass because of multiple sources of heat placed in the medium. A small heating strip is placed centrally apart from regular heating at the left surface. The height of the strip is varied to understand its effect on double diffusion in the medium. The mass diffusion occurs between the vertical surfaces of the cavity under the influence of concentration gradient as well as the thermal gradient. Finite element method (FEM) has been employed for solving the equations. Solution is obtained through an in-house code that works on FEM. The elements of the domain are selected such that they have triangular shape with three nodes. Solution of the equations is obtained following an iterative process, and the results are described using contour plots of concentration, isotherms, and streamlines.

Keywords Porous · Internal heating · Double diffusion

1 Introduction

Most of the engineering and scientific problems cannot be experimentally determined due to various restrictions imposed. Among those restrictions are the cost of experimentation as well as the time required to conduct such experiments. This is also true for the study of porous medium where many researchers have relied on the numerical investigation of different phenomenon such as natural convection [1–5], mixed convection [6–8], viscous dissipation [9, 10], and non-equilibrium [11, 12] to

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name but a few. Among such phenomenon, the double diffusion is also an important case where the heat and mass diffusion take place simultaneously due to gradients created by applied heat and mass concentration at different surfaces in the domain. The diffusion of mass can depend on its own variables such as Lewis number and buoyancy ratio apart from the parameters which are common for the heat transfer [13–17]. The mass transfer behavior is studied extensively for the cases where the single heat source in regular form exists. However, the effect of multiple heat sources on the behavior of especially mass transfer is yet to be reported. The current article is a step in that direction to know the scenario when the heat source is present at two places, i.e., one at the outside surface of the domain and another at the internal section of the porous medium. The high concentration exists at left surface and low concentration at right surface.

2 Analysis

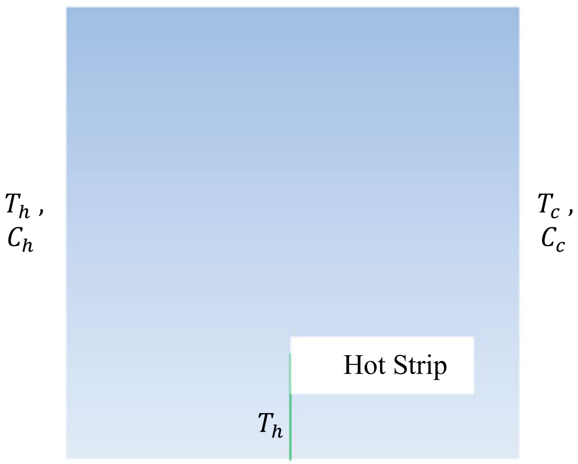
This work comprises the study of double diffusion due to multiple heat sources as shown in Fig. 1.

The related equations in porous domain can be derived from [18–22]

$$\frac{\partial u}{\partial x} + \frac{\partial \mathrm{v}}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial y} - \frac{\partial \mathrm{v}}{\partial x} = -\frac{gK}{\nu}\left(\beta_T \frac{\partial T}{\partial x} + \beta_c \frac{\partial C}{\partial x}\right) \tag{2}$$

Fig. 1 Porous cavity



$$u\frac{\partial T}{\partial x} + \mathrm{v}\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial x} \tag{3}$$

$$u\frac{\partial C}{\partial x} + \mathrm{v}\frac{\partial C}{\partial y} = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) \tag{4}$$

$$q_r = -\frac{4\sigma}{3\beta_r}\frac{\partial T^4}{\partial x} \tag{5}$$

Expanding T^4 [23–28]

$$T^4 \approx 4TT_\infty^3 - 3T_\infty^4 \tag{6}$$

Boundary conditions are

$$x = 0, T = 1, C = 1, u = 0, \mathrm{v} = 0 \tag{7a}$$

$$x = L, T = T_c, C = 0, u = 0, \mathrm{v} = 0 \tag{7b}$$

$$y = 0, \partial T/\partial y = 0, u = 0, \mathrm{v} = 0 \tag{7c}$$

$$\partial T/\partial y = 0, \partial C/\partial y = 0, T = T_h, u = 0, \mathrm{v} = 0 \tag{7d}$$

At the strip

$$\partial C/\partial y = 0 \tag{7e}$$

Finite element method as employed in references [29–35] is used to solve the equations.

