Global Stability Analysis of the Spatially Developing Boundary Layer: Effect of Wall Suction and Injection

Ramesh Bhoraniya and Vinod Narayanan

Abstract The laminar boundary layer's global temporal modes have been computed under the effect of suction and injection at the wall. The base velocity profile of the boundary layer has affected significantly by the mass transpiration in through the wall in the normal to flow direction. The governing stability equations have been derived using standard procedures. A spectral collocation method with the Chebyshev polynomials have been used for the discretization of the stability equations. The two-dimensional eigenvalues problem has been formed and solved using Arnoldi's algorithm. The different rates of suction and injection ($V_{\mathrm{w}} = 0.01, 0.025$, and 0.050) at $\mathrm{Re} = 226$ were considered to study the temporal and spatial growth of the small disturbances. It has been observed that an increased rate of suction has a stabilization effect, and injection has destabilization effect.

Keywords Global stability · Suction · Injection · Boundary layer

1 Introduction

The boundary layer stability and the transition is a wide field of research due to its significant impact on the fundamental physics and applications. The suction delays the flow separation extends the laminar regime and thus reduces drag on the solid surfaces of the propelling bodies through a viscous fluid. The suction and injection have a significant effect in industrial applications like heat exchangers, recovery of petroleum resources, catalytic reactors, chemical reactions, and chemical vapor deposition on the solid surfaces. It also has an important role to control the boundary layer flow over aircraft wings and turbine blades. The suction/injection in wall-normal direction can be made possible through discrete spanwise slots or porous strips.

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Reynolds and Saric [1] experimentally observed that the velocity profile moved toward the wall in a region of high dissipation due to suction [2]. Saric and Reed [3] experimentally investigated the effect of mass injection and suction and found that suction has stabilizing effect while weak blowing reduces the critical Reynolds number for a boundary layer [3]. Watanabe [4] studied the effect of suction and injection of uniform rate for an incompressible boundary layer subjected to the streamwise pressure gradient. The electrically conducting fluid was considered for this theoretical study. The author found that the skin friction increased, and displacement thickness reduced with increased suction/injection parameter. The critical Reynolds number increased with the increased suction/injection parameter, which makes the flow stable [4]. Lingwood [5] studied the onset of the boundary layer formed on a rotating disk with uniform suction and found that suction delay onset of the absolute instability while injection promotes the onset. At the same magnitude of suction/injection, the effect of suction is found more effective in stabilization than that of injection in destabilization. It is also found that suction damps both stationary and traveling modes [1]. Fransson and Alfredsson [6], experimentally and theoretically investigated the effect of suction on the transition of the boundary layer. They found that under the influence of free-stream turbulence, suction prevents boundary layer transition, while without suction, it gave rise to transition [6]. Aydln and Kaya [2] studied the effect of suction and injection on the heat transfer rate on a porous flat-plate using a similarity solution. They found an enhancement in heat transfer with suction and reduction with wall injection. Wang et al. [7] investigated the effect of cooling and suction on the supersonic boundary layer at a Mach number of 4.5. They found that cooling can delay the onset of the transition, and flow control becomes active with a reduction in wall temperature. The suction hampers the transition up to some intensity of it. However, it also increases the friction coefficient [7]. Huang and Wing [8] investigated the effect of local suction, and they found that a weak suction rate of 0.086 m/s has a significant impact on base flow, and T-S waves have been reduced to 90% which has a strong effect on the transition process. The total suction rate is a crucial parameter, and suction location near the lower branch of the neutral curve found more effective [8]. Hinvi et al. [9] studied the effect of small constant suction at lower and small constant injection at the upper wall in a plane Poiseuille flow with the porous wall. They found that the instability of a small disturbance can be studied by the modified Orr–Sommerfeld equation, and small suction or injection is important for control [9].

The main objectives of the present study are to investigate the effect of wall suction and injection on the global modes of the boundary layer. Many researchers have studied the effect of wall suction and injection on the boundary layer stability in the past using the local stability approach. To the best of our knowledge in all previous works, the local stability approach has been applied for the stability analysis.

