Determination of Unloading—Reloading Modulus and Exponent Parameters (m) for Hardening Soil Model of Soft Soil in Ho Chi Minh City

Trung Ngo Duc, Phan Vo, and Thanh Tran Thi

1 Introduction

In Ho Chi Minh City (HCMC) in recent years, the calculation of geotechnical works often uses finite element method with constitutive models. The biggest problem for design engineers is to properly analyze the behavior of the soil by selecting the right constitutive model and input parameters.

The Hardening Soil (HS) model is based on the Dun-can Chang model showing more advances than the Mohr-Coulomb (MC) model. Similar to the MC model, stress states of stress are expressed by the friction angle φ, the cohesion force c, the dilatation angle ψ, but the stiffness of the soil is expressed with greater precision by using 3 different input modulus variables: secant modulus; unloading-reloading stiffness and tangent oedometric modulus.

The HS model also explains the dependence of the stiffness on stress. The level of dependence of stress is below given by the exponent m. In order to simulate stress dependence according to the logarithmic law, Schanz et al. [4] investigated soft soils, the chosen exponent is $m = 1$. According to Janbu [2], the value of m is about 0.5 for sand and clay in Norway. Whereas von Soos (1980) has a m value of $0.5 < m < 1$ [7]. Usmani [6] proposed that $m = 0.67$ in stress-strain analysis of Delhi clay sand.

Thus, the choice of the m-parameter makes it difficult for engineers to correlate the empirical expressions, since the amplitude is still relatively wide and results in large discrepancies.

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This paper will identify the m parameter for soft soil Ho Chi Minh City on the basis of drained triaxial compression test as defined in the HS model.

2 Overview of Soft Soil in HCMC

HCMC belongs to the Saigon River delta, the stratigraphic structure of this area belongs to the Quaternary period—Cenozoic Era and the Neogene period accumulates which form a total of 6 layers of natural soil. Layer 1 and layer 2 consist of slurry and thick soils with a depth of 20 ÷ 30 m, high organic content, high water content of 85–104%, void ratio e = [1.5 ÷ 2.5] soft soil is very compact, high liquid IL index, reaching 1.85 [3] (Fig. 1).

Soft soil of HCMC is located in: Binh Thanh District, Can Gio District, District 6, District 7, District 4 and Binh Chanh District. Soft soils are highly compressive, with very low load capacity. One or more of the physical properties, durability and deformation of the soil are within the following range: Void ratio e = [1.5 ÷ 2.5]; Water content W ≥ 65%; Water unit weight $\gamma_{\text{-w}}$ = [14 ÷ 16] kN/m^3; undrained shear strength S_u < 50 kPa; standard penetration test N30 < 4; cohesion intercept c < 10 kPa; Settlement ratio a_{1-2} > 5 m^2/kN; Deformation modulus E < 5000 kPa.

This study was conducted on two soft clays of HCMC: very soft clay and soft clay with a depth of 4 to 30 m below groundwater, which often affects the stability and deformation of underground structures.

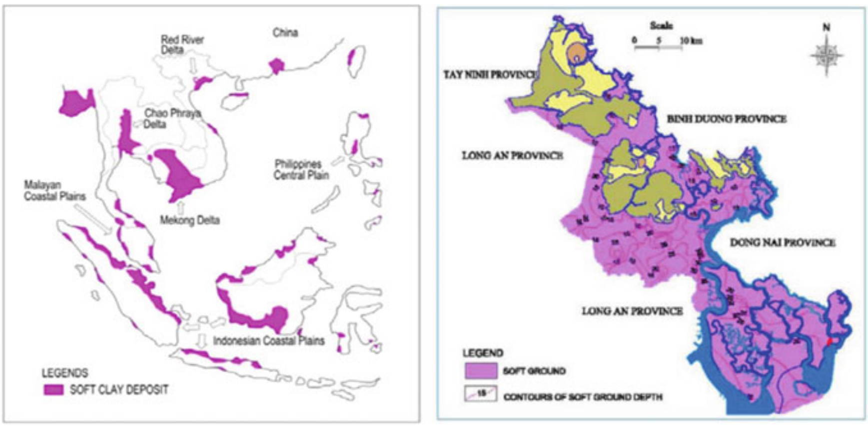


Fig. 1 Distribution of soft clay in Southeast Asia region and in HCMC [3]

