Chapter 33
Sterile Neutrino in Minimal Extended Seesaw with A_4 Flavour Symmetry

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Abstract In this work, we first review the status of evidences in support of existence of sterile neutrinos. Then, after revisiting one of the most minimal seesaw (MES) model that gives rise to a $(3 + 1)$ light neutrino mass matrix, we include A_4 flavour symmetry in the theory. Considering the generic vacuum alignments of A_4 triplet flavons, we classify the resulting mass matrices based on their textures, and predict interesting correlations between neutrino oscillation in the allowed cases. We also find that all of these allowed cases prefer normal hierarchical pattern of light neutrino masses over inverted hierarchy.

33.1 Introduction

Existence of non-zero neutrino masses and large mixings have now become a well established fact, as guided by several experimental results [1]. Along with precise values of the solar and atmospheric mixing angles and mass squared differences, relatively recent experiments like MINOS, T2K, NOνA, Double ChooZ, Daya-Bay and RENO (for details, please see [1]) have established the large value of reactor mixing angle. Apart from the currently unknown issues in the neutrino sector, like mass hierarchy, Dirac CP violating phase as the global fit data suggest, another interesting question in the neutrino sector is the possibility of additional neutrino species with eV scale mass. In fact, this has turned out to be not just a speculation, but has gathered considerable attention in the last two decades following some anomalies reported by a few experiments. The first such anomaly was reported by the Liquid Scintillator Neutrino Detector (LSND) experiment in their anti-neutrino flux measurements. The LSND experiment searched for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations in the appearance mode and reported an excess of $\bar{\nu}_e$ interactions that could be explained by incorporating at least

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A. Giri and R. Mohanta (eds.), *Workshop on Frontiers in High Energy Physics 2019*, Springer Proceedings in Physics 248,
https://doi.org/10.1007/978-981-15-6292-1_33

one additional light neutrino with mass in the eV range. This result was supported by the subsequent measurements at the MiniBooNE experiment. Similar anomalies have also been observed at reactor neutrino experiments as well as gallium solar neutrino experiments. These anomalies received renewed attention recently after the MiniBooNE collaboration reported their new analysis incorporating twice the size data sample than before (please see the [1]), confirming the anomaly at 4.8σ significance level which becomes $>6\sigma$ effect if combined with LSND. Although an eV scale neutrino can explain this anomaly, such a neutrino can not have gauge interactions in the standard model (SM) from the requirement of being in agreement with precision measurement of Z boson decay width at LEP experiment [1]. Hence such a neutrinos is often referred to as a sterile neutrino while the usual light neutrinos are known as active neutrinos. Status of this framework with three active and one sterile or $3+1$ framework with respect to such short baseline neutrino anomalies can be found in several global fit studies [2–4]. It is worth mentioning that the latest cosmology results from the Planck collaboration [1] constrains the effective number of relativistic degrees of freedom $N_{\text{eff}} = 2.99 \pm 0.17$ at 68% confidence level (CL), which is consistent with the SM prediction $N_{\text{eff}} = 3.046$ for three light neutrinos. Similarly, the constraint on the sum of absolute neutrino masses $\sum_i |m_i| < 0.12$ eV (at 95% CL) does not leave any room for an additional light neutrino with mass in eV order. Although this latest bound from the Planck experiment cannot accommodate one additional light sterile neutrino at eV scale within the standard ΛCDM model of cosmology, one can evade these tight bounds by considering the presence of some new physics beyond the standard model (BSM).

