# Dynamic Analysis of Beams on Nonlinear Foundation Considering the Mass of Foundation to Moving Oscillator

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## 1 Introduction

Analyzing dynamic behavior of beams on the ground bearing moving loads is one of the topics having been interested in for more than a decade and the special consideration in nonlinear foundation including analytical and numerical solutions has been indicated in many studies as Ding et al. [2], Froio et al. [3], Jorge et al. [4], Rodrigues et al. [10], Younesian et al. [11] and Zhou et al. [12]. Nowadays, the interest has been increasing rapidly thank to the improvement in transport system. This topic is applied in designing building structures like airport runway surface, train rails, bridge structure and horizontal fluid conduit etc. For some types of soil, modern high-speed trains could move at the same speed as the smallest phase velocity of the propagating wave in the elastic substrate [1, 5, 6], displacement due to vibrating causes can be significantly larger due to static load. Therefore, it is interesting to study the dynamic reaction of structures supporting motion mechanical systems to minimize the above-mentioned impacts.

Most models used in reality or research have a common feature not to mention the influence of the mass of foundation during the analysis. But the fact is that the ground has mass, so the mass of foundation will have some influence on the behavior of the structure. Therefore, the problem of analyzing the effect of the mass of foundation on the dynamic behavior of the structure interacting with the foundation is really necessary and deserves attention. But in most studies, this has not been really focused on. Therefore, there have been very few works published in recent years.

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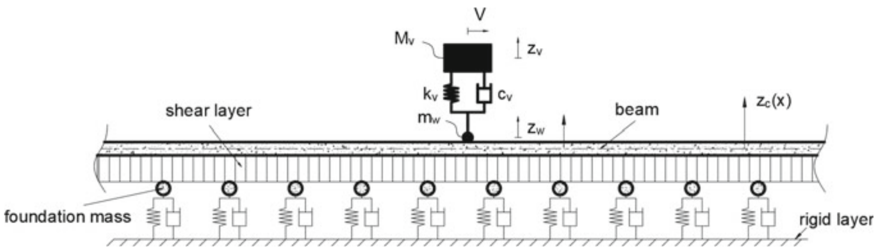



**Fig. 1** Problem model

Pham et al. [9] experiments determine the effect of the mass of foundation on the natural frequency of the plate on the elastic foundation, the experimental results show that the mass of foundation involved in vibration is a significant influence on dynamic characteristics of the plate. Nguyen et al. [8] proposed a new model for dynamic analysis of beams on a nonlinear foundation subject to moving mass. This model includes linear and nonlinear Winkler foundation parameters, Pasternak linear foundation parameters, viscous coefficient and special consideration of the influence of the mass of foundation parameters.

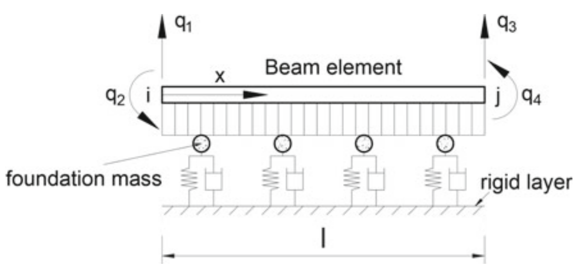
Through what the author has presented above, the problem of considering the effect of the mass of foundation on the behavior of beams is still quite new and there has not been much research on it (Fig. 1).

## 2 Formulation

Consider a simply supported beam of length $L$, height $h$, width $b$, Young's modulus $E$, density mass of the beam $\rho$ and the foundation mass $m_f$. Based on finite element method, the beam is discretized to $n$ element of length $l$. Each element has two nodes, two degrees of freedom per node as Fig. 2.

The generalized displacements and transverse of the element are as follows

**Fig. 2** Beam element



$$\mathbf{q}^e = \left\{ q_1 \ q_2 \ q_3 \ q_4 \right\}^{\mathrm{T}}$$

$$z_c(x,t) = \left\{ N_1(x) \ N_2(x) \ N_3(x) \ N_4(x) \right\} \begin{Bmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \\ q_1(t) \end{Bmatrix} = \mathbf{N}^e(x)\mathbf{q}^e(t) \tag{1}$$