3 Results and Discussion

Figure 2 depicts heat and mass diffusion with the help of the contours of temperature, concentration, and stream functions at $R_d = 0.5$, Ra = 100, Le = 5. $N = 0.1$, The left column shows the heating strip having the height 1/4th of cavity height and that of right column as ½ cavity height. The heat transfer should reduce from hot surface to the medium due to increase in the strip height which can be very much understood from the fact that the isotherm line with non-dimensional temperature of 0.95 has moved deep into the cavity. This ensures that the temperature gradient between two vertical surfaces and the cavity reduces, that in turn reduces the heat transfer. The mass transfer too is affected due to changing the strip height from 1/4th to ½ of cavity

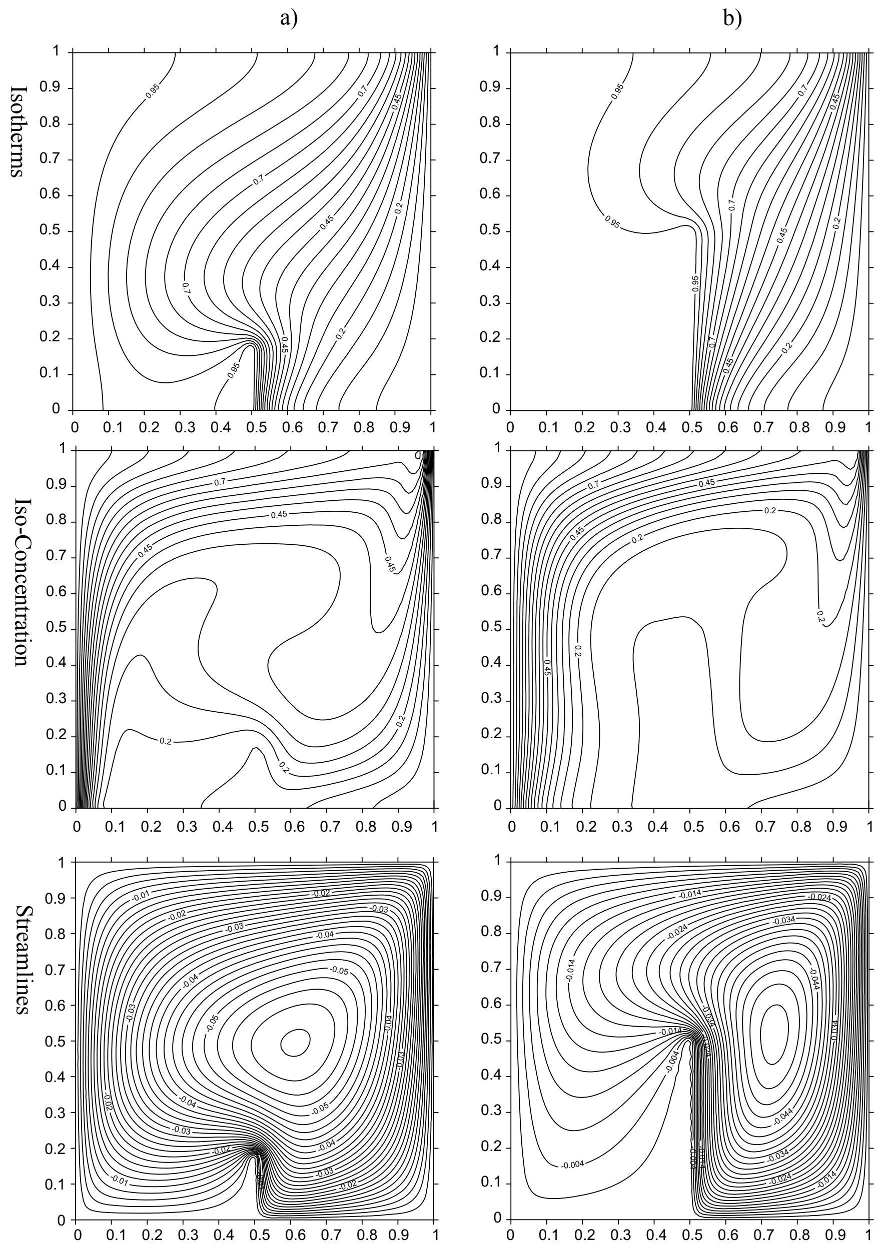


Fig. 2 Contours **a** St = 1/4, **b** St = ½

height. For the case of small strip height, the mass diffusion is more at the bottom of the cavity which can be observed through high concentration gradient at the lower part of the left surface and it decreases as the height of the left surface increases. This means that the mass transfer decreases from bottom of left surface until the top. However, the increase in the height of inside heater changes this scenario and the mass transfer across the left vertical surface remains almost constant which is indicated by almost straight concentration lines along that surface. There is large variation in concentration gradient in upper part of cavity due to increased strip height as compared to its smaller height. The streamlines mainly flow between the hot strip and the cold surface when strip height is more in comparison with lower height.

