Exotic Branes and Exotic Dualities in Supergravity

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Abstract We show how T-duality in string theory implies the presence of exotic branes, that is branes of the lower-dimensional theory that do not have a geometric higher-dimensional origin. We then move to discuss the potentials under which these branes are electrically charged. We show that these are mixed-symmetry potentials, and we discuss the duality relations among these potentials and the standard potentials of ten-dimensional supergravity. Finally, we discuss how such duality relations can be naturally described within the framework of double field theory, and we show one particular physical consequence of this description.

1 Introduction

Duality symmetries play a crucial role in our understanding of various aspects of string theory. In particular, S and U dualities relate BPS branes with tensions scaling with different powers of the string dilaton, and therefore allow us to gain information on non-perturbative aspects of the theory. In general, these duality symmetries act as discrete subgroups of the global symmetry groups of the low-energy supergravity theory. In this talk we are interested in theories with maximal supersymmetry, that arise as torus reductions of IIA/IIB string theories. The global symmetry group of the theory in $10-d$ dimensions is $E_{d+1(d+1)}$, and the non-perturbative U-duality symmetry of the full quantum theory is conjectured to be its discrete subgroup $E_{d+1(d+1)}(\mathbb{Z})$ [14].

The T-duality group $O(d,d;\mathbb{Z})$, which is a subgroup of U-duality, is a symmetry of the perturbative string spectrum of the theory dimensionally reduced on T^d. Correspondingly, in the low energy supergravity one can consider the maximal subgroup $\mathbb{R}^+ \times O(d,d)$ of $E_{d+1(d+1)}$, where $\mathbb{R}^+$ is a symmetry under shifts of the d-dimensional string dilaton, while $O(d,d)$ leaves the dilaton invariant and it is therefore a perturbative symmetry of the low-energy action. In four dimensions, the $\mathbb{R}^+$

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symmetry is enhanced to $SL(2,\mathbb{R})$, while in three dimensions the full $\mathbb{R}^+ \times O(7,7)$ is enhanced to $SO(8,8)$.

We quickly review how the $O(d,d)$ symmetry acts on the scalar fields of the maximal supergravity theory in $10-d$ dimensions. In particular we are interested in the scalars coming from the metric and the B field, that parametrise the coset space $O(d,d)/[O(d)\times O(d)]$ by forming the $O(d,d)$ matrix

$$\mathcal{M}_{MN} = \begin{pmatrix} g^{mn} & -g^{mp}B_{pn} \\ B_{mp}g^{pn} & g_{mn} - B_{mp}g^{pq}B_{qn} \end{pmatrix}. \tag{1}$$

Under an $O(d,d)$ transformation $\mathcal{O}$, this matrix transforms as

$$\mathcal{M} \to \mathcal{O}^T \mathcal{M} \mathcal{O}. \tag{2}$$

T-duality is the discrete subgroup $O(d,d;\mathbb{Z})$. That is, given background values for the G and B scalars, every $O(d,d;\mathbb{Z})$ transformation, that acts on these background fields as in (2), leaves the string spectrum invariant. One defines the $O(d,d)$ invariant tensor

$$\eta_{MN} = \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix}, \tag{3}$$

which identifies the "lightlike" $O(d,d)$ coordinates X and $\tilde{X}$. The coordinates X are precisely the coordinates of the d-dimensional torus, and one can ask what is the physical meaning of the coordinates $\tilde{X}$. To answer this question, one writes X in terms of the string coordinates $X_L(\sigma,\tau)$ and $X_R(\sigma,\tau)$ which describe the left and the right modes respectively, as

$$X = X_L + X_R. \tag{4}$$

The factorised T-duality transformation that maps IIA to IIB inverting the compactification radius corresponds to

$$X_L^a \to X_L^a \quad X_R^a \to -X_R^a, \tag{5}$$

where a is the direction one is T-dualising. On the other hand, such transformation is the $O(d,d)$ matrix that maps X^a to $\tilde{X}^a$. This means that the coordinates $\tilde{X}$ are the "winding" coordinates

$$\tilde{X} = X_L - X_R. \tag{6}$$

The fact that T-duality transformations exchange the metric and the B field implies that in string theory one has to generalise the concept of geometry. In particular one can consider compactifications on generalised manifolds such that the transition functions are T-duality transformations [13]. As a simple occurrence of non-geometry, we can consider the IIB theory compactified to six dimensions on the orbifold $T^4/\mathbb{Z}_2$.

The six-dimensional low-energy theory is $\mathcal{N} = (2, 0)$ supergravity coupled to 21 tensor multiplets. Can we interpret this as arising from IIA? We can, but from the point of view of IIA the $\mathbb{Z}_2$ involution will act non-geometrically.

