Optimal Design of Structure with Specified Fundamental Natural Frequency Using Topology Optimization

Kandula Eswara Sai Kumar and Sourav Rakshit

Abstract Resonance occurs when the natural frequency of the system matches with the vibrating frequency. It may cause structural instabilities. To avoid this, engineers maximize the first natural frequency of the system. In many applications, the natural frequency is pre-designed. Structural engineers aim to reduce the weight of structures subject to functional and safety constraints. This motivates us to modify the frequency optimization problem to weight minimization problem, for a specified fundamental natural frequency. In this paper, we solve for weight minimization using topology optimization subject to lower bound constraint on fundamental frequency.

Keywords Topology optimization · Eigen frequency · Resonance · Optimum volume fraction · Method of moving asymptotes

1 Introduction

The phenomenon of resonance causes structural instabilities and it has to be avoided while design of structures. To avoid resonance, the Eigen frequency is taken as the objective function to maximize, using structural topology optimization [1]. Frequency optimization is importance in designing the structures under dynamics loads.

In many engineering applications, the fundamental natural frequency of the structure is pre-designed [2]. In such cases, structural engineers aim to minimize the weight of structures. The structure of minimum weight saves the cost of the material and improves the efficiency when they used in machine components [3]. In this paper, we solve the weight optimization problem for a clamped beam using topology optimization subject to a lower bound constraint on natural frequency, i.e. the fundamental natural frequency of the structure is greater than the specified frequency.

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2 Topology Optimization Formulation

2.1 Problem Definition

In this paper, we pose an alternative approach to optimization of structures to avoid the resonance.

Minimization of the volume fraction:

$$\begin{aligned} &\min V = \sum_{e=1}^{N} \vartheta_e \rho_e \\ &\text{subject to } \left(\boldsymbol{K} - \omega_1^2 \boldsymbol{M}\right)\emptyset_1 = 0 \\ &\omega_g \leq \omega_1 \end{aligned} \tag{1}$$

where $\boldsymbol{K}$ and $\boldsymbol{M}$ are global stiffness and mass matrices, respectively, ω_1 is the fundamental natural frequency of the structure, ω_g is the specified frequency value, $\emptyset_1$ is the Eigen vector corresponding to fundamental natural frequency, ϑ_e is the volume of each element, N is number of elements, V is the volume fraction and ρ_e is the density (design variable).

2.2 Topology Optimization

Topology optimization is an iterative optimization method, and it finds the optimal material distribution in the design domain subjected to the constraints [4]. We supply the dimensions of the design domain, material properties and a specified frequency as an input to the topology optimization. After that, we discretize the design domain into finite elements and assign design variable i.e. density variable to each element. It calculates the fundamental Eigen frequency of the structure by the finite element analysis [5]. This paper uses the MMA as an optimization solver to update the design variable [6]. If the density value is 1, it represents the material point i.e. solid (in black color) and if it is 0, it represents the no material, i.e. void (in white color).

Topology optimization suffers with two kinds of mathematical instabilities, named checkerboard pattern and mesh dependency problem [7]. To avoid these, we apply filtering techniques to the sensitivities [8]. Figure 1 shows the flow chart of the topology optimization process [4].