Figure 3 depicts buoyancy ratio (N) at $\mathrm{Ra} = 100$, $\mathrm{R_d} = 0.5$, $\mathrm{St} = 1/3$, $\mathrm{Le} = 5$, which is one of the important parameter to determine mass transfer. Heat transfer slightly decreases with increase in N which can be inferred from the isotherms. However, the concentration lines are seen to be aligned more closely to the left surface when the value of buoyancy ratio is increased as shown by the iso-concentration on right column of Fig. 3. The fluid movement changes the direction to more on left to right from being top to bottom surface when the buoyancy ratio is increased.

Figure 4 depicts another important parameter from mass transfer perspective, i.e., Lewis number (Le) at $\mathrm{Ra} = 100$, $\mathrm{R_d} = 0.5$, $N = 0.3$, $\mathrm{St} = 1/3$. The left column of figure corresponds to $\mathrm{Le} = 1$ and that of right column to $\mathrm{Le} = 10$. The isotherms are not much affected though it comes slightly closer to hot surface when Le is increased. However, there is drastic change in the shape of iso-concentration lines when Le is increased from 1 to 10. At $\mathrm{Le} = 1$, the concentration lines are highly spread out from each other that shows that the concentration gradient is smaller at $\mathrm{Le} = 1$. The concentration lines come very close to the left surface at increased Le ($\mathrm{Le} = 10$) which create high concentration gradient. This leads to substantially enhanced mass transfer as compared to the case of $\mathrm{Le} = 1$. The fluid flow pattern is also affected when Le is changed. At higher values of Le, the fluid cell flows from top to bottom, whereas its orientation is more toward left to right surface at $\mathrm{Le} = 1$.

4 Conclusion

This study aims to know multiple heating effects on double diffusion in a porous material. The study evaluates the effect of hot strip, the Lewis number, buoyancy ratio. It is noted that the

- Heat transfer decreased when height of the strip is increased.
- The mass transfer attains almost constant value all along the vertical surface when strip is elongated.
- Fluid orientation changes due to increased strip height.
- Mass transfer is highly sensitive to the Lewis number.