In the following we will first discuss how T-duality implies that in string theory one has to consider, together with "standard" branes, that are the branes of the 10-dimensional IIA or IIB theory, also "exotic" branes, that are branes that arise in the lower-dimensional theory but do not have a clear higher-dimensional origin. We will then move to study the potentials under which these branes are electrically charged, and show that these are in general mixed-symmetry potentials related by "exotic" duality relations to the potentials of the ten-dimensional theories. Finally, we will show how these duality relations are unified in the framework of double field theory (DFT), and we will discuss what information can be gained from the DFT picture.

2 Exotic Branes

We start by considering the IIA or IIB theory compactified on a 2-torus to eight dimensions. In this case the perturbative global symmetry of the supergravity theory is $SO(2, 2)$, which is isomorphic to $SL(2, \mathbb{R}) \times SL(2, \mathbb{R})$. This means that the G and B scalars parametrise the coset manifold $(SL(2, \mathbb{R})/SO(2))^2$. The scalars can be grouped in two complex scalars τ and ρ each transforming under one of the two $SL(2, \mathbb{R})$'s in a linear fractional way. While the scalar τ is made purely in terms of the metric, the scalar ρ is

$$\rho = B_{89} + i\sqrt{\det G} \tag{7}$$

and therefore a transformation

$$\rho \to \frac{a\rho + b}{c\rho + d} \tag{8}$$

mixes the B field and the determinant of the internal metric.

The NS5-brane solution in the string frame is

$$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu + H(r)dy^m dy^m, \tag{9}$$

where the NS-NS 3-form field strength and the dilaton are related to the harmonic function $H(r)$ as

$$H_{mnp} = \epsilon_{mnpq}\partial_q H(r) \qquad e^\phi = H^{1/2}(r). \tag{10}$$

We want to T-dualise along the transverse directions 8 and 9. So we first have to smear the NS5 along these directions. After smearing, the harmonic function becomes logarithmic. The equation for B_{89} becomes $\frac{1}{r}\partial_\theta B_{89} = -\partial_r H(r)$. Hence B_{89} depends linearly on θ, that is

$$B_{89} = \frac{\theta}{2\pi}, \tag{11}$$

and if one rotates around the brane $B_{89} \to B_{89} + 1$. That is, the monodromy is the T-duality transformation

$$\rho \to \rho + 1, \tag{12}$$

which is a symmetry of the eight-dimensional theory.

One can ask what happens to this solution after a generic T-duality transformation. In particular, one can consider the transformation corresponding to two factorised T-dualities in the directions 8 and 9. The action of such transformation on the scalar ρ is

$$\rho \to -1/\rho. \tag{13}$$

Hence, one ends up with a solution with monodromy

$$\beta^{89} \to \beta^{89} + 1 \tag{14}$$

where

$$\beta^{89} = \mathrm{Re}(-1/\rho) = -B_{89}/(B_{89}^2 + \det G). \tag{15}$$

Because of the monodromy, the explicit solution [8] is such that the internal metric is not well-defined. This means that the resulting 5-brane is globally non-geometric, *i.e.* it is "exotic". It is called 5_2^2 in the literature, where the top number denotes the number of isometries (in this case directions 8 and 9), while the bottom number denotes the scaling of the tension with respect to the dilaton (in this case g_S^{-2}). Models constructed introducing these branes had already appeared in the literature [10] well before the work of [8]. In particular, the model of [10] describes IIA 5-branes localised on a 2-sphere S^2, with monodromy $SL(2,\mathbb{Z})_\rho \times SL(2,\mathbb{Z})_\tau$. If the monodromy is non-trivial only with respect to $SL(2,\mathbb{Z})_\tau$, the model has $\mathcal{N} = (1,1)$ supersymmetry and it is geometric, that is it is IIA on K3 where the K3 is elliptically fibered. If the monodromy is non-trivial only with respect to the other $SL(2,\mathbb{Z})$, the model has $\mathcal{N} = (2,0)$ supersymmetry and it is in general non-geometric. Finally, if the monodromy is non-trivial with respect to both groups, supersymmetry is broken to $\mathcal{N} = (1,0)$.