Several BSM proposals that can account for an eV scale sterile neutrino having non-trivial mixing with active neutrinos can be found in literature. While the usual seesaw mechanisms like type I, type II and type III [1] explaining the lightness of active neutrinos were studied in details for a long time, their extensions to the $3+1$ case was not very straightforward primarily due to the gauge singlet nature of the sterile neutrino. Yet, there have been several proposals to generate a 4×4 light neutrino mass matrix within different seesaw frameworks in recent times [5–12]. Here we adopt a minimal framework known as the minimal extended seesaw proposed in the $3+1$ neutrino context by [5, 6] and study different possible realisations within the framework of non-abelian discrete flavour symmetry A_4. Flavour symmetry is needed to explain the observed flavour structure of different particles of the standard model. In the original proposal [6] also, the A_4 flavour symmetry was utilised but within the limited discussion the issue of non-zero reactor mixing angle as well as different A_4 vacuum alignments were not addressed. In another recent work based on the same model with A_4 flavour symmetry (see [56] of [1]), some details of the associated neutrino phenomenology was discussed by sticking to the effective 3×3 active neutrino mass matrix which can be generated by integrating out the sterile neutrino. In our present work, we consider the full 4×4 mass matrix and do not integrate out the sterile neutrino as its mass may not lie far above the active ones always, as hinted by experiments mentioned above. We also classify different possible textures of the 4×4 neutrino mass matrix based on generic A_4 vacuum alignments for triplet flavons. Similar but not texture specific work in three neutrino cases to constrain different A_4 vacuum alignments from three neutrino data which was further constrained from successful leptogenesis (see [57] of [1]). Here we extend such studies to the $3+1$ neutrino cases. Texture zeros in $3+1$ neutrino scenarios were discussed in different contexts earlier using flavour symmetries like Z_N, $U(1)$ etc. [8, 10, 11] but here we show that some of these textures can be realised (upto a few more constraints) just from the vacuum alignment of A_4 triplet flavons. We first make the classifications for allowed and disallowed textures based on already known texture results in $3+1$ neutrino frameworks [8, 10, 13] and then numerically analyse some of the textures which have not been studied before. To be more specific, we categorise our textures based on $\mu-\tau$ symmetric cases, texture zero cases, hybrid cases and disallowed ones. Out of them, we numerically analyse all the textures belonging to $\mu-\tau$ symmetric and texture zero cases leaving the discussion on hybrid textures to future works. It should be noted that, although the discovery of non-zero reactor mixing angle has ruled out $\mu-\tau$ symmetry in the three neutrino scenarios, it is possible to retain it in a $3+1$ scenario where the 3×3 neutrino block retains this symmetry while the active-sterile sector breaks it. This interesting but much less explored idea to generate non-zero θ_{13} by allowing the mixing of three active neutrinos with a eV scale sterile neutrino was proposed earlier in [14, 15] and was also studied in details recently in [12]. We find that many of the textures belonging to these categories are already ruled out by neutrino data while the ones which are allowed give interesting correlations between neutrino parameters which can be tested at ongoing and future experiments. This article is organised starting with description of the model, then classification of textures, numerical analysis along with results and discussions, lastly we have given a conclusion.

33.2 The Model

As mentioned before, we adopt the model first proposed in [6] but discuss it from a more general perspective taking all the allowed terms in the Lagrangian and all possible generic vacuum alignments of A_4 triplets. We note that the discrete non-abelian group A_4 is the group of even permutations of four objects or the symmetry group of a tetrahedron. It has twelve elements and four irreducible representations with dimensions n_i such that $\sum_i n_i^2 = 12$. These four representations are denoted by $\mathbf{1}, \mathbf{1}', \mathbf{1}''$ and $\mathbf{3}$ respectively. The particle content of the model along with their transformations under the symmetries of the model are shown in Table 33.1. Apart from the SM gauge symmetry and A_4 flavour symmetry, an additional discrete symmetry Z_4 is also chosen in order to forbid certain unwanted terms. For example, the chosen Z_4 charge of the singlet neutrino S keeps a bare mass term away from the Lagrangian. This is important because a bare mass term will be typically large, at least of electroweak scale and hence will not help us generate a 4×4 light neutrino mass matrix with all terms at or below the eV scale. To have a seesaw mechanism at place, three right handed neutrinos $\nu_{Ri}, i=1,2,3$ are included into the model. Apart from the usual Higgs field H responsible for electroweak symmetry breaking,

