Performance of MIMO System—A Review

Sweta Sanwal, Aman Kumar, Md. Arib Faisal, and Mohammad Irfanul Hassan

Abstract The present communication system demands high data rate, spectral efficiency, and reliability. By employing numerous antennas in transmitter and receiver sides of a wireless channel, the spatial multiplexing or diversity gains can be explored. The modern communication network can be designed to attain a high data rate, enhanced link reliability, and improved range. MIMO technique can increase spectral efficiency without using extra bandwidth. This paper reviews recently published results on MIMO—Multiple Input Multiple Output. This paper describes the BER performance using Alamouti Space-Time Block Code and Average Channel Capacity has been discussed for different antenna system, i.e., SISO—Single Input Single Output, SIMO—Single Input Multiple Output, MISO—Multiple Input Single Output, and MIMO—Multiple Input Multiple Output systems under Rayleigh and Rician fading conditions. The simulated BER of MIMO has been compared with its theoretical result and with all other antenna configuration systems. Finally, the Average Channel Capacity for all the systems is analyzed and simulated under both Rayleigh and Rician Fading Channels.

Keywords Alamouti scheme · BER · Rayleigh channel · Rician channel · Space-time block coding · Space-time coding · Space-time trellis coding · Multiple input multiple output · Multiple input single output · Single input multiple output · Single input single output

---

S. Sanwal · A. Kumar · Md. Arib Faisal · M. I. Hassan (✉)
Department of Electronics and Communication Engineering, Graphic Era (Deemed to be University), Dehradun, Uttarakhand 248002, India
e-mail: irfu_hasan@rediffmail.com

S. Sanwal
e-mail: sawalsweety98@gmail.com

A. Kumar
e-mail: amankumar.8651@gmail.com

Md. Arib Faisal
e-mail: arib.faisal@gmail.com

G. S. Hura et al. (eds.), *Advances in Communication and Computational Technology*, Lecture Notes in Electrical Engineering 668,
https://doi.org/10.1007/978-981-15-5341-7_50

1 Introduction

The rapid growth of the next-generation of wireless mobile communication systems demands high-speed facilities by a large number of potential users [1]. Because of three particular limitations of wireless mobile communication systems, such as compounded and bleak channels, deficient usable radio spectrum, and limitation of the power and size of hand-held terminals [2], makes a challenging issue to fulfill the demand of high-speed facility. Therefore, to reduce those constraints [2], efficient spectral, and power fading alleviation techniques are required. MIMO antenna techniques provide the required spectral efficiency and communication reliability [3]. However, the MIMO system implementation increased the cost and hardware requirements [4]. To solve the implementation complexity of MIMO antenna system with retaining all its benefits, there are practical and effective antenna techniques used called antenna selection (AS) approach [3, 4]. The basic concept of AS is to choose an optimal set of well-organized transmit and/or receive antennas [5]. This is accomplished by using channel state information (CSI) feedback which maximizes spectral efficiency and improves the error performance in wireless mobile system networks [5]. By utilizing a number of antennas at both the transmitter and receiver ends of wireless mobile channels is to explore and analyses the spatial multiplexing or diversity gain, today mobile communication system can be designed to attain a high data rate, improved channel reliability, and range [6]. One important feature of MIMO technique is to provide increased spectral efficiency without using extra bandwidth. In fact, MIMO technique is compulsory by many wireless communication standards like IEEE 802.11n (WLAN), 802.16e (WiMAX), LTE (cellular), and other emerging applications [4].

In Fig. 1, MIMO system is shown with M_T transmitting antennas and M_R receiving antennas. After applying appropriate operation on the transmitter side, the input data is sent by using M_T antennas. The applied operation may include channel coding, modulation, space-time-encoding, spatial mapping [3]. Wireless mobile channel is used by each antenna to sends a signal. All antennas at left-hand side of Fig. 1 used

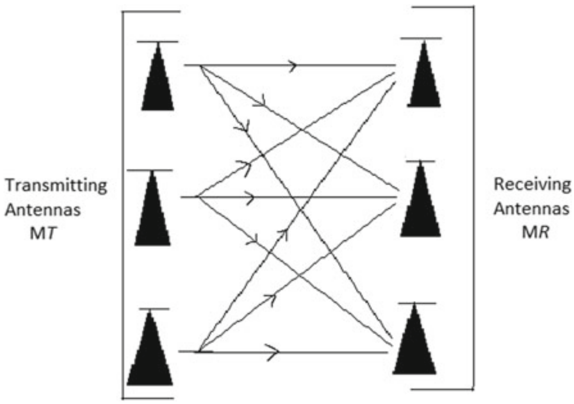

Fig. 1 A typical MIMO antenna configuration

as an entire transmitter. The radiated signals are shown by the column vector (x) that has $M_T \times 1$ dimensions. These radiated are collected by signals, M_R receiving antennas after passing through the wireless mobile channels [6].

The reminder of this paper is organized as follows. In Sect. 1.1, MIMO system is discussed. In Sect. 1.2, a different antenna configuration is defined. In Sect. 1.3, STC in MIMO system is defined. Whereas in Sect. 1.4, Alamouti Scheme for 2 × 1 MISO system, 2 × 2 MIMO system, BER performance, and Channel Capacity are derived and analyzed. Finally, the result analysis has been carried out and the conclusion has been drawn.

1.1 MIMO System Model

A typical MIMO configuration is shown below in Fig. 2. MIMO configuration is represented in space domain [6]. Let us suppose that a MIMO communication system has N_t transmit antennas and N_r receive antennas. This results in $N_t \times N_r$ different channels in between transmitter in receiver. A wireless channel in multipath environment may correspond to a complex Gaussian random variable. The MIMO system channel between N_t transmit and N_r receive antennas can be depicted as a $N_r \times N_t$ complex Gaussian random matrix, indicated by H [5]. A complete understanding of MIMO system channel model is essential to appropriately design and estimate the working of a wireless communication system using MIMO. Practically the MIMO channels are triply selective, i.e., a MIMO channel may exhibit fading across space, time, and frequency [7].

