# **Design Optimization of Go-Kart Chassis Frame Using Modal Analysis**



#### **A. A. Dere, Manpreet Singh, A. Thakan, Rajeev Kumar, and Harpreet Singh**

**Abstract** The chassis of automobile houses crucial mechanical component such as engine, suspension, steering and transmission system. Therefore, the chassis structure must be strong enough to absorb the static and dynamic loads generated by these mechanical components. In this work, the structural strength of go-kart chassis has been improved against static and dynamic loads through geometrical modifications. The geometrical modifications in the chassis structures were decided individually on each structural element where maximum deformation was analyzed in the modal analysis. This structural element was reanalyzed after making multiple variations in its geometry in attempt to minimize the deformation. When the minimum deformation was achieved in the structural element, then structure was finalized for stage 1. Similarly, other structural elements were also modified in the same continuous iterative process by keeping in consideration the weight constraints. After the termination of each modification torsion test, impact analysis was also carried out to examine torsional rigidity and crashworthiness. In five successive iterations, the optimum results for the chassis structure were obtained with little scope of further improvement. In the final structure, the lowest modal frequency was found to be shifted from 11.691 to 57.318 Hz to that of the initial structure. A significant reduction of 42% in maximum deformation along with a reduction in mode shapes was also witnessed in the final structure. The final structure was also found to be better in the results obtained from torsional analysis and impact testing.

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**Keywords** Modal analysis · Chassis structure · Resonance · Dynamic analysis · Vibration analysis

#### **1 Introduction**

Go-kart is a single-seated land-racing vehicle with or without bodywork and four nonaligned wheels [\[1\]](#page-14-0). This vehicle is prominently used in motorsport and recreation. A chassis is the backbone of a go-kart, as it holds various automobile components and is responsible to withstand dynamic and static loads without undue distortion [\[2\]](#page-14-1). Various studies on automobile chassis have shown that the engine movement and road profile are mainly responsible for dynamic excitations [\[3](#page-14-2)[–5\]](#page-14-3). When the frequency of excitation from engine or road profile matches with the natural frequency, it may lead to the devastating effect of resonance, which causes loosening of joints, unwanted noise, failure of parts and driver discomfort [\[6–](#page-14-4)[11\]](#page-14-5). A number of techniques are presently available to determine the dynamic characteristic of the structure, but among all techniques, modal analysis is one of the most reliable ones [\[12\]](#page-14-6). Modal analysis is the procedure to determine the intrinsic dynamic characteristic like the mode shapes, natural frequency and damping factor of a tubular space frame. These characteristics can be employed to formulate a mathematical model for its dynamic behavior. This mathematical modal can be computationally assessed by using FEM analysis for the purpose of improving design [\[13\]](#page-14-7). In complicated space frame structure due to a high number of degrees of freedom, a large number of mathematical models are formulated, it is difficult to access this model analytically thus, use of computational simulation-based method becomes adequate [\[14\]](#page-14-8). Also, the computational FEA proves to be a handy tool in the design process as it accelerates the design procedure by providing the freedom of making random modifications in the automotive structure and also minimizing physical test [\[15,](#page-14-9) [16\]](#page-14-10).

In improving the structural characteristics of chassis, mostly, researchers have changed either material or geometrical structure or both material and geometrical structure. Antonio et al. [\[14\]](#page-14-8) used a prototype of sports car modified chassis to carry out the modal analysis. They have identified that the static and dynamic properties differ with a change in geometrical structure. Patel and Patel have modified the structure of truck chassis using ANSYS software. The modified structure was found to have lighter in weight and with improved strength [\[16\]](#page-14-10). Archit and Dheer have changed the material of chassis structure from steel 52 to epoxy composite. This has not only witnessed the reduction in weight of the chassis but also improved the static and dynamic properties [\[17\]](#page-14-11). However, change in material can bring good outcomes in the structure, but it may incur extra cost to the manufacturer. Another effective way to improve the dynamic characteristics of the structure is a geometrical modification. Rao and Bhattu have relocated the members of the truck chassis. Along with relocation, they have also reduced the overall length and the thickness of certain members. So, with the geometrical modification, they have not only reduced the weight but also improved the structural strength of the chassis [\[6\]](#page-14-4). Several other

researchers have worked on the various truck chassis and have observed that by changing the geometry of the chassis structure the dynamic characteristic can be improved [\[18–](#page-14-12)[22\]](#page-15-0). However, in previous research work, it has been observed that geometrical modification improves dynamic properties significantly, but authors have modified the chassis structure through single modification. In addition to that they have not considered major constraint such as the weight, maximum deformation in structural members and numbers of modes.