2 Problem Formulation

The flow of incompressible fluid has been considered on a porous materials plate to allow the transpiration of mass trough the wall in the normal direction to the flow. The base flow is non-parallel due to the viscous effect of the fluid. The displacement thickness ($\delta*$) at the inflow boundary of the computational domain is considered as a length scale to compute the Reynolds number. The suction and injection velocities are normalized with the uniform inlet velocity. The suction and injection velocity have been normalized with the uniform inlet velocity (U_∞). The three different magnitudes of 0.01, 0.025, and 0.050 at Re = 226 have been considered for the present study. The governing stability equations have been derived in the rectangular coordinates for the small disturbances using standard procedure. The normal modes form of the disturbances is considered for the stability analysis. The spanwise direction (Z) is homogeneous, and thus, spanwise wavenumber (β) has zero magnitudes. The two-dimensional normal mode form of the disturbances is considered.

$$\mathrm{Re} = \frac{U_\infty \delta^*}{\nu} \tag{1}$$

The instantaneous flow quantities can be presented as the sum of basic and small disturbances as,

$$\overline{U} = U_b + u_p, \overline{V} = V_b + v_p, \overline{P} = P_b + p_p \tag{2}$$

The disturbance amplitudes are the functions of flow direction (x) and normal wall direction (y).

$$u_p(x, y, t) = \hat{u}_p(x, y)\mathrm{e}^{-i\omega t}, v_p(x, y, t) = \hat{v}_p(x, y)\mathrm{e}^{-i\omega t}, p(x, y, t) = \hat{p}(x, y)\mathrm{e}^{-i\omega t} \tag{3}$$

The linearized Navier–Stokes equations for the instability analysis are as follows.

$$\frac{\partial u_p}{\partial t} + U_b\frac{\partial u_p}{\partial x} + u_p\frac{\partial U_b}{\partial x} + V_b\frac{\partial u_p}{\partial y} + v_p\frac{\partial U_b}{\partial y} + \frac{\partial p_p}{\partial x} - \frac{1}{\mathrm{Re}}\left[\nabla^2 u_p\right] = 0 \tag{4}$$

$$\frac{\partial v_p}{\partial t} + U_b\frac{\partial v_p}{\partial x} + u_p\frac{\partial V_b}{\partial x} + V_b\frac{\partial v_p}{\partial y} + v_p\frac{\partial V_b}{\partial y} + \frac{\partial p_p}{\partial y} - \frac{1}{\mathrm{Re}}\left[\nabla^2 v_p\right] = 0 \tag{5}$$

$$\frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} = 0 \tag{6}$$

$$\text{where, } \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{7}$$

The following different boundary conditions are considered for the solution of linearized Navier–Stokes equations. No-slip and no-penetration conditions have been considered at the plate wall for velocity disturbances.

$$u_p(x, 0) = 0, \quad v_p(x, 0) = 0, \tag{8}$$

In the far-field, far away from the wall, velocity and pressure disturbances exponentially reduce and vanish to zero.

$$u_p(x, \infty) = 0, \quad v_p(x, \infty) = 0, \quad p_p(x, \infty) = 0 \tag{9}$$

The global stability analysis required boundary conditions in the flow direction at the inlet and outlet. In the present work, we considered that small disturbances are evolving within the flow domain only. Thus, homogeneous Dirichlet conditions have been considered for the disturbances at the inflow boundary [10].

$$u_p(0,\ y) = 0, \quad v_p(0,\ y) = 0, \tag{10}$$

At the outflow boundary, physically, it is difficult to predict how disturbances will leave the domain. Artificial numerical conditions in the form of an extrapolated type conditions have been considered for the velocity disturbances [11].

$$u_{p_{n-2}}(x_n - x_{n-1}) - u_{p_{n-1}}(x_n - x_{n-1}) + u_{p_n}(x_{n-1} - x_{n-2}) = 0 \tag{11}$$

$$v_{n-2}(x_n - x_{n-1}) - v_{p_{n-1}}(x_n - x_{n-1}) + v_{p_n}(x_{n-1} - x_{n-2}) = 0 \tag{12}$$

where x_{n-2}, x_{n-1}, and x_n are the most exterior grid points toward the downstream. No physically any conditions exist at a wall for the pressure perturbations. However, compatibility conditions are collocated at the wall from the governing stability equations itself.

$$\frac{\partial p_p}{\partial x} = \frac{1}{\mathrm{Re}} \frac{\partial^2 u_p}{\partial y^2} \tag{13}$$

$$\frac{\partial p_p}{\partial y} = \frac{1}{\mathrm{Re}} \frac{\partial^2 v_p}{\partial y^2} \tag{14}$$

The above-discussed boundary conditions and discretization of the linearized Navier–Stokes equations together form a general eigenvalues problem. Arnold's iterative algorithm has been used to compute a few selected eigenmodes and eigenfunctions to study the evolution of small disturbances. Readers are requested to refer [9] for a detailed description of the discretization and solution of the eigenvalues problem.

