State of Art on Load-Carrying Capacity and Settlements of Stone Columns

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Abstract Construction of structures over a soft clay deposit is probably the most challenging task for a geotechnical engineer. Structures built over the soft soil experience a huge settlement, which may become the main cause of failure. Thus, for any construction, improvement of the soft clay properties is necessary. Stone columns are frequently used to improve the soft soil nowadays. Stone columns speed up the rate of consolidation of the soft soil and thereby increase the load-carrying capacity and lower the settlement value. A lot of studies have been reported on the behavior of soft clay reinforced with stone columns; many theories have also been developed. An Indian Standard is also available to determine the bearing capacity and the settlement behavior of stone columns. However, the theories are not well accepted because of wide variations of results. The present study includes a review of published theories and a comparison of results based on a typical problem.

Keywords Stone column · Bearing capacity · Settlement · Ground improvement · Soft clay

1 Introduction

Construction of stone columns is a widely used technique for the improvement of soft clayey soil. Basically, the stone columns improve the stiffness of the soft clay, thereby improve the load-carrying capacity of the soft clay and also act as a drain for a speedy drainage of entrapped water, and thereby accelerate the consolidation settlement. Stone columns were first introduced by Moreau et al. in 1830 [1]. They used the stone column of diameter 0.2 m and length 2 m for the foundation of ironworks for the military purpose in Bayonne, France. Moreau declared a huge amount

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of improvement in soft soil with the use of stone columns in form of a reduction in settlement and increase in load-carrying capacity. Steuermann introduced the vibro-compaction technique of stone column construction for the first time in 1939. The use of the vibroflotation technique was introduced in USA and Germany in the 1940s. The depth of treatment of soft soil by using stone column was introduced to 20 m at the end of the 1950s. In 1955, the use of stone column spread to Japan, and later, this technique of ground improvement was widely used in China and other countries. In India, the use of stone columns started in the 1970s.

It is reported that the stone columns are able to support any structure constructed on very soft to firm clayey soils and loose silty sands with fines more than 15% [2, 3]. Stone columns are being applied to support many structures like embankments, raft foundations, liquid storage tanks and other low-rise structures. As per IS 15284-1 (2003), the stone columns work most effectively in clayey soils with undrained shear strength ranging from 7 to 50 kPa [4]. Depending upon the method of installation and the installation equipment, the stone columns can be formed up to a depth of 30 m [5]. However, as per IS 15284-1 (2003), the optimal value of length of a stone column is four times its diameter.

Maheshwari and Khatri discussed the nonlinearity of stone column-reinforced soil [6]. Saroglou et al. also discussed the properties of stone column-reinforced soft soil [7]. Rajesh and Jain (2015) studied the influence of permeability affecting stone column-reinforced ground [8]. As per the available records, the use of stone columns started in 1830. However, the first theory on the determination of the capacity of a stone column and the extent of improvement of properties of soft clay was published in 1983 [2]. Most of the studies were conducted in the laboratory, and a few studies were conducted in the field. The present paper discusses a state of art on determination of load-carrying capacity [4, 9–11] and estimation of settlement [12–14] for structures resting on stone columns. At the end, a comparison of results obtained from different theories is also presented.

2 Installation Techniques

There are two types of installation techniques for construction of stone columns, namely vibroflotation technique and rammed technique [15]. In both the techniques, bore holes are first made in the soft soil and are filled with well-graded stones in layers. The size of stones varies from 25 to 40 mm, and spacing of holes varies from 2 to 3 times the column diameter. In the vibroflotation technique, a vibroflot is used to compact the stone, and in rammed technique, a rammer is used to compact the stone. The insertion and compaction of stones are carried out simultaneously in layers. The column installation effect decreases with the increase in radial distance [16]. During compaction, the diameter of the hole increases by 33.33% of the original diameter [17].

3 Failure Mechanism

Up to 1974, researchers only studied on field experiments. After that the researchers started for laboratory model tests on stone column. In 1974, Hughes and Withers obtained the bulging effect up to a depth four times the column diameter [1]. They also obtained the bulging failure in case of a group column like a unit cell. As per unit cell concept, every single column in a group acts individually, and no interaction effects are considered. Therefore, the capacity of a group column is considered as the summation of the capacity of individual columns. But later it was found that there must be some interaction effect in between two adjacent columns. Therefore, the concept of unit cell was proved partially wrong [18]. In 1995, Hu conducted the study on laboratory model tests of group stone columns and concluded that the general shear failure occurs in case of a group stone column [19]. In 1997, Rao et al. conducted the laboratory model tests in single and group columns and observed the characteristics of spacing between the stone columns [20]. They reported that as the column comes closer to the bulging zone, the bearing capacity of a group column increases due to confinements. In 2002, Bae et al. conducted both laboratory model test and numerical analysis with FEM method and found the general shear failure in a conical shape in case of a group stone column [21]. In a group stone column, the failure mode was obtained as the combination of lateral deflection and bulging failure of stone columns [22]. From the previous studies of the failure of a stone column, we cannot conclude anything with full confirmation because of much confusion related to the failure in case of a group of stone columns. Therefore, elaborated studies are needed to know about the details of failure as per soil conditions and stone column pattern whether single or group pattern and in case of a group column, the different type of arrangements, i.e. triangular, rectangular, etc.

4 Bearing Capacity Determination

When stone columns are loaded, they deform by bulging near the surface. The adjoining soil imparts a confining effect which helps the stone column to take the vertical load. Moreover, a stone column is not a uniform composition of concrete but an assemblage of stone aggregates. Thus, the accepted formula of load-carrying capacity of a pile cannot be applied to a stone column.

There are various methods for the determination of the bearing capacity of a stone column. The methods are briefly discussed below.

