Stability of Parametrically Excited Active Magnetic Bearing Rotor System Due to Moving Base

Tukesh Soni, J. K. Dutt, and A. S. Das

Abstract Active magnetic bearings (AMBs) offer contact-less functioning and active vibration control capability while supporting and levitating a rotor. This is the reason that the AMBs are being progressively researched for novel and challenging applications in the industry. In application areas, such as ships, airplanes and space crafts, the rotor is mounted on a moving base, which causes parametric excitation to the system. This, in turn, is generally known to cause stability issues in a rotor shaft system. The present work thus attempts to conduct stability analysis of a rotor shaft system supported by an AMB and is parametrically excited due to the presence of periodically varying base motion. The finite element model for a generic rotor shaft system mounted on a moving base is first presented, and the time-periodic state matrix for the system is found. The Floquet–Liapunov method of analyzing stability of a periodically varying system is used to find the stability boundaries for the system with two widely used control laws for the AMB. The analysis reveals that it is important to consider the parametric excitation caused to the system when the AMBs are being designed for applications involving large base motions.

Keywords Active magnetic bearing · Parametric stability · Rotor dynamics

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1 Introduction

Active magnetic bearings (AMBs) offer some major advantages over conventional bearings, namely contact-less support and an opportunity to actively control vibrations in the system. This results in lesser vibration and noise levels in the rotor system. This is also the reason that the AMBs are researched for novel and challenging applications in the industry. Reference [1] provides an excellent introduction to the AMBs. A detailed review on recent research advancements in the application areas of the AMB can be found in [2]. Rotor shaft systems subject to large base motion have been shown to be parametrically excited [3]. It is also well known that parametric excitation to rotor shaft systems may also lead the system to instability [4]. It is therefore necessary to conduct a thorough stability analysis of an AMB levitated rotor system, which is mounted on a moving base.

The issue of stability has been a concern for the researchers in the field of rotor dynamics for many decades [5–8]. Two major sources of instabilities have been clearly identified and extensively studied in the literature on rotor dynamics, namely the instability due to internal material damping and the instability due to rotor–fluid interaction in the fluid film type-bearings. The varying parameters of a system for example varying stiffness lead to another type of instability in rotors called the parametric instability. When the cross section of the rotor shaft is not axisymmetric, then the bending stiffness of the rotor shaft varies with the rotation of the rotor shaft and thus leads to parametric instability in such systems [9, 10].

Kamel and Bauomy [11] analyzed the stability of an nonlinear AMB-supported rigid rotor system with varying stiffness. Investigations into the steady-state stability with varying parameters were carried out. Bauomi [12] considered a similar system and studied the effect of cubic and quadratic nonlinearity of the stiffness on the dynamics of a rigid rotor-AMB system. Duchemin et al. [3] and later Driot et al. [13] analyzed the dynamics and stability of a rotor shaft system subject to periodic base motion. The rotor was considered to be simply supported. The equations of motion for a generic rotor shaft system with base motion were derived. However, Rayleigh–Ritz method was used to simplify the equations of motion to conduct the stability analysis. Han and Chu [14] conducted the stability analysis of parametrically excited flexible rotor shaft system with conventional bearings mounted on a base with periodic angular motion. Stability boundaries were drawn for various cases of periodic base motion frequency and amplitude. The authors used dynamic state transition matrix (DSTM) method to find the instability regions of the rotor shaft system. An interesting problem of stability of an aircraft rotor during the maneuvering of the aircraft is analyzed by Hou et al. [15]. In the work, the bearings were modeled as a Duffing-type nonlinear spring and dampers and the aircraft maneuver was modeled as a sine wave. The variation in bifurcation diagram for the system with respect to the aircraft maneuver was reported.

To the best of authors knowledge, the existing literature has a shortcoming that these research activities do not consider the stability analysis of a rotor shaft system levitated on an AMB and subject to parametric excitation due to large generic base motion. Recently, Soni et al. [16] conducted parametric stability analysis of a rotor-AMB system subject to periodic base motion. However, the stability boundaries of the system with respect to base motion parameters were not reported. This research gap has inspired the authors to report the present work. This paper, therefore, conducts a stability analysis of a flexible rotor system which is levitated by an AMB and is aboard a moving base. Finite element model governing the motion of a generic rotor mounted on a moving base is first presented. Mathematical model of the force–current relationship of an AMB is then discussed. An efficient numerical algorithm based on the Floquet–Liapunov method of analyzing stability of a periodic system is introduced [17]. Results pertaining to the stability boundaries of the system are then presented.