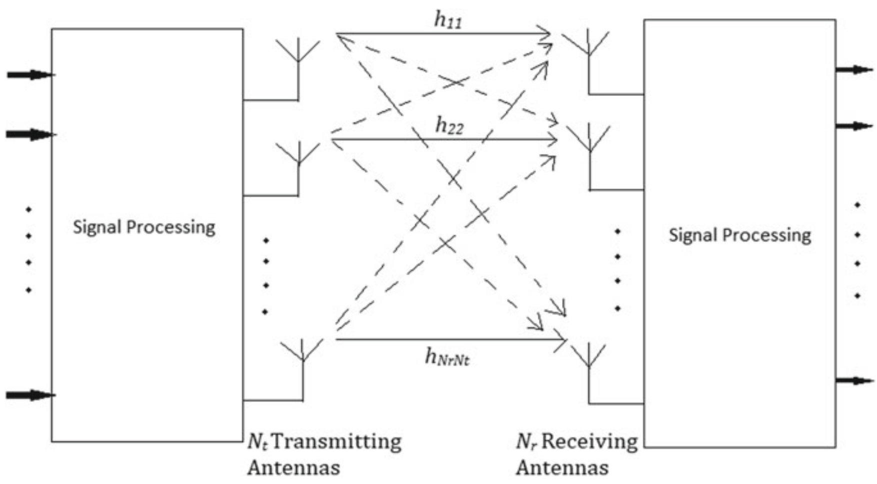

Fig. 2 A typical MIMO configuration

The input and output relation of a MIMO structure, assuming a time-invariant channel can be represented in vector notation as:

$$y = H + n \tag{1}$$

Here x is $(N_t \times 1)$ transmit vector, y is $(N_r \times 1)$ receiving vector, H is $(N_r \times N_r)$ channel gain matrix, and $n(N_r \times 1)$ is called Additive White Gaussian Noise (AWGN) vector. Assuming frequency flat fading (which implies that the MIMO channel is static and deterministic for the given time interval), a MIMO channel gain matrix for N_t transmit and N_r receive antennas could be represented as [6]:

$$H = \underbrace{\begin{bmatrix} h_{11} & \cdots & h_{1N_t} \\ \vdots & \ddots & \vdots \\ h_{N_r1} & \cdots & h_{N_rN_t} \end{bmatrix}}_{N_t \text{transmit antennas}} N_r \text{ receive antennas} \tag{2}$$

Generally, a single element of $(N_r \times N_t)$ channel matrix is represented by h_{ij} $(i = 1, 2 \ldots N_r$ and $j = 1, 2 \ldots N_t)$ which represents a complex channel gain between the jth sending and ith collecting antenna. The channel gains of $N_r \times N_t$ elements in MIMO channel matrix H are the function of characteristics of the propagation environment and antenna spacing in the transmitter and receiver (i.e., array characteristics) [5].

Let us say all the transmitter has an average power constraint P, overall the transmit antennas, and noise power is supposed unity (making power equivalent to SNR). After passing via the MIMO channel, at each receiver antenna, the signal that is received is a superposition of the N_t received signals (product of transmitted signals and channel gain coefficients), in addition to noise introduced in the channel. Then the received signal at antenna i can be written as [5]:

$$y_i = \sum_{j=1}^{N_t} h_{ij} x_j + n_i \quad i = 1, 2, \ldots N_r \text{ and } j = 1, 2, \ldots N_r \tag{3}$$

The term n_i denotes the additive complex channel noise added by the channel. Following the discrete-time model, it can be illustrated in matrix notation as:

$$\begin{bmatrix} y_1 \\ \vdots \\ y_{N_r} \end{bmatrix} = \begin{bmatrix} h_{11} & \cdots & h_{1N_t} \\ \vdots & \ddots & \vdots \\ h_{N_r1} & \cdots & h_{N_rN_t} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_{N_r} \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_{N_r} \end{bmatrix} \tag{4}$$

The above matrix can be written in vector notation as Eq. (1).

After reception, the signal is decoded to estimate the sent data. Generally, the maximum likelihood (ML) decoding algorithm is applied, which uses a decision metric that is based on the squared Euclidean distance between the received sequence and the actual sequence. It selects a code with a minimum value of the metric to determine the transmitted data. This concept is discussed previously in the paper of Sivash M. Alamouti [8].

1.2 Different Antenna Configurations

There are various antenna layouts can be applied to define space-time systems. Traditionally, a transmitter and a receiver have a sole antenna (i.e., $N_t = N_r = 1$), recognized as Single Input Single Output (SISO) system [6]. Another system, known as Single-Input-Multiple-Outputs (SIMO) has a sole antenna at the transmitter ($N_t = 1$) and N_r, a number of antennas at the receiver [6]. In the Multiple Input Single Output (MISO) system, the transmitter has N_t antennas, and receiver have a sole antenna ($N_r = 1$). On the other hand, MIMO uses multiple antennas at both ends [6]. Figure 3 shows four different antenna configurations. SISO is severely affected by multipath propagation, which increases error probability [6]. SIMO enables receiver diversity, where a suitable combining technique is used at the receiver to combat signal fading [6]. Use of combining technique requires the concept of the Channel State Information (CSI) at the receiver. MISO enables beamforming, which aims to focus transmission powers from different antennas in the desired direction [6]. Beamforming requires knowledge of the CSI at the transmitter [6].

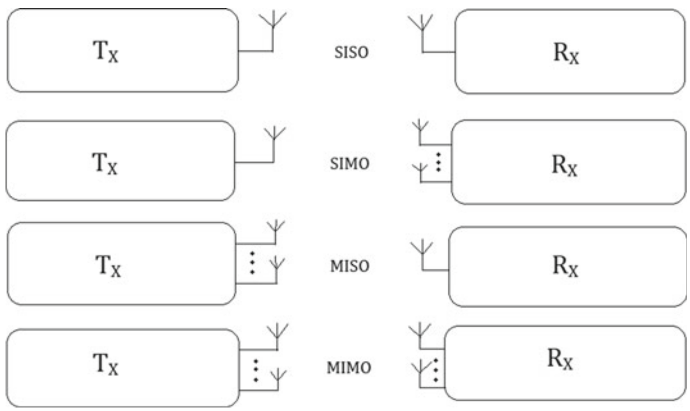

Fig. 3 Different antenna configuration

1.3 Space-Time Coding (STC) in MIMO

MIMO system offers a much higher capacity than a conventional system. The capacity increase in MIMO is obtained by appropriate coding in space and time domain before transmission, called time coding (STC) [9]. In STC, at first, the input data stream (i.e., source data bits and error correction bits) is split into a number of sub-streams, and then each sub-stream is mapped onto a sender antenna [6, 9]. In this way, a signal is encoded in space and time.