The total kinetic energy of the beam and foundation of the element

$$T = T_b + T_f = \int_0^l dT_b + \int_0^l dT_f = \frac{1}{2}\int_0^l m_f \dot{z}_c(x,t)^2 dx$$
$$+ \frac{1}{2}\int_0^l \rho A \dot{z}_c(x,t)^2 dx = \frac{1}{2}\left(\dot{\mathbf{q}}^e\right)^{\mathrm{T}} \mathbf{M}_b^e \dot{\mathbf{q}}^e + \frac{1}{2}\left(\dot{\mathbf{q}}^e\right)^{\mathrm{T}} \mathbf{M}_f^e \dot{\mathbf{q}}^e \tag{2}$$

where, $\mathbf{M}_b^e$ and $\mathbf{M}_f^e$ are the elementary consistent beam's and foundation's mass matrices given as

$$\mathbf{M}_b^e = \int_0^l \mathbf{N}^e(x)^{\mathrm{T}} \rho A \mathbf{N}^e(x)\mathrm{dx} = \frac{\rho A l}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix}$$
$$\mathbf{M}_f^e = \int_0^l \mathbf{N}^e(x)^{\mathrm{T}} m_f \mathbf{N}^e(x)\mathrm{dx} \tag{3}$$

The beam elastic strain energy is given as

$$U_b = \frac{1}{2}\int_V \sigma_{xx}\varepsilon_{xx} dV = \frac{1}{2}\int_0^l M \frac{\partial^2 z_c}{\partial x^2} dy = \frac{1}{2}\left(\mathbf{q}^e\right)^{\mathrm{T}} \mathbf{K}_b^e \mathbf{q}^e \tag{4}$$

in which $\mathbf{K}_b^e$ is the elementary beam's stiffness matrix as follows

$$\mathbf{K}_b^e = \frac{EI_y}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \tag{5}$$

Reaction on each unit of length is $F_f = F_l + F_{nl} = k_l z_c + k_S \nabla^2 z_c + k_{nl} z_c^3$, where $k_l$ is the Winkler linear elastic parameter, $k_S$ is the Pasternak shear layer parameter, and $k_{nl}$ is the nonlinear elastic parameter. So that, elastic strain energy on each unit of length is given as

$$u_f = \int_0^{z_c} F_f dz_c = \frac{1}{2}k_l z_c^2 + \frac{1}{2}k_S \nabla^2 z_c^2 + \frac{1}{4}k_{nl} z_c^4 \tag{6}$$

The elastic strain energy of the foundation and potential of the forces are as follows

$$\begin{aligned} U_f &= \int_0^l u_f dz_c = \int_0^l \left(\frac{1}{2}k_l z_c^2 + \frac{1}{2}k_S \nabla^2 z_c^2 + \frac{1}{4}k_{nl} z_c^4\right) dz_c \\ &= \frac{1}{2}\left(\mathbf{q}^e\right)^{\mathrm{T}} \mathbf{K}_l^e \mathbf{q}^e + \frac{1}{2}\left(\mathbf{q}^e\right)^{\mathrm{T}} \mathbf{K}_S^e \mathbf{q}^e + \frac{1}{4}\int_0^l k_{nl}\left(\mathbf{N}^e(x)\mathbf{q}^e\right)^4 dz_c \end{aligned} \tag{7}$$

$$V = -\int_0^l f_c \delta(x - x_0) z_c(x, t) \mathrm{dx} - \left(\mathbf{Q}^e\right)^{\mathrm{T}} \mathbf{q}^e = -f_c \mathbf{N}^e(x_0)\mathbf{q}^e - \left(\mathbf{Q}^e\right)^{\mathrm{T}} \mathbf{q}^e \tag{8}$$

where, $\delta(x - x_0) = \begin{cases} 1 \text{ if } x = a \\ 0 \text{ if } x \neq a \end{cases}$ is Dirac's delta function, $f_c$ is the contact reaction force between the moving oscillator and the beam, and $\mathbf{Q}^e$ is the vector of generalized forces. The governing equations of the system can be obtained from the Lagrange equations and Hamilton's principle as