As per IS code [4], the load-carrying capacity of the ground treated with stone columns is obtained by adding the contribution of three components:

(a) the resultant capacity obtained from the resistance provided by the surrounding soft soil against the lateral deformation, i.e. bulging under the axial load, Q_1 as expressed in Eq. (1),

(b) capacity resulting from an increase in the resistance provided by the surrounding soft soil because of the surcharge over it, Q_2 as expressed in Eq. (2) and
(c) the bearing support, which the intervening soil provides between the columns, Q_3 as expressed in Eq. (3).

$$Q_1 = \frac{\sigma_v \pi D^2/4}{\text{FOS}} \tag{1}$$

where

FOS factor of safety = 2,
σ_v limiting axial stress $= \sigma_{rL} K_{p_{\text{col}}}$,
σ_{rL} limiting radial stress $= \sigma_{r_0} + 4c_u$,
c_u undrained shear strength (undisturbed) of the soft clay surrounding the column,
σ_{r_0} initial effective radial stress,
$K_{p_{\text{col}}}$ coefficient of passive earth pressure $= \tan^2\left(45^\circ + \frac{\varphi_c}{2}\right)$,
D column diameter and
φ_c shearing resistance angle of stone.

$$Q_2 = \frac{K p_{\text{col}} \Delta\sigma_{r_\text{o}} A_\text{s}}{2} \tag{2}$$

where

$\Delta\sigma_{r_\text{o}}$ increase in mean radial stress of the soil and
A_s area of the stone column.

$$Q_3 = q_{\text{safe}} A_\text{g} \tag{3}$$

where

A_g Intervening soil area for each column $= 0.866S^2 - \frac{\pi D^2}{4}$ for triangular arrangement and
q_{safe} the safe bearing pressure of soil.

Thus, safe load on each column,

$$Q = Q_1 + Q_2 + Q_3. \tag{4}$$

Afshar and Ghazavi (2012) used an imaginary retaining wall such that it stretches vertically from the stone column edge and presented a theoretical expression as presented in Eq. (5) to find the bearing capacity of the column [10]. The bearing capacity of the column is expressed as

$$q_{\mathrm{ult}} = C_c N_{\mathrm{c}} \overline{q} N_{\mathrm{q}} + \frac{1}{2} W \gamma_{\mathrm{c}} N_{\mathrm{c}} \quad (5)$$

where, $N_{\mathrm{c}} = 2\frac{\cos\frac{\varphi_{\mathrm{c}}}{2}}{\cos\frac{\varphi_{\mathrm{s}}}{2}}\frac{\sqrt{K_{pc_{\mathrm{c}}}}}{K_{\mathrm{as}}}$, $N_{\mathrm{q}} = \frac{K_{p\mathrm{c}}}{K_{\mathrm{as}}}\frac{\cos\frac{\varphi_{\mathrm{c}}}{2}}{\cos\frac{\varphi_{\mathrm{s}}}{2}}$ and $N_{\gamma} = \tan\eta_{\mathrm{a}}\left(\frac{K_{p\mathrm{c}}}{K_{\mathrm{as}}}\frac{\cos\frac{\varphi_{\mathrm{c}}}{2}}{\cos\frac{\varphi_{\mathrm{s}}}{2}} - \frac{\gamma_{\mathrm{s}}}{\gamma_{\mathrm{c}}}\right)$
where $K_{p_{\mathrm{c}}}$ = lateral passive earth pressure coefficient, K_{as} = lateral active earth pressure coefficient, γ_{c} = unit weight of the stone material, γ_{c} = unit weight of soft soil, φ_{s} and φ_{c} are the friction angles of soft soil and stone column material, respectively, q = surcharge pressure on passive region surface, η_{a} = angle between active wedge and horizontal direction and K_{as} = lateral active earth pressure coefficient.

$$K_{\mathrm{as}} = \frac{\cos^2\varphi_{\mathrm{s}}}{\cos(\delta_1)\left[1 + \sqrt{\frac{\sin(\varphi_{\mathrm{s}}+\delta_1)\sin(\varphi_{\mathrm{s}})}{\cos(\delta_1)}}\right]^2}$$

$$K_{\mathrm{pc}} = \frac{\cos^2\varphi_{\mathrm{c}}}{\cos(-\delta_2)\left[1 - \sqrt{\frac{\sin(\varphi_{\mathrm{c}}+\delta_2)\sin(\varphi_{\mathrm{c}})}{\cos(-\delta_2)}}\right]^2}$$

$$K_{\mathrm{pc_c}} = K_{\mathrm{pc}}\left(1 + \frac{c_{\mathrm{w}}}{c_{\mathrm{c}}}\right)$$

$$\delta_1 = \frac{\varphi_{\mathrm{s}}}{2} \text{ and } \delta_2 = \frac{\varphi_{\mathrm{c}}}{2}$$

where

c_{w} is the wall–soil interface cohesion whose value varies between $0.3c_{\mathrm{c}}$ for stiff soil and c_{c} for soft soil. In the absence of experimental data, the recommended c_{w} value is $0.45c_{\mathrm{c}}$.

W = width of continuous strips for each row of the stone columns = $\frac{A_{\mathrm{s}}}{S}$
where A_{s} is the horizontal cross-sectional area of the column and S is the center to center distance between two subsequent stone columns.

$$\eta_{\mathrm{a}} = \varphi_{\mathrm{s}} + \tan^{-1}\left(\frac{-\tan\varphi_{\mathrm{s}} + C_1}{C_2}\right)$$

where

$$C_1 = \sqrt{\tan\varphi_{\mathrm{S}}(\tan\varphi_{\mathrm{s}} + \cot\varphi_{\mathrm{s}})(1 + \tan\delta_1\cot\varphi_{\mathrm{s}})}$$

and

$$C_2 = 1 + (\tan\delta_1[\tan\varphi_{\mathrm{s}} + \cot\varphi_{\mathrm{s}}])$$

Etezad et al. (2015) investigated the bearing capacity of stone columns (group) in soft soil [11]. They presented an analytical model for the prediction of the bearing capacity of the stone column-reinforced soft soil under rigid raft foundation considering general shear failure mechanism. The authors presented the forces which act on the soft soil section and composite soil section with the failure a mechanism as shown in Figs. 4 and 5. They expressed an equation for the ultimate bearing capacity of the stone column as shown in Eq. (6). They also presented the expression of bearing capacity factors (N_c, N_q and N_γ) as shown in Eqs. (7), (8) and (9).

