Bearing Capacity Estimation of Shallow Foundations on Dense Sand Underlain by Loose Sand Strata Using Finite Elements Limit Analysis

Pragyan Paramita Das and Vishwas N. Khatri

Abstract In the present paper, a statistical limit analysis was carried out to estimate the bearing capacity of the surface strip and the circular base resting on dense sand under the loose sand layer. The analysis was accompanied by a lower and upper bound limit analysis in combination with finite elements and second-order conic programming (SOCP). In this approach, the Mohr–Coulomb yield criterion was used to model soil behavior. Assuming an associated flow rule, rigorous lower and upper bounds on ultimate bearing capacity are obtained with the use of this technique. Comparisons were made with the available solutions from the literature wherever applicable.

Keywords Finite element limit analysis · Bearing capacity · Mohr–Coulomb · Conic programming

1 Introduction

Estimation of bearing capacity and settlement of the foundation has been a significant topic of interest since time immemorial. Although many research studies have been published in the literatures on layered soil media (Meyerhof 1974; Meyerhof and Hanna 1978; Hanna 1981, 1982, 1987; Georgiadis and Michalopoulos 1985; Oda and Win 1990; Michalowski and Shi 1995; Burd and Frydman 1997; Kenny and Andrawes 1997; Okamura et al. 1998; Shiau et al. 2003; Farah 2004; Kumar et al. 2007; Shoaei et al. 2012; Kumar and Chakraborty 2015), nevertheless, there are few experimental studies conducted by scientists in layered sand media (Meyerhof and Hanna 1978; Hanna 1981, 1982; Farah 2004; Kumar et al. 2007). Meyerhof and Hanna (1978) determined the ultimate bearing capacity of the strip and the circular

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footing under inclined load using the limit equilibrium method. The results of the theoretical analyses were compared with small-scale model tests. Hanna (1981, 1982) developed design charts for estimating the bearing capacity of the strip and circular footing placed on layered sand media. That is a dense sand layer on loose sand strata and a loose sand layer on dense sand. Farah (2004) derived expressions for strip footing resting on layered soil. According to Farah (2004), the bearing capacity depends on the shear strength parameter of the upper and lower layers, the thickness of the upper layer and the width of the base to the depth ratio. Kumar et al. (2007) determined the carrying capacity of the dense sand layer overlying the loose sand deposit with and without geogrid on dense sand layer.

It is clear from the available literature that research has been carried out to determine the bearing capacity of layered sand media employing laboratory tests or the use of traditional approaches involving several simplified assumptions. Hardly any computational studies are available to determine the ultimate bearing capacity of layered sand media. This paper is trying to fill the gap. In the present paper, the bearing capacity of the strip and circular footing resting on layered sand media, that is, dense on loose sand is estimated using the lower and upper bound finite element limit analysis in conjunction with the second-order conic programming (SOCP). The analyses were conducted by varying the thickness of the upper dense layer and the angle of the internal friction of the two layers. The results are presented in a dimensionless manner and the comparisons are made with the available literature.

1.1 Problem Definition

A rigid rough strip and a circular footing are placed over a dense sand layer underneath the loose sand layer for analysis, as shown in Fig. 1. H is the thickness of the top

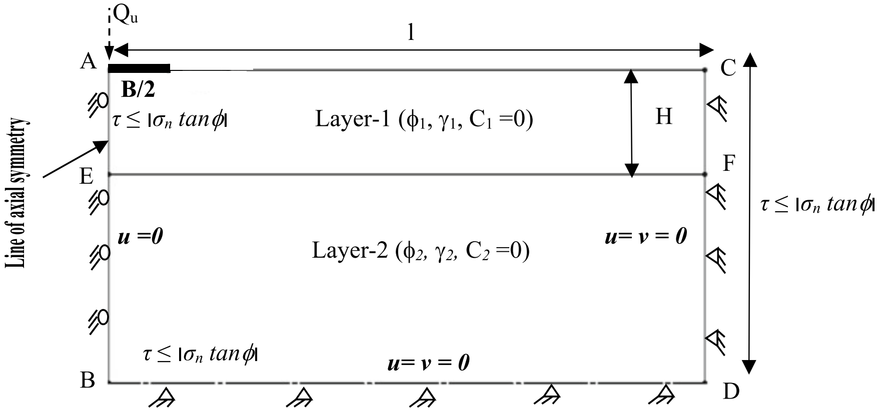

Fig. 1 Selected problem domain and associated boundary conditions

Table 1 The unit weights and the associated friction angles for sand in the analysis following Bowles (1977)