Table 33.1 Fields and their transformations under the chosen symmetries

	l	e_R	μ_R	τ_R	H	ϕ	$\phi^{'}$	$\phi^{''}$	ξ	$\xi^{'}$	χ	ν_{R1}	ν_{R2}	ν_{R3}	S
$SU(2)_L$	2	1	1	1	2	1	1	1	1	1	1	1	1	1	1
A_4	3	1	$1^{''}$	$1^{'}$	1	3	3	3	1	$1^{'}$	1	1	$1^{'}$	1	1
Z_4	1	1	1	1	1	1	i	-1	1	-1	$-i$	1	$-i$	-1	i

there are six flavon fields ϕ, $\phi^{'}$, $\phi^{''}$, ξ, $\xi^{'}$, χ responsible for spontaneous breaking of the flavour symmetries and generating the desired leptonic mass matrices. The leading order Lagrangian for the leptons can be written as

$$\mathcal{L}_Y \supset \frac{y_e}{\Lambda}(\bar{l}H\phi)_{\underline{1}}e_R + \frac{y_\mu}{\Lambda}(\bar{l}H\phi)_{\underline{1'}}\mu_R + \frac{y_\tau}{\Lambda}(\bar{l}H\phi)_{\underline{1''}}\tau_R + \frac{y_1}{\Lambda}(\bar{l}H\phi)_{\underline{1}}\nu_{R1} + \frac{y_2}{\Lambda}(\bar{l}H\phi')_{\underline{1''}}\nu_{R2} + \frac{y_3}{\Lambda}(\underline{l}H\phi'')_{\underline{1}}\nu_{R3} + \frac{1}{2}\lambda_1\xi\overline{\nu_{R1}^c}\nu_{R1} + \frac{1}{2}\lambda_2\xi'\overline{\nu_{R2}}^c\nu_{R2} + \frac{1}{2}\lambda_3\xi\overline{\nu_{R3}}^c\nu_{R3} + \frac{1}{2}\rho\chi\overline{S^c}\nu_{R1} + y_4\xi\overline{S^c}\nu_{R2} + y_5\chi^\dagger\overline{S^c}\nu_{R3} + \text{h.c.} \tag{33.1}$$

where Λ is the cut-off scale of the theory, $y_e, y_\mu, y_\tau, y_1, y_2, y_3, y_4, y_5, \lambda_1, \lambda_2, \lambda_3, \rho$ are the dimensionless Yukawa couplings. It is worth noting that the last two terms were not included in the original model [6] although they are allowed by the chosen symmetry of the model. We include them here as they contribute non-trivially to the neutrino mass matrix as well as the generation of correct neutrino mixing. We denote a generic vacuum alignment of the flavon fields as follows - $\langle\phi\rangle = v(n_1, n_2, n_3)$, $\langle\phi'\rangle = v(n_4, n_5, n_6)$,

$$\langle\phi''\rangle = v(n_7, n_8, n_9), \quad \langle\xi\rangle = \langle\xi'\rangle = v, \quad \langle\chi\rangle = u \tag{33.2}$$

where n_i, $i = 1 - 9$ are dimensionless numbers which we choose to take values as $n_i \in (-1, 0, 1)$, which are natural choices for alignments in such flavour symmetric models. Here v or u denotes the vacuum expectation value (VEV) of the flavon fields which typically characterises the scale of flavour symmetry breaking. Similar but more restricted alignments are chosen in the original proposal [6]. The charged lepton mass matrix can be written as

$$m_l = \frac{\langle H\rangle v}{\Lambda}\begin{pmatrix} n_1 y_e & n_2 y_\mu & n_3 y_\tau \\ n_3 y_e & n_1 y_\mu & n_2 y_\tau \\ n_2 y_e & n_3 y_\mu & n_1 y_\tau \end{pmatrix}. \tag{33.3}$$

The neutral fermion mass matrix in the basis (ν_L, ν_R, S) can be written as

$$\mathcal{M} = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & M_R & M_S^T \\ 0 & M_S & 0 \end{pmatrix} \tag{33.4}$$

where M_D, the Dirac neutrino mass matrix is

$$M_D = \frac{\langle H\rangle v}{\Lambda}\begin{pmatrix} y_1 n_1 & y_2 n_5 & y_3 n_7 \\ y_1 n_3 & y_2 n_4 & y_3 n_9 \\ y_1 n_2 & y_2 n_6 & y_3 n_8 \end{pmatrix} = \sqrt{A}\begin{pmatrix} y_1 n_1 & y_2 n_5 & y_3 n_7 \\ y_1 n_3 & y_2 n_4 & y_3 n_9 \\ y_1 n_2 & y_2 n_6 & y_3 n_8 \end{pmatrix} \quad (33.5)$$