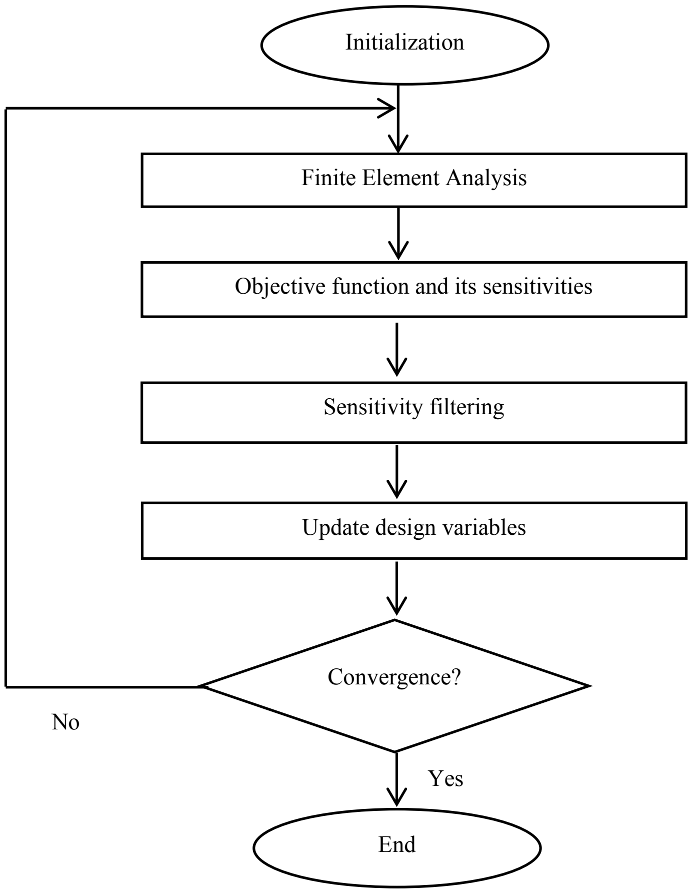

Fig. 1 Flow chart of topology optimization

2.3 Sensitivity Analysis

2.3.1 Sensitivity of Objective Function

Objective function

$$V = \sum_{e=1}^{N} \vartheta_e \rho_e \tag{2}$$

$$\frac{\partial V}{\partial \rho_e} = v_e \tag{3}$$

2.3.2 Sensitivity of Constraint

Equality Constraint is given as

$$(K - \omega_1^2 M)\phi_1 = 0 \tag{4}$$

Assume $\lambda_1 = \omega_1^2$ then, Eq. (4) becomes

$$(K - \lambda_1 M)\phi_1 = 0$$

$$\frac{\partial}{\partial \rho_e}(K - \lambda_1 M)\phi_1 = 0$$

$$\left(\frac{\partial K}{\partial \rho_e} - \lambda_1 \frac{\partial M}{\partial \rho_e} - \frac{\partial \lambda_1}{\partial \rho_e} M\right)\phi_1 + (K - \lambda_1 M)\frac{\partial \phi_1}{\partial \rho_e} = 0 \tag{5}$$

In the above equation, the second term will become zero. By pre-multiplying the Eq. (5) with ϕ_1^T becomes,

$$\phi_1^T\left(\frac{\partial K}{\partial \rho_e} - \lambda_1 \frac{\partial M}{\partial \rho_e} - \frac{\partial \lambda_1}{\partial \rho_e} M\right)\phi_1 = 0$$

$$\phi_1^T\left(\frac{\partial \lambda_1}{\partial \rho_e} M\right)\phi_1 = \phi_1^T\left(\frac{\partial K}{\partial \rho_e} - \lambda_1 \frac{\partial M}{\partial \rho_e}\right)\phi_1$$

$$\left(\frac{\partial \lambda_1}{\partial \rho_e}\phi_1^T M \phi_1\right) = \phi_1^T\left(\frac{\partial K}{\partial \rho_e} - \lambda_1 \frac{\partial M}{\partial \rho_e}\right)\phi_1 \tag{6}$$

but, $\phi_1^T M \phi_1 = 1$, then

$$\frac{\partial \lambda_1}{\partial \rho_e} = \phi_1^T\left(\frac{\partial K}{\partial \rho_e} - \lambda_1 \frac{\partial M}{\partial \rho_e}\right)\phi_1 \tag{7}$$

The sensitivities of objective function and equality constraint is given by Eqs. (3) and (7) respectively [9], [10].

2.4 Design Domain and Properties

Figure 2 shows a clamped beam, for which we minimize the weight using topology optimization of a mesh size 280 × 40. The material properties are, Young's Modulus, E = 25e7 Pa, Poisson's ratio, υ = 0.3, Mass density, ρ = 250 kg, Beam thickness, t = 0.1 m, Beam length, l = 280 m, Beam width, b = 40 m.