2 Finite Element Model of a Rotor with Moving Base

The equation of motion of a rotor with large generic base motion has been derived in [18]. The resulting finite element model for such a system is reproduced in this section. A schematic of a rotor shaft system levitated and supported by an AMB with moving base is shown in Fig. 1. Three coordinate frames are defined and are also shown in Fig. 1. Frame, F_i, is the inertial reference frame, and F_{AMB}(with unit vectors $i_A - j_A - k_A$) is the coordinate frame attached to the rotor base at the left AMB, coordinate frame F attached to the rotor shaft. The global finite element matrices are given as:

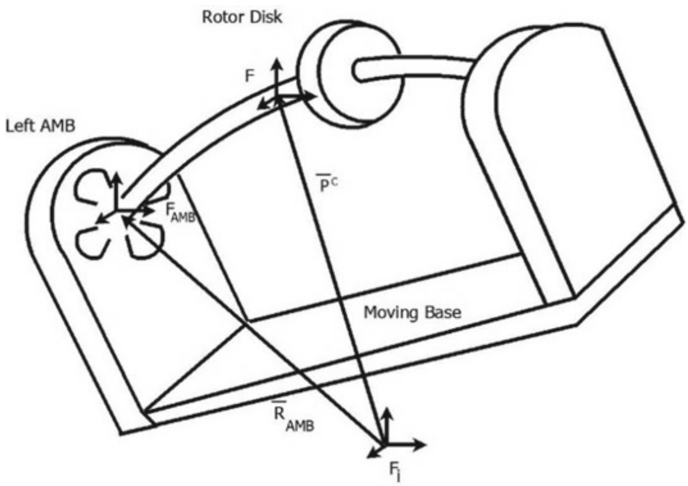

Fig. 1 Rotor disk system supported on AMB with base motion

$$[M] = \sum_D [M]_D + \sum_e [M]_S^e \tag{1}$$

$$[D] = \left(\Omega_{X_b}^b + \dot{\phi}\right)\left(\sum_D [G]_D + \sum_e [G]_S^e\right) + 2\Omega_{X_b}^b\left(\sum_D [C]_D + \sum_e [C]_S^e\right) + [D]_{\text{brg}} \tag{2}$$

$$\begin{aligned}[K] = -\Bigg\{ &\left(\ddot{\phi} + \dot{\Omega}_{X_b}^b\right)\left\{\sum_D [H]_D + \sum_e [H]_S^e\right\} - \dot{\Omega}_{X_b}^b\left\{\sum_D [C]_D + \sum_e [C]_S^e\right\}\Bigg\} \\ &\left\{\Omega_{X_b}^{b2}\left\{\sum_D [M]_D + \sum_e [M]_S^e\right\} + \Omega_{Z_b}^{b2}\left\{\sum_D [K_{p11}]_D + \sum_e [K_{p11}]_S^e\right\}\right. \\ &\left.\Omega_{Y_b}^{b2}\left\{\sum_D [K_{p22}]_D + \sum_e [K_{p22}]_S^e\right\} - \Omega_{Y_b}^b \Omega_{Z_b}^b\left\{\sum_D [K_{p12}]_D + \sum_e [K_{p12}]_S^e\right\}\right\} \\ &+ \left\{[K_B]_S^e + [K]_{\text{brg}}\right\}\end{aligned} \tag{3}$$

where $\dot{\phi}$ is the rotor spin speed and $\Omega^b = \Omega_{X_b}^b i_A + \Omega_{Y_b}^b j_A + \Omega_{Z_b}^b k_A$ is the angular velocity of the frame F_{AMB} with respect to inertial frame F_i.

Equation (1) gives the global mass matrix for the system $[M]$, $[D]$ is the global damping matrix and $[K]$ is the global stiffness matrix. From Eqs. (2) and (3), it is can be seen that global stiffness and damping matrix contain the time-varying base motion parameters, and this leads to parametric excitation to the system. The objective of the present work is to study the stability of the system; therefore, the global force vector acting on the system by the virtue of base motion is not given here. Details regarding the other matrices are given in the Appendix.