with $A = \frac{\langle H\rangle^2 v^2}{\Lambda^2}$. The right-handed neutrino mass matrix takes the diagonal form, and M_S in the basis (S, ν_R) is given by - $M_S = (\rho u, y_4 v, y_5 u)$. In the case where $M_R \gg M_S > M_D$, the effective 4×4 light neutrino mass matrix in the basis (ν_L, ν_s) can be written as given in [6]. Using the expressions for M_D, M_R, M_S mentioned above, the 4×4 active-sterile mass matrix can be written as

$$m_\nu^{4\times 4} = \begin{pmatrix} -Aa_7 & -Aa_8 & -Aa_9 & -\sqrt{A}a_1 \\ -Aa_{10} & -Aa_{11} & -Aa_{12} & -\sqrt{A}a_2 \\ -Aa_{13} & -Aa_{14} & -Aa_{15} & -\sqrt{A}a_3 \\ -\sqrt{A}a_4 & -\sqrt{A}a_5 & -\sqrt{A}a_6 & -a_0 \end{pmatrix} \quad (33.6)$$

where constants a_is can be expressed in terms of various yukawa couplings and VEVs [1]. This is a 4×4 complex symmetric mass matrix, in general having ten independent elements. However, depending upon the vacuum alignments or the specific values of $n_i \in (-1, 0, 1)$, the mass matrix can have interesting textures which we discuss in details in the next section.

33.2.1 Classification of Textures

We choose to work in the basis where the charged lepton mass matrix is diagonal. This allows the leptonic mixing matrix to be directly related to the diagonalising matrix of the light neutrino mass matrix. As discussed in the previous section, this corresponds to the VEV of the flavon field ϕ to be $\langle\phi\rangle = v(n_1, n_2, n_3)$, with $n_1 = 1$, $n_2 = n_3 = 0$. In the most general case of the vacuum alignments of the flavon fields ϕ' and ϕ'', each of $n_4, n_5, n_6, n_7, n_8, n_9$ can take 3 values, i.e. 0, 1, -1. Therefore we have $3^6 = 729$ possible cases of different vacuum alignments, which will generate 729 different 4×4 neutrino mass matrices. We first single out the disallowed textures based on the known results from previous analysis [8–10, 13] which are given in details in [1]. **Allowed cases:**

1. $\mu - \tau$ symmetry in 3×3 active neutrino block. Total number of such textures is 40.
2. One zero texture mass matrix. Total number of such textures is 96.
3. Two zero texture mass matrix. Total number of such textures is 64.
4. Three zero texture mass matrix. Total number of such textures is 8.
5. Hybrid texture mass matrix with no zeros but some constraints relating different elements. Total number of such textures is 296.

Total number of such allowed mass matrices is 504. We further classify each of these allowed categories into different sub-categories based on the constraints relating different elements of the light neutrino mass matrix [1].

33.2.2 Classification of Allowed Textures

33.2.2.1 ($\mu - \tau$) Symmetric Texture, Texture 1 Zero and Texture 3 Zero

The 40 $\mu - \tau$ symmetric textures can be classified into 4 sub-categories depending upon the constraints that they satisfy. We have also classified all the 96 texture 1 zero cases into 12 categories depending upon constraints satisfied by them. For representative purpose, we have mentioned one such VEV alignment and the corresponding mass matrix in [1]. All 8 texture 3 zero cases can be classified into the category with 3 complex constraints i.e. $M_{e\mu} = 0, \quad M_{e\tau} = 0, \quad M_{\mu\tau} = 0$.

33.2.2.2 Texture 2 Zero Case

All 64 texture 2 zero cases can be classified into 8 categories. For example, one of them is 8 matrices with 3 complex constraints i.e. $M_{e\mu} = 0, \quad M_{e\tau} = 0, \quad M_{\mu\mu} = M_{\mu\tau}$. For other categories, please see [1].

