# Breathing Crack Detection Using Linear Components and Their Physical Insight

**J. Prawin and A. Ramamohan Rao**

**Abstract** Identification of fatigue-breathing cracks at the time of initiation that develop in structures under repetitive loading is desirable for successful implementation of any health monitoring system. The presence of higher order harmonics and sidebands apart from the fundamental excitation harmonic in the Fourier power spectrum subjected to harmonic excitation are widely used as breathing crack damage indicators. The majority of the existing breathing crack detection and localization techniques use the amplitudes of the super harmonics/modulation components obtained spatially across the structure from the current and healthy states. In the present work, instead of using nonlinear harmonic components for breathing crack detection, we exploit the decrease in the Fourier power spectrum amplitude of the linear components due to the transfer of energy from linear components to nonlinear harmonic components in the presence of breathing crack. Two new indices quantifying the ratio of energy variations in linear and nonlinear components have been proposed to highlight the effectiveness of the proposed concept. Numerical simulation and experimental investigations established the fact that the energy variation in linear components between varied crack depths shows significantly higher sensitivity than the energy variation due to nonlinear harmonic components.

**Keywords** Breathing crack · Nonlinear · Intermodulation · Super-harmonics · Bitone

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## 1 Introduction

Detection of incipient damage at the earliest possible stage is desirable for successful implementation of any health monitoring system. The fatigue-breathing crack is one of the most common damages that occur in civil structures due to dynamic loading. These fatigue cracks exhibit repetitive crack open-close breathing like phenomenon and induce bilinear rather nonlinear behaviour. The bilinear dynamic behaviour of the structure with breathing cracks makes them difficult to detect when compared to their counterpart open cracks, using the traditional vibration-based damage detection techniques. Therefore, the detection, localization and severity quantification of fatigue cracks using nonlinear sensitive features poses a greater challenge and new damage diagnostic techniques are being developed. [1–4]. When the structure is healthy (i.e. intact and linear), and subjected to harmonic loading with excitation frequency $\omega_{exc}$, it vibrates at same excitation frequency. In contrast, the cracked response contains not only input excitation frequency $\omega_{exc}$, but also their super harmonics ($n\omega_{exc}$, where n is an integer). Similarly, when the system is excited simultaneously with two different frequencies $\omega_{prob}$ and $\omega_{pump}$ (bitone: $\omega_{prob} > \omega_{pump}$), the cracked response contains not only input frequencies but also their harmonics and modulation. These modulations lead to the presence of side bands ($\omega_{prob} \pm n\omega_{pump}$,). The presence of super harmonics (i.e. higher order harmonics) and sidebands apart from fundamental excitation harmonics confirms the bilinear behaviour of the structure due to breathing crack.

The majority of the existing breathing crack detection and localization techniques are based on the comparison of the amplitudes of the first one or two super harmonics and sidebands components obtained spatially across the structure from the current and healthy states [1–4]. Further, these nonlinear harmonics (i.e. higher order or super harmonics) rather depends heavily on the spatial location and the depth of the crack, excitation frequency and as well the amount of damping. However, extracting these super harmonics and sidebands may mislead the damage diagnostic process due to highly corrupted noisy measurements and environmental variability. It may be noted here that the Fourier power spectrum amplitude of the linear components is of the higher order of magnitude than that of the higher order harmonics and sideband components. Hence, alternatively in this paper, instead of using nonlinear harmonic components for breathing crack detection, we exploit the decrease in Fourier power spectrum amplitude of the linear components due to the transfer of energy from linear components to nonlinear harmonic components in the presence of breathing crack.

## 2 Breathing Crack Detection Using Linear Harmonic Components

The uniqueness of the present study is based on the identification of breathing crack using the nonlinearity reflected on linear responses rather than the nonlinear harmonic/modulation components being used popularly. The presence of breathing crack causes reduction in Fourier power spectrum amplitude of the linear responses and increase in Fourier power spectrum amplitude of higher order harmonics and sidebands. Therefore, there occurs the transfer of energy from the fundamental linear excitation harmonic component to higher order superharmonic and sideband harmonics components due to nonlinear behaviour of the structure in the presence of breathing crack. Therefore, the energy decrease in the linear component response is definitely larger than the energy increase in the super harmonic and sideband components generated due to nonlinearity.