3 Active Magnetic Bearing (AMB)

Active magnetic bearing is a mechatronic device which provides contact-less levitation of a rotor shaft system and has the active vibration control capability. The basic components of an AMB are the electromagnet pole pairs (radially arranged around the rotor shaft), proximity displacement sensors, power amplifiers, data acquisition system and the controller. A schematic of an AMB is shown in Fig. 2.

Linearized expressions for the force exerted by the AMB electromagnet can be written as [1],

$$F_Y = k_i i_{cY} - k_Y y_{\text{AMB}};\ F_Z = k_i i_{cZ} - k_Z z_{\text{AMB}} \tag{1}$$

where k_i, k_Y and k_Z are constants for an electromagnet and depend upon the bias current (i_0) and the nominal air gap (g_0). Expressions for k_i, k_Y and k_Z are given

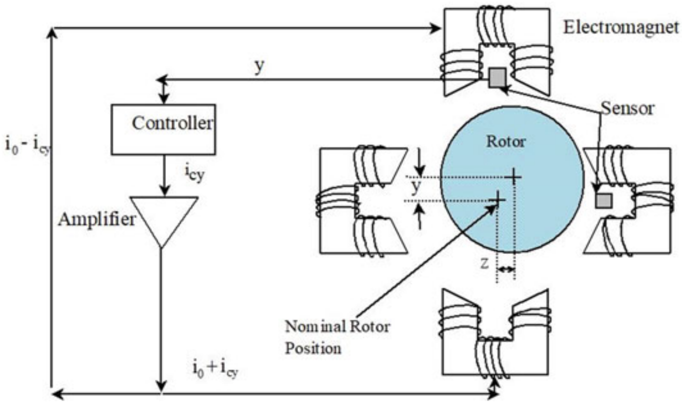

Fig. 2 Schematic representation of an AMB

in Eq. (5). i_{cY} and i_{cZ} are the control current provided to the vertical and horizontal electromagnet pairs. y_{AMB} and z_{AMB} are the corresponding excursions of the rotor at the location of the AMB proximity sensor (assumed to be collocated with the AMB, in the present study).

$$k_i = 4k_{\text{mag}}\frac{i_0}{g_0^2};\ k_Y = k_Z = k_s = -4k_{\text{mag}}\frac{i_0^2}{g_0^3} \tag{2}$$

where $k_{\text{mag}} = \frac{\mu_0 A_p N^2}{4}$ is a constant for an electromagnetic actuator. A_p is the face area of the electromagnetic pole (m^2), N is number of coil turns and μ_0 is magnetic permeability of air. The control law decides the relationship between the control current i_{cY} and the displacement at the AMB location. For the case of a simple PID control law,

$$i_{cY} = -\left(k_p y_{\text{AMB}} + k_I \int y_{\text{AMB}} dt + k_d \dot{y}_{\text{AMB}}\right);\ i_{cZ} = -\left(k_p z_{\text{AMB}} + k_I \int z_{\text{AMB}} dt + k_d \dot{z}_{\text{AMB}}\right) \tag{3}$$

where k_p, k_I and k_d are the proportional, integral and derivative gains of the PID control law. Then, the assembled equations of motion for the rotor shaft system levitated by an AMB and mounted on a moving base can written as,

$$[M]\{\ddot{\Gamma}\} + [D]\{\dot{\Gamma}\} + [K]\{\Gamma\} + [\mathcal{C}]\{\Gamma\} = \{0\} \tag{4}$$

where $[\Gamma]$ is the global displacement vector and the $[\mathcal{C}]$ matrix represents the contribution of force on the rotor shaft due to the AMB. At the AMB node, the elemental $[\mathcal{C}]$ matrix is given as,

$$[\mathcal{C}]^e_{\mathrm{AMB}} = \begin{bmatrix} k_i\left(k_\mathrm{p} + k_\mathrm{I}\frac{1}{\mathcal{D}} + k_\mathrm{d}\mathcal{D}\right) - k_Y & 0 & 0 & 0 \\ 0 & k_i\left(k_\mathrm{p} + k_\mathrm{I}\frac{1}{\mathcal{D}} + k_\mathrm{d}\mathcal{D}\right) - k_Z & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (5)$$

where $\mathcal{D}$ is the differentiation operator, equal to $\frac{\mathrm{d}}{\mathrm{d}t}$.