3 Overview of Hardening Soil Model

The HS model developed by Schanz et al. [4] is based on the classical elastic-plastic theory to simulate the resilient and flexible behavior of the soil. Its elasticity uses two stiffness modules, are the secant modulus E_{50} and unloading-reloading stiffness E_{ur}. Plasticizers follow the nonlinear flow rule and the directional re-orientation standard, to describe the relationship between stress and strain of the soil in a hyperbolic curve (Figs. 2 and 3).

Yield surface:

$$f_1 = \frac{q_a}{E_{50}} \frac{(\sigma_1 - \sigma_2)}{q_a - (\sigma_1 - \sigma_2)} - \frac{2(\sigma_1 + \sigma_2)}{E_{ur}} - \gamma_p \quad (1)$$

$$f_2 = \frac{q_a}{E_{50}} \frac{(\sigma_1 - \sigma_3)}{q_a - (\sigma_1 - \sigma_3)} - \frac{2(\sigma_1 - \sigma_3)}{E_{ur}} - \gamma_p \quad (2)$$

$$f_3 = \frac{q_a}{E_{50}} \frac{(\sigma_2 - \sigma_3)}{q_a - (\sigma_2 - \sigma_3)} - \frac{2(\sigma_2 - \sigma_3)}{E_{ur}} - \gamma_p \quad (3)$$

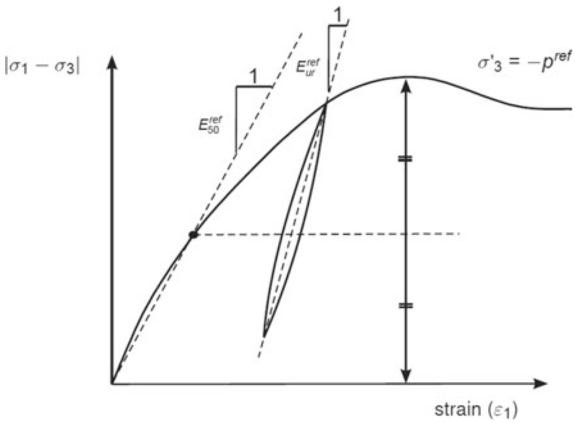


Fig. 2 Definition of E_{50} and E_{ur} in drained triaxial test [4]

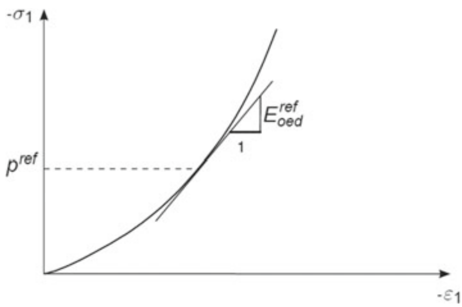


Fig. 3 Definition of E_{oed}^{ref} in oedometer test [4]

With q_a, E_{50} and E_{ur} are defined by formulas (4), (5), (6) and the notation γ_p for plastic stress.

$$E_{50} = E_{50}^{ref}\left(\frac{c\cot g\varphi - \sigma_3'}{c\cot g\varphi + p^{ref}}\right)^m \quad (4)$$

$$E_{ur} = E_{ur}^{ref}\left(\frac{c\cot g\varphi - \sigma_3'}{c\cot g\varphi + p^{ref}}\right)^m \quad (5)$$

$$q_f = (c\cot\varphi - \sigma_3)\frac{2\sin\varphi}{1-\sin\varphi}, q_a = \frac{q_f}{R_f} \quad (6)$$

In the un-loading and re-loading stress paths, the stress-strain relationships are still in the form of hyperbolic, and empirical studies [1] show that modulus E_{50} in the unloading and reloading experiments is larger than in the conventional triaxial compression tests many times and different from each soil type. In this study, we focused on the E_{ur}/E_{50} ratio for the soft clay in HCMC.