First, a damage index proposed based only on the linear components of the healthy and cracked structure response is defined as

$$DI_{\text{linear}} = \sum_{i=1}^{n} \left| 1 - \frac{A_{i,\text{linear}}^{\text{cracked}}}{A_{i,\text{linear}}^{\text{healthy}}} \right| / n; \tag{1}$$

where $A_{i,\text{linear}}^{\text{cracked}}$ is the response amplitude of linear harmonics of $\text{i}^{\text{th}}$ degree of freedom of the structure with breathing crack and n indicates the total number of degree of freedom. Similarly $A_{i,\text{linear}}^{\text{healthy}}$ corresponds to the healthy structure. The value of the damage index lies in the range $0 \leq DI_{\text{linear}} \leq\approx 1$. The value of $DI_{\text{linear}}$ corresponding to 0 and close to 1 indicate the healthy and the completely damage state of the structure. The value of $DI_{\text{linear}}$ increases with increase in crack depth. It can be easily explained by the fact that due to the generation of nonlinear harmonics or modulations, there will be reduction in the amplitude/energy of linear components in the case of cracked structure when compared with the healthy structure. The energy reduction occurs due to transfer of energy from linear component to nonlinear harmonic/modulation components in the case of structure with breathing crack. It may be noted here that the amplitude of linear harmonics includes both amplitudes of probing and pumping frequencies in the case of structure subject to simultaneous excitation of two different frequencies.

In order to demonstrate that the linear components are more sensitive to breathing crack than nonlinear harmonics or intermodulation components, a damage index based on the ratio of the energy of amplitude of nonlinear harmonics to linear harmonics, widely used by the researchers is used [1–4]. It is defined as

$$DI_{\text{harmonics/modulations}} = \sum_{i=1}^{n} \frac{A_{i,\text{nonlinear}}^{\text{cracked}}}{A_{i,\text{linear}}^{\text{cracked}}} / n. \tag{2}$$

## 3 Numerical Investigations

The steel cantilever beam, considered by popular researchers earlier [1, 4] for identification of breathing crack in engineering structures is used in the present work. We consider the same dimensions as the length of the beam as 0.3 m and the area of the beam as 2.5e−4 $m^2$. The finite element discretization of the cantilever beam along with the node numbers and degrees of freedom per node is shown in Fig. 1.

Standard 1D Euler-Bernoulli beam elements are employed to model the beam and shear deformation is neglected. There are two nodes per element and each node has three degrees of freedom: longitudinal displacement, translational displacement and bending rotation as shown in Fig. 1. The bilinear behaviour of the breathing crack is shown in Fig. 2. Rayleigh damping is considered. The beam is idealized with 10 elements. The fundamental frequencies of the uncracked beam are found to be 92.172, 577.650, 1617.8 and 3172.4 Hz.

The damaged element stiffness matrix of a plane beam element, with the breathing crack simulated by the Heavy side step function is given below

$$K_d = K_u - H(\theta_i - \theta_j)K_c : \begin{cases} H(\theta_i - \theta_j) = 1, \theta_i > \theta_j \\ H(\theta_i - \theta_j) = 0, \theta_i < \theta_j \end{cases} \tag{3}$$

$$K_c = \frac{\mu E}{l}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ & \frac{12I_u}{l^2} & \frac{6I_u}{l} & 0 & -\frac{12I_u}{l^2} & \frac{6I_u}{l} \\ & & 4I_u & 0 & -\frac{6I_u}{l} & 2I_u \\ & & & 0 & 0 & 0 \\ & sym & & & \frac{12I_u}{l^2} & -\frac{6I_u}{l} \\ & & & & & 4I_u \end{bmatrix}, \mu = \frac{(I_u - I_d)}{I_u} \tag{4}$$

where E, L, K, I, H indicates the Young's Modulus, span, element stiffness, moment of inertia and Heaviside step function. The subscripts 'u' and 'd' indicate the