S. no.	Unit weight (γ) kN/m^3	Friction angle (ϕ) degrees
1	13.5	30
2	14.5	32
3	15	34
4	16	36
5	16.5	38
6	17.5	40
7	18	42
8	19	44
9	20	46

densified sand layer and B is the width/diameter of the base. It is intended to determine the ultimate bearing capacity of the base with variation in the frictional angle of the upper and lower layers and the thickness of the top dense layer. ϕ_1, ϕ_2, and γ_1, γ_2 are the friction angles and unit weight of the top and bottom sand layers, respectively. Bowles (1977) provided the relationship between unit weight (γ) and friction angle (ϕ). These values were considered for analysis. The values are presented in Table 1. The soil is assumed to be completely plastic, comply with the associated flow rule and the Mohr–Coulomb failure criterion.

Problem domain and mesh details

The selected problem domain with associated boundary conditions is shown in Fig. 1. l and d are the length and depth of the chosen problem domain, respectively. Considering the corresponding stress and velocity conditions for the lower and upper bound analysis, one-half of the chosen domain on the x–y plane was used for analysis. At the base, at the bottom horizontal and the right vertical boundary (BC and CD), $\tau \leq |\sigma_n \tan\phi|$ was applied.

No horizontal velocity (u) along the base and line of symmetry was allowed during the upper bound analysis. The horizontal (u) and the vertical velocity (v) are kept to zero for the selected horizontal and vertical boundary. Only uniform vertical velocities were imposed along the footing base. The problem domain was selected by performing several trials so that patterns of failure lie within the domain without touching the horizontal bottom and the vertical boundaries right. l and d values of 7B and 5B, respectively, are sufficient to satisfy the conditions. Adaptive meshes for footing with H/B = 0.25, 0.75 and 1.25 for $\phi_1 = 42°$ and $\phi_2 = 32°$ are shown in Fig. 2. The meshes are continuously updated based on the shear dissipation of the domain.