Equations (4), (5) defines E_{50}, E_{ur}, and E_{oed} is defined by the following equation:

$$E_{oed} = E_{oed}^{ref}\left(\frac{c\cot g\varphi - \sigma_3^{'}}{c\cot g\varphi + p^{ref}}\right)^m \quad (7)$$

E_{oed}^{ref} is tangent oedometric modulus in oedometer test at the vertical stress $-\sigma_1^{'} = p^{\mathrm{ref}}$.

The advantage of the HS model over the MC model is not only the use of hyperbolic strain curves instead of linear relations but also the control of the stiffness dependence on the stress load. When using the MC model, the user must select a fixed Young module value while the real stiffness level depends on the pressure level. It is then necessary to estimate the pressure level in the soil and use that pressure level to obtain the appropriate stiffness value. With the HS model the difficult selection of input parameters is no longer necessary. Instead, the modulus is defined by the smallest stress $\sigma_3 = p^{ref}$ as the default value in Plaxis is p^{ref}=100 (kN/m^2).

However, defining the parameters E_{ur}^{ref}, E_{oed}^{ref} in Plaxis generally chooses the default word for all types of soil as formulas (8) often make calculations difficult [5]:

$$E_{oed}^{ref} = E_{50}^{ref};\ E_{ur}^{ref} = 3E_{50}^{ref} \quad (8)$$

4 Determination of M and E_{ur}/E_{50} of the HS Model for Soft Soil in HCMC

4.1 Drained Triaxial Compression Test

To determine the parameter m, which depicts the dependence of the stiffness on the stress for soft soil in HCMC, the author carried out experiments on 12 clay samples at depths from 4 to 30 m below the groundwater, with drained triaxial tests have unloaded and reloaded at the cell pressure level σ_3' are 50, 100, 200 and 400 kPa. The samples are located in Binh Chanh District. Results of the analysis of mechanical properties are given in Table 1.

The results of the experiment for the two clay layers are shown in Figs. 4, 5, 6, 7, 8, 9, 10 and 11.

From the stress-strain diagram (q, ε_1), we define c', φ' and the parameters as Table 2. With $(\sigma_{1f}' - \sigma_{3f}')$ is deviator stress.

Table 1 Physical characteristics of soft soils in HCMC

Parameters	Symbol	Very soft clays	Soft clays
Water content (%)	W_n	70 ÷ 100	60 ÷ 70
Void ratio	e	2.0 ÷ 3.0	1.30 ÷ 2.0
Water unit weight (kN/m^3)	γ_n	14.0 ÷ 16.0	16.0 ÷ 17.0
Dry unit weight (kN/m^3)	γ_d	7.5 ÷ 8.5	8.5 ÷ 12.0
Liquid limit (%)	W_L	70 ÷80	45÷70
Plastic limit (%)	W_P	30 ÷ 40	20 ÷ 30
Saturation (%)	S	95 ÷98	99 ÷100