$$q_{ult} = C_{comp} N_c + q N_q + \frac{1}{2} B \gamma_{comp} N_\gamma \tag{6}$$

where

$$N_\gamma = 2\cos(\psi - \varphi) \left\{ \frac{\frac{F}{\gamma_{comp} B^2} \cos\delta \left[(a-1)\cdot(\tan\psi - \cot\theta_1) + \left(\frac{a \cdot e^{\theta_1 \tan\varphi_{comp}}}{\sin\theta_1} \right) \right]}{\frac{1}{3}\cos(\psi - \varphi_{comp}) + \sin(\psi - \varphi_{comp}) \left(\frac{1}{3}\tan\psi - \frac{\tan\psi - \cot\theta_1}{2} \right)} - \frac{\frac{1}{12\sin^3\theta_1 \left(9\tan^2\varphi_{comp} + 1\right)} \left[e^{\theta_1 \tan\varphi_{comp}} - \left(3\tan\varphi_{comp}\sin\theta_1 + \cos\theta_1\right) \right] - \frac{\tan\psi - \cot\theta_1}{24}}{\frac{1}{3}\cos(\psi - \varphi_{comp}) + \sin(\psi - \varphi_{comp}) \left(\frac{1}{3}\tan\psi - \frac{\tan\psi - \cot\theta_1}{2} \right)} \right\} - \frac{\tan\psi}{2}, \tag{7}$$

$$\frac{F}{B^2} = \gamma_c \left\{ \frac{\frac{H1^2 \sin\theta^* \tan(90 - \theta^*) \cos\varphi_c}{3B^2 \sin^2\theta_2} \left[-\cos(\theta^* + \theta_2) + \cos\theta^* e^{\theta_2 \tan\varphi_c} \right]^2 \cdot \left[\frac{1}{2}\cos(\theta^* + \theta_2) + \cos\theta^* e^{\theta_2 \tan\varphi_c} \right]}{\cos\delta \left[a \cdot \sin\theta_2 + \sin\theta^* \cos(\theta^* + \theta_2) \right] - \sin\delta \cos\theta^* \cos(\theta^* + \theta_2)} + \frac{\frac{\xi \cdot H1^2 \cos^3\theta^*}{3B^2 \sin^2\theta_2 \left(9\tan^2\varphi_c + 1\right)} - \frac{H1^2 \cos^2\theta^* \cos^2(\theta^* + \theta_2)}{3B^2 \sin\theta_2}}{\cos\delta \left[a \cdot \sin\theta_2 + \sin\theta^* \cos(\theta^* + \theta_2) \right] - \sin\delta \cos\theta^* \cos(\theta^* + \theta_2)} \right\},$$

$$N_q = \cos(\psi - \varphi_{comp}) \frac{\frac{A}{B} \cdot \cos\delta \left[\frac{2H_1 \cdot m}{B} - (\tan\psi - \cot\theta_1) \right]}{\left[\frac{1}{4}\cos(\psi - \varphi_{comp}) + \frac{1}{2}\sin(\psi - \varphi_{comp}) \left(\frac{-\tan\psi}{2} + \cot\theta_1 \right) \right]} \tag{8}$$

and

$$N_c = \tan\psi + \cos(\psi - \varphi_{comp}) \frac{\frac{D \cdot C_c}{B \cdot C_{comp}} \cdot \cos\delta \left[\frac{2H_1 \cdot m}{B} - (\tan\psi - \cot\theta_1) \right] + \frac{1}{4\sin^2\theta_1 \tan\varphi_{comp}} \left(e^{2\theta_1 \tan\varphi_{comp}} - 1 \right) - C_{comp}}{\left[\frac{1}{4}\cos(\psi - \varphi_{comp}) + \frac{1}{2}\sin(\psi - \varphi_{comp}) \left(\frac{-\tan\psi}{2} + \cot\theta_1 \right) \right]} \tag{9}$$

By using all these equations, the bearing capacity of a stone column can be found out. The researchers also developed some design charts for N_c, N_q and N_γ against the friction angle of stone column materials. The values of these charts are converted into tabular form and presented in Tables 1, 2 and 3.

The angles θ_1, θ_2, θ^*, φ_c, φ_{comp} and δ can be better understood from Figs. 3 and 4 of reference [11].

Table 1 Variation of N_c against friction angle of stone column material (φ_s) for various native soil friction angles (φ_c) (after Afshar and Ghazavi [10])

φ_s	φ_c										
	30	32	34	36	38	40	42	44	46	48	50
0	8.5	9.2	9.99	10.3	11.8	12.48	13.1	14.97	15.18	17.54	20
5	9.85	10.04	11.2	12.4	12.58	13.7	15	16.3	18.05	20.05	22.5
10	11	11.7	12.49	13.1	14	15.85	17.29	18.4	20.5	22.72	25.95
15	12.6	12.75	13.75	14.9	16.4	17.6	19.04	21.92	23.90	26.73	29.95
20	13.98	14.99	15.98	17.35	18.10	20.05	22.1	24.92	27.5	30.03	33.8

Table 2 Variation of N_q versus friction angle of stone column material (φ_s) for various native soil friction angles (φ_c) (after Afshar and Ghazavi [10])

φ_s	φ_c										
	30	32	34	36	38	40	42	44	46	48	50
0	3.05	4.01	4.09	4.55	5	5.54	6.02	6.52	7.1	7.5	7.85
5	4.85	5.0	5.11	5.9	6.5	6.8	7.1	7.55	8.05	8.95	10
10	5.95	6.25	6.8	7.1	7.5	7.82	8.3	9.7	10.04	11.95	12.55
15	7.48	7.55	7.85	8.05	9.85	10.04	11.45	12.5	13.4	15.03	17.2
20	9.02	9.95	10.25	11.9	12.49	13.6	15	16.9	17.95	20.05	22.51

Table 3 Variation of N_γ versus friction angle of stone column material (φ_s) for various native soil friction angles (φ_c) for $\left(\frac{\gamma_s}{\gamma_c}\right) = 1.2$. (after Afshar and Ghazavi [10])