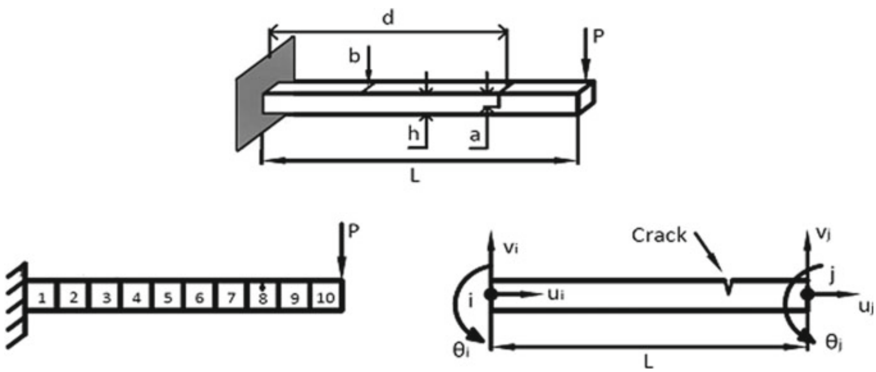



**Fig. 1** Finite element model of the cracked Cantilever beam

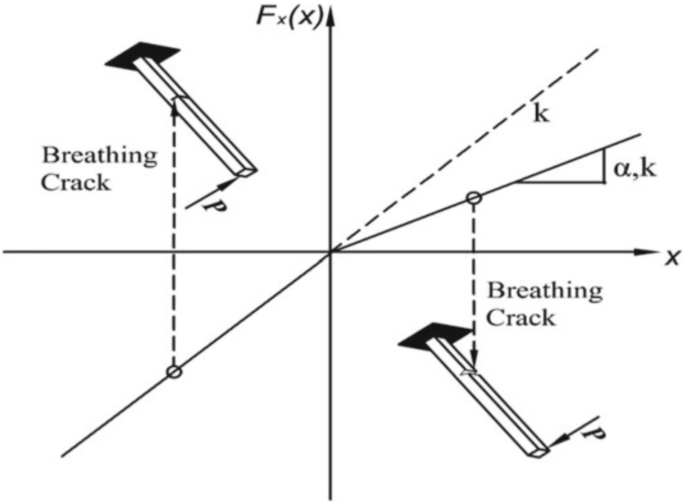


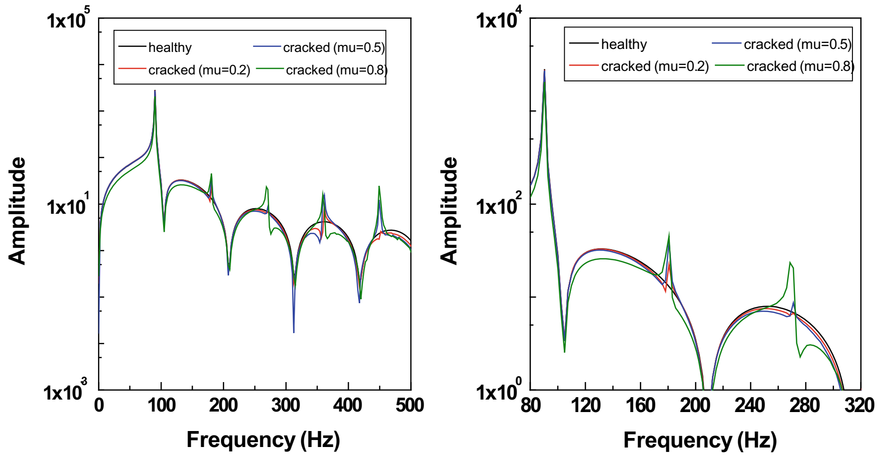

**Fig. 2** Bilinear behaviour of breathing crack

undamaged and damaged states of the structure. The index '$\mu$' represents the non-dimensional flexural damage [3, 5]. The acceleration time history responses before post-processing are polluted with 15% noise level (i.e. SNR = 25) to test the applicability of the proposed approach in the presence of noise.