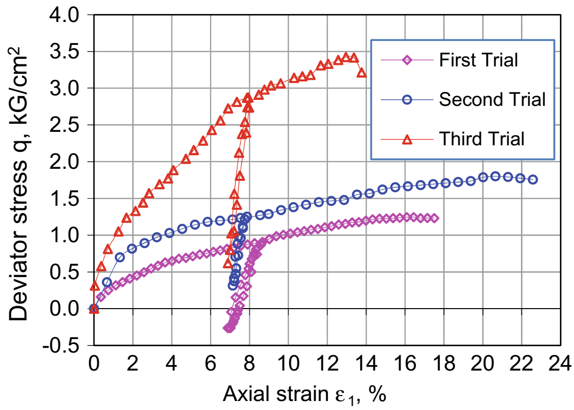


Fig. 4 The relationship (ε_1 − q) of sample No. 1, 2, 3

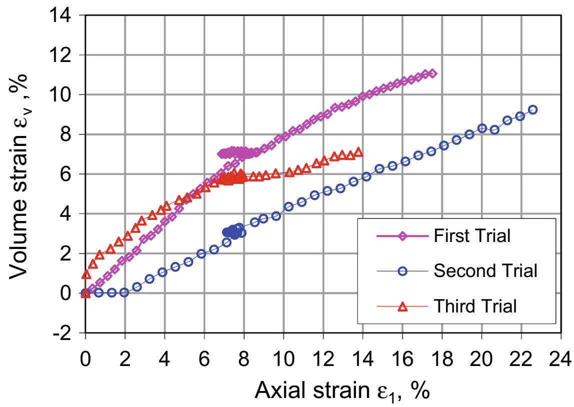


Fig. 5 The relationship ($\varepsilon_1 - \varepsilon_v$) of sample No. 1, 2, 3

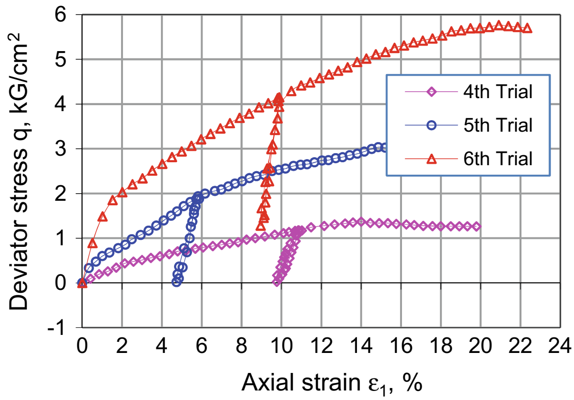


Fig. 6 The relationship ($\varepsilon_1 - q$) of sample No. 4, 5, 6

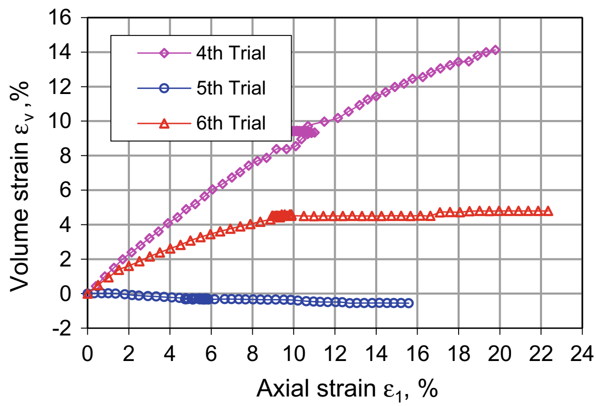


Fig. 7 The relationship ($\varepsilon_1 - \varepsilon_v$) of sample No. 4, 5, 6

Fig. 8 The relationship ($\varepsilon_1 - q$) of sample No. 7, 8, 9

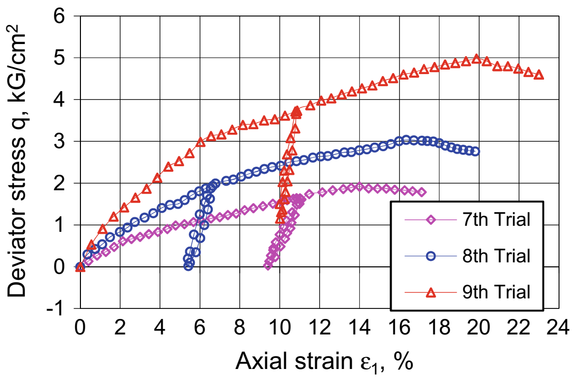


Fig. 9 The relationship ($\varepsilon_1 - \varepsilon_v$) of sample No. 7, 8, 9

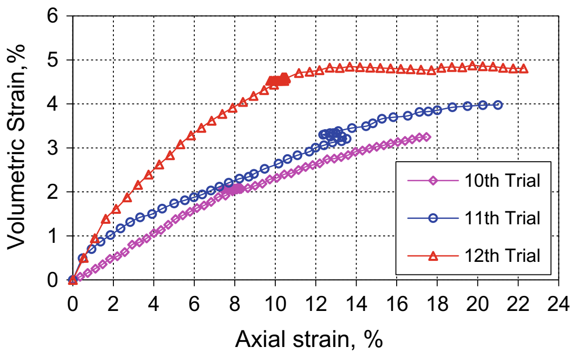
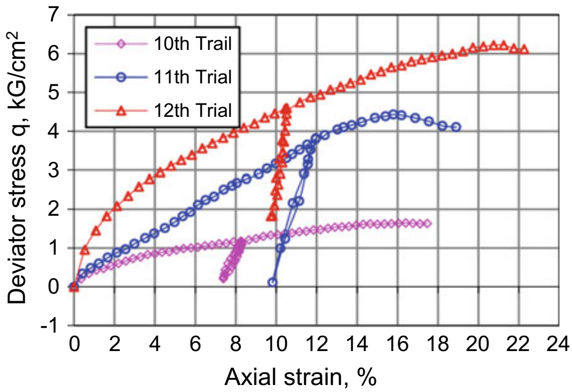


Fig. 10 The relationship ($\varepsilon_1 - q$) of sample No. 10, 11, 12



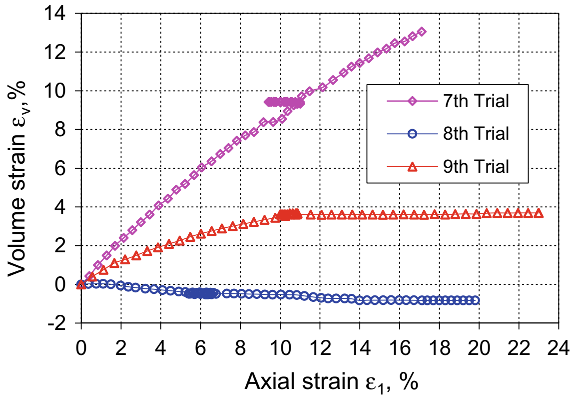


Fig. 11 The relationship ($\varepsilon_1 - \varepsilon_v$) of sample No. 10, 11, 12

Table 2 Shear strength parameters