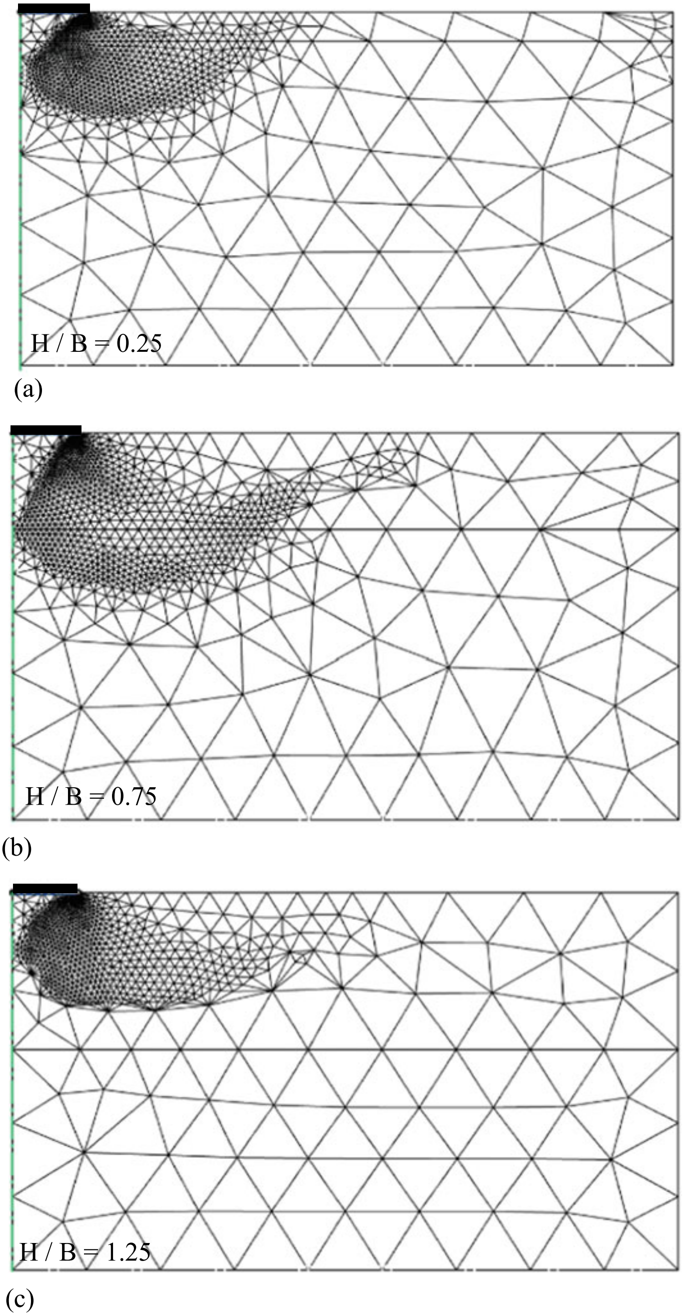

Fig. 2 Mesh details for footing on sand with $\phi_1 = 42°$, $\phi_2 = 32°$, with **a** H/B = 0.25, **b** H/B = 0.75 and **c** H/B = 1.25

2 Methodology

The numerical lower and upper bound finite element limit analyses were carried out using the lower and upper bound formulations given by Makrodimopoulos and Martin (2006, 2007) and Krabbenhoft et al. (2007, 2008), respectively. For the lower bound analysis, three-noded triangular elements with nodal stress σ_x, σ_y, τ_{xy} were used to represent the problem of the plane strain and the nodal stress σ_r, σ_z, σ_θ and τ_{rz} were used to represent the problem of the axisymmetrical strain. Statically admissible stress discontinuities have been introduced along with the interfaces of all triangular elements so that there will be a change in normal stress and shear stress at the nodes, but the stress remains continuous along the interface path. The main objective of the lower bound analysis is to maximize the collapse load. Six-noded triangular elements with horizontal and vertical velocity (u and v) were considered for the upper bound analysis. The interfaces have introduced kinematically admissible velocity discontinuities. The main objective of the upper bound analysis is to minimize the collapse. Lower and upper bound values of bearing capacity were obtained for dense on loose sand by varying the top (ϕ_1) and bottom friction angle (ϕ_2) and changes in the values of H/B. The results thus obtained are compared with the available literature, wherever applicable.

The methodology of the lower and upper bound analysis is explained in Sloan (1988), Sloan and Kleeman (1995), Lyamin (1999), Lyamin and Sloan (2002a, b), Krabbenhoft et al. (2007), Krabbenhoft et al. (2008), Makrodimopoulos and Martin (2006), (2007); Kumar and Chakraborty (2014) and Kumar and Mohapatra (2017). In the present study, numerical computations were carried out using the Optum G2 computer program.

3 Results and Comparison

After the determination of collapse load (Q_u), the ultimate bearing capacity (q_u) for circular and strip footing was determined following Eqs. 1 and 2.

$$q_u = Q_u/(\pi B^2/4) \tag{1}$$

$$q_u = Q_u/B \tag{2}$$

Before presenting the results for two-layered sand, it was thought of validating the model and software used for the present studies with the literatures for homogeneous

