Numerical Procedures for Simulation of Wave Propagation in Plates

Mohammed Aslam, Praveen Nagarajan, and Mini Remanan

Abstract Structural health monitoring (SHM) using wave propagation technique is an emerging method that can be used to detect, locate, and quantify the structural damages before catastrophic failures. Advancement of several finite-element simulation programs has helped scientists and engineers in validating the numerical solutions with the experimental results. Most of the researchers use explicit procedures for wave propagation problems. However, for electromechanical problems where piezoelectric materials are used for exciting waves, the explicit procedure is not available in most cases. Hence, implicit procedures are used to account for the piezoelectric effect. It becomes necessary to choose which procedure is apt for obtaining sufficient accuracy and to run the problem within reasonable computational time. This paper presents a comparative study of different finite-element procedures for modeling wave propagation in plates. Three different analysis procedures are studied, namely implicit analysis, explicit analysis, and implicit–explicit co-simulation analysis. The results show that the co-simulation model is more reliable and efficient compared to other models.

Keywords Lamb waves · Finite-element method · Implicit · Explicit · Co-simulation

1 Introduction

Wave propagation technique is well known for its ability in structural health monitoring and damage detection. The elastic waves can be generated and captured using piezoelectric transducers (PZT) adhesively bonded to the structures [1]. Plate structures are used in various fields of engineering. They find applications in aircraft, bridges, industrial buildings, storage vessels, ships, warehouses, and oil rigs. Specific types of elastic waves, namely "Lamb waves," propagate in plate structures. These waves are called as guided waves and can travel a large distance from a single source

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[2]. One of the major challenges facing with Lamb waves is that they are multimodal and dispersive [3]. In SHM, the presence of a defect is identified by quantitative wave parameters, like frequency, wavelength, amplitude, wave velocity, and phase. The progress of many finite-element analysis programs has led to growing interest in whether data obtained from the simulation can be verified by experiments. A brief review of different methods for simulating wave propagation can be found in Willberg et al. [4]. In transient wave propagation problems, direct time integration is used for obtaining finite-element solutions. In general, there are two types of time integration schemes: implicit scheme and an explicit scheme. If the solution procedure requires factorization of an effective stiffness matrix, it is called implicit; else it is explicit [5]. Both schemes can be used for transient analysis. However, for wave propagation problems explicit method is widely used [6]. When piezoelectric materials are used to generate and receive wave signals, the only option is to use an implicit integration method because coupled piezoelectric finite elements are not available for the explicit procedure. Therefore, a suitable analysis procedure is essential to obtain enough accuracy and to solve the problem with minimum cost.

This paper presents a comparative study of different finite-element procedures for modeling wave propagation in plates. Three different solution procedures are studied, namely implicit method, explicit method, and combined implicit–explicit method. The modeling aspects in each case are explained in subsequent sections. The simulation is carried out using commercially available software package Abaqus CAE 2016. The wave parameters, like group velocity, wave amplitude, and mode shapes, obtained from different simulation methods are compared. The group velocities calculated using numerical data are also compared with the theoretical dispersion curve.

2 Numerical Model

For the present study, a steel plate having a uniform thickness of 1.6 mm is used. The geometry of the plate and the location of the PZT actuator is shown in Fig. 1. A distance of 500 mm is provided between the actuator and sensor for capturing both fundamental wave modes S0 and A0. The plate and PZT are assumed to be of infinite extent in the z-direction (plane strain condition). A four-node bilinear plane strain