In STC, at each instant, a block of m data bits are given into the space-time encoder. The m data bits make a set of $M = 2^m$ symbols [9]. The space-time encoder maps these symbols on N_t transmitting antennas. Say the symbol in each antenna is denoted by as x_j, where $j = 1, 2 \ldots . N_t$ then the transmission code vector is $x = [x_1, x_2, \ldots, x_{Nt}]$. A serial to parallel converter, i.e., Multiplexer converts the incoming data stream to $N_t \times 1$ column vector, to be transmitted simultaneously by N_t transmit antennas [9]. These parallel data streams pass through the MIMO channel matrix, where each individual channel may have an independent channel gain coefficient [9].

In STC, the code matrix is designed such that the rows and columns are orthogonal to each other [9]. This yields that the inner product of each row with any other row results to zero, and therefore the rows of the matrix are independent eigenvalues, helping to realize full transmit diversity [9]. Full transmit diversity implies that each transmit antenna contributes to one row in the matrix (this is called full rank matrix) [10]. The orthogonality enables the decoupling of the various signals transmitted from different antennas at the receiver [10, 11]. This permit using simple ML-based decoding, using linear processing at the receiver, which simplifies the reception process [10, 11].

CSI is not required by STC at the transmitter, thus simplifies the transmission process. STC can be easily combined with channel coding, offering coding gain furthermore spatial diversity gain [9]. STC facilitates MIMO to realize significant improvements in error rate performance (Compared to SISO), and therefore enables to minimize outage probability (or equivalently maximize outage capacity) [9]. As a result, in a few years only, STC has progressed from invention to adoption in major wireless standards [9]. Figure 4 shows a MIMO communication system where STC is used as a part of it [9].

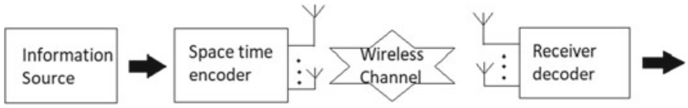

Fig. 4 STC in MIMO

1.4 STC and Pre-coding

STC and pre-coding are two different encoding concepts in MIMO. STC assumes no knowledge of CSI at the transmitter; on the other hand, pre-coding essentially needs knowledge of CSI at the transmitter side [6]. In fact, knowledge about CSI at the transmitter antenna makes a significant difference in system performance, by enabling the transmitter to adapt appropriately the power and rate of data in accordance with channel states [9, 10]. This concept is used in pre-coding [9, 10]. However, STC simplifies implementation by avoiding the need for CSI [9, 10]. The STC is an open-loop approach, whereas pre-coding is a closed-loop approach [9, 10].

Pre-coding is different from beamforming also; however, both require knowledge of CSI at the transmitter [10]. Beamforming offers a well-defined directional beam pattern, which maximizes the received signal power [10]. However, just as multiple antennas are present at the receiver, the received signal power cannot be maximized by beamforming at all the receiver antennas simultaneously [10]. Then pre-coding is required. Pre-coding can be combined along with spatial multiplexing to increase the rate of data performance [11]. It can be mixed with spatial diversity to enhance the reliability of decoding [11]. Pre-coding has been successfully implemented in the IEEE 802.16e standard for broadband WMAN networks [11].

1.5 Spatial Multiplexing and Spatial Diversity

STC uses two different approaches to improve MIMO system performance, which are the spatial multiplexing (SM) and spatial diversity (SD) [5]. SM aims to enhance transmission data rate (in fact, spectral efficiency) and SD aims to enhance transmission reliability [5]. These are two important motivations for using MIMO [5].

1.5.1 Spatial Multiplexing (SM)

Intuitively, if a receiver can differentiate between two streams (using STC), then it can also differentiate between two streams carrying different data [6]. In SM, various data signals are sent over N_t transmitting antennas [6]. It may correspond to a situation, where a high rate of data stream is bifurcated into several lower rate of data streams, and then are sent parallelly through different antennas [6]. At the receiver, these parallel streams are combined to acquire the indigenous rate of data [6]. Therefore, it makes it possible to realize a high data rate performance [8]. The SM gain obtained, simply indicates the increased data rate over the entire given bandwidth (i.e., the upgraded spectral efficiency) and is represented as a function of

SNR (at a specified BER) [8]. The maximum SM gain ($r_{\max}$) is represented as a ratio between the spectral efficiency (S) at given SNR to the logarithmic value of SNR when the SNR is assumed to be asymptotically high, as given below [8]:

$$r_{\max} = \lim_{\mathrm{SNR}\to\infty} \frac{S(\mathrm{SNR})}{\log_2 \mathrm{SNR}} \tag{5}$$

It is essential that to reliably separate the streams of data received, the number of collecting antennas must be at least same as the number of transmitting antennas ($N_r \geq N_t$) [6]. SM is considered to be a powerful technique to enhance Channel Capacity in high SNR conditions [6]. It can be utilized to offer a high rate of data to the users near the base station (where SNR is high). SM could be utilized with or without CSI at the transmitter [6].

1.5.2 Spatial Diversity (SD)

SD does not aim to enhance data rate; rather it aims to enhance the reliability of communication made across the fading channel [10]. In SD, a stream of data is sent through all the transmitter antennas [10]. This contrasts with SM, where different data streams are sent through different antennas [10]. SD employs orthogonal or near orthogonal coding using STC [10]. At the receiver end, duplicate of the same stream of data are collected and mixed to yield improved SNR [10]. This improves reliability and the gain so realized is called SD gain [10]. The negative of the maximum SD gain ($-d_{\max}$) is a ratio of the log of the probability of error (P_e at a given SNR) to the log of SNR when the SNR is assumed to be asymptotically high, as given below [10]:

$$-d_{\max} = \lim_{\mathrm{SNR}\to\infty} \frac{\log P_e(\mathrm{SNR})}{\log \mathrm{SNR}} \tag{6}$$

In the above expression, the log can be of any base (since it cancels out as the ratio of the two logs having the same base). SD is used when CSI is not present at the transmitter.

1.6 STTC and STBC

STC (Space-Time Coding) can be classified into two parts which depend on the way of transmission of signal in the wireless channel [6]:

- STTC (Space-Time Trellis Coding)
- STBC (Space-Time Block Coding).

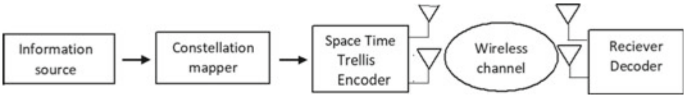

Fig. 5 STTC in MIMO

1.6.1 STTC

It is an extended version of conventional Trellis codes to MIMO systems [11]. In STTC, the symbols are transmitted serially, encoding is done in the transmitter end and the processing of signal is done in the receiver end [11]. Significant diversity and coding gains over fading channels are realized by STTC [11]. The delay diversity can be considered as a simple form of STTC because in delay diversity same code is transmitted from N_t transmit antennas and viewed as $1/N_t$ repetition code [11]. To combat fading, error control coding, and diversity scheme (joint design of modulation scheme) are performed in STTC to outline an effective signaling scheme [11]. ML sequence estimation is used by STTC via the Viterbi algorithm for decoding at the receiver end. Figure 5 shows STTC in MIMO [11].

