

# Modification and Modeling of Experiments with Bi-directional Loading on Reinforced Concrete Columns



Subhadip Naskar, Sandip Das, and Hemant B. Kaushik

**Abstract** Capacity evaluation of bi-directionally loaded column is important not only for the performance-based seismic design of structures but also for the estimation of structural damage. In this paper, an experimental study has been carried out on full-scale columns with different axial stress ratios, followed by the development of an analytical model to predict the lateral load response of the column under bi-directional loading after taking care of the effect of the functional interactions between the two loading actuators and the column specimen. These interactions, if not taken into account, result in apparent underestimation of ultimate strength and overestimation of maximum displacement capacity of the test specimen, thereby demanding unnecessary changes in model parameters for the purpose of calibration. It is also found that the analytical model accounting for the aforementioned functional interactions leads to a more realistic and different dynamic structural response from that obtained using the analytical model ignoring the interactions.

**Keywords** Bi-directional loading on RC columns · Capacity evaluation · Pseudo-static cyclic load · Cyclic degradation of strength · Seismic damage index

## 1 Introduction

Performance-based seismic design of any civil structure requires capacity parameters, such as lateral displacement capacity and lateral strength capacity. Estimation of structural damage (or any other response-dependent performance parameter) due to seismic ground motions also needs the capacity evaluation of the structures. Therefore, since the last few decades, many researchers have done experiments on

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S. Naskar · S. Das (✉) · H. B. Kaushik  
Civil Engineering Department, IIT Guwahati, Guwahati 781039, Assam, India  
e-mail: [sandip.das@iitg.ac.in](mailto:sandip.das@iitg.ac.in)

S. Naskar  
e-mail: [s.naskar@iitg.ac.in](mailto:s.naskar@iitg.ac.in)

H. B. Kaushik  
e-mail: [hemantbk@iitg.ac.in](mailto:hemantbk@iitg.ac.in)

cantilever column subjected to uniaxial or biaxial lateral force under the action of constant or varying axial load to study the cyclic behavior, ultimate strength, and ultimate displacement of RC members. Vertical cantilever specimen fixed at a concrete base [1, 2], strong beam-weak column sub-assemblages [3–5], was used to study the effect of uniaxial flexural load in presence of constant axial load. But the inelastic response of column is greatly influenced by the variation of axial forces during the cyclic response, depending on the relative magnitude of lateral and gravity loads and the proportioning of the force members of the frame. Therefore, experiments were carried out under different axial load conditions varying with respect to transverse displacement or transverse load [6–10]. Later to get a more realistic prediction of response of RC structural members during two-dimensional seismic excitations, many researchers have studied the behavior of cantilever column [11–21] and beam-column sub-assemblages [22–25], subjected to biaxial flexure and constant axial load. Very few researchers carried out experimental studies of structural response due to biaxial lateral loading condition with varying axial force [26–29]. These experimental studies have been used further for validation of many different types of numerical modeling such as material level modeling at the point-by-point basis, member-by-member type of modeling considering one-to-one correspondence between elements of the model and members of the structure, relatively simple few degrees-of-freedom models. A detailed report on such validation of different modeling can be found in the state-of-the-art report on RC frames under earthquake loading [30]. It is understood that under large deformation or high axial load ratio the subtle change in loading directions (of the actuators) might affect the load-deformation behavior and warrant some interaction among the actuators and the specimen. It is important to decouple such interaction effect by means of some essential kinematic corrections before developing analytical models from the experimental data. However, no such modifications of the experimental results have been reported for bi-directionally loaded cantilever column subjected to large lateral deformation or high axial load ratio.