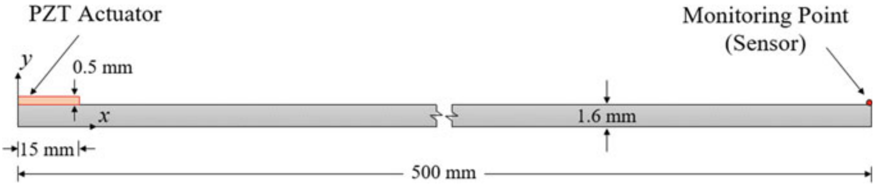

Fig. 1 The geometry of the plate and location of PZT actuator

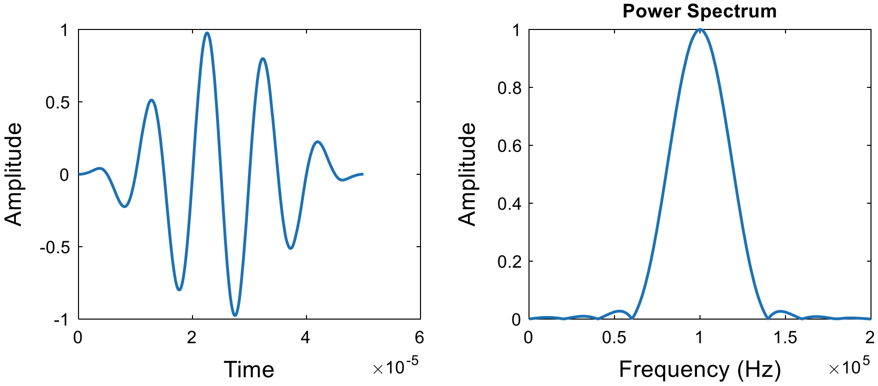

Fig. 2 Time history and power spectrum of a five-count tone burst voltage signal with a central frequency of 100 kHz

quadrilateral element (CPE4) is used to mesh steel plate and a four-node bilinear plane strain piezoelectric quadrilateral element is used to mesh PZT layer.

The PZT is excited with a five-count tone burst voltage signal with an amplitude of 10 V. The central frequency is swept from 25 to 500 kHz. A limit of 500 kHz is chosen to avoid higher wave modes. Figure 2 shows the excitation waveform centered at 100 kHz and its corresponding power spectrum. The material properties of PZT-5H used for the study can be found in Aslam et al. [7].

2.1 Implicit Model

Coupled piezoelectric finite elements are available in Abaqus implicit. In implicit method, Newton's method is used to solve the equilibrium equation. An implicit operator, Hilbert-Hughes-Taylor is used. The integration for displacement and velocity is based on Newmark's formulae [8]. In the implicit method, there is no limit on time step size as they are unconditionally stable. However, a maximum increment of $T/10$ (where T is the period) is chosen for obtaining reasonable results [8]. The implicit model is shown in Fig. 3. Here PZT is bonded to steel plate assuming that there is no slip between the layers.

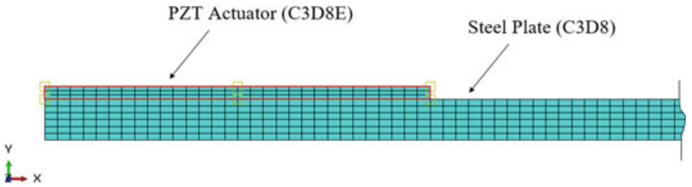

Fig. 3 Implicit finite-element model

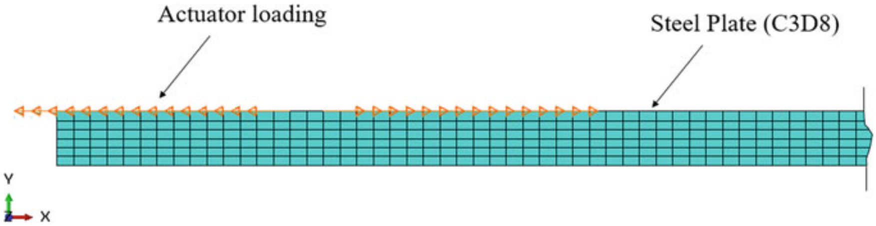

Fig. 4 Explicit finite-element model

2.2 Explicit Model

In the explicit method coupled piezoelectric finite elements are not available. Here displacement boundary conditions are specified on related nodes in contact with the piezoelectric actuator. In most cases, it is assumed that the PZT plate exerts a force in the x-direction as shown in Fig. 4 [9]. This force is a line load per unit length. Explicit solution imposes an upper limit on time step size. Explicit method is conditionally stable. Hence, choosing a suitable time step is vital for obtaining accurate solutions. The time step (Δt) is chosen such that $\Delta t < L_{\min}/C$, where $L_{\min}$ is the smallest element size and C is the wave speed. The mesh size for all the models is taken such that the spatial resolution of propagating waves is achieved. It is suggested that at least 10 elements are required per wavelength [10].