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\mathbf{q}}^e} - \frac{\partial T}{\partial \mathbf{q}^e} + \frac{\partial (U + V)}{\partial \mathbf{q}^e} = \mathbf{Q}_{nc}^e \tag{9}$$

where $\mathbf{Q}_{nc}^e = -\mathbf{C}^e \dot{\mathbf{q}}^e$, $\mathbf{C}^e = a_0 \mathbf{M}_b^e + a_1 \mathbf{K}_b^e + \mathbf{C}_f^e$, $\mathbf{C}_f^e = \int_0^{l_e} \mathbf{N}^e(x)^{\mathrm{T}} c_f \mathbf{N}^e(x) \mathrm{dx}$, $a_0 = 2\zeta_f \sqrt{\frac{2k_l}{\rho A}}$; Eq. (9) is derived as

$$\left(\mathbf{M}_b^e + \mathbf{M}_f^e\right)\ddot{\mathbf{q}}^e + \mathbf{K}_b^e \mathbf{q}^e + \mathbf{Q}_{nl}^e\left(q^e\right) = f_c \mathbf{N}^e(x_0) + \mathbf{Q}^e + \mathbf{Q}_{nc}^e \tag{10}$$

in which $\mathbf{Q}_{nl}^e(\mathbf{q}^e) = \frac{\partial U_f}{\partial \mathbf{q}^e} = \mathbf{K}_l^e \mathbf{q}^e + \mathbf{K}_S^e \mathbf{q}^e + \int_0^l \mathbf{N}^e(x)^{\mathrm{T}} k_{nl} (\mathbf{N}^e(x)\mathbf{q}^e)^3 \mathrm{dx}$ is a force vector including the Pasternak and nonlinear elastic parameters; beam's stiffness matrix derives as $\frac{\partial^2 U}{\partial \mathbf{q}^e \partial \mathbf{q}^e} = \mathbf{K}_b^e + \mathbf{K}_l^e + \mathbf{K}_S^e + 3\int_0^l \mathbf{N}^e(x)^{\mathrm{T}} k_{nl} (\mathbf{N}^e(x)\mathbf{q}^e)^2 \mathbf{N}^e(x) \mathrm{dx}$. From Eq. (10), the governing equation of motion is obtained as

$$\left(\mathbf{M}_b + \mathbf{M}_f\right)\ddot{\mathbf{q}} + \mathbf{C}_f\, \dot{\mathbf{q}} + \mathbf{K}_b \mathbf{q} + \mathbf{K}_l \mathbf{q} + \mathbf{K}_S \mathbf{q}$$

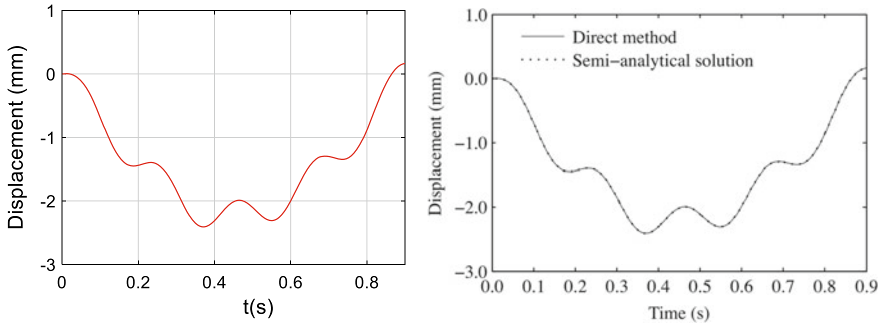



**Fig. 3** The displacements of the midpoint of the beam: present (left) and Neves (right)

$$+\int_0^l \mathbf{N}(x)^{\mathrm{T}} k_{nl} (\mathbf{N}(x)\mathbf{q})^2 \mathbf{N}(x) \mathrm{d}x\mathbf{q} = f_c \mathbf{N}(x_0) \tag{11}$$

This equation is solved by the step by step of Newmark algorithm in the time domain based on the program written in MATLAB language.

# 3 Numerical Results

## 3.1 Verified Examples

In the first verifiable numerical example, the solutions of displacement at the mid-beam (no foundation) subjected to a moving oscillator along a simple beam with constant velocity are derived and compared with ones of Neves et al. [7]. The results show the agreements expressed in Fig. 3. It can be seen that program using MATLAB code is completely reliable and used to investigate the parameters in the following section.