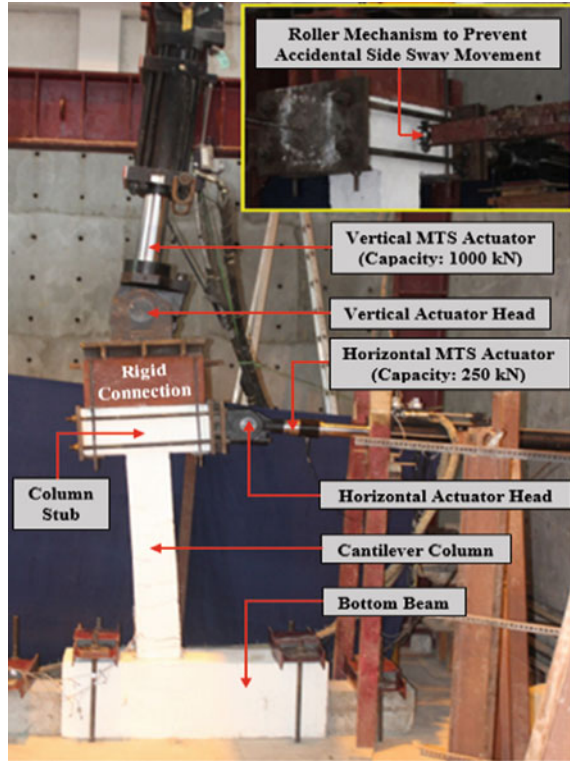
In the present study, different nonlinear monotonic and pseudo-static cyclic experiments on cantilever column specimens under the action of different constant axial stress ratios are conducted. A methodology of kinematic correction is proposed to account for the aforementioned interactions before developing an analytical model. Further, the effects of such interactions on the analytical model and the structural response thereof, including structural damage, under the action of seismic motion are also studied.

## 2 Experimental Details

### 2.1 *Experimental Setup*

Monotonic and cyclic experiments on a full-scale cantilever column have been carried out, by simulating the lateral load under the action of different levels of axial load

**Fig. 1** Detailed view of the typical experimental setup



on the specimen. A displacement controlled MTS actuator of 250 kN capacity and another displacement controlled MTS actuator of 1000 kN capacity has been used to simulate the horizontal force and vertical axial force condition, respectively (Fig. 1).

## 2.2 Loading Characteristics and Material Details

Full-scale cantilever column has been used to study the behavior of column of moment resisting frame to reduce both cost and time of carrying out the experiments, as the deflected shape of a cantilever column, subjected to a transverse load, resembles the same of both ends fixed column considering the fixed end to the point of contraflexure. A vertical axial load is given to the free end of the cantilever specimen to simulate the effect of gravity load. Table 1 shows the different loading characteristics of monotonic and cyclic experiments. Mix design of concrete for the test specimens has been carried out as per IS:10262-2009. The yield stress, ultimate stress, modulus of elasticity of longitudinal reinforcement of the columns have been found as 535.28 MPa, 641.40 MPa, and 215840.32 MPa, respectively, from test results.

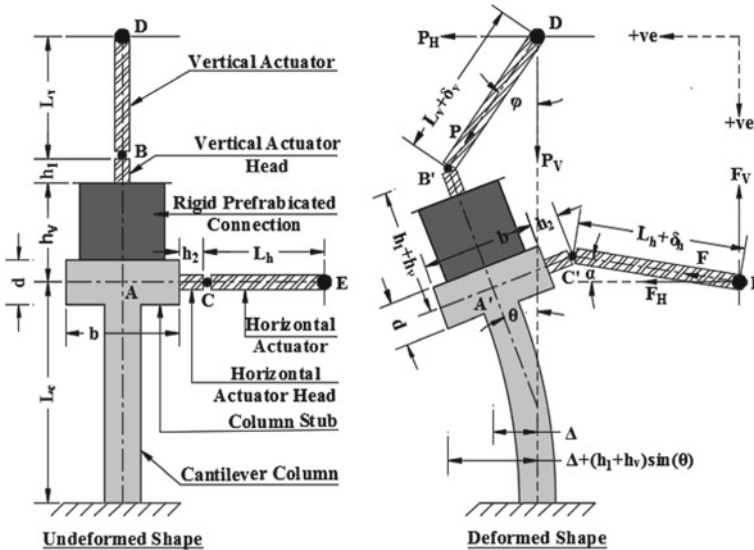
**Table 1** Loading characteristics and characteristic strength of test specimens

Test no.	Nomenclature	Loading type	Axial stress ratio	Characteristic strength, $f_{ck}$ (MPa)	Axial load, $P_u$ (kN)
1	M1	Monotonic	0.05	32	100.0
2	M2	Monotonic	0.20	24	300.0
3	C1	Cyclic	0.05	32	100.0
4	C2	Cyclic	0.10	32	200.0

### 3 Modification of Experimental Results

During the experiment, both horizontal and vertical actuators make an angle with the horizontal or vertical axes (see Fig. 2), due to which actual horizontal and vertical forces become the sum of horizontal and vertical components of both actuators, respectively. Similarly, the horizontal component of horizontal actuator should be used as the horizontal displacement of the cantilever column. Also, an extra moment, generated due to the inclined position of the actuators, is responsible for additional deformation of the cantilever column.