3.1 Footings on Homogeneous Sand

The analysis was performed for strip and circular base resting on a homogeneous sand layer. The angle of internal friction of sand varied from 30° to 45°. The ultimate bearing capacity was related to bearing factor Nγ (Eq. 3). The Nγ values are shown in Figs. 3 and 4 for strip and circular footing, respectively, following Terzaghi's equations as

$$N_\gamma = q_u \big/ 0.5\gamma \mathrm{B} \tag{3}$$

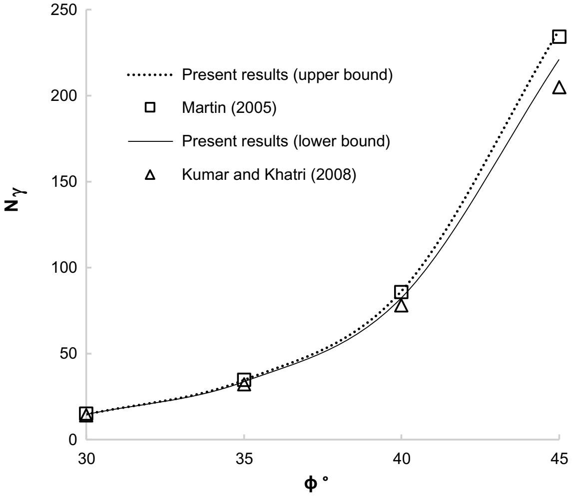

Fig. 3 Comparison of N_γ values for strip footing on homogeneous sand layer with literatures

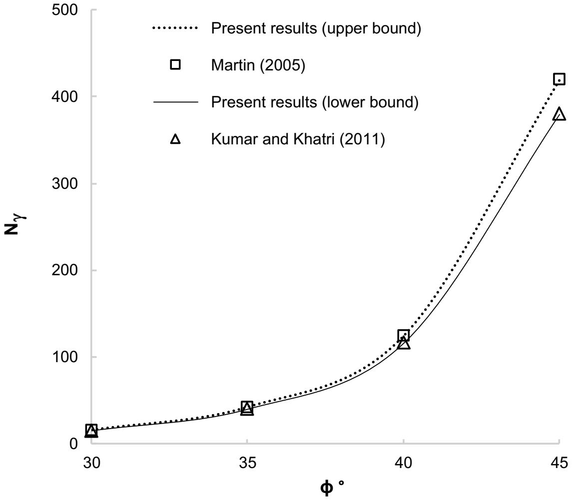

Fig. 4 Comparison of N_γ values for circular footing on homogeneous sand layer with literatures

The N_γ values presented in Figs. 3 and 4 show that the present analysis is quite comparable with those reported by Kumar and Khatri (2008, 2011) by using the lower bound limit analysis and linear optimization. Further, the N_γ values given by Martin (2005) by using the method of stress characteristics were found to lie between the lower and upper bound values of the present analysis.

3.2 Footings on Layered Sand

After carrying out the analysis for homogeneous cases, the analysis was carried out for layered sand media. The friction angle of the top dense layer varied from 42° to 46° and the bottom loose layer varied from 32° to 36°. The H/B ratio also varied until the bearing capacity became constant. The bearing capacity was expressed in terms of the efficiency factor, which is defined as the ratio of bearing capacity of the layered sand media to the ratio of bearing capacity of the homogeneous sand layer. The friction angle of the top dense sand is ϕ_1 and bottom loose sand is ϕ_2. The efficiency factor was calculated by considering the average of lower and upper bound bearing capacity values.

3.3 Variation of Efficiency Factor with H/B

The variation of efficiency factor with H/B for different values of ϕ_1 and ϕ_2 is shown in Figs. 5 and 6 for strip and circular footings, respectively. From these figures, it can be observed that due to the inclusion of dense sand layer the efficiency factor increases with an increase in H/B and later it becomes constant with certain H/B values. It was also noticed that for the same values of H/B, ϕ_1 and ϕ_2, the efficiency factor was found to be greater for circular footing in comparison to strip footing. For example, for H/B = 2, ϕ_1 and $\phi_2 = 42°$ and 32°, the efficiency factor for circular and strip footings was found to be 8.41 and 3.95, respectively. It was also observed that the efficiency factor increased with the increase in ϕ_1 and decrease in ϕ_2. From Figs. 5 and 6, it was also observed that the efficiency factor increases with H/B value. Further, the efficiency factor continues for a higher H/B value for strip footing compared to the circular footing. A similar observation was revealed by Meyerhof and Hanna (1978).