2.3 Implicit–Explicit Co-simulation Model

In implicit–explicit method Abaqus allows the user to make use of both implicit and explicit procedures. Here piezoelectric analysis is performed using the implicit method and the output is used as the input for explicit analysis. The whole finite-element model is subdivided into the implicit and explicit model. Then by using an interactive interface, the data are exchanged in a synchronized manner. The co-simulation model is illustrated in Fig. 5.

3 Results and Discussion

The sensor response obtained from the three numerical models were compared and studied. The comparison of the magnitude of displacement response at the sensor when PZT is actuated at 125 kHz central frequency is shown in Fig. 6. It is observed that all the three models captured both S0 and A0 wave modes. However, the implicit model showed a slight time delay. The magnitude of implicit and co-simulation models is comparable, whereas the magnitudes of the explicit model were found to

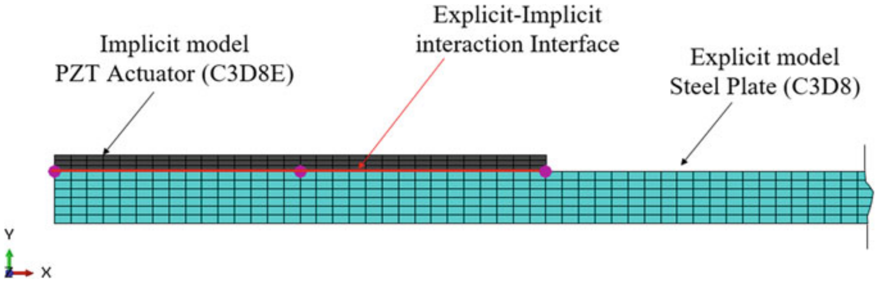

Fig. 5 Co-simulation model

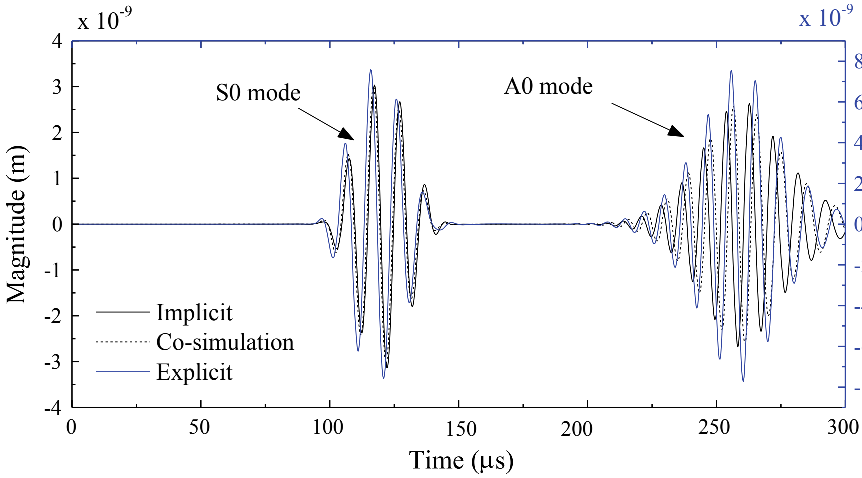

Fig. 6 Sensor response at 125 kHz

be higher. This might be due to the reason that the prescribed effective displacement may be higher than the actual displacement. By knowing the time of flight of S0 and A0 wave modes, the group velocities were calculated and compared with that of the theoretical dispersion curve of steel plate. Figure 7 illustrates the comparison of group velocities obtained from the numerical study with the theoretical group velocity [11]. It is noted that at lower frequencies, the group velocity predicted is well matching with the theoretical curve. For higher frequencies, the implicit models show lower velocities which are due to slight delay in phase. This delay can be adjusted by reducing the time increment and the element size, but the computational cost would become excessive.