In general, using chains of S and T dualities one finds all the non-geometric solutions of the type of the 5_2^2-brane [16]. Moreover, using the same dualities one derives also the expression for the tension of all such branes as functions of the string coupling and the compactification radii [9, 17, 18]. Following [17], one can consider instead of the tension the mass that arises when one compactifies the brane to a particle in three dimensions. So for instance for the D7-brane one gets $m_{D7} \sim g_s^{-1} R_3 \ldots R_9$, while for its S-dual we have $m_{SD7} \sim g_s^{-3} R_3 \ldots R_9$. The NS5 gives a mass $g_s^{-2} R_3 \ldots R_7$ and the 5_2^2 gives $g_s^{-2} R_3 \ldots R_7 R_8^2 R_9^2$. The fact that the exotic brane gives a mass proportional to a power of the radius higher than one is completely general and implies that the tension of the exotic brane diverges in the decompactification limit.

We want to associate to each brane the potential under which the brane is electrically charged. We use the following notation: if tension scales like g_s^{-n}, with $n = 1, 2, 3, 4 \ldots$, we denote the potentials with letters $C, D, E, F, \ldots$. That is, n is associated to the order in the alphabet. The indices of these potentials correspond to the directions contributing to that mass formulae for the three-dimensional particles above (plus the time direction). This means that the wrapped D7-brane above is charged with respect to the component $C_{03456789}$ of the RR 8-form C_8, its S-dual is charged with respect to $E_{03456789}$, which is a component of the 8-from E_8, and the NS5 gives D_{034567} (potential D_6). The square dependence on the radii R_8 and R_9 for the 5^2_2 give a potential $D_{03456789,89}$, which is a component of the mixed-symmetry potential $D_{8,2}$ (that is a field in a hook Young Tableau representation made of two columns, one with 8 boxes and one with 2). This gives a precise mapping between exotic branes and mixed-symmetry potentials [5]. What we want to analyse in the following is what are these mixed-symmetry potentials and how can they be related to the standard potentials of supergravity.

3 Exotic Dualities

We start by considering the NS5-brane. This brane is electrically charged under the potential D_6, which is the electromagnetic dual of the NS-NS 2-form B_2. We know how to dualise the NS-NS 2-form potential B_{ab}. We start from the kinetic term of the 2-form,

$$S[B] = \int d^{10}x\Big(-\frac{1}{12}H_{abc}H^{abc}\Big), \tag{16}$$

where $H_3 = dB_2$, and we write the parent action

$$S[D, H] = \int d^{10}x\Big(-\frac{1}{12}H_{abc}H^{abc} - \frac{1}{6}\epsilon^{a_1\dots a_6 abcd}D_{a_1\dots a_6}\partial_a H_{bcd}\Big), \tag{17}$$

where now the 3-form H_3 is treated as an independent field. The equation for D_6 gives the Bianchi identity $dH_3 = 0$, which implies $H_3 = dB_2$ and plugging this back into the action (17) gives back Eq. (16). On the other hand, the equation for H_3 gives the duality relation

$$H_{a_1\dots a_7} = 7\partial_{[a_1}D_{a_2\dots a_7]} = \frac{1}{6}\epsilon_{a_1\dots a_7 abc}H_{abc}, \tag{18}$$

and solving this for H_3 in terms of D_6 in Eq. (17) gives the dual action for D_6.

In the full supergravity theory, this potential turns out not only to transform with respect to its own gauge transformations, but also with respect to the gauge transformations of the RR potentials. As a result, the NS5 brane effective action contains couplings to the RR potentials which give information on which type of brane can end on the NS5. In particular, in the IIA theory the NS5 Wess-Zumino term has the form

$$\int [D_6 + \mathcal{G}_1 C_5 + \mathcal{G}_3 C_3 + \mathcal{G}_5 C_1], \tag{19}$$

where $\mathcal{G}_1$ and $\mathcal{G}_5$ are the field strengths of a world-volume scalar and its dual, while $\mathcal{G}_3$ is the field strength of a world-volume self-dual 2-form. The NS5 in IIA is the end-point of D0, D2 and D4 branes. Similar considerations apply to the IIB NS5-brane.

We want to repeat the same analysis in eight dimensions. We want 6-form potentials that couple to the NS5, the KK monopole and the 5^2_2. These potentials are in the $(\mathbf{3},\mathbf{1}) \oplus (\mathbf{1},\mathbf{3})$ of $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$, which is as we already mentioned the perturbative symmetry of the eight-dimensional theory, and they come from the 10-dimensional mixed-symmetry potentials

$$D_6 \qquad D_{7,1} \qquad D_{8,2}. \tag{20}$$

We want to identify the last two potentials as dual to the standard fields of the ten-dimensional theory. As we will show, the $D_{7,1}$ is the dual of the graviton, while the $D_{8,2}$ is the exotic dual of B_2.