1.6.2 STBC

It transmits data in blocks and involves three design parameters [6]:

- N_t (number of transmitter antennas which defines the transmission matrix size).
- K (number of transmitted symbols per time slot).
- T (number of time slots used to transmit one block of data or encoded symbols).

In STBC, blocks are nothing but the divided data streams [10]. A block is transmitted over T time slots in STBC [10]. During each and every time slot K symbols are encoded and then transmitted parallelly using N_t transmitters which develops a transmission matrix S of size $N_t \times T$ as [10]:

$$\begin{bmatrix} s_{11} & \cdots & s_{1N_t} \\ \vdots & \ddots & \vdots \\ s_{T1} & \cdots & s_{TN_t} \end{bmatrix}$$

Here s_{ij} is encoded symbol which is sent in time slot "i" from "j" transmitter antenna where $i = 1, 2, 3, \ldots T$ and $j = 1, 2, \ldots N_t$ [10]. STBC assumes N_r receiver antennas and can be designed to exploit full diversity order (N_r x N_t), but it is not designed for full diversity order, rather be able to reduce fading effectively it is designed for a sufficiently high diversity order [10, 11]. Channel State Information (CSI) is not required at transmitter [10].

STBC could use a square transmission matrix (complex orthogonal design) which satisfies the conditions of orthogonality both in time and space [10]. It can also

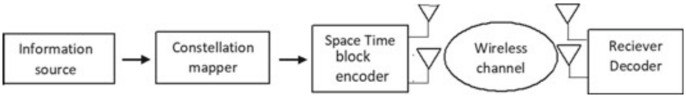

Fig. 6 STBC in MIMO

use non-orthogonal design which has a non-square transmission matrix that satisfies orthogonality only in time, not in space [10]. Using simple linear processing symbols can be detected at the receiver in STBC [6]. In STBC same data is transmitted through different antennas so it can be viewed as repetition code over space and time. Figure 6 shows STBC in MIMO [6].

1.6.3 Comparison Between STBC and STTC

STTC offers both diversity as well as coding gain whereas STBC provides only diversity gain [6]. STBC gives a lower performance as compared to STTC [6]. STBC is less complex in implementing [6]. STBC requires simple decoding at the receiver end to retrieve data whereas STTC requires complex decoding techniques to retrieve data at the receiver end [6].

1.7 Alamouti Scheme

The Alamouti scheme provides a simple transmit diversity technique that uses space-time coding [8]. Two transmit antennas and a single receiver antenna are used to achieve transmit diversity when CSI is not available at the transmitter [8]. This is the simplest form of STBC. Complex orthogonality is satisfied with a square transmission matrix in the space domain as well as time domain [8]. Alamouti STBC gives STBC with rate one and offers full diversity gain without compromising with data rate performance. The Alamouti code matrix is [8]:

$$S = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \text{ space} \rightarrow$$
$$\text{Time} \downarrow \tag{7}$$

S is a complex orthogonal matrix known as transmission matrix, where * denotes complex conjugate [8]. There are two-time slots used to send two symbols s_1 and s_2 using two antennas [8]. Since, there are two-time slots that are needed for sending two symbols, using two antennas, thus $k = 2$ and $T = 2$. Hence, the code rate is 1. It is supposed that the gain of channel remains unchanged over the same time interval [8].

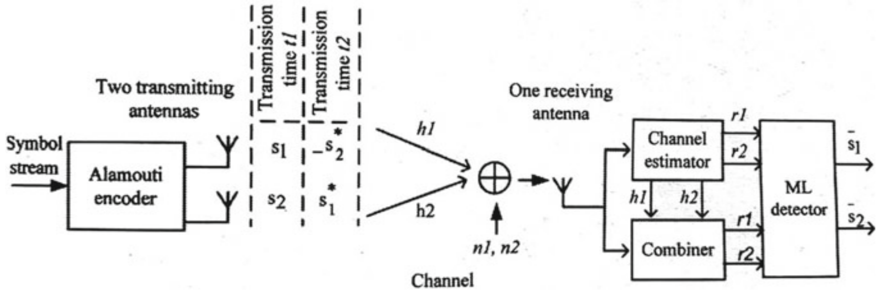

Fig. 7 An illustration of the Alamouti scheme

In code matrix, row at the first represents the first transmission period, where s_1 is sent by first antenna and s_2 is sent by second antenna [8]. Row at the second represents second transmission period, where $-s_2^*$ is sent by first antenna and the s_1^* is sent by second antenna. This implies that the transmission takes place in space (using two antennas) as well as time (at two-time instant) [8]. The information sequence received by first antenna is $[s_1, -s_2^*]$ and by second antenna is $[s_2, s_1^*]$ [8]. This satisfies the condition of orthogonality, both in space and time domain [8, 12]. It is inferred that the two completely orthogonal streams are collected by the receiver [12], giving transmit diversity of two. The approach used in the Alamouti scheme is shown in Fig. 7.

Let us assume, $h_1(t)$ and $h_2(t)$ represents the fading coefficients, respectively from antenna 1 and antenna 2. It is supposed that the fading coefficients are constant during the symbol interval, then [12]:

$$h_1(t) = h_1(t+T) = h_1 = |h_1|e^{j\theta_1} \tag{8}$$

And

$$h_2(t) = h_2(t+T) = h_2 = |h_2|e^{j\theta_2} \tag{9}$$

Here, T denotes the symbol duration, amplitude gains, and phase shifts, respectively, are $|h_i|$ and θ_i (for $i = 1, 2$). During the first and the second symbol periods the received signals are r_1 and r_2 after passing through the channel that can be expressed as [12]:

$$r_1 = h_1 s_1 + h_2 s_2 + n_1 \tag{10}$$

$$r_2 = -h_1 s_2^* + h_2 s_1^* + n_2 \tag{11}$$

where n_1 and n_2 are independent complex variable AWGN samples having zero mean and unit variance that are being added to the transmitted signal during the interval of transmission [12]. Euclidian distance between the received symbol and the possible transmitted symbol are estimated after which the received signals are passed through the ML detector [12]. The decision rule is used to identify the symbol which is detected is the symbol which has the minimum Euclidian distance is identified as the transmitted symbol [12].