φ_s	φ_c										
	30	32	34	36	38	40	42	44	46	48	50
0	4	4.1	5	5.11	6.1	7.45	9.1	10	12.02	14.05	16.95
5	5	5.06	6.11	7.15	8.05	10	11.2	12.98	15.05	18.1	22.5
10	6	7.45	8.3	9.95	10.07	12.55	14.95	17.5	20.03	24.95	30
15	9.05	10	11.8	12.64	15	17	20	24.02	27.7	32.55	40
20	12.4	13.97	15.02	17.48	20	23.12	22.45	32.41	37.5	44.95	55

where $c_{\text{comp}} = A_s c_s + (1 - A_s)c_c$,
$\gamma_{\text{comp}} = A_s\gamma_s + (1 - A_s)\gamma_c$,
and A_s = replacement ratio

$$= \frac{A_{\text{col}}}{s^2}$$

for a square column pattern.

$$\varphi_{\text{comp}} = \tan^{-1}[A_s\mu_s \tan\varphi_s + (1 - A_s)\mu_c \tan\varphi_c],$$

where

A_{col} cross section area of the column,
s spacing between the columns,
c_s cohesion of stone column's material,
c_c cohesion of clay,

$$\mu_s = \frac{n}{1 + (n - 1)A_s}$$

and

$$\mu_c = \frac{1}{1 + (n - 1)A_s},$$

where n = stress ratio

$$= \frac{\text{vertical stress in the granular material}}{\text{vertical stress in cohesive soil.}}$$

The authors presented the following charts for the solution:

(a) N_γ versus shearing resistance angles for the reinforced soil for $\frac{\gamma_{\text{comp}}}{\gamma_c} = 1$, $\frac{\gamma_{\text{comp}}}{\gamma_c} = 1.2$, $\frac{\gamma_{\text{comp}}}{\gamma_c} = 1.4$, $\frac{\gamma_{\text{comp}}}{\gamma_c} = 1.6$, $\frac{\gamma_{\text{comp}}}{\gamma_c} = 1.8$ and $\frac{\gamma_{\text{comp}}}{\gamma_c} = 2$.
(b) N_q versus shearing resistance angles for the reinforced soil.
(c) N_c versus shearing resistance angles for the column-reinforced soil for $c_{\text{comp}}/c_c = 0.2$, $c_{\text{comp}}/c_c = 0.4$, $c_{\text{comp}}/c_c = 0.6$ and $c_{\text{comp}}/c_c = 0.8$.

The charts for the determination of N_q were converted into tabular form which is presented in Table 4.

The remaining charts can be obtained from reference [11].

By using these charts, the bearing capacity factors and thereby the bearing capacity of stone columns for different friction angles can be found out.

Table 4 Variation of N_q with the shearing resistance angles for the reinforced ground (after Etezad et al. [11])

ϕ_c	ϕ_{comp}						
	15	20	25	30	35	40	45
0	2.5	2.53	2.6	3.2	4.9	6.4	7.5
6	2.51	2.8	3.5	5	6.1	7.8	10
12	3.5	4.1	5	6.2	7.45	10.13	13.1
18			7.3	8.5	11.3	14.84	17.9
24				12.65	15.5	19.99	24.98
30						30	39.85

5 Settlement Calculation of Stone Columns

Various investigators conducted the theoretical study on settlement of stone columns and developed some solutions [15, 23–26]. As per IS 15284 (part 1) 2003 [4], the settlement of the treated ground can be calculated with the use of reduced stress method. As per IS code, by using the expression presented in Eqs. (10), (11) and (12), the settlement of stone columns can be calculated. This method is based on the replacement ratio (a_s) and the stress concentration factor (n). The stress concentration factor can be estimated as

$$n = \frac{\text{vertical stress in the composite soil}, \sigma_g}{\text{vertical stress in soft soil}, \sigma_s} \quad (10)$$

And, the replacement ratio is the ratio of the area of the stone column to the equivalent area represented by a stone column, which is estimated as

$$a_s = \frac{A_s}{A_s + A_g} \quad (11)$$

where A_s represents the area of the stone column and A_g represents the plan area of the soil for the column.

In case of a stone column, the settlement of stone column mainly occurs due to the consolidation of the soil. IS code suggests the consolidation settlement (S_t) as

$$S_t = m_v \sigma_g H \quad (12)$$

where

m_v modulus of volume decrease of soil,
σ_g vertical stress in surrounding soil, and
H thickness of treated soil.

Castro (2016) conducted an analytical study and obtained the expression for the settlement of stone columns below rigid footings [23]. He considered a horizontal slice at a depth of 'z' of the unit cell as shown in Fig. 1 and investigated the settlement over there.

He found out the settlement of each layer and obtained the total settlement of the stone columns by summing up all these values. To find out the total settlement, he used Eq. (13).

$$S_z = \sum_{1}^{i} \varepsilon_{z,i} \Delta t_i \tag{13}$$

where

Δt_i thickness of ith slice and
$\varepsilon_{z,i}$ vertical strain at ith slice.

For stone column,
oedometric (constrained) modulus,

$$E_{\mathrm{mc}} = E_{\mathrm{c}} \frac{(1 - \vartheta_{\mathrm{c}})}{(1 + \vartheta_{\mathrm{c}})(1 - 2\vartheta_{\mathrm{c}})}$$

For soft soil, oedometric modulus,

$$E_{\mathrm{ms}} = E_{\mathrm{s}} \frac{(1 - \vartheta_{\mathrm{s}})}{(1 + \vartheta_{\mathrm{s}})(1 - 2\vartheta_{\mathrm{s}})}$$

where E_{c} and E_{s} are Young's modulus of stone column materials and soft soils, respectively.

ϑ_{s} and ϑ_{c} are Poisson's ratio of soft soil and stone column, respectively.