We first consider the dual graviton. We dualise linearised gravity in the frame formulation, *i.e.* we dualise the linearised vielbein $e_\mu{}^a = \delta_\mu{}^a + h_\mu{}^a$ [22]. One starts with the linearised EH action written as

$$S_{\rm EH}[h] \;=\; \int d^dx \left[f_{ab}{}^b f^{ac}{}_c - \tfrac{1}{2} f_{abc} f^{acb} - \tfrac{1}{4} f_{abc} f^{abc} \right], \tag{21}$$

where

$$f_{ab}{}^c = \partial_a h_b{}^c - \partial_b h_a{}^c. \tag{22}$$

In terms of f, the linearised Einstein equations are

$$\partial^c f_{c(ab)} + \partial_{(a} f_{b)c}{}^c - \eta_{ab}\, \partial^c f_{cd}{}^d \;=\; 0, \tag{23}$$

where f satisfies the Bianchi identity

$$\partial_{[a} f_{bc]}{}^d \;=\; 0. \tag{24}$$

One then moves to a first order formulation and considers the parent action adding the lagrange multiplier $D_{d-3,1}$ that imposes the Bianchi identity,

$$\int d^dx\, \epsilon^{a_1\ldots a_{d-3}bcd} D_{a_1\ldots a_{d-3},e} \partial_b f_{cd}{}^e. \tag{25}$$

Observe that now you cannot impose that the $(d-3,1)$ potential is irreducible: there is also a completely antisymmetric part. The equation for D gives the Bianchi identity, while the equation for f gives the duality relation, and using the latter to solve for f in terms of $D_{d-3,1}$ and plugging this back in the action gives the linearised action for the dual graviton. In ten dimensions the potential is $D_{7,1}$.

We now move on to discuss the potential $D_{8,2}$, and show that it is related to B_2 by an exotic duality relation. By suitably integrating by parts, we write the B_2 kinetic action as

$$S[B] = -\frac{1}{4}\int d^dx\Big(Q_{a,bc}Q^{a,bc} - 2Q_a{}^{ab}Q^c{}_{cb}\Big), \tag{26}$$

where $Q_{a,bc} = \partial_a B_{bc}$ (only antisymmetric in bc). We then introduce the parent action

$$S[Q,D] = -\frac{1}{4}\int d^dx\Big(Q^{a,bc}Q_{a,bc} - 2Q_a{}^{ab}Q^c{}_{cb} + \epsilon^{a_1\ldots a_{d-2}ab}D_{a_1\ldots a_{d-2},cd}\partial_a Q_b{}^{cd}\Big) \tag{27}$$

where the $D_{d-2,2}$ potential imposes the Bianchi identity $\partial_{[a}Q_{b]cd} = 0$, and as before it is in a reducible representation. The equation for D gives the Bianchi identity, while the equation for Q gives the duality relation, and plugging this back into the action one then recovers the second order equation for the dual field [6]. In ten dimensions the exotic dual potential is precisely $D_{8,2}$.

4 Exotic Dualities in DFT

The duality relations described in the previous section have a natural unified description in the framework of double field theory (DFT) [15, 20, 21]. In DFT the coordinates X and $\tilde{X}$ discussed in the introduction are treated on the same footing, and are grouped together in $X^M = (X^m, \tilde{X}_m)$, where M is an $SO(10,10)$ index. The fields can depend in principle on both sets of coordinates, provided that they satisfy the section condition, that is on any pair of fields on the doubled space one must impose

$$\eta^{MN}\partial_M \otimes \partial_N = 0. \tag{28}$$

We are only interested in linearised field equations, and we employ the formulation of [1, 2], which is the DFT extension of the vierbein formulation of gravity. One introduces the generalised fluxes

$$\mathcal{F}_{ABC} = 3\,\partial_{[A}h_{BC]}\,, \qquad \mathcal{F}_A = \partial^B h_{BA} + 2\,\partial_A\phi, \tag{29}$$

where $A, B, \ldots$ are $SO(1,9)\times SO(1,9)$ indices, h_{AB} is the generalised vierbein and ϕ is the dilaton. The linearised action is

$$S_{DFT} = \int d^{2d}X\, e^{-2\bar{\phi}} \left(S^{AB} \mathcal{F}_A \mathcal{F}_B + \frac{1}{6} S^{ABCDEF} \mathcal{F}_{ABC} \mathcal{F}_{DEF} \right), \qquad (30)$$

where S^{AB} and S^{ABCDEF} are invariant tensors of $SO(1,9) \times SO(1,9)$.