## 3.2 Numerical Investigation

In this section, a simple beam of length $L = 5$m, cross-sectional area $A = 0.1\text{m}^2$, density mass $\rho = 7860\text{kg/m}^3$, Young's modulus $E = 206 \times 10^9\text{N/m}^2$ on the dynamic foundation has $K_L = 50$, $K_S = 1$, $K_{NL} = 10^7$, $c_f = 100\text{Ns/m}^2$, $m_f = \beta\rho$ is used to dynamic analysis. The moving oscillator has dynamic characteristics as damping factor $\zeta_v = 10\%$, velocity $V = 50$m/s, $\kappa = 0.5$, $\gamma = 0.5$. The dimensionless parameters are defined as $K_L = \frac{k_l L^4}{EI}$, $K_{NL} = \frac{k_{nl} L^6}{EI}$, $K_S = \frac{k_S L^2}{\pi^2 EI}$, $\kappa = (M_v + m_w)/M_b$, $\gamma = \frac{\omega_v}{\omega_b}$. From the Fig. 4 to Fig. 9, the behavior of the beam in this system based

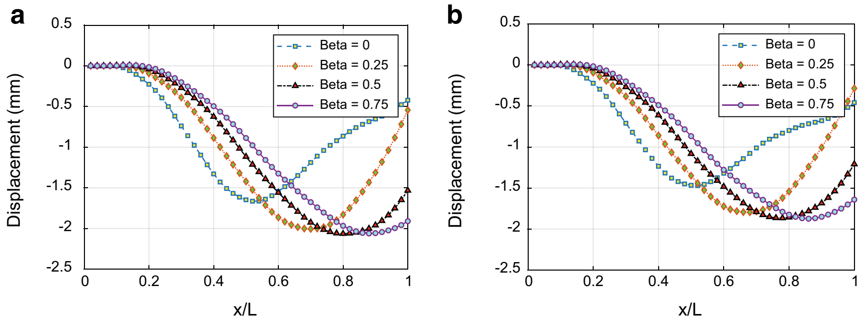



**Fig. 4** The displacements of the midpoint: **a** $K_L = 25$, **b** $K_L = 50$

on the time history of vertical deflection at the midpoint and dynamic magnification factor (DMF), defined as the ratio of the maximum dynamic displacement at the midpoint and the maximum static displacement, is determined. The foundation mass has significantly effects on the dynamic response of the beam, shown from Fig. 4 to Fig. 11. In the many cases, the foundation mass is more increasing or decreasing the time history displacement of the beam than without effects of the foundation mass with various parameters. These results can be analyzed as follows. Due to the total mass of the system including the beam, foundation and moving oscillator, increased and the global stiffness remaining constant, then the dynamic properties of the system are also changed as natural frequencies reduced. So that, the dynamic behavior of the structure must be changed in the various of the range of values of velocity. These behavior also depend on the various foundation parameters clearly expressed from Fig. 4 to Fig. 11. Next, the time history of vertical displacement at mid point of the beam due to varying velocity of moving oscillator and multiple moving oscillators is also studied. These responses of the beam are respectively plotted in Figs. 12 and 13 with various of foundation mass, defined as β parameter. Similarly, it can be seen that the dynamic displacement of the beam increased significantly and clearly compared to without effects of the foundation mass.