4 Comparisons

To compare the results of the present analysis with those available in the literature, the magnitude of bearing capacity was expressed in non-dimensional manner by dividing with (g1B). The analysis was carried out by considering $\phi_1 = 47.7°$, $\phi_2 = 34°$, γ_1

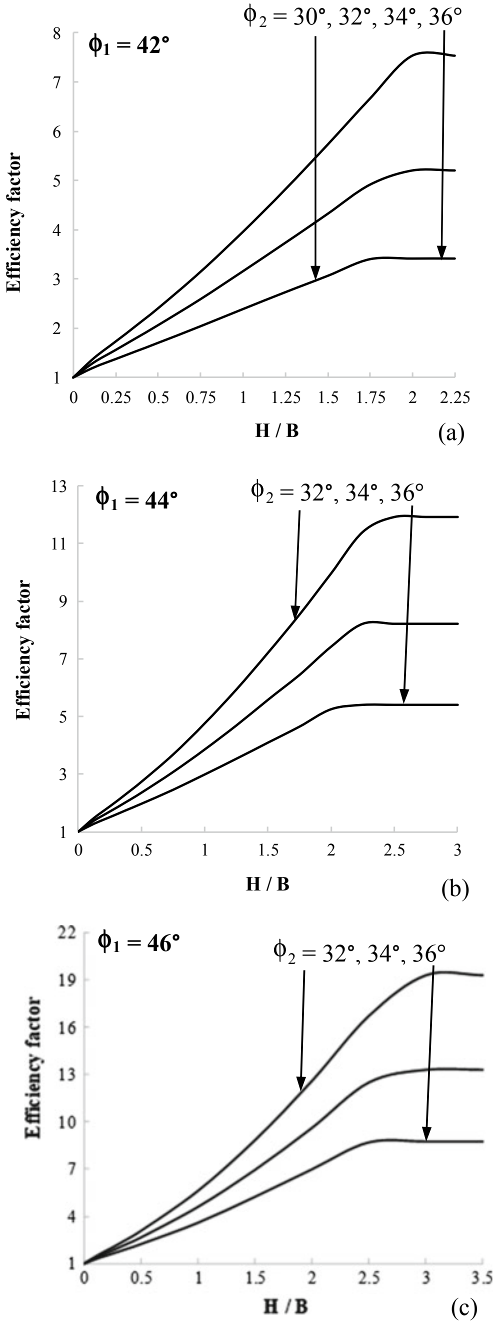

Fig. 5 Variation of efficiency factor with H/B and ϕ_2 for **a** $\phi_1 = 42°$, **b** $\phi_1 = 44°$ and **c** $\phi_1 = 46°$ for rough strip footing