3 Base flow Solution

The steady two-dimensional Navier–Stokes equations are solved numerically using finite volume code, ANSYS Fluent. The second-order upwind scheme has been used for the spatial discretization of the N-S equations. The numerical solution of the base flow has been checked for grid convergence. The steady two-dimensional Navier–Stokes equations are solved numerically using finite volume code, ANSYS Fluent. The second-order upwind scheme has been used for the spatial discretization of the N-S equations. The numerical solution of the base flow has been checked for grid convergence.

$$U_b\frac{\partial U_b}{\partial x} + V_b\frac{\partial U_b}{\partial y} = \left(-\frac{\partial P_b}{\partial x}\right) + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 U_b}{\partial x^2} + \frac{\partial^2 U_b}{\partial y^2}\right) \tag{15}$$

$$U_b\frac{\partial V_b}{\partial x} + V_b\frac{\partial V_b}{\partial y} = \left(-\frac{\partial P_b}{\partial y}\right) + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 V_b}{\partial x^2} + \frac{\partial^2 V_b}{\partial y^2}\right) \tag{16}$$

$$\frac{\partial U_b}{\partial x} + \frac{\partial V_b}{\partial y} = 0 \tag{17}$$

The following boundary conditions have been considered to close the above problem.

$$\text{Inflow boundary: } U_b(0, y) = 1; \quad V_b(0,\ y) = 0;$$
$$\text{Solid wall: } U_b(x, 0) = 0; \quad V_b(x, 0) = \pm V_w$$

$$\text{Free-stream: } U_b(x, \infty) = 1; \quad V_b(x, \infty) = 0;$$
$$\text{Outflow: } \frac{\partial U_b}{\partial x} = 0; \ \frac{\partial V_b}{\partial x} = 0; \quad P_b = 0$$

where V_w is the wall suction or injection, the positive V_w has an injection effect, and negative V_w has a suction effect. Figures 1 and 2 show the effect of wall suction and injection on the streamwise velocity (U) and its first derivative ($\mathrm{d}U/\mathrm{d}y$) at streamwise location $x = 1.00$ m from the leading edge of the boundary layer. The base velocity profile U moves toward the wall under the effect of suction and away from the wall under the effect of the injection. The magnitude of the first derivative ($\mathrm{d}U/\mathrm{d}y$) increases with the increased suction rate and decreases with the increased injection rate. Thus, the characteristics of the basic state of the velocity profile are directly affected by wall suction and injection. It is having a significant effect on the stability characteristics of the boundary layer.

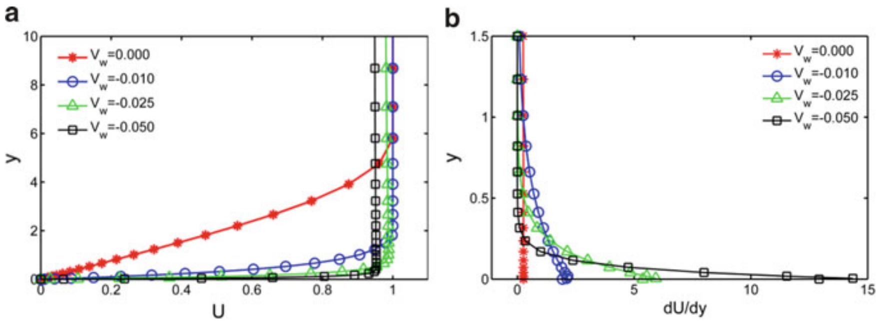

Fig. 1 Variation of **a** U and **b** dU/dy for different suction velocity (V_w) at streamwise location $x =$ 1.00 m for Re = 226