1.8 Alamouti STBC (2 × 1 System Model)

A simple method for achieving spatial diversity with two transmit antennas which are described as A simple transmit diversity presented in the paper of Alamouti [7] and in [13]. For the channel model, refer to Sect. 1.4. The channel gain matrix is shown in Table 1. And the antenna configuration is shown in Fig. 8.

1.8.1 Alamouti STBC Receiver

The signal received in the first time slot is:

$$y_1 = h_1 x_1 + h_2 x_2 + n_1 = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1 \tag{12}$$

The signal received in the second time slot is:

$$y_2 = -h_1 x_2^* + h_2 x_1^* + n_2 = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} -x_2^* \\ x_1^* \end{bmatrix} + n_2 \tag{13}$$

where:

1. y_1, y_2 is the symbol received, respectively, in the first and second time slot.

Table 1 2 × 1 Alamouti channel model

	Time t	Time $t + T$
Antenna 1	x_1	$-x_2^*$
Antenna 2	x_2	x_1^*

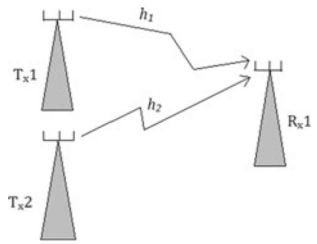

Fig. 8 2 × 1 Alamouti STBC

2. h_1, h_2 is the gain of channel by first and second transmitting antennas.
3. x_1, x_2 are the transmitted symbols.
4. n_1, n_2 is the noise in 1st and 2nd-time slots, noise terms are identically and identically distributed.

$$E\left\{\begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}\begin{bmatrix} n_1^* \ n_2 \end{bmatrix}\right\} = \begin{bmatrix} |n_1|^2 & 0 \\ 0 & |n_2|^2 \end{bmatrix} \tag{14}$$

Equation 15 can be depicted in matrix as:

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix} \tag{15}$$

where, $H = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}$, then we want to solve for $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, for this the inverse of H would be found.

And the pseudo-inverse for a $m \times n$ matrix is:

$$H^+ = \left(H^H H\right)^{-1} H^H \tag{16}$$

$$\left(H^H H\right) = \begin{bmatrix} h_1^* & h_2 \\ h_2^* & -h_1 \end{bmatrix}\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} = \begin{bmatrix} |h_1|^2 + |h_2|^2 & 0 \\ 0 & |h_1|^2 + |h_2|^2 \end{bmatrix} \tag{17}$$

Since, inverse of a diagonal matrix (Eq. 17) is just the inverse of diagonal elements that is:

$$\left(H^H H\right)^{-1} = \begin{bmatrix} \frac{1}{|h_1|^2+|h_2|^2} & 0 \\ 0 & \frac{1}{|h_1|^2+|h_2|^2} \end{bmatrix} \tag{18}$$

Approximately, the transmitted symbol is:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \left(H^H H\right)^{-1} H^H \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} \tag{19}$$

$$= \left(H^H H\right)^{-1} H^H \left(H \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}\right)$$

$$= \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \left(H^H H\right)^{-1} H^H \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}$$

1.9 *Alamouti STBC (2 × 2 System Model)*

By getting some motivation from Alamouti 2 × 1 channel model, our further work is on 2 × 2 (a simple transmit and receiver diversity) which provides spatial diversity and less BER [14, 15]. For the channel model, refer to Sect. 1.4. The channel gain matrix is same as the 2 × 1 model and is shown in Table 1. And the antenna configuration is shown in Fig. 9.

1.9.1 Receiver in Alamouti 2 × 2

The signal received in the first time slot,

$$\begin{bmatrix} y_{11} \\ y_{12} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_{11} \\ n_{12} \end{bmatrix} \tag{20}$$

The signal received in the second time slot,

$$\begin{bmatrix} y_{21} \\ y_{22} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} -x_2^* \\ x_1^* \end{bmatrix} + \begin{bmatrix} n_{21} \\ n_{22} \end{bmatrix} \tag{21}$$

where

y_{ij} = received signal in ith time slot by jth antennasx_1, x_2 = modulated symbolsh_{ij} = channel gainn_{ij} = AWGN noise during ith time slot.

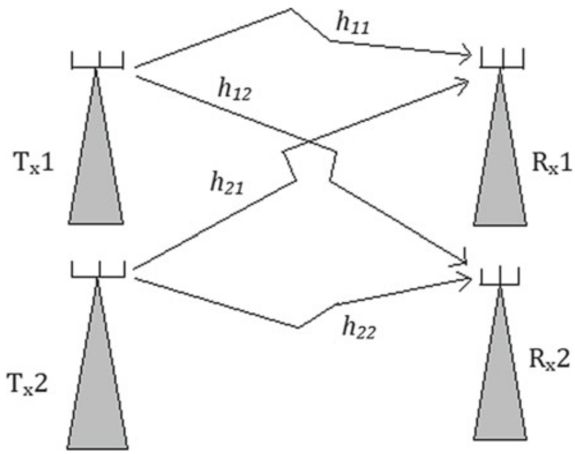

Fig. 9 Alamouti 2 × 2 STBC

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{12}^* & -h_{11}^* \\ h_{22}^* & -h_{21}^* \end{bmatrix}$$

Then, we want to solve for $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, for this, we would have to get H inverse. It is known that the pseudo-inverse for a $m \times n$ matrix is:

$$H^+ = (H^H H)^{-1} H^H \tag{22}$$

$$(H^H H) = \begin{bmatrix} |h_{11}|^2 + |h_{21}|^2 + |h_{12}|^2 + |h_{22}|^2 & 0 \\ 0 & |h_{11}|^2 + |h_{21}|^2 + |h_{12}|^2 + |h_{22}|^2 \end{bmatrix} \tag{23}$$

Since, inverse of a diagonal matrix (Eq. 23) is just the inverse of diagonal elements that is:

$$\left(H^H H\right)^{-1} = \begin{bmatrix} \frac{1}{|h_{11}|^2+|h_{21}|^2+|h_{12}|^2+|h_{22}|^2} & 0 \\ 0 & \frac{1}{|h_{11}|^2+|h_{21}|^2+|h_{12}|^2+|h_{22}|^2} \end{bmatrix} \tag{24}$$

Approximately, the transmitted symbol is:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \left(H^H H\right)^{-1} H^H \begin{bmatrix} y_{11} \\ y_{12} \\ y_{21}^* \\ y_{22}^* \end{bmatrix} \tag{25}$$

1.10 BER Calculation

In this paper, simulation of BER performance of MISO (2 × 1 Model) and MIMO (2 × 2 Model) has been brought for BPSK modulation under Rayleigh as well as Rician channel.