The system is excited at free end with 90 Hz excitation and as well as bitone with frequencies of 90 and 1710 Hz. The breathing crack is induced in element no.5 and three different breathing crack depths considered for investigation are about 7%, 20% and 41% of the overall depth of the beam (i.e. with non-dimensional flexural damage $\mu = 0.2$, 0.5and 0.8). Three different crack depths have been considered to demonstrate the sensitivity of the above proposed two damage indices.

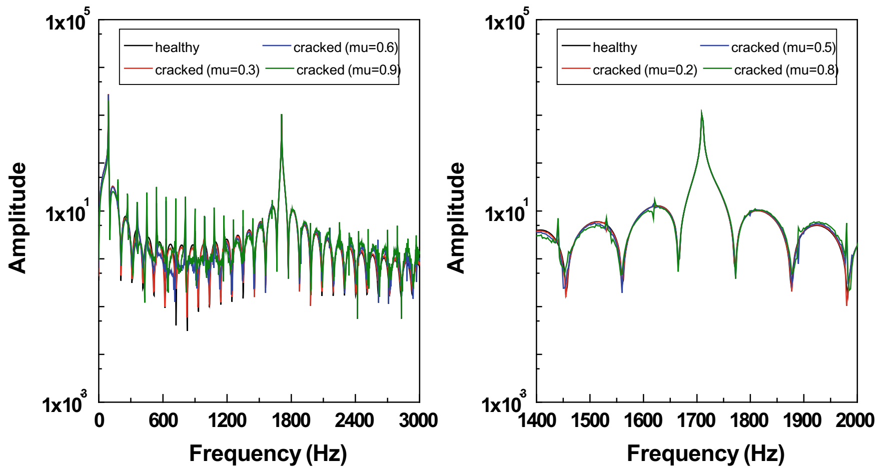
The free end power spectrum response of the healthy and damaged beam subjected to harmonic excitation of 90 Hz is shown in Fig. 3. Similarly, the Fourier Power spectrum responses of the structure measured at free end when excited simultaneously with two different frequencies of 90 and 1710 Hz is shown in Fig. 4. While Subplots (a) of Figs. 3 and 4 indicate the complete Fourier power spectrum and (b) indicate the zoomed power spectrum. The following are the observations

1. The Fourier power spectrum shows peaks with high amplitude only at excitation frequency (i.e. 90 Hz in case of harmonic excitation with single frequency and 90 Hz and 1710 with respect to simultaneous excitation of structure with two frequencies) for the healthy beam.
2. The Fourier power spectrum of the cracked structure, shown in Fig. 3 shows peaks at higher order harmonics of 90 Hz (i.e. 180, 270, 360 Hz and so on apart from excitation frequency.
3. The Fourier power spectrum of the response shows more super harmonics with the increase in crack depth




*(a) Complete spectrum* *(b) zoomed spectrum*

**Fig. 3** Power Spectrum— single tone—numerical cantilever beam




*(a) Complete spectrum* *(b) zoomed spectrum*

**Fig. 4** PowerSpectrum—bitone— numerical cantilever beam

4. The Fourier power amplitude of super harmonics increases and amplitude of linear harmonics decreases with increase in crack depth.
5. Overall, irrespective of any crack depth, the amplitude of higher order harmonics is of less order in magnitude than that of the linear harmonic component. Further, the Fourier power spectrum amplitude of higher order harmonics is of very low

**Table 1** Energy Variation Indices

| Harmonic excitation | Single tone | | | Bitone | | |
|---|---|---|---|---|---|---|
| Crack depth ($\mu$) | 0.2 | 0.5 | 0.8 | 0.2 | 0.5 | 0.8 |
| .. | 0.0371 | 0.0664 | 0.275 | 0.0528 | 0.0802 | 0.314 |
| $DI_{\text{harmonics/modulations}}$ | 0.0128 | 0.0255 | 0.0576 | 0.0026 | 0.0082 | 0.0175 |

magnitude with respect to the fundamental harmonics even for higher crack depth, which is evident from Fig. 3.