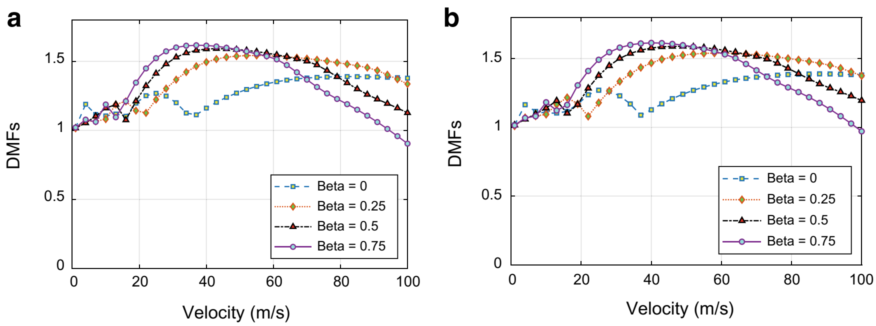



**Fig. 5** Dynamic magnification factor: **a** $K_L = 25$, **b** $K_L = 50$

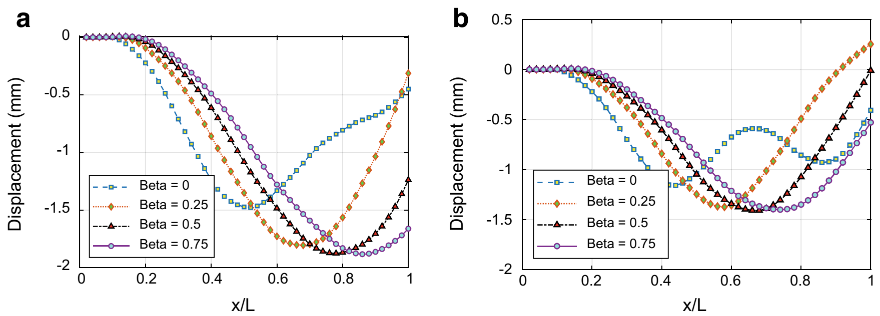



**Fig. 6** The displacements of the midpoint: **a** $K_{NL} = 10^5$, **b** $K_{NL} = 10^9$

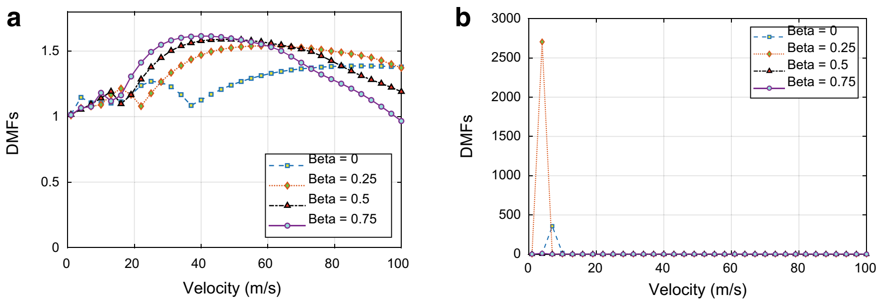



**Fig. 7** Dynamic magnification factor: **a** $K_{NL} = 10^5$, **b** $K_{NL} = 10^9$

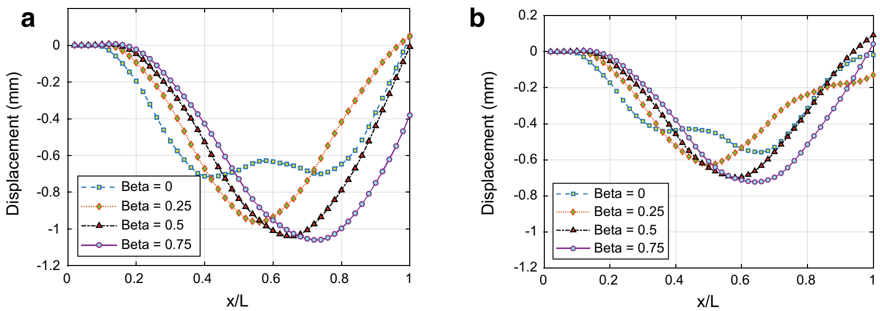



**Fig. 8** The displacements of the midpoint: **a** $K_S = 3$, **b** $K_S = 5$

## 4 Conclusion

The numerical results indicate that the mass of foundation has a certain of influence on the dynamic behavior of the beam. The mass of foundation in most cases increases the behavior of beams such as: increase displacement, increase dynamic coefficient.