Therefore, some rigorous correction, depending on the geometry of the experimental setup, kinematic constraint, and basic equilibrium conditions at each time



**Fig. 2** Schematic diagram of undeformed and deformed test specimen

step, is required to estimate the exact value of horizontal force, vertical force, horizontal displacement, and nullify the effect of the additional moment before using these experimental values further.

### 3.1 Correction Due to Vertical Actuator

In Fig. 2, **A** represents the column tip, **B** represents the hinge joint between the vertical actuator and rigid prefabricated connection of the column before deformation and **D** represents the hinge joint between the vertical actuator and the strong wall. **A'** and **B'** represent the location of **A** and **B** after deformation. From the geometry, moment due to inclined vertical actuator ( $M_V$ ), force components of vertical actuator ( $P_H$ ,  $P_V$ ), and the actual horizontal displacement ( $\Delta$ ) can be expressed as

$$M_V = P(h_1 + h_v) \sin(\theta + \phi); \quad P_H = P \cdot \sin \phi; \quad P_V = P \cdot \cos \phi; \quad \Delta = \delta_h \cdot \cos \alpha \quad (1)$$

$$\phi = \sin^{-1} \left\{ \frac{\Delta + (h_1 + h_v) \sin \theta}{L_v + \delta_v} \right\} \quad (2)$$

where  $h_1$  = length of the vertical actuator head,  $h_v$  = distance between the center of column stub and bottom end of the vertical actuator,  $P$  = force reading of the vertical actuator,  $\delta_h$ , and  $\delta_v$  = displacement reading of the horizontal and the vertical actuator, respectively;  $L_v$  = initial length of the vertical actuator before test,  $\theta$  = tip rotation of the column, and  $\phi$  = angle between the vertical actuator and the vertical axis.

### 3.2 Correction Due to Horizontal Actuator

In Fig. 2, **C** represents the hinge joint between the horizontal actuator and rigid prefabricated connection of column before deformation and **E** represent the hinge joint between the horizontal actuator and the strong wall. **C'** represents the location of **C** after deformation. Similarly, the moment due to the inclined horizontal actuator ( $M_H$ ) and force components of the horizontal actuator ( $F_H$ ,  $F_V$ ) can be expressed as given below:

$$M_H = F(h_2 + b/2) \sin(\theta + \alpha); \quad F_H = F \cdot \cos \alpha; \quad F_V = F \cdot \sin \alpha \quad (3)$$

$$\alpha = \sin^{-1} \left\{ \left( \frac{h_2 + b/2}{L_h + \delta_h} \right) \sin \theta \right\} \quad (4)$$

where  $h_2$  = length of the horizontal actuator head,  $b$  = width of the column stub,  $F$  = force reading of the horizontal actuator,  $L_h$  = initial length of the horizontal actuator before test, and  $\alpha$  = angle between the horizontal actuator and the horizontal axis. Angle parameters  $\alpha$  and  $\phi$  should be known to estimate actual horizontal force and displacement components. But they are a function of unknown tip rotation  $\theta$ , which can be expressed as the sum of elastic tip rotation ( $\theta_e$ ) and plastic tip rotation ( $\theta_p$ ). Further,  $\theta_e$  is the resultant effect of both horizontal force ( $H = P_H + F_H$ ) and moment ( $M = M_V + M_H$ ) acting at the tip of the column. Therefore,

$$\theta = \theta_e + \theta_p = (\theta_e^H + \theta_e^M) + \theta_p; \quad \theta_e^H = \frac{HL_c^2}{2E_cI_e}; \quad \theta_e^M = \frac{ML_c}{E_cI_e} \quad (5)$$

where  $L_c$  = length of the column,  $E_c$  = elastic modulus of concrete and  $I_e$  = effective moment of inertia of the column. Now,  $\Delta$  can be also be expressed in terms of the elastic portion due to horizontal force ( $\Delta_e^H$ ), moment ( $\Delta_e^M$ ), and the plastic portion ( $\Delta_p$ ) as

$$\Delta = \Delta_e + \Delta_p = (\Delta_e^H + \Delta_e^M) + \Delta_p; \quad \Delta_e^H = \frac{HL_c^3}{3E_cI_e}; \quad \Delta_e^M = \frac{ML_c^2}{2E_cI_e} \quad (6)$$