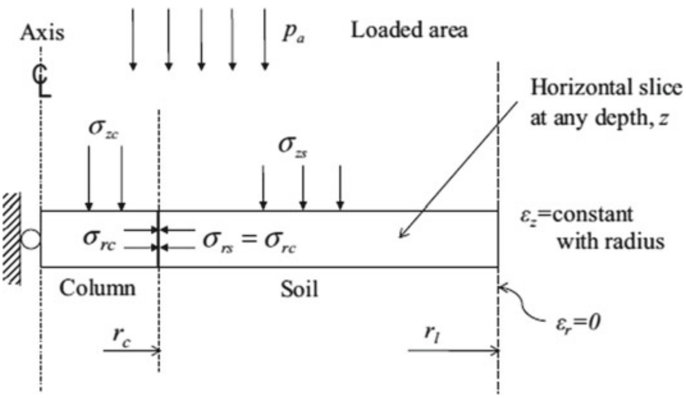

Fig. 1 Analytical model showing the horizontal slice (after Castro [23])

Shear modulus of stone column materials, $G_c = \frac{E_s}{2(1+\vartheta_c)}$.
Shear modulus of soft soil, $G_s = \frac{E_s}{2(1+\vartheta_s)}$.
Lame's constant for stone column materials, $\lambda_c = E_{mc} - 2G_c$.
Lame's constant for soft soil, $\lambda_s = E_{ms} - 2G_s$.
Active earth pressure coefficient
for the stone column, $K_{ac} = \frac{1-\sin\varphi_c}{1+\sin\varphi_c}$
and for dilatancy angle, $K_c = \frac{1-\sin_c}{1+\sin_c}$,
where ψ is the dilatancy angle.
Column constant, $C_E = \frac{3\lambda_c + 2G_c}{1+2K_{ac}K_c + \frac{\lambda_c}{G_c}(1-K_{ac}-K_c+K_{ac}K_c)}$.
For each slice,
area replacement ratio, $a_r = \frac{Nd_c^2}{\left(D+\frac{Z}{2}\right)^2}$,

where N is the number of stone columns in a group pattern, d_c is the stone column diameter and Z is the depth of the slice considered.

The average vertical stress at each depth,

$$p_a(z) = p_a(0)\frac{D^2}{\left(D + \frac{Z}{2}\right)^2},$$

where $p_a(0)$ is the applied load and D is the diameter of footing.

$$F^* = \frac{\lambda_c - \lambda_s}{a_r(\lambda_c + \lambda_s + G_c + G_s) + (\lambda_c + G_c - G_s)}$$

$$E_{ml}^e = a_r E_{mc} + (1-a_r)E_{ms} + F^* a_r(\lambda_s(1-a_r) - \lambda_c(1+a_r))K^*$$

$$= \frac{K_{ac} - \frac{1}{C_E}\lambda_s}{\frac{a_r(\lambda_s + G_s) - G_s}{C_E(1+a_r)} + K_{ac}K_c}$$

$$E_{ml}^p = (1-a_r)E_{ms} + \frac{(1-a_r)}{(1+a_r)}a_r\lambda_s K^* + \frac{a_r}{K_{ac}}J^*,$$

where $J^* = \lambda_s + \frac{a_r(\lambda_s + G_s) - G_s}{1+a_r}K^*$.

$$Y = \frac{\left(K_{0s}\gamma_s' - K_{ac}\gamma_c'\right)E_{ml}^e}{G_c[2K_{ac} + F^*(1+a_r)] - \lambda_c(1-K_{ac})[1 - F^*(1+a_r)]}$$

$$p_a^y = Yz$$

If $p_a \le p_a^y$, then $\varepsilon_z = \frac{p_a}{E_{ml}^e}$.
If $p_a > p_a^y$, then $\varepsilon_z = \frac{p_a^y}{E_{ml}^e} + \frac{p_a^p}{E_{ml}^p}$.

Zahmatkesh and Choobbasti (2010) defined a term called the settlement reduction ratio (SRR) as SRR $= \frac{\text{settlement of the composite ground}}{\text{settlement of ground without stone column}}$ for evaluating the settlement of soft clay reinforced with stone columns [25]. SRR is also defined as SRR $= \frac{E_o}{E_{eq}}$, where

E_O Young's modulus of ground without stone column and
E_{eq} Young's modulus of the composite ground.

Zhang et al. (2013) suggested the settlement of stone columns as the summation of the total compression deformation of the stone column (S_p) and the settlement of the underlying unreinforced layers (S_s) as shown in Eq. (14) [26]. Therefore, they divided the unit cell into N elements and found out the settlement of each element differently and obtained the total settlement by summing up settlements of all the elements. The total compression deformation of the column (S_p) and the settlement of the underlying unreinforced layers (S_s) can be obtained from Eqs. (15) and (16). Thus, the total settlement becomes

$$S = S_p + S_s \tag{14}$$

where

$$S_p = \sum_{i=1}^{N} \Delta s_{p,i} \tag{15}$$

and

$$S_s = \sum_{i=1}^{N_s} \frac{q_i}{E_{si}} H_i \tag{16}$$

q_i = vertical stress due to the transfer of the applied stress (q) down into the ith subjacent unreinforced soil layer,

H_i is the thickness, E_{si} is the compression modulus of the ith subjacent unreinforced soil layer,

and N_s represents number of the soil layers.

$$\Delta s_{p,i} = l_i \times \frac{\sigma_{zp,i}}{E_p} \times \frac{1 - \mu_p - 2\mu_p^2}{(1 - \mu_p) - 2\mu_p k_i}$$

where μ_p is Poisson's ratio of the column and E_p is Young's modulus of the column.

$$\sigma_{zp,i} = \text{uniform vertical stress}$$
$$= \frac{E_p}{1 - \mu_p - 2\mu_p^2}\left[(1 - \mu_p) - 2\mu_p k_i\right]\varepsilon_{z,i}$$

$\varepsilon_{z,i}$ is the vertical strain of the column at the ith slice.