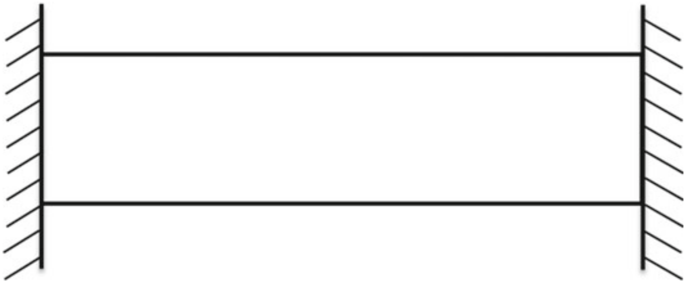

Fig. 2 Design domain for clamped beam of mesh size 280 × 40

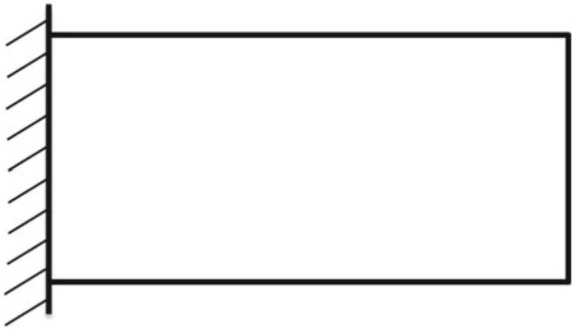

Fig. 3 Design domain for cantilever beam of mesh size 150 × 75

Figure 3 shows a cantilever beam, for which we minimize the weight using topology optimization of a mesh size 150 × 75. The material properties are, Young's Modulus, E = 25e7 Pa, Poisson's ratio, $\upsilon = 0.3$, Mass density, $\rho = 250$ kg, Beam thickness, t = 0.1 m, Beam length, l = 150 m, Beam width, b = 75 m.

3 Results

3.1 Example 1: Clamped Beam

We solve Eq. (1) for different specified frequency values, starts from 10 to 270 rad/s with a range of 20 rad/s. Table 1 lists the optimal volume fractions and the corresponding converged fundamental natural frequency of the optimal topologies for the above-mentioned cases. From Table 1, it is observed that in some cases, the converged natural frequency is more than the specified frequency, and in others, it is less than the specified frequency. Later case violates the frequency constraint of optimization problem. For the specified frequencies (in rad/s) of 10, 30, 50, 70, 90, 110, 130, 150, 170, 190, 210 the optimization converges, whereas for the specified frequencies (in rad/s) of 230, 250 and 270 the optimization diverges because it violates the lower bound constraint on Eigen frequency. From the above all solutions, we choose the solution obtained for a specified natural frequency of 170 rad/s as design because the objective is to minimize the volume and it has the minimum volume fraction value.

Table 1 Optimal volume fraction and converged fundamental natural frequencies for different specified frequencies starts from 10 to 270 rad/s with a range of 20 rad/s for a clamped beam

Optimal volume Fraction 'V'	Converged fundamental natural frequency 'ω_1' (in rad/s)	Specified fundamental natural frequency 'ω_g' (in rad/s)
0.6593	214.5122	10
0.6495	214.1080	30
0.6684	219.9565	50
0.6198	214.8531	70
0.6581	215.3216	90
0.6286	215.8456	110
0.6531	215.1164	130
0.6159	219.4630	150
0.6152	219.9838	170
0.6384	217.7039	190
0.6441	216.1638	210
0.6446	215.9406	230
0.6421	216.3036	250
0.6374	217.9464	270

Fig. 4 Optimal topology of clamped beam with an optimized volume fraction of 0.6152 with natural frequency of 219.9838 rad/s for a specified frequency of 170 rad/s

Figure 4 shows the optimal topology obtained for a specified fundamental natural frequency of 170 rad/s and the convergence history of both volume fraction and converged fundamental natural frequency is shown in Fig. 5.

3.2 Example 2: Cantilever Beam

We solve Eq. (1) for different specified frequency values, starts from 40 to 520 rad/s with a range of 40 rad/s for the cantilever domain. Table 2 lists the optimal volume fractions and the corresponding converged fundamental natural frequency of the optimal topologies for the above-mentioned cases. From the above all solutions, we choose the solution obtained for a specified natural frequency of 360 rad/s as design because the objective is to minimize the volume and it has the minimum