4 Parametric Stability Analysis of Periodic Systems

The assembled equations of motion of the rotor-AMB system in Eq. (4) can be transformed into the state space form as follows,

$$\dot{x}(t) = A(t)x(t) \quad (6)$$

which represents a time-varying linear system. For the case of periodic base motion, the state matrix $A(t)$ is also periodic with $A(t + \mathcal{T}) = A(t)$ and $\mathcal{T}$ is the fundamental time period of the state matrix. Efficient numerical method for ascertaining the stability of periodic system has been described by Friedmann et al. [17]. The method can be considered as a numerical counterpart to the Floquet–Liapunov's theory of stability of linear periodic systems. The method is based on the eigenvalues of the state-transition matrix of the system. However, for a complex system, it may not be always possible to find the state-transition matrix; therefore, in this numerical method an estimated state-transition matrix is found. The method to find the estimated state-transition matrix is outlined in Fig. 3.

After the estimated state-transition matrix has been deduced, the stability regions can be found using the eigenvalues of the estimated state-transition matrix. The system is considered to be in a stable state if the following condition is satisfied,

$$\max(|\sigma + j\omega|) < 1 \quad (7)$$

where $\sigma + j\omega$ is the eigenvalue of the estimated state-transition matrix of the system.

5 Results and Discussion

5.1 System Details

An overhung rotor shaft system is considered for the stability analysis in this work [19]. The details of the rotor shaft system and the AMB used for simulation in this work are given in Table 1. Finite element discretization of the rotor shaft system is depicted in Fig. 4. The periodic base motion analyzed in this work is (a) periodic

Fig. 3 Steps to find the estimated state-transition matrix

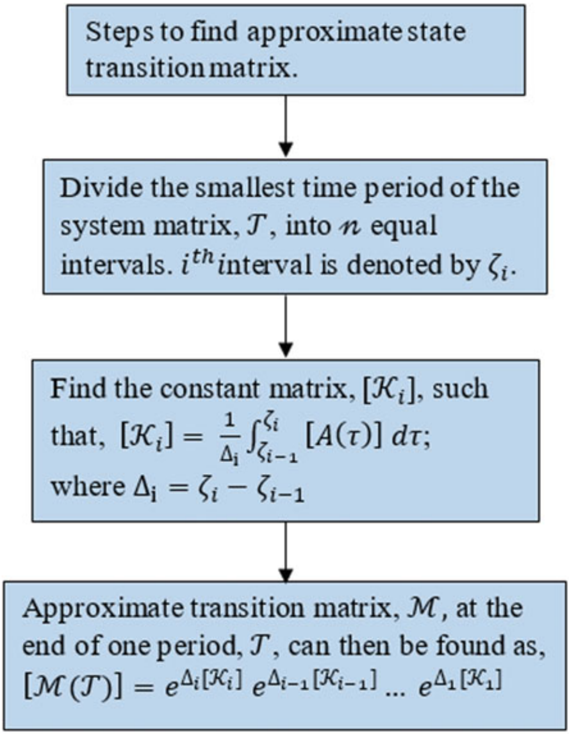

Table 1 Rotor shaft disk and AMB details

Rotor shaft disk details		AMB details	
Shaft length	1.5 m	Pole face area, A_p	500 mm^2
Shaft diameter	0.03 m	Current stiffness, k_i	177.4 N/A
Disk diameter	0.5 m	Displacement stiffness, k_s	−1.388 kN/mm
Disk thickness	0.07 m	Bias current, i_0	5 A
Young's modulus of elasticity	211 GPa	Radial air gap, g_0	2.5 mm
Density	7810 kg/m^3	AMB node location	1.5
Rotor spin speed	1000 rev/min		

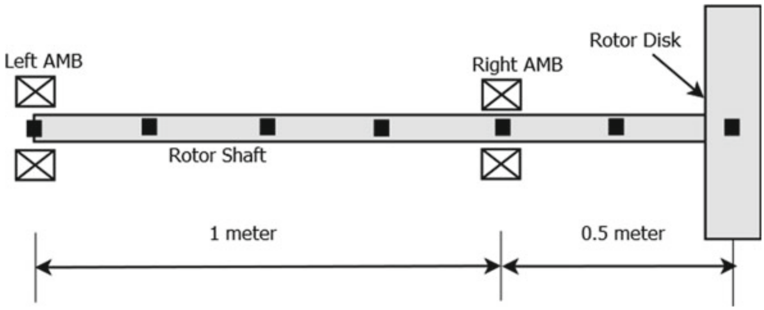

Fig. 4 Overhung rotor shaft AMB system with finite element discretization

pitch motion of the base (A_P—amplitude, ω_P—frequency), (b) periodic roll motion of the base (A_R—amplitude, ω_R—frequency) and (c) periodic yaw motion of the base (A_Y—amplitude, ω_Y—frequency).