The fluxes obey Bianchi identities, which in a first order formulation we want to obtain as equations for the dual fields. We thus consider a parent action with Lagrange multipliers D_{ABCD} and D_{AB},

$$\int d^{2d}X\, [D^{ABCD}\, \partial_A \mathcal{F}_{BCD} + D^{AB} \big(\partial^C \mathcal{F}_{CAB} + 2\, \partial_A \mathcal{F}_B \big) + D\, \partial^A \mathcal{F}_A], \qquad (31)$$

whose field equations are the linearised Bianchi identities

$$\begin{aligned} \partial_{[A} \mathcal{F}_{BCD]} &= 0 \\ \partial^C \mathcal{F}_{CAB} + 2\, \partial_{[A} \mathcal{F}_{B]} &= 0 \\ \partial^A \mathcal{F}_A &= 0. \end{aligned} \qquad (32)$$

The equations for the fluxes give the duality relations, and plugging this back in the parent action gives the linearised action for the dual fields. The potentials D_6, $D_{7,1}$ and $D_{8,2}$ of the previous section are the components D^{abcd}, $D^{abc}{}_d$ and $D^{ab}{}_{cd}$ of the DFT potential D_{ABCD}, and this analysis reproduces exactly the duality relations of D_6, $D_{7,1}$ and $D_{8,2}$ [3]. In particular, the standard dualisation and the exotic dualisation of B_2 are unified in DFT.

To go back to the brane effective actions, we want to write down a DFT equivalent of the WZ term in Eq. (19). To do this, one needs a DFT formulation of the RR potentials. This formulation was given in [12], and it consists in collecting the RR potentials in a chiral spinor of $SO(10, 10)$

$$\chi = \sum_{p=0}^{10} \frac{1}{p!} C_{m_1 \ldots m_p}\, \Gamma^{m_1 \cdots m_p} |0\rangle, \qquad (33)$$

with the Clifford vacuum $|0\rangle$ annihilated by all the gamma matrices Γ_m. The field strengths of the world-volume potentials describing D-branes ending on the NS5-brane and their T-duals is also a chiral spinor $\mathcal{G}$, and the DFT expression for the WZ term is [4]

$$S_{WZ} = \int d^6\xi\; Q_{MNPQ} [D^{MNPQ} + \overline{\mathcal{G}} \Gamma^{MNPQ} \chi], \qquad (34)$$

where $\overline{\mathcal{G}} \Gamma^{MNPQ} \chi$ is an $SO(10, 10)$ spinor bilinear. The charge Q_{MNPQ} selects the type of brane one is considering. In particular, Q_{mnpq} corresponds to the NS5, $Q_{mnp}{}^q$ to the KK monopole and $Q_{mn}{}^{pq}$ to the 5^2_2-brane, while the remaining charges correspond to branes whose solutions are not even locally geometric.

As a nice application of this framework, we can consider the form of this effective action when the IIA Romans mass [19] is turned on. It is known [7] that massive couplings in WZ terms give anomalous creation of branes. For instance, for the D0-brane, one has in the WZ term:

$$S_{\text{massive D0-brane}} \sim \int m\, b_1 \tag{35}$$

which implies that when a D0 crosses a D8, a fundamental string is created:

$$\begin{array}{l|l|l} \text{D0}: & \times & - - - - - - - - - \\ \text{D8}: & \times & \times \times \times \times \times \times \times \times - \\ \text{F1}: & \times & - - - - - - - - \times \end{array}$$

Similarly, for the NS5-brane, one has

$$S_{\text{massive NS5-brane}} \sim \int m\, c_6 \tag{36}$$

giving rise to the creation of a D6 brane when a D8 crosses an NS5:

$$\begin{array}{l|l|l} \text{NS5}: & \times & \times \times \times \times \times - - - - \\ \text{D8}: & \times & \times \times \times \times \times \times \times \times - \\ \text{D6}: & \times & \times \times \times \times \times - - - \times \end{array}$$

What our WZ term shows is that one can similarly consider the T-dual picture, in which a 5^2_2 crosses a D8 giving rise to a D6 [4]:

$$\begin{array}{l|l|l} 5^2_2: & \times & \times \times \times \times \times - - \otimes \otimes \\ \text{D8}: & \times & \times \times \times \times \times \times \times - \times \\ \text{D6}: & \times & \times \times \times \times \times - - \times - \end{array}$$

To conclude, we have shown that at least at the linearised level one can introduce mixed-symmetry potentials which couple to exotic branes and are related to the standard potentials by exotic duality relations. We have also shown how DFT provides a unified framework in which standard dualities and exotic dualities are treated on the same footing. One can then write down unified effective actions. It would be extremely interesting both from a conceptual point of view and from the point of view of model building to understand whether this descriptions could be extended at the interacting level.

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