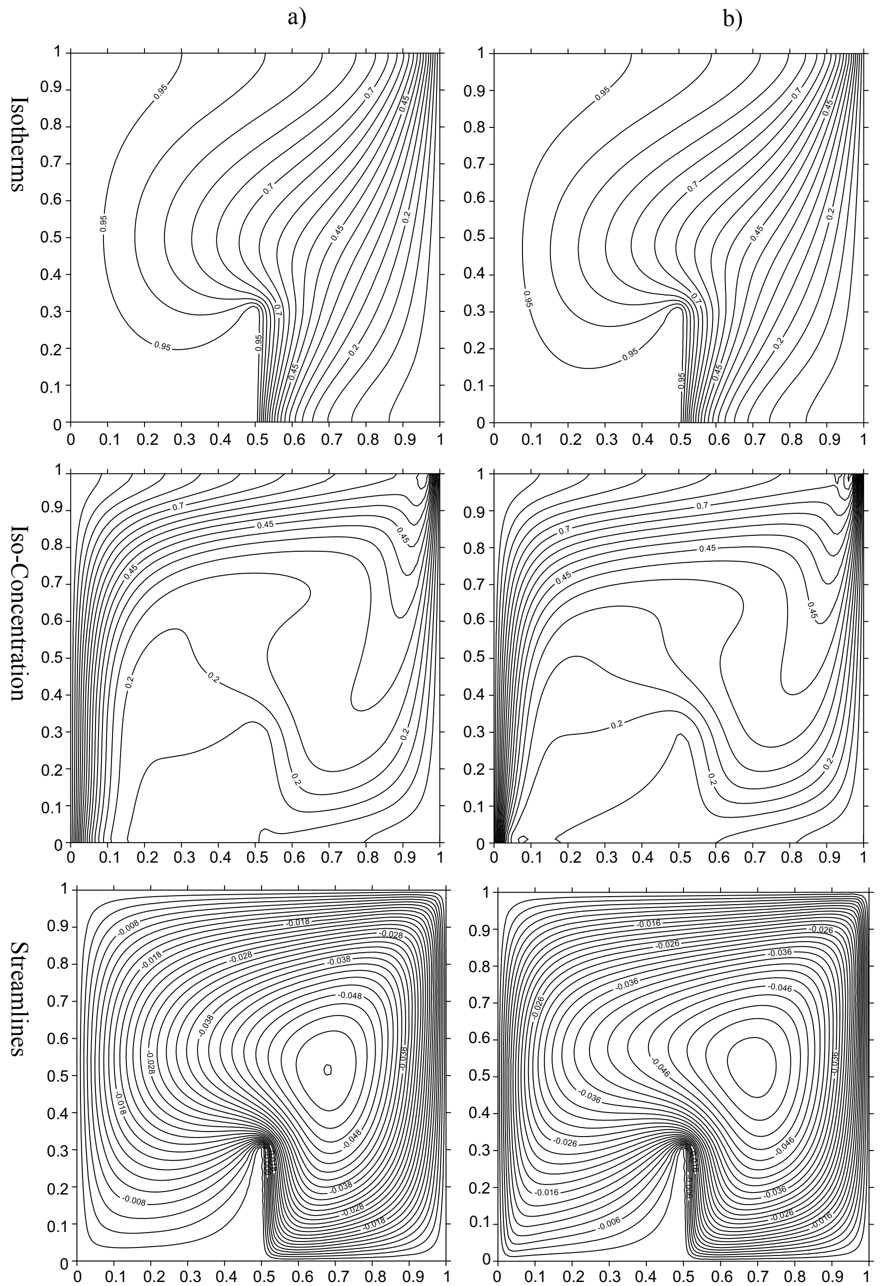


Fig. 3 Contours **a** $N = 0.1$, **b** $N = 0.5$

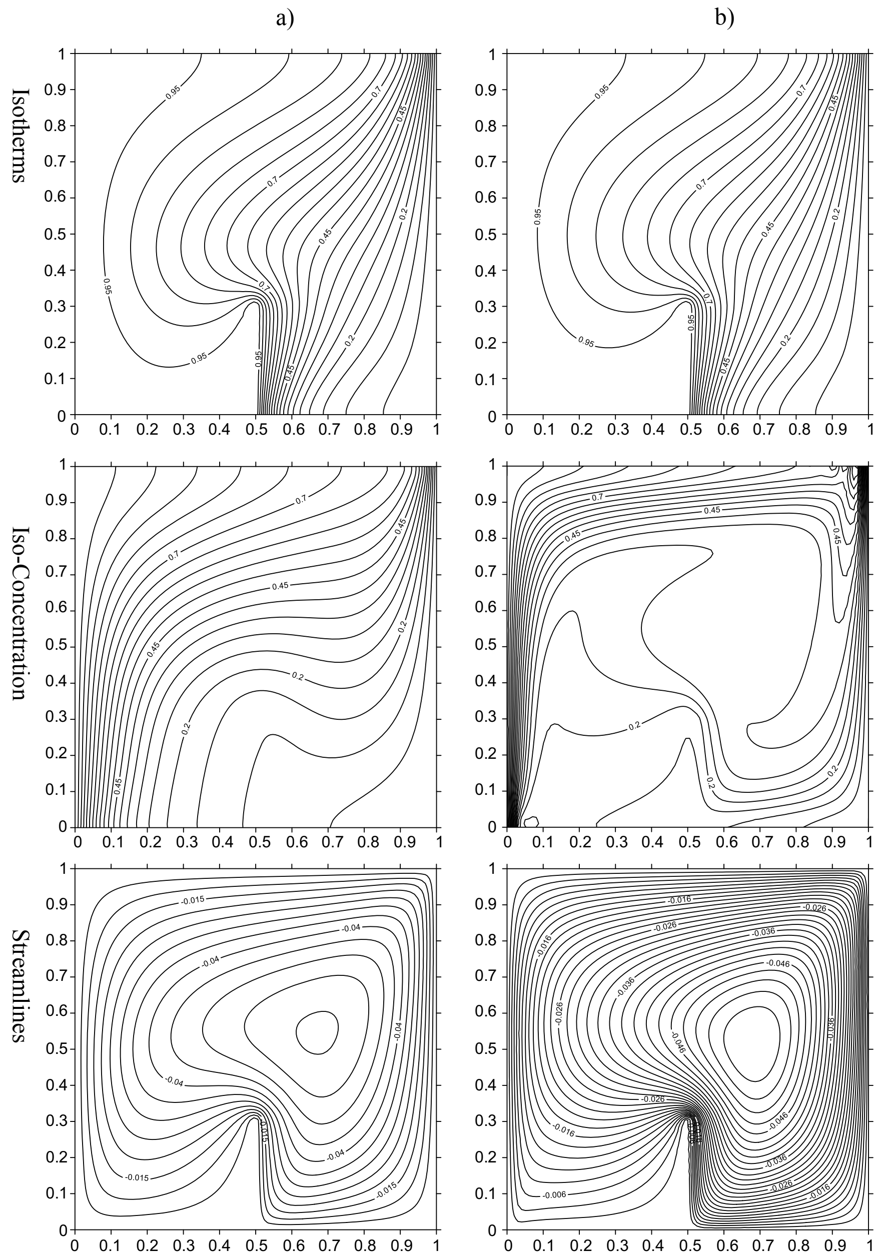


Fig. 4 Contours **a** Le = 1, **b** Le = 10

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