A typical plot showing the contour of resultant displacement of A0 and S0 modes is illustrated in Fig. 8. The plot corresponds to co-simulation model. Similar contour is obtained for the other two models. The first mode arriving the sensor is the S0 mode followed by A0 mode. As expected, it can be observed that the particle displacement

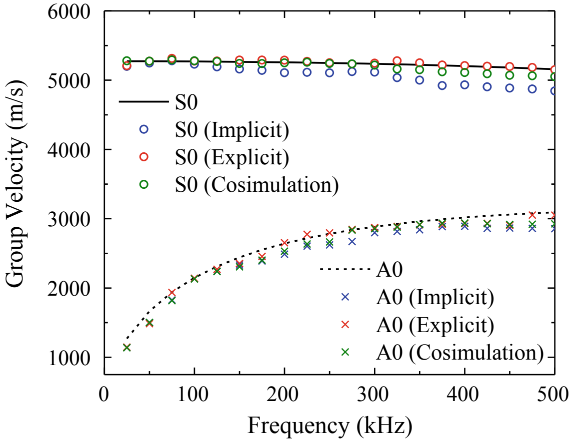

Fig. 7 Comparison of group velocity dispersion curve for 1.6 mm steel plate

Fig. 8 Typical finite-element record showing displacement amplitude of A0 and S0 mode at time step 8.1e-5 s

in the *x*-direction (U1) is more compared to the particle displacement in the *y*-direction (U2) for S0 mode and vice versa in case of A0 mode.

For the present study, a desktop computer with an Intel *R* Core™ i5-5200U processor is used. Table 1 presents the computation time taken for each analysis models. The time is normalized with respect to minimum time. It is noted that as frequency increases the run time increases for both implicit and co-simulation models. However, the time required for co-simulation is less compared to implicit. The average run time for implicit, explicit, and co-simulation analysis is 3463, 22.7, and 1040 s, respectively. This indicates that explicit analysis is computationally effective compared to the other two models.

The variation in amplitude with respect to excitation frequency is plotted in Fig. 9. It is observed that both implicit and co-simulation models show a similar trend for both the wave modes. In an explicit model, an extra peak is observed in S0 mode near to 250 kHz. In A0 mode, the amplitude of the second peak is close to the first peak for the explicit model. This might be due to the assumption of line loading in the explicit model.

Figure 10 presents the displacement components along *x* and *y* directions (U1 and U2) through the thickness profile. All the displacement components are normalized with respect to their maximum values. The graph reveals that the mode shape predicted by all the models are similar. For S0 mode, the displacement along *x*-direction is symmetric with respect to the midplane, while the displacement along

Table 1 Comparison of CPU time

Frequency (kHz)	Normalized run time		
	Implicit	Explicit	Co-simulation
25	110.84	1.68	54.74
50	122.84	1.32	44.37
75	132.21	1.26	42.26
100	128.89	1.16	37.68
125	141.63	1.16	43.16
150	154.58	1.16	40.58
175	154.58	1.16	33.79
200	117.79	1.11	49
225	130.42	1.16	32.42
250	131	1.42	25.95
275	184.32	1.11	49.74
300	200.53	1.21	35.84
325	248.47	1.16	45
350	212.63	1.11	67.89
375	307.79	1.16	85
400	225.95	1.32	54.63
425	266.16	1	49.47
450	231.05	1.05	99.53
475	226.84	1.11	95.47
500	221.74	1.11	104.11