In this paper, the design and development of go-kart chassis have been carried out by using ANSYS-based modal analysis. The chassis structure was updated using the computational methodology, and optimum design has been determined after five continuous and progressive iterations within the weight constraints. A dynamic and static characteristic such as deformation in structural elements, crashworthiness and torsional stiffness has been analyzed for all the updated designs.

#### **2 Chassis Design**

The chassis of the go-kart vehicle has been constructed with a tubular space frame structure by following the rule book of the competition. However, there is always the provision of structural customization as per the participant's requirement [\[23\]](#page-15-1). Generally, these customizations are made to improve the structural and fatigue strength to sustain the dynamic and static loading with low deformation. The other factor such as dimensional limit, manufacturing restriction, weight and financial constraint must also be considered while carrying out the customization [\[24\]](#page-15-2). The initial go-kart

<span id="page-2-0"></span>**Fig. 1** Original go-kart participated in 2018



<span id="page-3-0"></span>



and the chassis structure drafted in Solid Edge software are shown in Figs. [1](#page-2-0) and [2,](#page-3-0) respectively.

# **3 Finite Element Method**

Finite element method (FEM) is a numerical method for solving designing problems of complex structures [\[25\]](#page-15-3). In this, complex structure is firstly discretized into small elements to form nodes, and then, each element is represented by a separate equation to form the complete solution of the structure [\[26,](#page-15-4) [27\]](#page-15-5). The prevalent steps of FEM which are used in the analysis are as follows  $[25]$ :

- 1. Discretize the body and select the type of element.
- 2. Select the displacement function.
- 3. Define the stress–strain and strain–displacement relationship.
- 4. Derive the stiffness matrix for each element.
- 5. Derive the global stiffness matrix.
- 6. Solve for unknown degree of freedom.
- 7. Determine the stress and strain components and interpret the results.

# **4 Chassis Analysis**

The analysis of go-kart chassis was carried out to check the strength and stability of the structure for static and dynamic loads. Firstly, the existing go-kart chassis structure was drafted in Solid Edge software, and then, modal analysis was carried out on the drafted structure using ANSYS software. Initially, input constraints such

as material properties and meshing size were defined. After the complete introduction of structure to the software, the modes were determined by fixing all the faces of four-wheel mountings. The modes and the natural frequency can be determined by solving the following equation analytically for eigenvalues and eigenvectors [\[28,](#page-15-6) [29\]](#page-15-7).

$$
[M] \cdot {\tilde{X}(t)} + [C] \cdot {\tilde{X}(t)} + [K] \cdot {X(t)} = 0
$$
 (1)

Here, each eigenvalue is corresponding to one mode shape and respective natural frequency of the structure. But, for higher numbers of the degrees of freedom, it becomes cumbersome to find the solution analytically [\[14\]](#page-14-8).

In software, the solution can be found by transforming Eq. [\(1\)](#page-4-0) from time to frequency domain by using the Fourier transform [\[28\]](#page-15-6).

<span id="page-4-1"></span><span id="page-4-0"></span>
$$
[A(j\omega)]\{x(j\omega)\} = \{0\} \tag{2}
$$

where  $[A(j\omega)]$  represents system matrix and is given by  $[A(j\omega)] = [M](j\omega)^2 +$  $[c](j\omega) + [k]$  and vector of Fourier transformation is represented by  $\{x(j\omega)\}.$ 

The solution of Eq. [\(2\)](#page-4-1) is a set of unique complex values and eigenvectors; the eigenvalues are in the complex conjugate pair, where the real part presenting the modal damping and the imaginary part as modal frequency. The corresponding eigenvector associated with the eigenvalue represents the mode shape.

So, each mode is defined by a complex conjugate pair of eigenvalues and eigenvector (mode shape). The important conceptual conclusions that could be drawn from the above definition are:

- Modes are unique and inherent to structure.
- Modes are independent of external loads.
- Modes will change only with a change in mass, damping and stiffness property of the structure.