33.2.3 Numerical Analysis, Results and Discussion

Next, we present the method adopted for numerical analysis for ($\mu - \tau$) symmetric textures, texture 1, texture 2 and texture 3 zero cases, in order to check their consistency with 3 + 1 neutrino data. It is well known that 4 × 4 unitary mixing matrix can be parametrised [1, 15] as

$$U = R_{34}\tilde{R}_{24}\tilde{R}_{14}R_{23}\tilde{R}_{13}R_{12}P \tag{33.7}$$

$$\tilde{R}_{14} = \begin{pmatrix} c_{14} & 0 & 0 & s_{14}e^{-i\delta_{14}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{14}e^{i\delta_{14}} & 0 & 0 & c_{14} \end{pmatrix} \tag{33.8}$$

with $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, δ_{ij} being the Dirac CP phases, and

$$P = \text{diag}(1, e^{-i\frac{\alpha}{2}}, e^{-i(\frac{\beta}{2}-\delta_{13})}, e^{-i(\frac{\gamma}{2}-\delta_{14})})$$

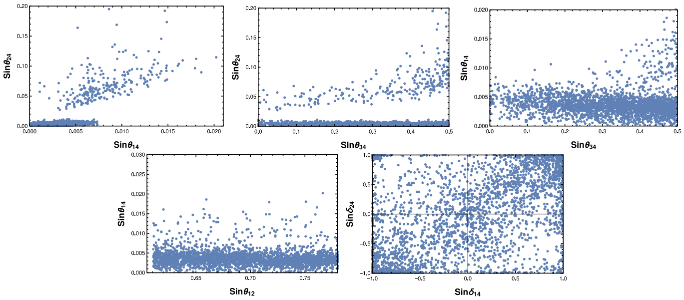

Fig. 33.1 Neutrino oscillation parameters in active-sterile sector for case (i) from texture 2 zero category for NH

is the diagonal phase matrix containing the three Majorana phases α, β, γ. In this parametrisation, the six CP phases vary from $-\pi$ to π. The 4×4 complex symmetric Majorana light neutrino mass matrix is written in [1]. One can analytically write down the 4×4 light neutrino mass matrix in terms of three mass squared differences, lightest neutrino mass $m_1(m_3)$, six mixing angles i.e., θ_{13}, θ_{12}, θ_{23}, θ_{14}, θ_{24}, θ_{34}, three Dirac type CP phases i.e., δ_{13}, δ_{14}, δ_{24} and three Majorana type CP phases i.e., α, β, γ [1]. For each class of neutrino mass matrix with textures that we analyse, there exists several constraints relating the mass matrix elements or equating some of them to zero. Depending upon the number of constraints, we choose the set of input parameters and solve for the remaining ones. We have varied our input parameters for the usual three neutrino part in the 3σ allowed range as given in the global analysis of the world neutrino data [1] and varied $\Delta m^2_{\rm LSND}$ from 0.7 eV^2 to 2.5 eV^2. Only some of the sub-classes give solutions and correlations in the range of parameters allowed by the global best fit values. For details, please see [1]. As an example, for the two zero texture case, the correlations corresponding to the solutions for subclass (i) are shown in Fig. 33.1.

33.3 Conclusion

To summarise, we first reviewed the status of existence of sterile neutrino, with reference to various data available in literature. Then, we summarised the findings of our work [1], in which we had studied the viability of different possible textures in light neutrino mass matrix within the framework of $3+1$ neutrino scenario by considering an A_4 flavour symmetric minimal extended seesaw mechanism (with an additional discrete symmetry Z_4). While the minimal extended seesaw mechanism naturally explains $3+1$ light neutrino scenario in an economical way predicting the lightest neutrino to be massless, presence of the A_4 flavour symmetry dictates the flavour structure of the 4×4 light neutrino mass matrix. We chose general VEV alignments for the triplet flavon VEVs, and obtained texture zero and $\mu - \tau$ symmetry forms of neutrino mass matrices. Though the existence an additional light neutrino having mass around the eV scale is yet to be confirmed by other neutrino experiments, our analysis show how difficult it is to realise such a scenario in the minimal extended seesaw if A_4 flavour symmetry with generic vacuum alignment is present. If the existence of such light sterile neutrino gets well established later, the predictions for unknown neutrino parameters obtained in our analysis can be tested for further scrutiny of the model, in a way similar to [16] where the possibility of probing texture zeros in three neutrino scenarios at neutrino oscillation experiments was studied.

Acknowledgements We sincerely thank the organisers of the International workshop FHEP 2019, for giving the opportunity to give a talk, and for warm hospitality. Financial support from Govt. of India, under the DST-SERB project $DST/SERB/2014/000296$ is deeply acknowledged. We also thank Debasish Borah, with whom this work was done.

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