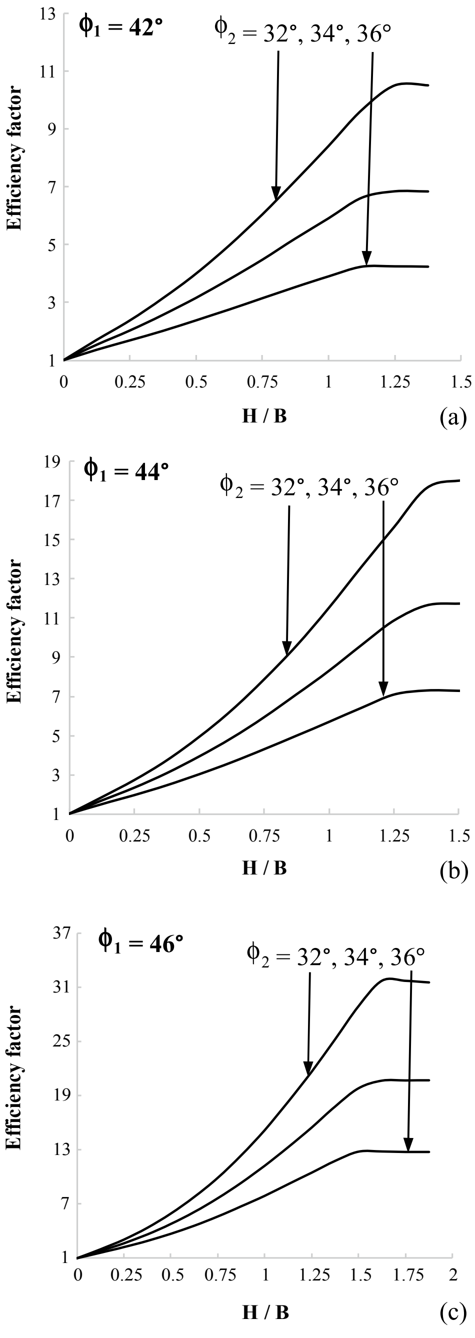

Fig. 6 Variation of efficiency factor with H/B and ϕ_2 for **a** $\phi_1 = 42°$, **b** $\phi_1 = 44°$ and **c** $\phi_1 = 46°$ for rough circular footing

Table 2 Comparison of $q_u/(\gamma_1 B)$ values with Hanna (1981) and Farah (2004) for strip footing on layered sand strata

H/B	Present results		Hanna (1981)	Farah (2004)
	Lower bound	Upper bound		
0	11.92	12.4	18.79	20.38
0.25	19.15	19.93	23.5	25.71
0.5	30.76	32.04	31.16	34.82
1	54.31	56.54	44.92	50.56
1.5	83.16	87.03	67	77.14
2	108.33	113.86	89.09	101.35

Table 3 Comparison of $q_u/(\gamma_1 B)$ values with Hanna (1981) for circular footing on layered sand strata

H/B	Present results		Hanna (1981)
	Lower bound	Upper bound	
0	16.04	17.18	22.41
0.25	22.05	23.58	24.49
0.5	34.41	39.50	36.75
1	90.76	95.35	61.23
1.5	109.47	116.32	94.95
2	156.73	162.83	141.87

$= 16.33$ KN/m^3 and $\gamma_2 = 13.78$ KN/m^3. Table 2 shows a comparison of the present lower and upper bound results for strip footings with the corresponding experimental results of Hanna (1981) and Farah (2004). From Table 2 it can be noticed that the bearing capacity reported by Hanna (1981) and Farah (2004) was found to be either higher or close to the present results for H/B values up to 1. However, for values of H/B greater than 1, bearing capacity of the present analysis was found to be greater than those reported in the literature. Similar trends were also observed for circular footings. Table 3 shows the comparisons of the present analysis for circular footings with the corresponding experimental results of Hanna (1981).

5 Conclusions

The bearing capacity of the strip and circular base on dense sand, overlaid by loose sand strata, was numerically determined using lower and upper bound finite element limit analysis with conic optimization. The results are presented in terms of non-dimensional efficiency factors. The study concludes with the following conclusions:

1. For a given friction angle of top dense layer and bottom loose layer, the efficiency factor increases with an increase in the thickness of the top dense layer.

2. The increase in efficiency factor with the inclusion of a dense layer of specific thickness becomes more significant for a circular base than for a strip. That is to say, for the same thickness of the top dense layer, the circular base has more bearing capacity than the strip foot.
3. The increase in efficiency factor with the inclusion of a dense layer of specific thickness becomes more significant for the circular base compared to the strip foot, that is, for the same thickness of the top dense layer circular footing has more bearing capacity than the strip footing.

The generated adaptive mesh shows the variation of the bearing capacity with the thickness of the top dense layer.

The results of the present study suggest that with the inclusion of a thin, dense layer just below the footing resting on layered sand strata, the bearing capacity can be improved.

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