Considering a new factor  $c = \theta_e L_c / \Delta_e$ , it can be obtained from Eqs. (5) and (6):

$$c = \frac{3L_c + 6\beta}{2L_c + 3\beta}; \quad \beta = \frac{M}{H} \quad (7)$$

Since no additional tip moment is encountered just at the start of the experiment, the initial value of factor  $c$  can be taken as  $c_0 = 1.5$ .

### 3.3 System Kinematics

From the geometry of the experimental setup, the following expressions can be obtained:

$$\Delta_e = \delta_h^e \cdot \cos \alpha; \quad \cos \alpha = \frac{\theta_e L_c}{c \delta_h^e}; \quad \Delta_p = \frac{\delta_h^p \theta_e L_c}{c \delta_h^e} \quad (8)$$

where  $\delta_h^e$  and  $\delta_h^p$  = elastic and plastic portion of  $\delta_h$ , respectively. But, initially only total displacement of the horizontal actuator ( $\delta_h$ ) is known, so an initial ratio  $r = \delta_h^e / \delta_h$  is assumed. The assumed value of  $r$  has been cross-checked with the estimated value by established elastic theory and iterated accordingly until the exact value of  $r$  has been found. Therefore,

$$\Delta_p = \frac{(1-r)\theta_e L_c}{cr}; \theta_p = \frac{\Delta_p}{L_0} = \frac{(1-r)\theta_e L_c}{cr(L_c - L_p)} \quad (9)$$

where  $L_0$  = length of the test specimen participating in plastic deformation and  $L_p$  = the plastic hinge length [31], as given below:

$$L_p = 0.08z + 0.022d_b f_y \quad (10)$$

where  $z$  = the distance from the critical section to the point of contraflexure,  $d_b$  = the largest dia. of the longitudinal reinforcement and  $f_y$  = the yield strength of steel. Since  $\delta_h$  comprises only elastic portion just at the start of the experiment, the initial value of  $r$  can be taken as  $r_0 = 1.0$ . Now, from Eqs. (4) and (8), it can be found that

$$a_1 \sin^2(a_2\theta_e) + a_3\theta_e^2 = 1 \quad (11)$$

$$a_1 = \left(\frac{h_2 + b/2}{L_h + \delta_h}\right)^2; \quad a_2 = 1 + \frac{(1-r)L_c}{cr(L_c - L_p)}; \quad a_3 = \left(\frac{L_c}{c\delta_h^e}\right)^2 \quad (12)$$

Further, Eq. (11) can be simplified as follows by assuming  $a_2\theta_e = x$ :

$$a_1 a_2^2 \sin^2 x + a_3 x^2 - a_2^2 = 0 \quad (13)$$

Since the exact solution of the above transcendental equation is not possible, Maclaurin series of  $\sin x$  [32] has been used to get the following expression:

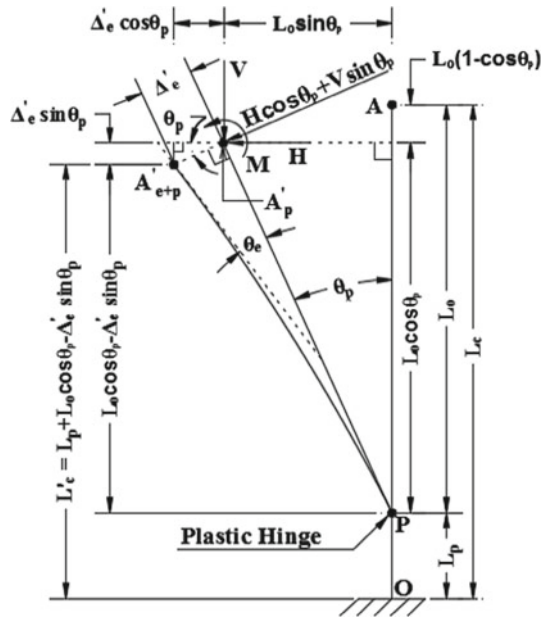
$$a_1 a_2^2 \left[ \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} \right]^2 + a_3 x^2 - a_2^2 = 0 \quad (14)$$

Newton–Raphson method has been used in this paper to get  $\theta_e$  by solving the above equation, from which the complete orientation of the test specimen during a particular time step can be found and therefore, the actual horizontal force ( $H$ ), actual vertical force ( $V = P_V - F_V$ ), and additional tip moment ( $M$ ) can be evaluated.