Christian and Carrier (1978) carried out model tests with stone columns and compared the load-settlement behavior with that obtained through commercially available software PLAXIS [27]. In a theoretical study, they used the following set of equations to find out the settlement ratio (SR) which is the ratio of the settlement of the composite soil to the settlement of the soft soil without a stone column.

$$s_{\mathrm{E}} = \mu_0 \mu_1 \frac{qB}{E} \tag{17}$$

$$E_{\mathrm{eq}} = \frac{\sigma}{s} \tag{18}$$

$$\varepsilon = \frac{s}{L}$$

$$\mathrm{SR} = \frac{E_0}{E_{\mathrm{eq}}},$$

where q represents the applied footing load, E represents elastic modulus of the soil and μ_1 and μ_0 are the constant values which depend on the thickness between the footing base and hard strata and the depth of the footing, respectively. σ is the average applied stress, s is the settlement of the footing, ε is the average strain and L is the thickness of the clay bed. E_0 represents Young's modulus of the unreinforced ground, and E_{eq} is the equivalent secant modulus with the assumption the whole soil medium is homogeneous.

Greenwood and Thompson (1984) presented one chart of area ratio versus settlement ratio to find out the settlement of treated soil for a constant undrained strength of soil [15]. The chart is shown in Fig. 2.

ΔH ettlement of untreated soil,

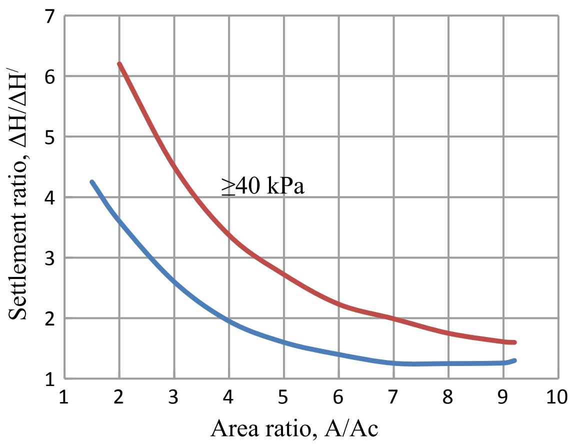

Fig. 2 Approximate settlement reduction for ground reinforced with stone columns (after Greenwood and Thompson [15])

$\Delta H'$ Settlement of stone column treated soil,
A_c Area of stone column and
A Area supported by column.

6 Design Example Problem

Problem statement:
To obtain the bearing capacity and settlement of a footing of 1 m diameter resting on a group of stone columns of diameter 0.5 m and length of 4 m arranged in a triangular pattern with spacing at 2.5 times the diameter of column,
friction angle of the column, $\varphi = 43°$,
undrained cohesion of clay, $c_u = 25$ kPa,
unit weight of stone column material, $\gamma_s = 20\text{kN/cu} \cdot \text{m}$,
unit weight of clay, $\gamma_c = 17\frac{\text{kN}}{\text{cu}} \cdot \text{m}$,
liquid limit, $w_L = 55\%$ and
water content $= 34\%$.
Solution:

A. Bearing Capacity Determination
Bearing capacity of untreated soil as per IS code

$$q_u = c_u N_c s_c d_c i_c + q N_q s_q d_q i_q = 25 \times 5.14 \times 1.3 \times 1 \times 1 + 17 \times 0 = 167.05\,\text{kPa}$$

The bearing capacity of a group of stone columns is obtained by different methods as discussed below:

1. IS 15284 (part 1) 2003 [4]:

(a) Bearing capacity due to bulging of column:

$$Q_1 = \frac{\left(\sigma_v \pi D^2\right)/4}{2}$$

$K_0 = 0.6$ for clays.
$\sigma_{v0} = 17$ kPa.
$\sigma_{r0} = 20.4$ kPa.
$\sigma_{rL} = 20.4 + 4 \times 25 = 120.4$ kPa.
$k_{p_{c0l}} = (\tan(45 + 21.5))^2 = 5.29$
$\sigma_v = 120.4 \times 5.29 = 636.916$ kPa.
Yield load $= 636.916 \times \frac{\pi \times 0.5^2}{4} = 125.058\,\text{kN}$
Safe load on column, $Q_1 = \frac{125.058}{2} = 62.53\,\text{kN}$

(b) Capacity due to a surcharge:
Increase in mean radial stress, $\Delta\sigma_{r0} = \frac{q_{\text{safe}}}{3}(1 + 2K_0)$.
For $\varphi = 43°$,

$N_c = 5.7$.
So, $q_{\text{safe}} = \frac{25 \times 5.14}{2.5} = 51.4$,

$$A_s = \frac{\pi \times 0.5^2}{4} = 0.1963\,\text{m}^2,$$

$$\Delta\sigma_{r0}\frac{51.4 \times (1 + 2 \times 0.6)}{3} = 37.7\text{ kPa and}$$

$$Q_2 = \frac{k_{p_{\text{col}}}\Delta\sigma_{r0}A_s}{2}$$

$= \frac{5.29 \times 37.7 \times 0.1936}{2} = 19.3kN.$

(c) Bearing capacity provided by the intervening soil:

$$Q_3 = q_{\text{safe}} \cdot A_g$$

$$A_g = 0.866s^2 - (\pi \times d^2)/4.$$

$$= 0.866 \times (1.25)^2 - \frac{\pi}{4} \times 0.5^2 = 1.157\,\text{m}^2$$

$$Q_3 = 51.4 \times 1.157 = 59.47\,\text{kN}$$

Therefore, the total bearing capacity, $Q = Q_1 + Q_2 + Q_3$
$= 62.53 + 19.3 + 59.47 = 141.3\,\text{kN}$.
Therefore, ultimate bearing capacity,

$$Q_u = 282.6\,\text{kN} = \frac{282.6}{0.866 \times (1.25)^2} = 208.85\text{kPa}.$$

2. By Etezad et al. [11]:
The ultimate bearing capacity of stone column,
$q_{\text{ult}} = C_{\text{comp}}N_c + qN_q + \frac{1}{2}B\gamma_{\text{comp}}N_\gamma$.
Replacement ratio,
$A_s = \frac{\frac{\pi}{4} \times (0.5)^2}{0.866 \times (1.25)^2} = 0.58$ for a triangular column pattern.