After finding the natural frequency, the modes of the system the software set the structure to vibrate at these natural frequencies to encounter the resonance effect. For each corresponding mode, the maximum deformation in the structural element has been noted, and the geometrical modifications were deciding there in the structure. This process was repeated until the maximum deformations for the structural elements were minimized within the weight constraints defined by the rule book. The methodology adopted for this work is shown in Fig. [3.](#page-5-0)

To check the structural strength of the chassis against the static load, torsional stiffness test was preferred and performed due to its better relevance with the current structure  $[14]$ . In this test, the rear wheel mountings were fixed, and a known twisting moment is applied at front wheel mounting to check the deformation [\[30\]](#page-15-8). The low value of the deformation accounts for the better structural strength against static loads.

The torsional stiffness for computational and experimental analysis is shown below [\[31\]](#page-15-9).

<span id="page-5-0"></span>

Twist angle  $\varnothing$  is given by the formula,

$$
\varnothing = \sin^{-1}\left(\frac{2D}{L}\right) \tag{4}
$$

After finding no scope for further reduction in maximum deformation, the final structure for go-kart chassis was recommended. The applied methodology for the chassis analysis is shown in Fig. [3.](#page-5-0)

**Fig. 3** Methodology

### **5 Geometrical Modification**

The frame of the chassis is made up of AISI 1020 tubes with 1-in. diameter and 2 mm wall thickness. The mechanical properties of AISI 1020 are presented in Table [1.](#page-6-0) The orthographic projections of go-kart chassis along with geometrical specifications are shown in Fig. [4.](#page-6-1)

Modal analysis on the go-kart chassis was carried out, and on the basis of excessive deformations, multiple geometrical modifications were tried on the structural elements, and on the basis of best outcomes, stage 1 structure was finalized. Similarly, the modal analysis was carried out successively on the modified structures without varying diameter and thickness of the tube. With no room for further improvement in dynamic characteristic after stage 3, the thickness of the tube in certain members was varied. The dotted line in the top view of Fig. [4](#page-6-1) represents the members in which thickness was varied from 2 to 3 mm. The variation of thickness was done in the structure by keeping the weight restrictions of the chassis as 28.5 kg. After two more successive iterations, the optimum structure was finalized after ensuring no further scope of improvement within the aforementioned weight constraints. The geometrical modifications made on the basis of modal analysis are represented in Table [2](#page-7-0) with the required dimensions.

As seen in Fig. [4,](#page-6-1)

AISI 1020 properties		
Young's modulus $[N/m^2]$	$2.00 \times 10^{11}$	
Poisson ratio	0.29	
Density [ $\text{kg/m}^3$ ]	7850	
Yield strength $[N/m^2]$	$3.5 \times 10^{8}$	
Tensile strength $[N/m^2]$	$4.2 \times 10^{8}$	
Shear modulus $[N/m^2]$	$8.0 \times 10^8$	

<span id="page-6-0"></span>**Table 1** Mechanical properties of chassis material



<span id="page-6-1"></span>**Fig. 4** Orthographic top (**a**) and side view (**b**) of go-kart chassis with geometrical specifications



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# **6 Result and Discussion**

#### *6.1 Modal Analysis*

The CAD model of the go-kart chassis structure was imported in ANSYS software for carrying out modal analysis. The material properties referred for AISI 1020 were defined to the imported structure. To define the structure completely for the purpose of modal analysis, meshing was generated with having a minimum edge length of  $4.2952 \times 10^{-4}$  m. The structure was then fixed at the wheel mounting faces, and a frequency of 8000 RPM was applied to the chassis by keeping into consideration all the possible frequencies. The fixed wheel mounting faces are shown in Fig. [5.](#page-9-0)

Firstly, the modal analysis was carried on original go-kart chassis structure, and in this analysis, 25 modes were obtained along with their corresponding natural frequencies. The values of the deformations at different structural elements were then checked at each of the corresponding natural frequency and are shown in Fig. [7.](#page-10-0) The maximum deformation was observed as 1.4082 m at 24th mode shape having a corresponding natural frequency as 104.12 Hz. The animated view of total deformation for 24th mode is shown in Fig. [6.](#page-9-1)

The CAD model was then geometrically modified by following the methodology explained earlier for five times in successive iterations. The total number of mode shapes, maximum deformation with its corresponding mode frequency and frequency of the first mode for all the iterations performed are presented in Table [3.](#page-10-1) The values of the deformations for the final structure at different modes and the corresponding frequency are shown in Fig. [8.](#page-11-0) The maximum deformation of 0.8172 m in the final structure was obtained at 8th mode (113.61 Hz).