5.2 Parametric Stability Boundaries

Figure 5 shows the stability boundaries for the overhung rotor system levitated by an AMB and subject to periodic base pitching. The condition of stability as per the Floquet–Liapunov method detailed in the previous section is that the maximum of absolute eigenvalue of the estimated state-transition matrix must be less than one (see Eq. 7). Therefore, it can be seen from Fig. 5, that for the case of periodic base pitching, the rotor shaft becomes unstable at the lowest frequency of around 15 rad/s for 0.4 rad base amplitude. Similar plots for the case of periodic base roll and yaw motion are shown in Figs. 6 and 7, respectively. For the case of base roll motion, the rotor-AMB system becomes unstable for a base roll frequency value as low as 3 rad/s for a 0.25 rad amplitude.

5.3 Free Vibration Response

To validate that estimated eigenvalue correctly predicts the stability of the rotor-AMB system, response due to initial conditions (free vibration) is simulated. To this end, an initial displacement of 1 mm in both horizontal and vertical directions is imposed at the rotor disk, and consequent response of the rotor disk is numerically simulated using the Newmark-beta method. The free vibration plot for the case of periodic base pitching with a frequency of 24.5 rad/s and an amplitude of 0.3 rad is shown in Fig. 8. As predicted by the value of the approximate eigenvalue (refer Fig. 5), unstable response at the rotor disk is observed. It must be noted the rotor-AMB system has