6. The Fourier power spectrum of the cracked structure shown in Fig. 4. shows high amplitudes at two excitation frequencies (i.e. 90 and 1710 Hz) an low amplitude at its corresponding sidebands frequencies (both left and right sidebands) of probing frequency (i.e. 1800, 1890, 1980, 1620, 1530 Hz and so on).
7. The Fourier power spectrum amplitude of both left and sidebands are also of lower magnitude when compared to the Fourier power spectrum amplitude of probing frequency.
8. The presence of super harmonics (i.e. higher order harmonics) and sidebands apart from fundamental excitation harmonics confirms the bilinear behaviour of the structure due to breathing crack.

The frequency domain responses shown in Figs. 3 and 4 indicate that there is amplitude reduction in linear components and amplitude increase in nonlinear harmonic/modulation components with increase in crack depth for both single as well as bitone excitation. The results of the damage indices are presented in Table 1. For the crack depth of 41%, the energy variation in the case of harmonic excitation with single frequency is found to be 27.5% for damage index based on linear components alone, while it is around 5.76% for damage index based on nonlinear harmonics. Similarly, the energy variations corresponding to bitone harmonic excitation are found to be 31.4 and 1.75% for damage index based on linear harmonics and nonlinear harmonics. This clearly demonstrates that the damage index based on reduction in energy of the linear components due to breathing crack is more sensitive than the increase in energy of the nonlinear harmonics/modulations. It can be concluded that the bitone harmonic excitation can be preferred due to their higher sensitivity in contrast to the harmonic excitation with single frequency.

## 4 Experimental Validation

A cantilever aluminium beam with and without breathing crack have been experimentally validated to demonstrate the proposed concept of breathing crack identification using linear harmonic components apart from the numerical investigation. Two aluminium alloy beams (i.e. two, three or many pieces) are bonded to form a single beam in contrast to the single unified beam. Bonding is carried out using Araldite epoxy adhesive. There may be two or three plates at the top depending upon the number of crack, where in the faces of the plates are in contact but not bonded to induce breathing behaviour. The faces of the plate in contact helps in simulating opening and closing behaviour of the beam under dynamic loading. Similarly, the healthy beam is formed by only bonding two plates (i.e. bottom and top) instead of a single plate to have equivalent uncracked beam as same as that of cracked beam. The experimental set up in the present work has been followed as per the guidelines by Prime et al. [6] and Douka et al. [7] for validation of their damage diagnostic techniques. The schematic illustration of the instrumentation set up of the PZT accelerometers (PCB393B04) placed longitudinally across the test cantilever beam and experimental specimen is presented in Fig. 5a. The beam is tightly clamped using C- clamp with the steel test bench for cantilever action as shown in Fig. 5b. The Data acquisition and spectrum analyzer used during experimentation is shown in Fig. 5c. The spacing of the accelerometers is given in Fig. 6. The cross section of the beam is 25.4 mm × 9.505 mm and the span is 1000 mm. The distance of the various aluminium plate pieces on the top dictates the spatial location and number of the crack. In the present test setup, the crack is located at 10 mm from the fixed end of the cantilever beam. The thickness of the top plate is 3.175 mm, which results in