$$c_{\text{comp}} = 0.58 \times 0 + (1 - 0.58) \times 25 = 10.5\text{ kN/m}$$

$$\gamma_{\text{comp}} = 0.58 \times 20 + (1 - 0.58) \times 17 = 22.1\text{ kN/m}$$

$$n = \frac{\sigma_s}{\sigma_g} = \frac{636.916}{167.05} = 3.812$$

$$\mu_s = \frac{n}{1+(n-1)A_s} = \frac{3.812}{1+(3.812-1)\times 0.58} = 1.45$$

$$\mu_c = \frac{1}{1+(n-1)A_s} = 0.38$$

So,

$$\varphi_{comp} = \tan^{-1}[A_s\mu_s \tan\varphi_s + (1-A_s)\mu_c \tan\varphi_c]$$
$$= \tan^{-1}\left[0.58\times 1.45\times \tan 43° + (1-0.58)\times 0.38\times \tan 0\right] = 38.1°$$

From the design charts, they presented the bearing capacity factors are obtained as shown below:
For $\frac{\gamma_{comp}}{\gamma_c} = 1.3$ and $\varphi_{comp} = 38.1°,\ N_\gamma = 7.05$
For $\frac{C_{comp}}{C_c} = \frac{10.5}{25} = 0.42$ and $\varphi_{comp} = 38.1°,\ N_C = 33$
For $\varphi_{comp} = 38.1°$, $N_q = 5.38$
So, the ultimate bearing capacity,

$$q_u = 10.5\times 33 + 0 + 0.5\times 22.1\times 1\times 7.05 = 424.4 \text{ kPa}.$$

3. By Afshar and Ghazavi [10]:
The bearing capacity of stone column, $q_{ult} = C_c N_c + \bar{q} N_q + \frac{1}{2} W\gamma_c N_\gamma$.
Here,

$$\delta_1 = \frac{\varphi_s}{2} = \frac{43°}{2} = 21.5°$$

and $\delta_2 = \frac{\varphi_c}{2} = \frac{0°}{2} = 0°$.
$c_w = C_c$ for soft soil $= 25$ kPa.
$K_{pc} = \frac{\cos^2 0°}{\cos(-0°)\left[1-\sqrt{\frac{\sin 0°\cdot\sin 0°}{\cos 21.5°}}\right]} = 1.$

$$K_{pc_c} = 1(1+1) = 2.$$

$K_{as} = \frac{\cos^2 43°}{\cos(21.5°)\left[1+\sqrt{\frac{\sin(43+21.5)°\sin(43°)}{\cos(21.5°)}}\right]^2} = 0.1748.$
So,

$$N_c = 2\frac{\cos\frac{0°}{2}}{\cos\frac{43°}{2}}\frac{\sqrt{2}}{0.1748} = 17.39,$$

$$N_q = \frac{1\times 1}{0.1748\times\cos 21.5°} = 6.15,$$

$$C_1 = \sqrt{\tan 43°(\tan 43° + \cot 43°)(1+\tan 21.5°.\cot 43°)} = 1.6307$$

and $C_2 = 1 + (\tan 21.5°[\tan 43° + \cot 43°]) = 1.8$.
So, $\eta_a = 43° + \tan^{-1}\left(\frac{-\tan 43° + C_1}{1.8}\right) = 64.374°$ and
$N_\gamma = \tan 43^0\left(\frac{1}{0.1748}\frac{cos0°}{cos21.5°} - \frac{20}{17}\right) = 10.366$.
Now, W = width of continuous strips for each row of stone columns $= \frac{A_s}{S}$ $= \frac{0.196}{1.25} = 0.1571$.
So, $q_{\text{ult}} = 25 \times 17.39 + 0 + \frac{1}{2} \times 0.1571 \times 17 \times 10.366 = 448.592$ kPa.

B. Settlement Determination

For untreated soil,
settlement,

$$S = \frac{C_c H}{1 + e_0}\log\left(\frac{\sigma_0 + \Delta\sigma}{\sigma_0}\right)$$
$$= 230.1\text{mm}.$$

For soils treated with stone column, the following methods are applied for calculation:

i. IS 15284, PART-1, 2003 [4]:

The settlement due to consolidation is

$$S_t = m_v \sigma_g H$$

Initial stress, $\sigma_0^{'} = 10 \times 4/2 = 20\,\text{kPa}$ and
$\sigma_v^{'}$ = change in stress $= \frac{167.05 \times \frac{\pi}{4} \times 1^2}{3 \times \frac{\pi}{4} \times 3^2} = 6.185\,\text{kPa}$.
Compression index, $c_c = 0.009(w_L - 10\%)$

$$= 0.009 \times (55 - 10)$$

$= 0.405$.
So, $e = 0.405 \times \log_{10}\frac{5.28+20}{20} = 0.0412$.
Again, water content = 34%,
$e_0 = w_s \times G = 0.34 \times 2.6 = 0.884$ and
$m_v = \frac{0.0412}{(1 + 0.884) \times 5.28} = 4.14 \times 10^{-3}$.
Again, $\sigma_g = \mu_g$,
$n = 3.812$,
$a_s = 0.58$,

$$\mu_g = \frac{1}{1 + (n - 1)A_s} = \frac{1}{1 + (3.812 - 1) \times 0.58} = 0.38 \quad \text{and}$$

Settlement of stone column, $S_t = 4.14 \times 10^{-3} \times 6.185 \times 4 = 0.08744\,\text{m} = 102.43\,\text{mm}$..