Depth (m)	Sample no.	C′ (kG/cm^2)	φ′ (°)	σ'_{1f} (kG/cm^2)	$\sigma'_{3f}(kG/cm2)$	$\frac{\sigma_y}{p^{ref}}$
Vert soft clays						
4 ÷ 6	1	0.12	25.85	1.40	0.36	0.485
	2			3.05	1.09	1.074
	3			5.29	1.86	1.690
12 ÷ 14	4	0.09	26.28	2.16	0.86	0.885
	5			5.01	1.90	1.758
	6			9.48	3.72	3.300
Soft clays						
18÷20	7	0.10	26.02	2.17	0.88	0.899
	8			3.92	1.81	1.674
	9			8.94	3.89	3.394
24÷26	10	0.11	26.32	2.37	0.98	0.982
	11			4.02	1.91	1.747
	12			9.64	3.99	3.442

4.2 To Determine the Power M from the Drained Triaxial Compression Test

The parameter m represents the dependence of the stiffness on the stress state of the ground. In this section, the author proceeds to define the exponent m from the modulus of the distortion in the HS model according to expressions (4, 5).

On the stress-strain diagram ($q - \varepsilon_1$), draw the secant-line E_{50} as defined by the E_{50} deformation modulus of the HS model. From there, the secant modulus E_{50} can be identified as shown in Table 2.