*(a) Test cantilever Beam setup*

*(b) Clamped end and crack*

*(c) Data acquisition system*

**Fig. 5** Experimental cantilever beam

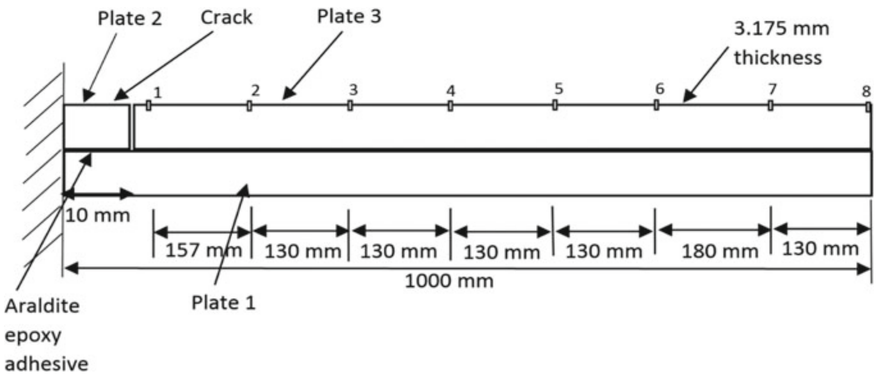
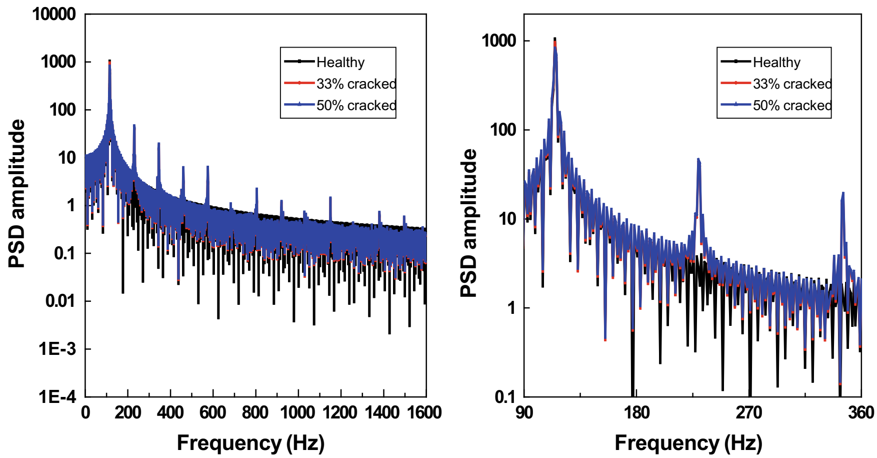



**Fig. 6** Instrumentation Setup

crack depth equal to 33% of total depth of the beam in the case of the first cracked specimen.

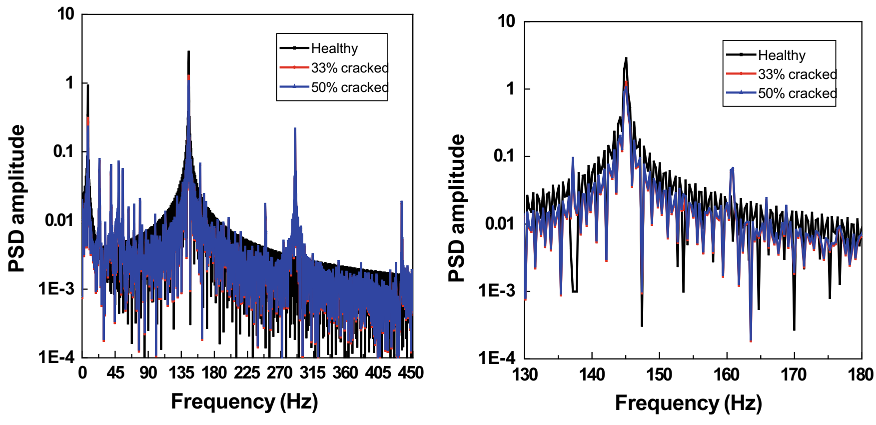
The second cracked specimen is formed similarly with both top and bottom plates having thickness 4.7525 mm each. This results in crack depth of 50% of total depth of the beam. The other properties of the beam and crack location are same as that of the previous cracked specimen. The natural frequencies (i.e. first four) of the healthy experimental beam are 7.782, 48.77, 136.61, and 267.85 Hz.