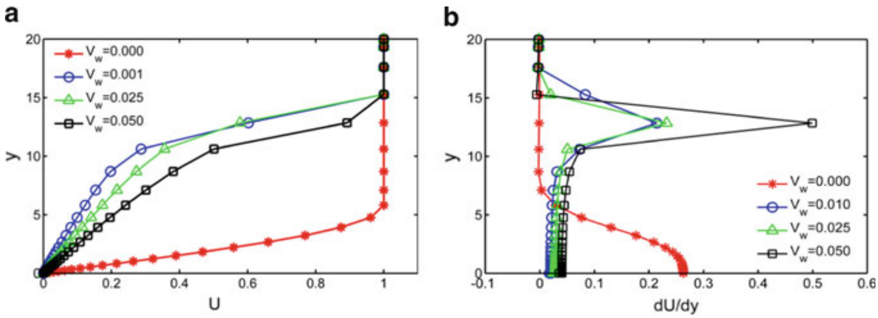

Fig. 2 Variation of **a** U and **b** dU/dy for different injection velocity (V_w) at streamwise location $x = 1.00$ m for Re = 226

4 Results and Discussions

In the present stability analysis of the boundary layer, we considered Reynolds number of 226 based on the displacement thickness along with the suction and injection rates of 0.01, 0.025, and 0.050 (normalized with inlet velocity). The computational domain of size 550 and 20 have been considered. In the flow direction, 241 and wall-normal direction 61 collocation points have been considered. The base flow and eigenvalues problem solution have been tested for the grid convergence. Sponging with appropriate strength and length has been applied near the outlet region to avoid any non-physical solution.

Figure 3a shows the comparison of the eigenspectra for the different suction velocities at Re = 226. Spectra with $V_w = 0$ is for a boundary layer without any suction. It has been found from the spectra that the increase in suction reduces the magnitudes of ω_i, which makes flow more stable. In the present computations, the imaginary part of the global modes is negative. Thus, flow is globally stable. The pattern of the discrete part of the spectra has been found different from the suction effect. Two distinct branches exist with the suction effect. The distribution of the

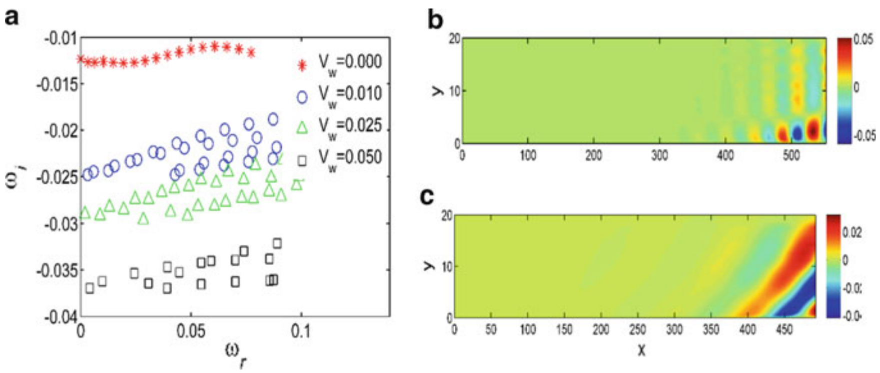

Fig. 3 **a** Comparison of the spectra at different suction velocity. **b** Contour plots of the streamwise velocity disturbances (u_p) for eigenmode, $\omega = 0.0607 - 0.0109i$ and $V_w = 0$. **c** Contour plot of velocity disturbance (u_p) for the eigenmode for $\omega = 0.0668 - 0.0245i$ and $V_w = -0.025$. The associated Reynolds number is considered as 226.