### 3.4 Effect of Large Displacement of Horizontal Actuator

During cyclic loading with small displacement amplitude, the cantilever tip movement can be analyzed by following a straight path. But the tip movement starts to follow a curvilinear path with subsequently increasing the amount of lateral drift, especially during monotonic tests.

**Fig. 3** Vertical lowering of column tip due to large lateral displacement



In Fig. 3 the column tip, A moves to the point A'p after a plastic rotation  $\theta_p$  about the plastic hinge, P and further, it moves to the point A'e+p after an elastic rotation  $\theta_e$ . Thus, the modified vertical length of the cantilever column ( $L'_c$ ) can be expressed as

$$L'_c = L_c - \Delta_{L_c} \tag{15}$$

$$\Delta_{L_c} = L_0(1 - \cos \theta_p) + \Delta'_e \sin \theta_p; \Delta'_e = \frac{(H \cos \theta_p + V \sin \theta_p)L_c^3}{3E_c I_e} + \frac{ML_c^2}{2E_c I_e} \tag{16}$$

where  $\Delta_{L_c}$  = total vertical shortening due to both plastic and elastic rotation,  $\Delta'_e$  = elastic displacement due to the loads acting normally to the direction of PA'p and the moment acting at A'p. Further  $L'_c$  can be used to evaluate the displacement removing the effect of the additional tip moment ( $\Delta'$ ) as given below:

$$\Delta' = \Delta - \frac{ML_c^2}{2E_c I_e} \tag{17}$$



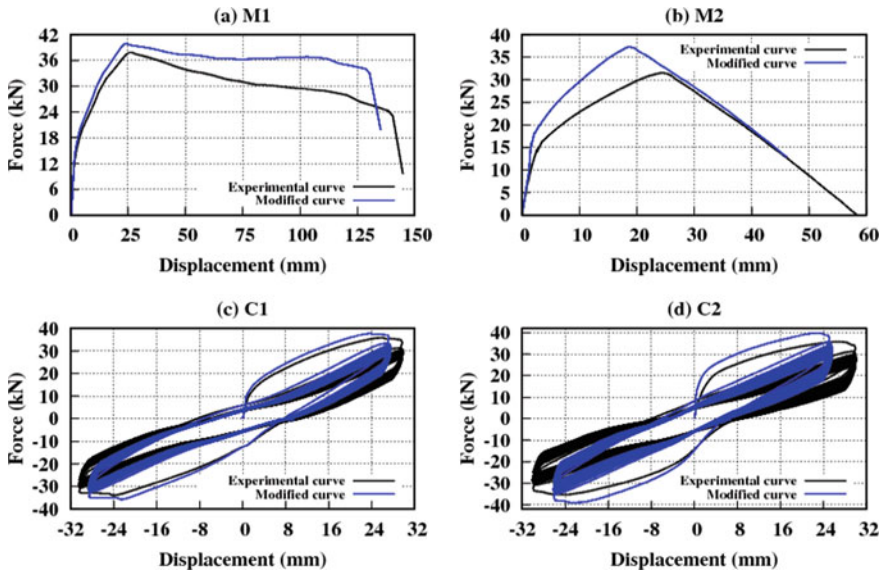


Fig. 4 Experimental and modified monotonic and cyclic pushover curves

## 4 Results and Discussions

### 4.1 Modification of Experimental Data

Figure 4 clearly shows that direct incorporation of experimental results will underestimate the lateral strength and overestimate the lateral displacement capacity of the system. Neglecting the horizontal force component of the vertical actuator and displacement due to additional tip moment generated by the inclined position of both actuators are the main reason for underestimation of lateral strength and overestimation of the lateral displacement capacity of the system, respectively. Clearly, the modification scheme makes the original system stiffer. Further, this modification has been found to be more significant in the presence of high axial load acting at the column (see Table 2). During cyclic experiments, a lesser amount of pinching and considerably higher amount of strength degradation has occurred in the presence of higher axial load.