Based on the definition of E_{50} in the HS model, formula (4), we have:

$$E_{50} = E_{50}^{ref}\left(\frac{\sigma_y}{p^{ref}}\right)^m = E_{50}^{ref}\left[\frac{c\cot\varphi' - \sigma_3^{'}}{c\cot\varphi' + p^{ref}}\right]^m \Rightarrow m = \log_{\left[\frac{\sigma_y}{p^{ref}}\right]}\left[\frac{E_{50}}{E_{50}^{ref}}\right] \quad (9)$$

The power m can be determined as E_{50} as shown in Table 3, with $p^{ref} = 100$ kPa (Fig, 12).

From there, the value of parameter m determined from the triaxial compression test through the secant modulus E_{50} is as follows:

Table 3 Modulus E_{50}, E_{50}^{ref} and m-parameter parameters from the triaxial compression test

Depth (m)	Sample no.	E_{50} (kG/cm^2)	E_{50}^{ref} (kG/cm^2)	$\frac{E_{50}}{E_{50}^{ref}}$	$\frac{\sigma_y}{p^{ref}}$	M (−)
Very soft clay						
4 ÷ 6	1	17.29	33.88	0.51	0.485	0.93
	2	33.88		1.00	1.074	–
	3	49.44		1.46	1.690	0.72
12 ÷ 14	4	21.15	21.15	1.00	0.885	–
	5	33.72		1.59	1.758	0.83
	6	60.51		2.86	3.300	0.88
Soft clay						
18 ÷ 20	7	20.72	20.72	1.00	0.899	–
	8	29.12		1.41	1.674	0.76
	9	52.05		2.51	3.394	0.75
24 ÷ 26	10	22.93	22.93	1.00	0.982	–
	11	30.66		1.48	1.747	0.72
	12	64.69		2.81	3.442	0.84

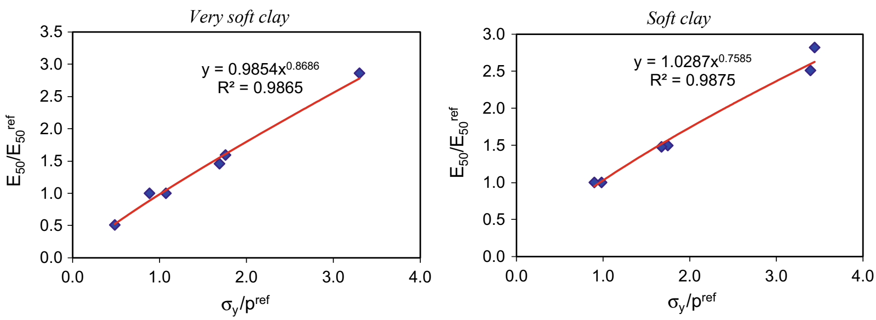


Fig. 12 The regression equation correlates between $\frac{E_{50}}{E_{50}^{ref}}$ and $\frac{\sigma_y}{p^{ref}}$

$$\text{Very soft clay:m} = [0.72 \div 0.93];\ \frac{E_{50}}{E_{50}^{ref}} = 0.9854\left(\frac{\sigma_y}{p^{ref}}\right)^{0.8686},\ R^2 = 0.9865 \tag{10}$$

$$\text{Soft clay: m} = [0.72 \div 0.84];\ \frac{E_{50}}{E_{50}^{ref}} = 1.0287\left(\frac{\sigma_y}{p^{ref}}\right)^{0.7585},\ R^2 = 0.9875 \tag{11}$$

This value is consistent with the experimental results of von Soos [7] that m is between 0.5≤ m ≤ 1.0 with the lower catchment as sand and the upper margin is soft clay.

From the stress-strain diagram obtained from the experiment, draw the tangents line E_{ur} as defined by the modulus E_{ur} of the HS model to determine the loading and re-loading E_{ur}, resulting in present in Table 4.