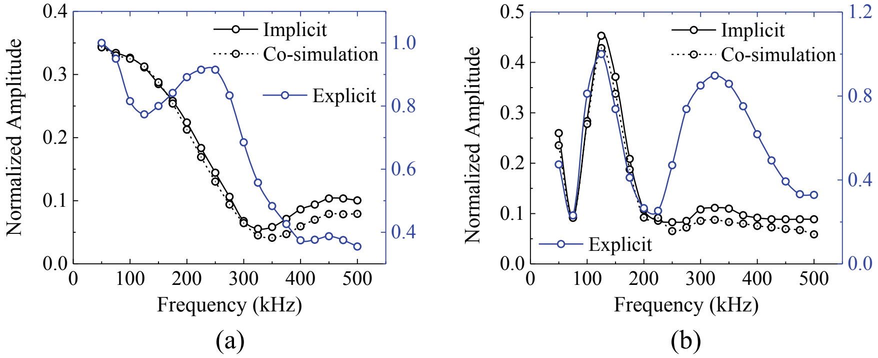

Fig. 9 Predicted variation of amplitude with excitation frequency **a** S0 mode, **b** A0 mode

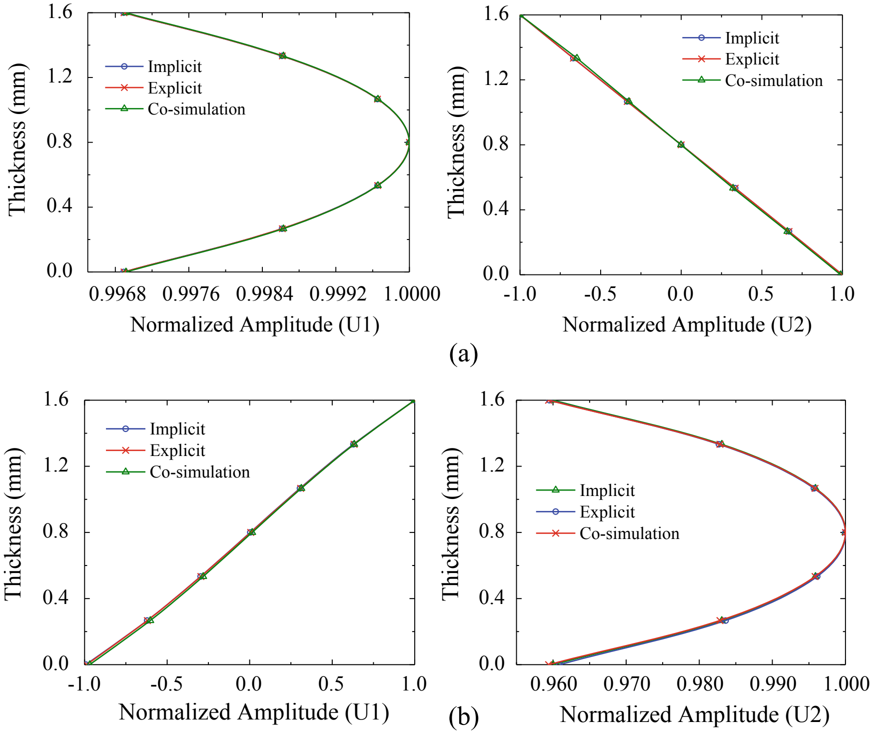

Fig. 10 Mode shapes of wave modes at 125 kHz **a** S0 mode, **b** A0 mode

y-direction is antisymmetric. For A0 mode, the displacement along *x*-direction is antisymmetric with respect to the midplane, while the displacement along *y*-direction is symmetric.

4 Conclusions

The study presents a comparison of three different analysis procedures models, namely implicit, explicit, and co-simulation method for wave propagation in plates. All the three models are implemented in software package Abaqus. The elastic waves were excited using piezoelectric transducers. In the co-simulation model, it combines an implicit model, which includes piezoelectric finite elements and explicit model for wave propagation. The results show that the co-simulation model is more reliable and efficient compared to other models. The implicit model is found to be unsuitable for modeling wave propagation as it shows considerable time delay. The computational time required for the analysis is also higher for implicit models. The explicit model is found to be cost-effective compared to the other two models. However, the magnitude of displacement is found to be higher in this case. The maximum percentage difference obtained is 80%. This can be modified by an improved actuator loading condition.

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