### 4.2 Analytical Model Using OpenSees Software

Two different analytical models have been developed in OpenSees platform by calibrating with experimental data as well as modified data. The analytical model with exact material properties has been found to produce similar results as the modified

**Table 2** Comparison of underestimation of lateral strength and overestimation of lateral displacement capacity for different tests

		M1	M2	C1		C2	
Experimental results	$F_{\max}$ (kN)	37.88	31.52	-34.00	35.88	-35.36	35.84
	$\delta_{\max}$ (mm)	144.80	58.32	-30.41	29.80	-30.08	30.08
Modified results	$H_{\max}$ (kN)	39.86	37.30	-35.75	37.81	-39.04	39.95
	$\Delta'_{\max}$ (mm)	135.1	46.14	-28.66	27.54	-26.17	25.65
% increase in $F_{\max}$		5.23	18.34	5.15	5.38	10.41	11.47
% decrease in $\delta_{\max}$		5.75	20.88	5.75	7.58	13.00	14.73

**Table 3** Details of different ground motions [33]

Sl. no.	Ground motion name	Location	Magnitude	PGA (cm/s <sup>2</sup> )	Nearest dist. to fault (km)
1.	Bhuj, India (2001)	23.02° N, 72.38° E	7.0 M <sub>b</sub>	-103.82	239.0
2.	Loma Prieta, USA (1989)	32.05° N, 181.80° W	7.0 M <sub>w</sub>	469.38	2.8

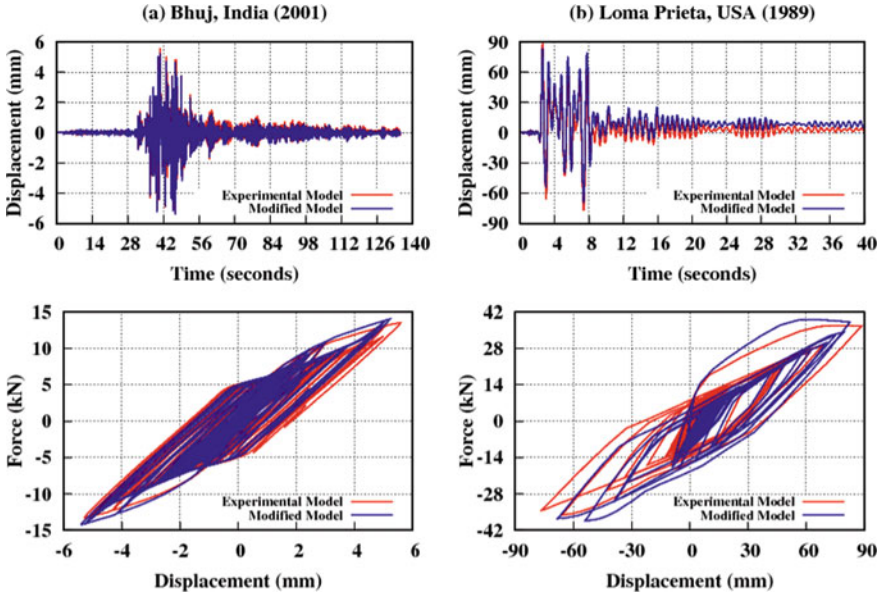
data rather than the experimental data, which definitely signifies the importance of the described modification scheme. Further, a column, with one end fixed and another end rotationally constrained (to achieve the deformed shape with double curvature), of length equal to 3 m has been considered and the analytical models have been used to find the behavior of the column subjected to two different ground motions (see Table 3).

Though the responses of two different analytical models are found to be similar when subjected to a far-field ground motion with lesser PGA value, two models behave differently being subjected to a near-field ground motion with higher PGA value, as shown in the Fig. 5. Experimentally calibrated analytical model overestimates the displacement response, but underestimates the lateral force demand and the residual inelastic displacement, due to the less inherent stiffness of the model compared to the analytical model calibrated with the modified data.