The theoretical BER of MRC system [15, 16] is given below:

$$P_{e,\mathrm{MRC}} = p_{\mathrm{MRC}}^2[1 + 2(1 - p_{\mathrm{MRC}})] \tag{26}$$

where,

$$p_{\text{MRC}} = \frac{1}{2} - \frac{1}{2}\left(1 + \frac{1}{E_b/N_0}\right)^{-1/2} \tag{27}$$

And BER for STBC case, i.e., two transmitters with two receivers or one receiver is given below:

$$P_{e,\text{STBC}} = p_{\text{STBC}}^2[1 + 2(1 - p_{\text{STBC}})] \tag{28}$$

where,

$$p_{\text{MRC}} = \frac{1}{2} - \frac{1}{2}\left(1 + \frac{1}{E_b/N_0}\right)^{-1/2} \tag{29}$$

1.11 Channel Capacity Calculation

1.11.1 SISO Channel Capacity

The SISO system has gain of channel h, therefore, signal to noise ratio at the receiving antenna, then without knowing the CSI the capacity is [17]:

$$C = \log_2\left(1 + \text{SNR}|h|^2\right) \text{ Bits} \tag{30}$$

And the theoretical capacity of this system will be:

$$C_{\text{Theoritical}} = \log_2\left(1 + \text{SNR} \cdot E\left(|h|^2\right)\right) \tag{31}$$

Or $E\left(|h|^2\right) = 1$
Thus:

$$C_{\text{Theoritical}} = \log_2(1 + \text{SNR}) \tag{32}$$

1.11.2 SIMO Channel Capacity

SIMO system has a transmitting antenna and M receiving antenna. the complex gain between the transmit antenna is h_i and the ith receiver, then the Channel Capacity of the system is [17]:

$$C = \log_2\left(1 + \mathrm{SNR} \cdot \sum_{i=1}^{M} |h|^2\right) \tag{33}$$

And $\sum_{i=1}^{M} |h|^2 = M^2$, its Shannon capacity is given by:

$$C_{\text{Theoritical}} = \log_2\left(1 + \mathrm{SNR} \cdot M^2\right) \tag{34}$$

1.11.3 MISO Channel Capacity

Channel Capacity of MISO system having M_T transmitting antenna and a receiver antenna is given by:

$$C = \log_2\left(1 + \mathrm{SNR}|h|^2/M_T\right) \tag{35}$$

1.11.4 MIMO Channel Capacity

In the MIMO system, multiple transmitting and receiving antennae are used. N transmitting and M receiving antennas are connected using a wireless link called MIMO channel. It contains $N \times M$ MIMO channel coefficients. For MIMO system matrix is H (refer Eqs. (4) and (5)) [17].

Where the complex gain of channel is h_{ij} between the jth transmitting antenna and ith receiving antenna:

$$C = \log_2\left(\det\left[I_M + \frac{\mathrm{SNR}}{N}\mathrm{HH}^H\right]\right) \tag{36}$$

where "det" is determinant, I_M depicts $N \times M$ identity matrix and H^H is transposed conjugate of a matrix [17].

2 Numerical and Result Analysis

In this part, we have discussed the simulation result for BER performance of 2 × 1 Alamouti scheme model with the theoretical 2 × 1 Alamouti model, 1 × 1 model (with no diversity) and 1 × 2 theoretical MRC system under Rayleigh as well as Rician Fading Channels. Then, the simulation result for BER performance of 2 × 2 system model with the theoretical 2 × 1 Alamouti model, 1 × 1 model (with no diversity) and 1 × 2 theoretical MRC system under Rayleigh as well as

Rician Fading Channels. This simulation is being done in MATLAB using a digital modulation scheme called as BPSK (Binary Phase Shift Keying) with Zero Forcing (ZF) Equalizer using MATLAB. The values of BER as a function of Eb/No (dB) has been found for all other antenna configuration models and compared with each other under Rayleigh as well as Rician Fading Channels.

Finally, the Channel Capacity of all antenna configuration systems (SISO, SIMO, MISO, MIMO) has been determined. The Average Channel Capacity per unit Bandwidth values as a function of SNR—Signal to Noise Ratio in dB scale has been determined and compared under Rayleigh as well as Rician Fading Channels.

In Figs. 10 and 11, we have simulated the BER curve for 2 × 1 system model, and the rest curves were plotted using the theoretical values of BER. 1 × 2 MRC system has the very lowest BER with respect to other system models. Alamouti 2 × 1 model has a slightly higher BER than of 1 × 2 MRC system. 1 × 1 system model with no diversity has very poor BER performance. It can be found that BER of simulated Alamouti 2 × 1 model is better at higher average SNR (dB) in Rician condition than in Rayleigh condition.

In Figs. 12 and 13, we have simulated the BER curve for the 2 × 2 system model and the rest curves were plotted using the theoretical values of BER, where the ratio of bit energy to the noise power density is called as Eb/No. We can see that the 2 × 2 system model has the best BER performance than the other model. 1 × 2 MRC system has lower BER performance than that of the 2 × 2 model. 2 × 1 Alamouti scheme has lower BER performance than that of 1 × 2 MRC system. Finally, the 1 × 1 system has the poorest BER performance.

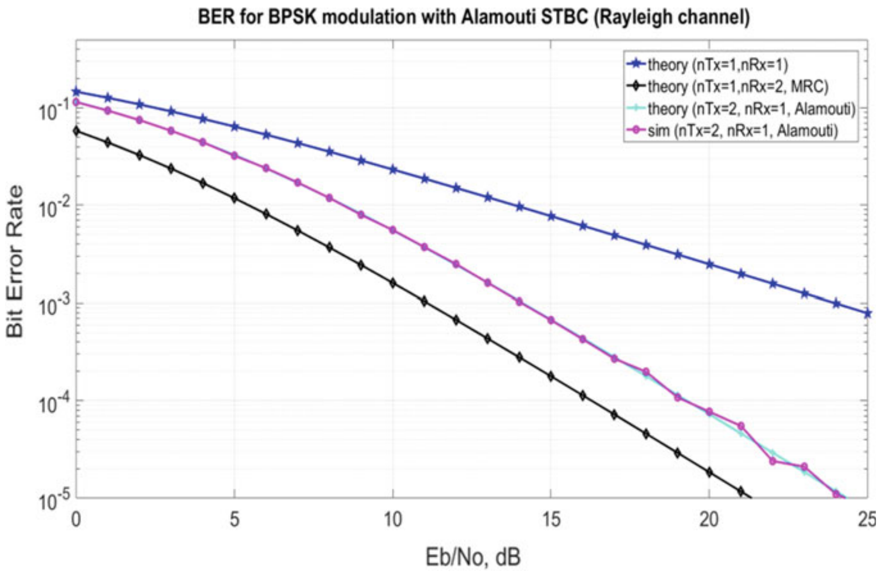

Fig. 10 Bit error rate versus Eb/No (dB) curve for simulation of 2 × 1 Alamouti Scheme under Rayleigh condition