Based on the definition of E_{ur} in the HS model, formula (5), we have:

$$E_{ur} = E_{ur}^{ref}\left(\frac{\sigma_y}{p^{ref}}\right)^m = E_{ur}^{ref}\left[\frac{c\cot\varphi' - \sigma_3'}{c\cot\varphi' + p^{ref}}\right]^m \Rightarrow m = \log_{\left[\frac{\sigma_y}{p^{ref}}\right]}\left[\frac{E_{ur}}{E_{ur}^{ref}}\right] \tag{12}$$

From Eq. (12), the parameter m is determined according to the unloading module E_{ur} as shown in Table 4.

From the relationship between E_{ur}/E_{ur}^{ref} and σ_y/p^{ref} (formula 12), the regression line of TRENDLINE as shown in Fig. 13, we have the following results:

Table 4 E_{ur}, E_{ur}^{ref} and m from test results

Depth (m)	Sample no.	E_{ur} (kG/cm^2)	E_{ur}^{ref} (kG/cm^2)	$\frac{E_{ur}}{E_{ur}^{ref}}$	$\frac{\sigma_y}{p^{ref}}$	M (−)
Very soft clay						
4 ÷ 6	1	68.98	127.27	0.54	0.485	0.85
	2	127.27		1.00	1.074	–
	3	205.90		1.62	1.690	0.92
12 ÷ 14	4	106.09	106.09	1.00	0.885	–
	5	177.23		1.67	1.758	0.91
	6	284.03		2.68	3.300	0.82
Soft clay						
18 ÷ 20	7	99.39	99.39	1.00	0.899	–
	8	149.33		1.50	1.674	0.79
	9	276.73		2.78	3.394	0.84
24 ÷ 26	10	108.31	108.311	1.00	0.982	–
	11	173.42		1.60	1.747	0.84
	12	298.81		2.76	3.442	0.82

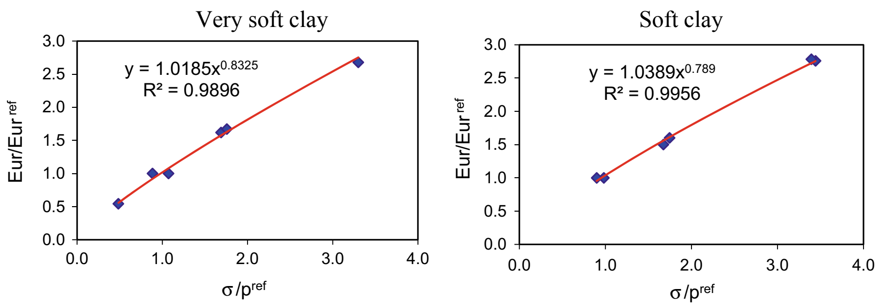


Fig. 13 The regression equation correlates between E_{ur}/E_{ur}^{ref} and σ_y/p^{ref}

$$\text{Very soft clays: m} = [0.82 \div 0.92];\ \frac{E_{ur}}{E_{ur}^{ref}} = 1.0185\left(\frac{\sigma_y}{p^{ref}}\right)^{0.8325},\ R^2 = 0.9896 \tag{13}$$

$$\text{Soft clays: m} = [0.79 \div 0.84];\ \frac{E_{ur}}{E_{ur}^{ref}} = 1.0389\left(\frac{\sigma_y}{p^{ref}}\right)^{0.789},\ R^2 = 0.9956 \tag{14}$$

4.3 Determination of Correlation Coefficient E_{ur}/E_{50} for Soft Soil in HCMC

With the default set of parameters of the HS model in Plaxis, $E_{ur}^{ref}/E_{50}^{ref} = 3$ is often chosen [4]. However, the actual ratio is very different for each soil type. From the results of experiments on soft soil in HCMC. The authors propose this coefficient as Table 5 (Fig. 14).