### 4.3 Estimation of Seismic Damage Index

Most widely used modified Park and Ang damage index [34], as given below, has been estimated in this paper:

$$DI = \frac{\delta_m - \delta_y}{\delta_u - \delta_y} + \frac{\beta}{Q_y \delta_u} \int dE \quad (18)$$



**Fig. 5** Different responses of two analytical models subjected to different ground motions. **a** Bhuj, India (2001). **b** Loma Prieta, USA (1989)

where  $\delta_m$  = the max. deformation under seismic loading condition,  $\delta_y$  and  $\delta_u$  = the yield and the ultimate deformation under monotonic loading condition,  $\beta$  = non-negative strength degrading parameter (taken as 0.15 in this paper),  $\int dE$  total absorbed hysteresis energy, and  $Q_y$  = yield strength under monotonic loading. Table 4 shows the estimation of considered damage index for two different analytical models. It has been found that the experimentally calibrated analytical model estimates incorrect seismic damage index by a significant amount, which is not desirable.

**Table 4** Comparison of damage indices for both experimental and modified model subjected to two different ground motions

	Parameters from static pushover			Bhuj, India (2001)			Loma Prieta, USA (1989)		
	$Q_y$ (kN)	$\delta_y$ (mm)	$\delta_u$ (mm)	$\delta_m$ (mm)	$\int dE$ (kN, mm)	DI	$\delta_m$ (mm)	$\int dE$ (kN, mm)	DI
Exp model	31.76	15.49	144.79	5.58	450.58	0.0147	88.47	9759.75	0.8828
Mod model	33.43	14.74	135.77	5.40	427.23	0.0141	82.69	11010.30	0.9253
% change				-3.23	-5.18	-4.08	-6.53	+12.81	+4.81

## 5 Conclusions

Nonlinear monotonic experiments and pseudo-static cyclic experiments have been carried out in the present study for various axial load ratios. A methodology of kinematic correction has been proposed to account for the kinematic interaction between inclined horizontal and vertical actuators, to refine the response of reinforced concrete cantilever columns under the action of bi-directional loading. It has been found that such interactions, if not taken into account, adversely affect the load-deformation behavior and the analytical model calibrated by using it and subsequent response prediction. The material model developed based on the proposed refined experimental data has been found to be more realistic as it can address the issue with underestimation of strength and overestimation of deformability associated with kinematic interactions.

## References

1. Otani S, Cheung V (1981) Behavior of reinforced concrete columns under biaxial lateral load reversals test without axial load. Department of Civil Engineering, University of Toronto, Canada
2. Saatcioglu M, Ozcebe G (1989) Response of reinforced concrete columns to simulated seismic loading. *Struct J* 86:3–12
3. Zagajeski SW, Bertero VV, Bouwkamp JG (1978) Hysteretic behavior of reinforced concrete columns subjected to high axial and cyclic shear forces. University of California, Berkley, p 78
4. Park R, Zahn F, Falconer T (1984) Strength and ductility of reinforced and prestressed concrete columns and piles under seismic loading. In: Proceedings of 8th world conference on earthquake engineering, San Francisco, USA
5. Rabbat B, Daniel J, Weinmann T, Hanson N (1986) Seismic behavior of lightweight and normal weight concrete columns. *J Proc* 83:69–79
6. Gilbertsen ND, Moehele JP (1980) Experimental study of small scale R/C columns subjected to axial and shear force reversals. University of Illinois Engineering Experiment Station, College of Engineering, University of Illinois at Urbana-Champaign
7. Kreger M, Linbeck L (1986) Behavior of reinforced concrete columns subjected to lateral and axial load reversal. In: Proceedings of 3rd US national conference on earthquake engineering, vol 11. Charleston, South Carolina, USA
8. Ristic D et al (1986) Effects of variation of axial forces to hysteretic earthquake response of reinforced concrete structures. In: Proceedings of 8th European conference on earthquake engineering, vol 4. Lisbon, Portugal
9. Abrams DP (1987) Influence of axial force variation on flexural behavior of reinforced concrete columns. *Struct J* 84:246–254
10. Saadeghvaziri MA, Foutch DA (1990) Behavior of RC columns under nonproportionally varying axial load. *J Struct Eng* 116:1835–1856
11. Takizawa H, Aoyama H (1976) Biaxial effects in modelling earthquake response of R/C structures. *Earthq Eng Struct Dyn* 4:523–552
12. Otani S, Cheung V, Lai S (1979) Behavior and analytical models of reinforced concrete columns under bi-axial earthquake loads. In: 3rd Canadian conference on earthquake engineering, Montreal
13. Zahn F, Park R, Priestly M (1989) Strength and ductility of square reinforced concrete column sections subjected to biaxial bending. *Struct J* 86:123–131