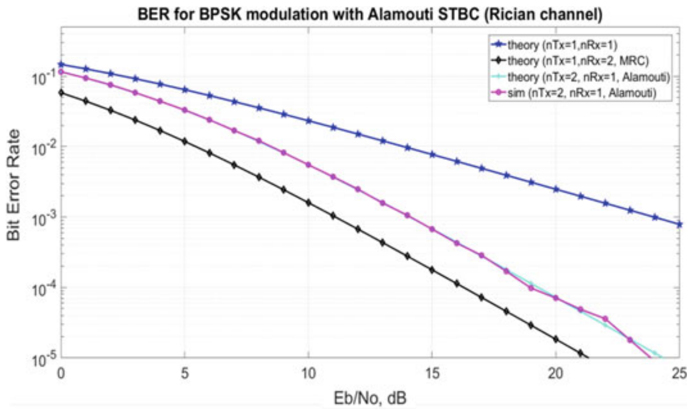

Fig. 11 Bit error rate versus Eb/No (dB) curve for simulation of 2 × 1 Alamouti scheme under Rician condition

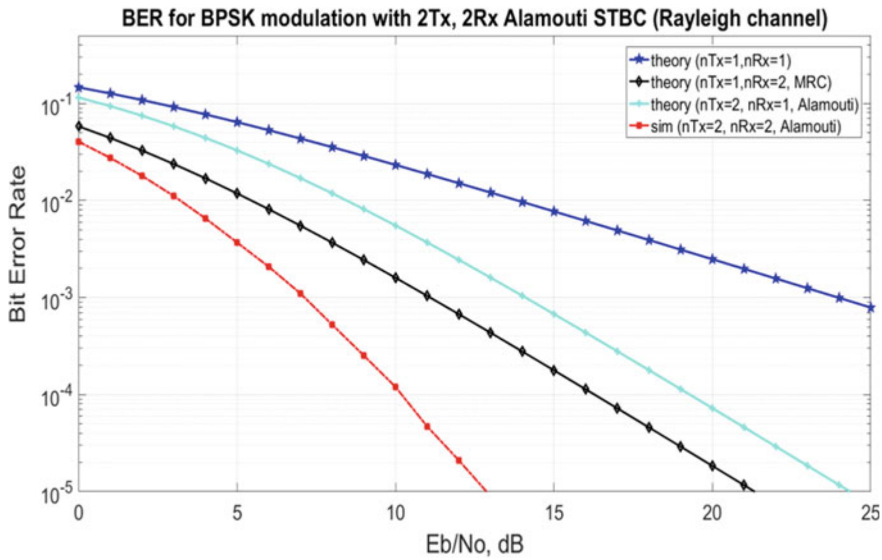

Fig. 12 Bit error rate versus Eb/No (dB) curve for simulation of 2 × 2 system model under Rayleigh condition

In Figs. 14 and 15, for various antenna configuration, i.e., SISO—one transmitter one receiver, SIMO—one transmitter and two receivers, MISO—two transmitters and one receiver and MIMO—two transmitters and two receivers, Average Channel Capacity per unit Bandwidth as a function of SNR is plotted under Rayleigh as well Rician condition. It can be found that the 2 × 2 system, i.e., MIMO has the highest capacity and 1 × 1 SISO has lowest Channel Capacity in both fading conditions. 1× 2 SIMO system has better capacity than the 2 × 1 system, i.e., MISO.

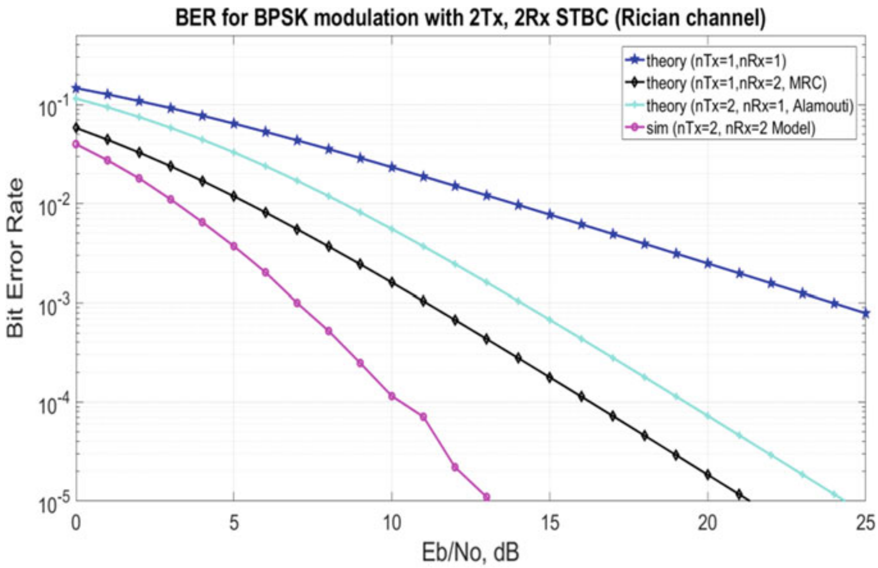

Fig. 13 Bit error rate versus Eb/No (dB) curve for simulation of 2×2 system model under Rician condition