In the final structure, the total modes have been reduced to 8 from 25 that of in the initial structure. This signifies that the problem of resonant frequencies has reduced in the final structure. The final structure's maximum deformation has also reduced to



<span id="page-9-0"></span>**Fig. 5** Isometric view of the chassis which demonstrate the boundary condition applied



<span id="page-9-1"></span>**Fig. 6** Isometric view of the original chassis representing the maximum deformation at mode 24

0.8172 m from 1.4082 m (initial structure); therefore, the final structure can be said to have better stability from the past. The first mode frequency in the final structure has also shifted from 11.691 to 57.318 Hz. This ensures that the final structure will not be facing the problem of resonance until the moderate speeds.



<span id="page-10-0"></span>**Fig. 7** Graph of maximum deformation and corresponding mode number of initial chassis

Step number	Total number of modes	Maximum deformation (m)	Corresponding frequency (Hz) for maximum deformation (mode number)	The frequency of first mode $(Hz)$
Initial chassis	25	1.4082	104.12(24)	11.691
	21	1.3592	94.306 (14)	25.954
$\mathfrak{D}$	17	1.2714	85.536 (9)	45.428
3	16	1.1454	91.038(8)	46.38
4	8	0.9692	118.24(7)	54.773
5	8	0.8172	113.61(8)	57.318

<span id="page-10-1"></span>**Table 3** Result of modal analysis after each modification

# *6.2 Torsional Analysis*

To calculate torsional rigidity of the go-kart chassis, Thompson methodology was adopted [\[9\]](#page-14-13). According to this methodology, the forces of equal magnitude and opposite directions have to be applied on the front wheel mountings by keeping rear wheel mountings fixed. The amount of force to be applied for carrying out torsional analysis can be calculated with reference to the weight of go-kart as defined in the rule book of the competition. In our analysis, the magnitude of force was calculated as 3532 N (magnitude 2G) by considering the maximum weight of go-kart as 180 kg.



<span id="page-11-0"></span>**Fig. 8** Graph of maximum deformation and corresponding mode number of stage 6 chassis

The isometric view of the go-kart chassis with applied directional forces and fixed faces is shown in Fig. [9.](#page-11-1) After the application of the forces, deflection was calculated on the frontal wheel mountings to know the static strength of the structure. This has been done for all the modified structures, and their respective deflections are plotted and shown in Fig. [10.](#page-12-0)

The values of deformations calculated are showing a downward trend against stage modifications in the structure. This indicates that static structural strength has improved with every geometrical modification.



<span id="page-11-1"></span>**Fig. 9** Isometric view to demonstrate the boundary condition of the torsional test on the initial chassis



<span id="page-12-0"></span>**Fig. 10** Graph of deformation due to torsional loading

# *6.3 Impact Testing*

This test is carried out to determine the crashworthiness of an automobile. The members of the go-kart chassis which are prone to crash such as front, rear and side sections are considered for the impact test. In this test, impact loads were applied on each section, and deformation in the same was calculated by fixing the opposite side wheel mountings. The simulation carried out for frontal impact test is shown in Fig. [11.](#page-12-1) The impact forces considered for frontal and rear impact test were of magnitude 4G (7064 N), and for side impact test, the magnitude was 2G (3532 N) as per the rule book guidelines [\[20\]](#page-15-10). The deformations calculated for each section in



<span id="page-12-1"></span>**Fig. 11** Boundary condition of frontal impact



#### **IMPACT TEST**

<span id="page-13-0"></span>**Fig. 12** Graph of deformation due to the impact

the initial chassis structure and after the successive geometrical modified structure are plotted and shown in Fig. [12.](#page-13-0)

The deformations calculated against impact loads for each section in the structure are decreasing with every modification made in the structure.

# **7 Conclusion**

In this paper, the modal analysis was carried out on go-kart chassis structure, and on the basis of excessive deformation, geometrical modifications were made for five times. The original chassis structure was compared with the final recommended structure, and on the basis of this comparison, the following conclusions can be drawn:

- The maximum deformation in the final structure was found to reduce by 41.96%; this ensures the structural stability against the dynamic loads.
- The problem of the number of resonant frequency in the final structure was reduced significantly as the modes in it were found to be 8 as that of 25 in the initial structure.
- In the final structure, the first natural frequency was found to be shifted from 11.691 to 57.318 Hz. This ensures that the final structure will not be facing the problem of resonance till moderate speeds.
- The torsional rigidity was also found to improve by 57.77%. This means that structural strength of the chassis against static loads has increased in the final structure.

• The results of frontal, rear and side impact have also shown an improvement assuring the increase in crashworthiness of the chassis.

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