From there, the mean value of the correlation coefficient $E_{ur}^{ref}/E_{50}^{ref}$ for soft soil is given HCMC is:

$$\text{Very soft clay } \frac{E_{ur}^{ref}}{E_{50}^{ref}} = [3.99 \div 5.26] \tag{15}$$

$$E_{ur}^{ref} = 4.5462\,E_{50}^{ref} - 2.077,\ R^2 = 0.9371 \tag{16}$$

$$\text{Soft clay } \frac{E_{ur}^{ref}}{E_{50}^{ref}} = [4.62 \div 5.32] \tag{17}$$

$$E_{ur}^{ref} = 4.8383\,E_{50}^{ref} + 2.5745,\ R^2 = 0.9766 \tag{18}$$

This coefficient differs considerably from the default value in Plaxis according to Vemeer [6] for all soil types:

Table 5 Correlation coefficient E_{ur}/E_{50} for soft soil in HCMC

Depth (m)	σ_c (kG/cm^2)	E_{50} (kG/cm^2)	E_{ur} (kG/cm^2)	$\frac{E_{ur}}{E_{50}}$
Very soft clay				
4 ÷ 6	0.5	17.29	68.98	3.99
	1.0	33.88	127.27	3.76
	2.0	46.97	205.90	4.16
12 ÷ 14	1.0	21.15	106.09	5.02
	2.0	33.72	177.23	5.26
	4.0	60.51	284.03	4.69
Soft clay				
18 ÷ 20	1.0	20.72	99.39	4.80
	2.0	29.12	149.33	4.87
	4.0	52.05	276.73	5.32
24 ÷ 26	1.0	22.93	108.31	4.72
	2.0	34.35	173.42	5.05
	4.0	64.69	298.81	4.62

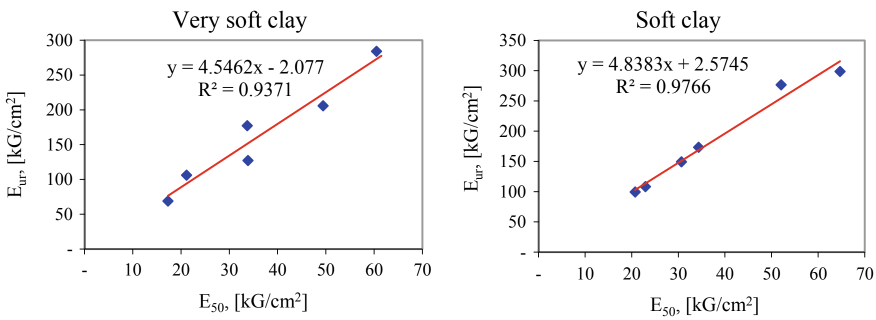


Fig. 14 The regression equation correlates $E_{ur}^{ref} - E_{50}^{ref}$ of soft soil HCMC

$$E_{ur}^{ref} \approx 3E_{50}^{ref} \tag{19}$$

5 Conclusions

- The stiffness of the soil depends on the state of stress, the dependence of the hardness on the stress state of soft soil of HCMC is in the range:
  - Determined from the drained triaxial compression test through E_{50}:

$$\text{Very soft clay : m} = [0.72 \div 0.93];\ \text{Soft clay : m} = [0.72 \div 0.84]$$

− Determined from the drained triaxial compression test through E_{ur}:

$$\text{Very soft clay : m} = [0.82 \div 0.92];\ \text{Soft clay : m} = [0.79 \div 0.84]$$

− Mean m value for soft soil of HCMC:

$$\text{Very soft clay : m} \approx 0.86;\ \text{Soft clay : m} \approx 0.80.$$

- Soil has a large modulus and non-linear in the stress path of loading and unloading, the actual stiffness of the soil is much higher than the modulus of deformation obtained from conventional test. With soft soil of HCMC ratio $E_{ur}^{ref}/E_{50}^{ref}$ as follows:

$$\text{Very soft clay : } \frac{E_{ur}^{ref}}{E_{50}^{ref}} = [3.99 \div 5.26];\ \text{Soft clay : } \frac{E_{ur}^{ref}}{E_{50}^{ref}} = [4.62 \div 5.32]$$

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