14. Bousias S, Verzelletti G, Fardis M, Magonette G (1992) RC columns in cyclic biaxial bending and axial load. In: 10th world conference on earthquake engineering, Madrid
15. Kim JK, Lee SS (2000) The behavior of reinforced concrete columns subjected to axial force and biaxial bending. *Eng Struct* 22:1508–1528
16. Qiu F, Li W, Pan P, Qian J (2002) Experimental tests on reinforced concrete columns under biaxial quasi-static loading. *Eng Struct* 24:419–428
17. Tsuno K, Park R (2004) Experimental study of reinforced concrete bridge piers subjected to bi-directional quasi-static loading. *Struct Eng/Earthq Eng* 21:11S–26S
18. Kawashima K, Ogimoto H, Hayakawa R, Watanabe G (2006) Effect of bilateral excitation on the seismic performance of reinforced concrete bridge columns. In: 8th US national conference on earthquake engineering, San Francisco, CA, USA
19. Chang SY (2009) Experimental studies of reinforced concrete bridge columns under axial load plus biaxial bending. *J Struct Eng* 136:12–25
20. Rodrigues H, Arede A, Varun H, Costa AG (2013) Experimental evaluation of reinforced concrete column behavior under biaxial cyclic loading. *Earthq Eng Struct Dyn* 42:239–259
21. Campione G, Cavaleri L, Di Trapani F, Macaluso G, Scaduto G (2016) Biaxial deformation and ductility domains for engineered rectangular rc cross-sections: a parametric study highlighting the positive roles of axial load, geometry and materials. *Eng Struct* 107:116–134
22. Suzuki N, Otani S, Kobayashi Y (1984) Three-dimensional beam-column sub-assemblages under bi-directional earthquake loading. In: 8th world conference on earthquake engineering, San Francisco, USA
23. Monti G, Nuti C (1992) Nonlinear cyclic behavior of reinforcing bars including buckling. *J Struct Eng* 118:3268–3284
24. Li L, Mander JB, Dhakal RP (2008) Bidirectional cyclic loading experiment on a 3D beam-column joint designed for damage avoidance. *J Struct Eng* 134:1733–1742
25. Akguzel U, Pampanin S (1987) Effects of variation of axial load and bidirectional loading on seismic performance of retrofitted reinforced concrete exterior beam-column joints. *J Compos Constr* 14:94–104
26. Low SS, Moehle JP (2004) Experimental study of reinforced concrete columns subjected to multi-axial cyclic loading. University of California, Berkeley
27. Li KN, Aoyama H, Otani S (1988) Reinforced concrete columns under varying axial load and bi-directional lateral load reversals. In: 9th world conference on earthquake engineering, Tokyo–Kyoto, Japan
28. Bechtoula H, Kono S, Watanabe F (2005) Experimental and analytical investigations of seismic performance of cantilever reinforced concrete columns under varying transverse and axial loads. *J Asian Archit Build Eng* 4:467–474
29. Rodrigues H, Furtado O, Arede A (2015) Behavior of rectangular reinforced-concrete columns under biaxial cyclic loading and variable axial loads. *J Struct Eng* 142
30. Comitè Euro-International Du Béton (CEB) (1996) RC frames under earthquake loading: vol. 2. ASCE, Publication Sales Department, Thomas Telford, London
31. Paulay T, Priestley MN (1992) Seismic design of reinforced concrete and masonry buildings. Wiley New York, USA
32. Zwillinger D (2011) CRC standard mathematical tables and formulae. CRC Press, USA
33. <http://www.strongmotioncenter.org/vdc/scripts/earthquakes.plx>. Accessed 27 Oct 2017
34. Kunnath SK, Reinhorn AM, Lobo R (1992) IDARC version 3.0: a program for the inelastic damage analysis of reinforced concrete structures. Technical Report NCEER, US National Center for Earthquake Engineering Research, 92