The beams with and without breathing crack are subjected to harmonic excitation of 115 Hz and as well simultaneously excited with two frequencies of 8 Hz and 145 Hz. The free end power spectrum response of the healthy and the cracked beam subjected to harmonic excitation of 100 Hz is shown in Fig. 7. The results




(a) Complete spectrum (b) zoomed spectrum

**Fig. 7** Power Spectrum—single tone—experimental cantilever beam




(a) Complete spectrum (b) zoomed spectrum

**Fig. 8** Power Spectrum—bitone—experimental cantilever beam

corresponding to responses measured under harmonic excitation of two frequencies of 8 and 145 Hz is shown in Fig. 8. The response of the healthy beam shows a single maximum peak at 115 Hz only. While the plot related to damaged beam, (i.e. both 33% and 50% cracked beam) shows peaks at 115, 230, 345 Hz and so on. The cracked beam exhibits more number of higher order harmonics for higher crack depth beam. Further, the amplitude of superharmonics increases and amplitude of linear harmonics decreases with increase in crack depth. The energy of higher order harmonics is very low with respect to the linear harmonic even in the case of beam with higher crack depth. The Fourier power spectrum reveals two peaks at 8 and 145 Hz for the healthy beam which is evident from Fig. 8. While the plot related to the cracked beam shows peaks at sidebands ($\omega_{prob} \pm n\omega_{pump}$) apart from the peaks at each individual excitation frequencies. Similar to single tone harmonic excitation case, the Fourier power spectrum amplitude of both left and right sidebands is of very low magnitude when compared to the Fourier power spectrum amplitude of probing frequency excitation component.

The results of the damage indices estimated for the cracked experimental specimens are given in Table 2. Table 2 establishes the fact that the energy variation in linear components (i.e.$DI_{\text{linear}}$) between varied crack depths shows significantly higher sensitivity than energy variation due to nonlinear components

**Table 2** Energy Variation Indices

| Harmonic excitation | Single tone | | Bitone | |
|---|---|---|---|---|
| Crack depth ($\mu$) | 0.2 | 0.5 | 0.2 | 0.5 |
| $DI_{\text{linear}}$ | 0.03688 | 0.122 | 0.062 | 0.148 |
| $DI_{\text{harmonics/modulations}}$ | 0.00824 | 0.0218 | 0.00104 | 0.00785 |

(i.e. $DI_{\text{harmonics/modulations}}$). This clearly emphasis the fact that the damage index based on reduction in energy of linear components due to breathing crack is more sensitive than increase in energy of nonlinear harmonics/modulations.

## 5 Conclusions

A unique breathing crack damage diagnostic technique exploiting the possibility of utilizing linear components instead of conventional nonlinear components is developed. Numerical investigations on a cantilever beam with breathing crack near the centre of the beam with varied crack depth are carried out to understand the transfer of energy from linear components to super harmonics and sideband components in the presence of nonlinearity due to breathing crack. Two damage indices; one based on linear components and the other considering nonlinear harmonics and modulations/sidebands, have been presented and compared to investigate the sensitivity of the response components for fatigue-breathing crack identification. This is further validated through experimental investigations. Both the numerical and experimental investigations conclude that there occurs transfer of energy from the fundamental linear excitation harmonic components to nonlinear harmonic components induced by the breathing crack. The decrease in Fourier power spectrum amplitude in the linear components due to energy transfer is highly sensitive than the Fourier power spectrum amplitude increase in the case of higher order harmonics and sideband nonlinear harmonic components. Therefore, the linear components can be effectively employed for damage diagnosis of breathing crack. Further, this eliminates the complex process of reliable extraction of nonlinear harmonics under harsh noisy environment.

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