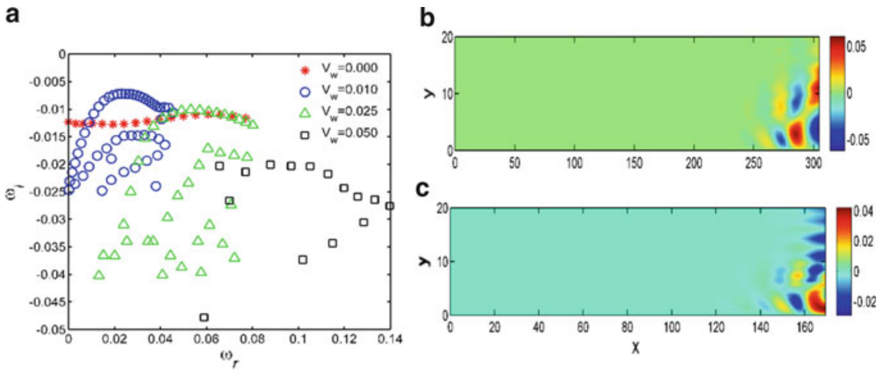

Fig. 4 **a** Comparison of the spectra at different injection velocity. **b** Contour plots of the streamwise velocity disturbance (u_p) for eigenmode, $\omega = 0.0232 - 0.0072i$ and $V_w = 0.010$. **c** Contour plot of u_p velocity disturbances of the eigenmode at $\omega = 0.0532 - 0.0101i$ and $V_w = 0.025$. The Reynolds number considered is 226

frequency even changed and a cluster of two frequencies have appeared with the suction. We have also plotted the 2D spatial structure of the temporal eigenmodes nearly $\omega \cong 0.06$. Figure 3b, c presents contour plots of u disturbance velocity with $V_w = 0$ and $V_w = 0.025$. The comparison clearly shows that the largest values of disturbance amplitudes have been reduced for $V_w = 0.025$. The elongated structure of the disturbances is found with the effect of suction. Wave-like behavior of the disturbance amplitude has been observed with and without suction. Thus, the study of temporal and spatial eigenmodes computed with ($V_w \neq 0$) and without ($V_w = 0$) the effect of suction prove that the effect of suction is to stabilize the boundary layer.

Figure 4a presents a comparison of the eigenspectra for different injection velocity at Re $= 226$. As shown, for $V_w = 0.01$, the temporal growth of the disturbances has been found, and a further increase in injection velocity reduces the temporal growth. However, $\omega_i < 0$ found for all the injection velocities considered here. Thus, flow is globally stable. The structure of spectra and distribution of the frequency has been found different from the effect of the injection. Even it is observed that the discrete part of the spectra has shifted toward the right side as injection velocity has increased. Similar to the previous case, more than one branch is found in the spectra also. Figure 4b, c present the 2-D spatial structure of the eigenmodes corresponding to the least stable temporal eigenmodes (largest values of $\omega_{i)}$ for injection velocity of 0.010 and 0.025. It is observed that the magnitudes of the disturbances are larger to increase injection velocity. The wave-like nature of the disturbances is found in the streamwise direction; however, the structure is not elongated in the streamwise direction like an under the effect of suction. One important and consistent observation found in the variation of the $\mathrm{d}U/\mathrm{d}y$ at the wall for suction and injection both and its significant effect has been reflected in the stability of the boundary layer. The magnitude of the $\mathrm{d}U/\mathrm{d}y$ increases with the increased suction.

5 Conclusions

Bi-global stability analysis of the laminar flat-plate boundary layer has been performed to study the effect of wall suction and injection on the temporal and spatial stability properties of the laminar boundary layer at Re $= 226$. The global temporal modes have been computed at different wall suction and injection rate, and spatial structure of the least stable temporal eigenmodes has been extracted. It has been found that at the considered value of Re and different wall suction/injection rate, the computed eigenmodes are found stable, and thus, flow is temporally stable. However, the growth of the small disturbance amplitudes has been found in the streamwise direction for the least stable eigenmodes, which indicates that the flow is spatially unstable. The increased wall suction rate from 0.0 to 0.050 has moved the profile of U velocity toward the wall and increased the magnitudes of the $\mathrm{d}U/\mathrm{d}y$ at the wall. The effect of the modified base flow has been found directly on the temporal and spatial growth of the small disturbances. The magnitudes of the temporal growth $(\omega_{i)}$ and spatial growth of the disturbance amplitudes have been found reduced with the increased suction rate. However, the increased rate of injection initially moves the U velocity profile away from the wall, then as injection increases profile moves toward the wall. However, the curvature of the profile is not the same as that of suction. A similar trend has been found for the $\mathrm{d}U/\mathrm{d}y$ at the wall—this modified base flow due to the wall injection reflected the boundary layer's stability characteristics. The growth of the disturbances initially increases with the increased injection rate and then reduces. Thus, overall suction has stabilized, and injection has a destabilizing effect on the laminar boundary layer.

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