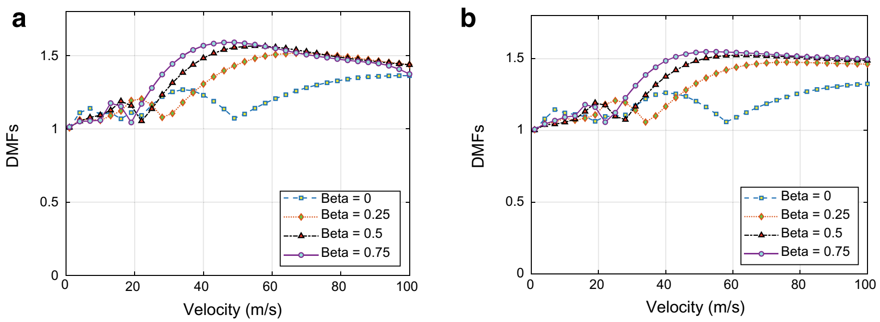



**Fig. 9** Dynamic magnification factor: **a** $K_S = 3$, **b** $K_S = 5$

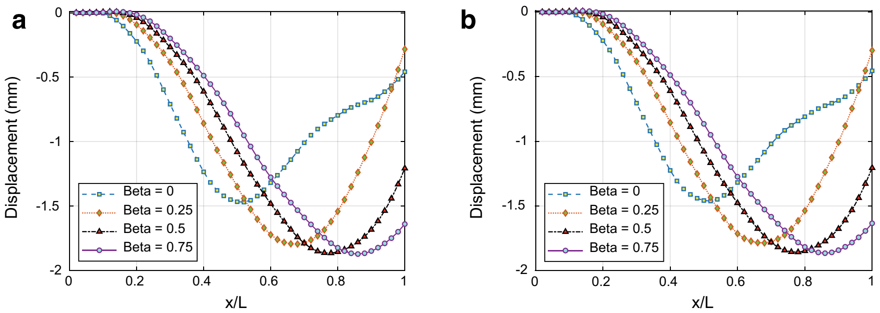



**Fig. 10** The displacements of the midpoint: **a** $c_f = 0$ (Ns/m$^2$), **b** $c_f = 10^3$ (Ns/m$^2$)

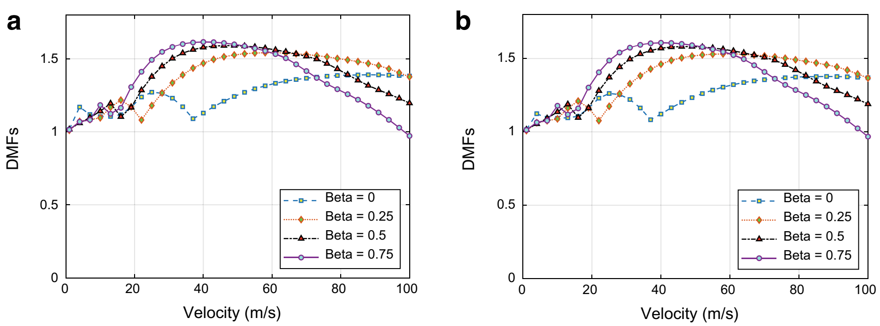



**Fig. 11** Dynamic magnification factor: **a** $c_f = 0$ (Ns/m$^2$), **b** $c_f = 10^3$ (Ns/m$^2$)

The foundation model is more suitable than the existing foundation models, because the model has considered the influence of the mass of foundation by including the kinetic energy contributing to the total kinetic energy of the system.

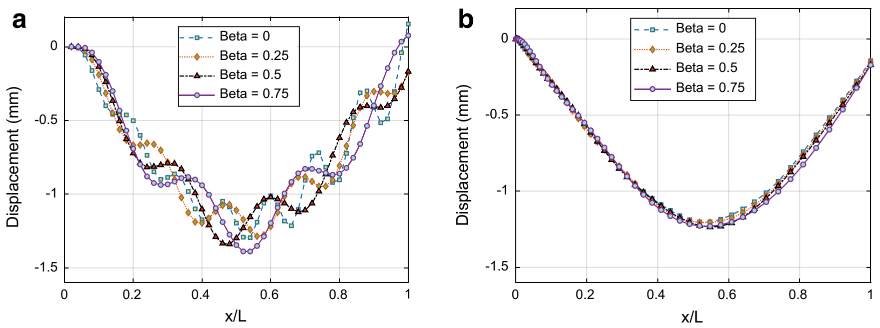



**Fig. 12** The displacements of the midpoint: **a** a = 0 (m/s$^2$), $v_0$ = 10 (m/s), **b** a = 40 (m/s$^2$), $v_0$ = 0 (m/s)

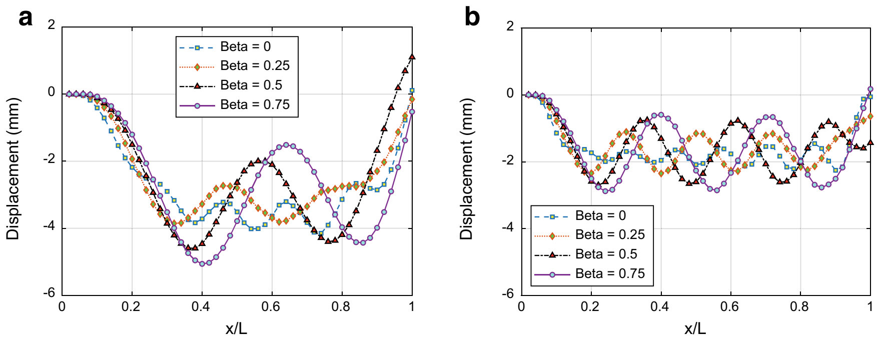



**Fig. 13** The displacements of the midpoint: **a** 10 moving oscillators, distance between two oscillators of 1 m, **b** 10 moving oscillators, distance between two oscillators of 2 m

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