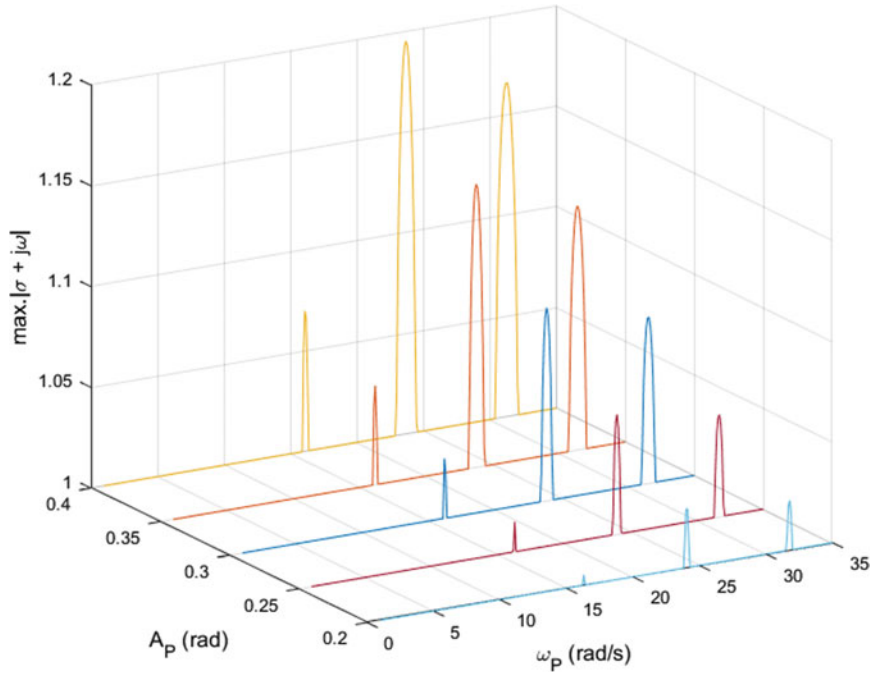

Fig. 5 Stability regions for case of base pitching

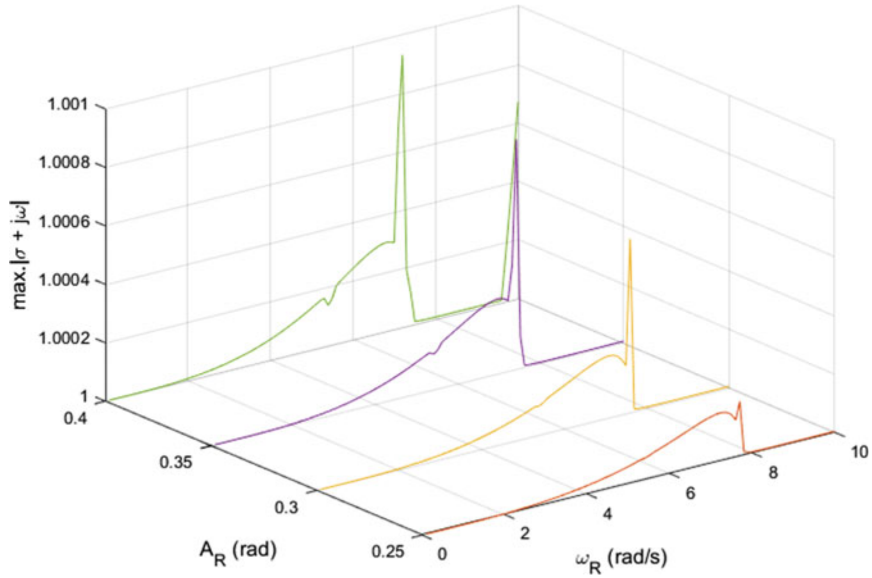

Fig. 6 Maximum absolute eigenvalue for periodic roll base motion

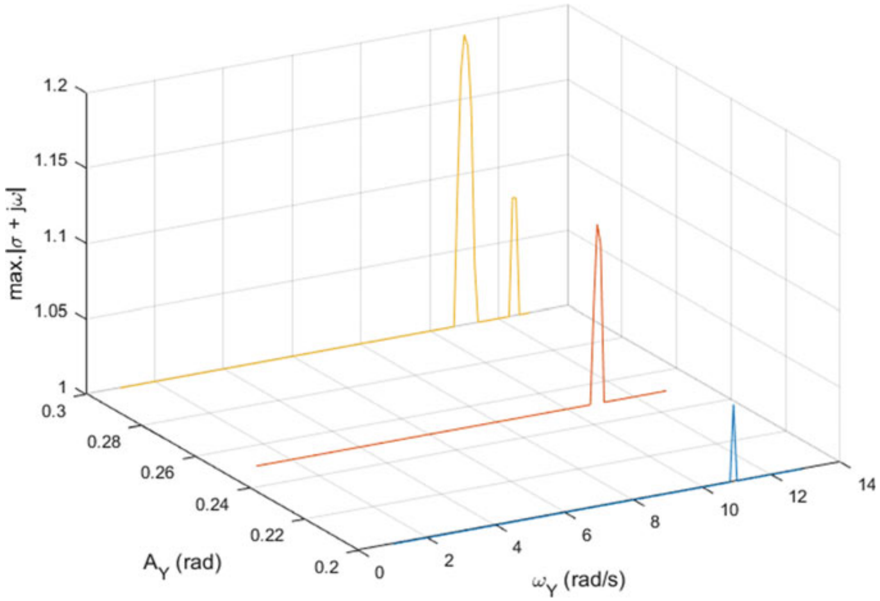

Fig. 7 Maximum absolute eigenvalue for periodic yaw base motion

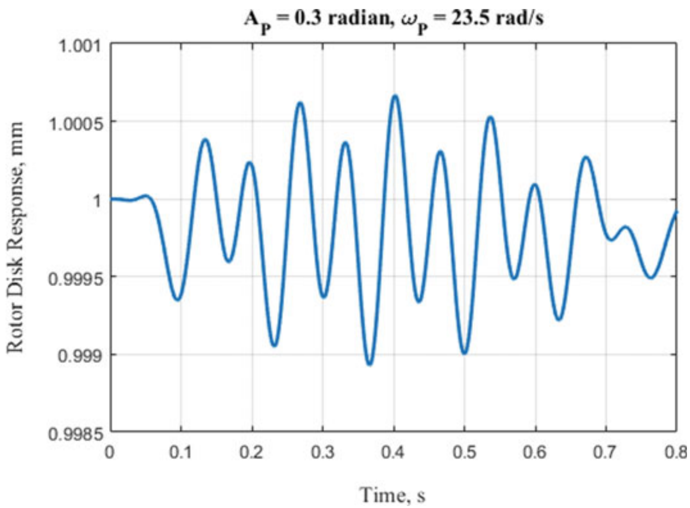

Fig. 8 Free vibration unstable response at the rotor disk due an initial displacement

been considered to be reasonably balanced, and only, the parametric excitation to the system is considered.