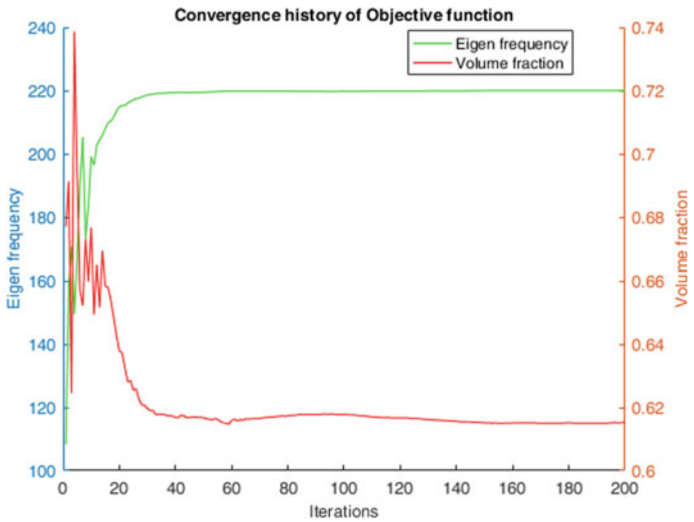

Fig. 5 Convergence history of Eigen frequency and volume fraction for 200 iterations for clamped beam

Table 2 Optimal volume fraction and converged fundamental natural frequencies for different specified frequencies starts from 40 to 520 rad/s with a range of 40 rad/s for a cantilever beam

Optimal volume Fraction 'V'	Converged fundamental natural frequency 'ω_1' (in rad/s)	Specified fundamental natural frequency 'ω_g' (in rad/s)
0.2947	525.1562	40
0.2845	532.9132	80
0.2797	502.6193	120
0.2884	520.9923	160
0.291	512.3198	200
0.2944	535.881	240
0.2811	540.9427	280
0.2804	536.5597	320
0.2784	542.1122	360
0.2919	502.2156	400
0.2791	519.8919	440
0.2832	531.7349	480
0.2849	509.5215	520

volume fraction value. Figure 6 shows the optimal topology obtained for a specified fundamental natural frequency of 360 rad/s and the convergence history of both volume fraction and converged fundamental natural frequency is shown in Fig. 7.

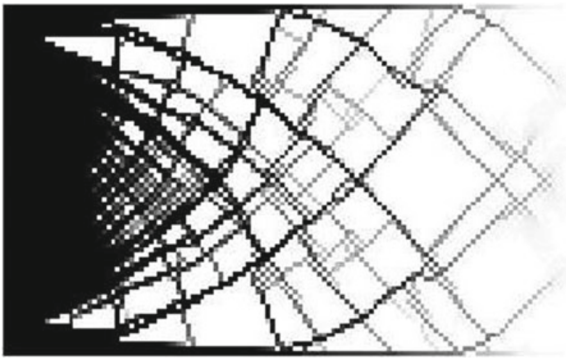

Fig. 6 Optimal topology of cantilever beam with an optimized volume fraction of 0.2784 with natural frequency of 542.1122 rad/s for a specified frequency of 360 rad/s

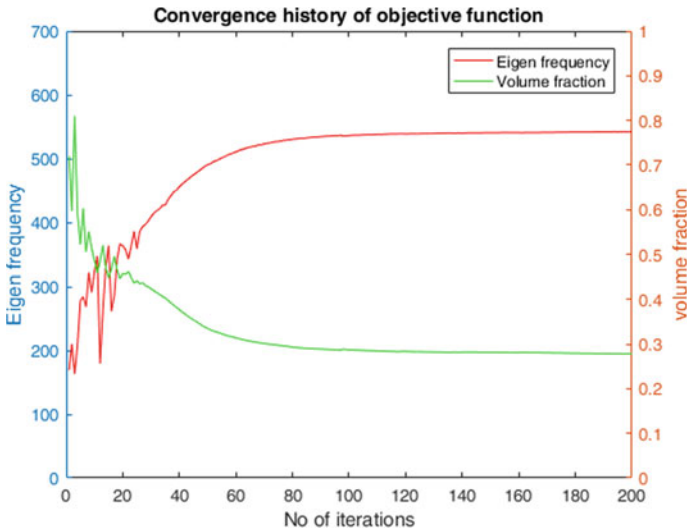

Fig. 7 Convergence history of Eigen frequency and volume fraction for 200 iterations for cantilever beam

4 Conclusions

This paper presents a new method to avoid resonance while designing the structures using topology optimization of weight minimization problem. A clamped beam of mesh size 280 × 40 and a cantilever beam of mesh size 150 × 75 solved by using this proposed method. The optimal volume fraction is 0.6152 and the converged fundamental natural frequency is 219.9838 rad/s found for a specified natural frequency of 170 rad/s for the clamped beam. The optimal volume fraction for cantilever beam is 0.2784. The converged fundamental natural frequency is 542.1122 rad/s found for a specified natural frequency of 360 rad/s.

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