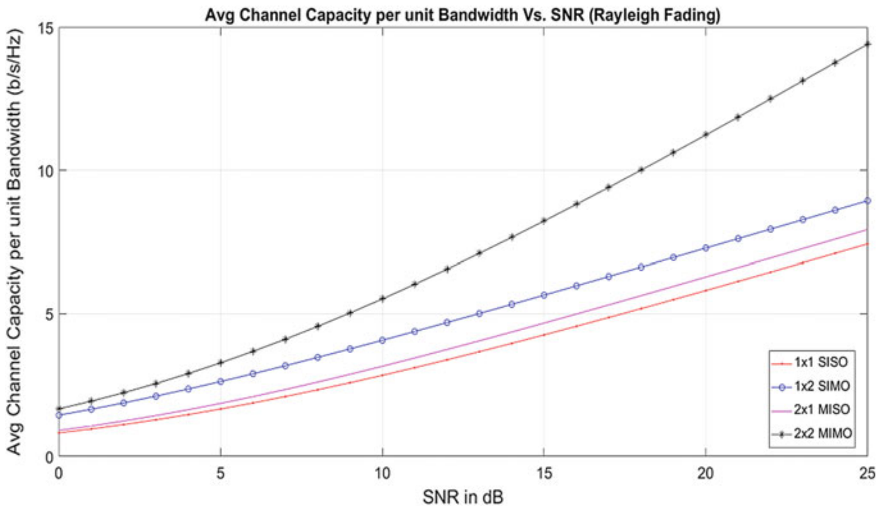

Fig. 14 Channel capacity curve for SISO, SIMO, MISO and MIMO system under Rayleigh fading

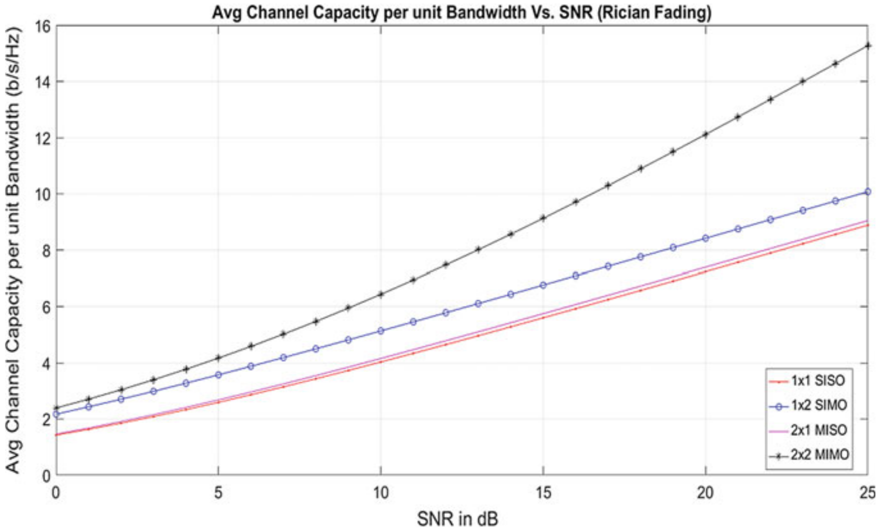

Fig. 15 Channel capacity curve for SISO, SIMO, MISO and MIMO system under Rician fading

As expected capacity in Rician fading condition is found to be better than Rayleigh fading condition as Rician has one line of sight signal. In Fig. 15, the plot is plotted for $k = 5$, where, k is Rician Factor which is defined by the ratio of the power of LOS components to the power of NLOS components.

On comparing Figs. 13 and 14, It was found that Average Channel Capacity per unit Bandwidth of 2×2 system, i.e., MIMO is 14.46 bits/s/hz and 15.39 bits/s/hz under Rayleigh and Rician fading respectively.

3 Conclusions

Through the BER performance of different antenna configuration systems, we found that the BER performance 1×1 system is worst. 1×2 theoretical MRC system has better BER performance than 2×1 system (Both simulated and theoretical) and 1×1 system in Rayleigh fading. At the same moment, in Rician fading all antenna configuration system has better performance than in Rayleigh fading condition. And when we move towards more antenna in the transmitter and receiver side, i.e., 2×2 MIMO system, it has better BER performance in comparison to 1×2 theoretical MRC system.

In terms of Channel Capacity, it increases if we increase the number of transmitting and receiving antenna. Thus, 2×2 MIMO has the best Channel Capacity. It can also be found that the Channel Capacity is more in Rician fading condition than Rayleigh fading condition.

References

1. Simon MK, Alouini MS (2002) Digital communication over fading channels—a unified approach to performance analysis, 1st edn. Wiley, USA
2. Molisch AF (2001) Wideband wireless digital communications. Pearson Education Inc (2001)
3. Goldsmith A (2005) Wireless communications. Cambridge University Press
4. Dalal U (2011) Wireless communications. Oxford University Press
5. Biglieri E, Calderbank R, Constantinides A, Goldsmith A, Paulraj A, Poor HV (2007) MIMO wireless communications. Cambridge University Press
6. Kumar S (2015) Wireless communication—the fundamental and advanced concepts. River Publishers, Denmark
7. Tarokh V (ed) (2009) New directions in wireless Communication research. Springer, Berlin
8. Alamouti SM (1998) A simple transmit diversity technique for wireless communications. IEEE J Sel Areas Commun 16(8)
9. Jankikaran M (2004) Space-time codes and MIMO systems. Artech House Inc, Boston, London (2004)
10. Luo T, Teng Y, Tong X-j, Yue G-x (2004) Space-time block codes design in OFDM system. J Beijing Univ Posts Telecommun 27
11. Tarokh V, Jafarkhani H, Calderbank AR (1999) Space-time block coding for wireless communications: performance results. IEEE J Sel Areas Commun 17(3)
12. Jiang M, Hanzo L (2007) Multiuser MIMO-OFDM for next-generation wireless systems. In: Proceeding of the IEEE, vol 95, issue no 7. IEEE
13. Lalitha Kumari D, Giri Prasad MN (2017) A review paper on performance analysis of MIMO based OFDMA system under fading channel. Int J Electr Commun Eng Technol (IJECET) 8(1):32–42
14. Bhagya R, Ananth AG (2012) Study of transmission characteristics of 2 × 2 Mimo system for OFDM multiplexing and Bpsk modulation with ZF equalizer and MMSE receivers. Int J Soft Comput Eng (IJSCE) 2(3)
15. Mindaudu AS, Miyim AM (2012) In: BER performance of MPSK and MQAM in 2X2 Alamouti MIMO systems. Int J Inf Sci Tech (IJIST) l&2(5)
16. Gupta S, Bhardwaj L, Anand A, Rajpoot AS (2016) Comparison of BER performance & capacity analysis of MIMO system with STBC & MRC to improve link performance in fading environment. Int J Sci Eng Appl Sci (IJSEAS) 2(5)
17. Ghayoula E, Bouallegue A, Ghayoula R, Chouinard J-Y (2014) Capacity and performance of MIMO systems for wireless communications. J Eng Sci Technol Rev (3)