6 Conclusion

The following conclusions are drawn from this work:

1. Base motion in a rotor shaft bearing system causes parametric excitation to the system, which may result in excessive vibrations even in a reasonably balanced rotor.
2. While designing AMBs for the applications involving base motion, an exhaustive parametric stability analysis of the system must be conducted.
3. Depending upon the type of base motion, the rotor shaft system levitated on an AMB may become unstable at lower combinations of amplitude and frequency of the base motion. For example, for the case of periodic base rolling motion, the rotor-AMB system becomes unstable for frequency values as low as 3 rad/s and amplitude 0.25 radians.

Appendix

Details of the matrices used for finding the global matrices given in Eqs. (1)–(3).

Shaft Inertia matrix: $[M]_S^e = \int_0^l m[\psi(x)]^T[\psi(x)]\mathrm{d}x + \int_0^l i_d\left[\psi'(x)\right]^T\left[\psi'(x)\right]\mathrm{d}x$ where $[\psi]$ is the shape function matrix, gyroscopic matrix: $[G]_S^e = \int_0^l i_p\left[\psi'\right]^T\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\left[\psi'\right]\mathrm{d}x$; $[H]_S^e = \int_0^l i_p\left[\psi'\right]^T\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}\left[\psi'\right]\mathrm{d}x$ bending stiffness matrix: $[K_B]_S^e = \int_0^l EI\left[\psi''\right]^T\left[\psi''\right]\mathrm{d}x$; $\left[\psi''\right] = \frac{d^2[\psi(x)]}{\mathrm{d}x^2}$.

Circulatory matrix: $[K_C]_S^e = \int_0^l EI\left[\psi''\right]^T\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\left[\psi''\right]\mathrm{d}x$

Coriolis matrix: $[C]_S^e = \int_0^l m[\psi(x)]^T\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}[\psi(x)]\mathrm{d}x + \int_0^l i_d\left[\psi'(x)\right]^T\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\left[\psi'(x)\right]\mathrm{d}x$

Parametric stiffness matrix due to base motion:

$$\left[K_{p11}\right]_S^e = \int_0^l m[\psi(x)]^T\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}[\psi(x)]\mathrm{d}x + \int_0^l i_p\left[\psi'(x)\right]^T\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}\left[\psi'(x)\right]\mathrm{d}x$$

$$\left[K_{p22}\right]_S^e = \int_0^l m[\psi(x)]^T \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} [\psi(x)]\mathrm{d}x + \int_0^l i_p \left[\psi'(x)\right]^T \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \left[\psi'(x)\right]\mathrm{d}x$$

$$\left[K_{p12}\right]_S^e = \int_0^l m[\psi(x)]^T \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} [\psi(x)]\mathrm{d}x + \int_0^l \left(i_p - i_d\right) \left[\psi'(x)\right]^T \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \left[\psi'(x)\right]\mathrm{d}x$$

Rotor disk finite element matrices

Inertia matrix: $[M]_D = \begin{bmatrix} m_D & 0 & 0 & 0 \\ 0 & m_D & 0 & 0 \\ 0 & 0 & I_d & 0 \\ 0 & 0 & 0 & I_d \end{bmatrix}$;

Gyroscopic matrix: $[G]_D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I_p \\ 0 & 0 & I_p & 0 \end{bmatrix}$;

Coriolis effect matrix: $[C]_D = \begin{bmatrix} 0 & -m_D & 0 & 0 \\ m_D & 0 & 0 & 0 \\ 0 & 0 & 0 & -I_d \\ 0 & 0 & I_d & 0 \end{bmatrix}$; Parametric stiffness matrix: $\left[K_{p11}\right]_D = \begin{bmatrix} m_D & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I_p & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$; $\left[K_{p22}\right]_D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & m_D & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_p \end{bmatrix}$; $\left[K_{p12}\right]_D = \begin{bmatrix} 0 & m_D & 0 & 0 \\ m_D & 0 & 0 & 0 \\ 0 & 0 & 0 & I_p - I_d \\ 0 & 0 & I_p - I_d & 0 \end{bmatrix}$

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