(ii) By Castro [23]:

Let us consider, Young's modulus for soft soil, $E_s = 25,000\text{kPa}$,

Young's modulus for column materials, $E_c = 55,000\text{kPa}$,
Poisson's ratio for soft soil, $\vartheta_s = 0.35$ and
Poisson's ratio for stone column material, $\vartheta_c = 0.33$.
Since vertical stress in surrounding soil, $\sigma_g = 167.05\text{kPa}$,
applied load, $p_\text{a}(0) = \left(167.05 \times \frac{1.157}{3}\right)\text{kN} = 64.43\,\text{kN}$.
For stone column,
oedometric (constrained) modulus, $E_\text{mc} = \frac{55000\times(1-0.33)}{(1+0.33)(1-2\times 0.33)} = 81490.49\,\text{kPa}$.
For soft soil, oedometric modulus, $E_\text{ms} = \frac{25000\times(1-0.35)}{(1+0.35)(1-2\times 0.35)} = 40123.456\,\text{kPa}$,
where E_c and E_s are Young's modulus of stone column materials and soft soils.
Shear modulus of the column materials, $G_\text{c} = \frac{55000}{2(1+0.33)} = 20676.69$.
Shear modulus of soft soil, $G_s = \frac{25000}{2(1+0.35)} = 9259.26$.
Lame's constant for stone column materials,

$$\lambda_\text{c} = 81490.49 - 2 \times 20676.69 = 40137.11.$$

Lame's constant for soft soil,

$$\lambda_\text{s} = 40123.456 - 2 \times 9259.26 = 21604.94$$

Coefficient of active earth pressure
for stone column, $K_\text{ac} = \frac{1-\sin 43°}{1+\sin 43°} = 0.189$
and for dilatancy angle, $K_\text{c} = \frac{1-\sin 0°}{1+\sin 0°} = 1$.
Column constant, $C_\text{E} = 45335.5$.
The whole soil profile is divided into 4 slices of 1 m thickness.
For the slice whose distance from top of the stone column, $z = 2$ m,
area replacement ratio, $a_r = 0.0625$.
The average vertical stress at each depth, $p_\text{a}(z) = 64.43 \times \frac{1^2}{\left(1+\frac{2}{2}\right)^2} = 16.106\,\text{kPa}$.

$$F^* = 0.328.$$

$$E_{ml}^{e} = 16666.82\,\text{kPa}.$$

$$K^* = 0.76.$$

$$E_{ml}^{p} = 7874.60\,\text{kPa}.$$

$J^* = 2060.95\,\text{kPa}$.

$$Y = 34.14.$$

Table 5 Settlement value for stone column

Depth in m	p_a in kPa	a_r in %	E^e_{ml} in kPa	E^p_{ml} in kPa	p^y_a in kPa	ε_z in %	S_Z in mm
0–1	64.43	0.25	4835.077	5867.3	0	3.59	35.9
1–2	28.64	0.111	4391.2	4839.76	32.375	1.998	19.98
2–3	16.1	0.0625	86666.82	16666.82	68.28	3.16×10^{-1} %	3.16
3–4	10.3	0.04	46355.51	18178.7	20.36	1.83×10^{-1}	1.83

Yielding load, $p^y_a = 34.14 \times 2 = 68.28\,\text{kPa}$.
Here, $p_a < p^y_a$.
So, strain at this layer, $\varepsilon_z = 3.16 \times 10^{-3}\%$,
Δt_i = thickness of ith slice = 1 m and $\varepsilon_{z,i}$ = vertical strain at ith slice $= 3.16 \times 10^{-3}\%$.
Settlement, $S_z = 3.16 \times 10^{-3} \times 1\,\text{m} = 3.16 \times 10^{-3}\,\text{m} = 3.16\,\text{mm}$.
Similarly, for other slices, the settlement values are listed in Table 5.
The total footing settlement becomes $S_z = \sum_1^i \varepsilon_{z,i} t_i$.
So, the total settlement is 60.87 mm.

iii By Zhang et al. [26]:
Total settlement,
S = total compression deformation of the stone column + settlement of the underlying unreinforced layers $= s_p + s_s$.
For the first layer, $\sigma_{zp,i} = 64.43\,\text{kPa}$.

$$\sigma_{zp,i} = 64.43 = \frac{E_p}{1 - \mu_p - 2\mu_p^2}\left[\left(1 - \mu_p\right) - 2\mu_p k_i\right]\varepsilon_{z,i}$$
$$= \frac{25000}{1 - 0.35 - 2 \times 0.35^2}[(1 - 0.35) - 2 \times 0.35 \times k_i]\varepsilon_{z,i}$$

$k_i = 0.922$
Now, $\Delta s_{(p,i)} = 1 \times \frac{49.6}{55000} \times \frac{(1-0.33-2\times0.33^2)}{(1-0.33)-2\times0.33\times k_i)} = \frac{0.00173}{(0.67-0.66k_i)} = \frac{0.00173}{0.05752} = 0.00735$.
So, $s_p = 29.5$ mm
Again, $S_s = \sum_{i=1}^{N_s} \frac{q_i}{E_{si}} H_i$
For the first layer, $q_1 = 64.43\,\text{kPa}$.
So, settlement of the first layer, $s_1 = \frac{64.43}{25000} \times 1 = 2.58 \times 10^{-3}\,\text{m}$.
For the second layer, $q_2 = q_{2i} + q_2$,
overburden pressure, $q_{2i} = \gamma z_2 = 19 \times 1 = 19\,\text{kPa}$,
pressure due to external load, $q_2 = \frac{64.43}{(0.5+1\times\tan 30°)^2} = 55.51\,\text{kPa}$ and
$q_2 = 19 + 55.51 = 74.51\,\text{kPa}$.

Settlement of the second layer, $s_2 = \frac{74.51}{25000} \times 1 = 2.98 \times 10^{-3}$ m.
For the third layer, $q_3 = q_{3i} + q_3$,
overburden pressure, $q_{3i} = \gamma z_3 = 19 \times 2 = 38$ kPa,
pressure due to external load, $q_3 = \frac{64.43}{(0.5+2\times\tan 30^\circ)^2} = 23.53$ kPa and
$q_2 = 38 + 23.53 = 61.53$ kPa.
Settlement of the third layer, $s_3 = \frac{61.53}{25000} \times 1 = 2.46 \times 10^{-3}$ m.
For the fourth layer, $q_4 = q_{4i} + q_4$,
overburden pressure, $q_{4i} = \gamma z_3 = 19 \times 3 = 57$ kPa,
pressure due to external load, $q_4 = \frac{64.43}{(0.5+3\times\tan 30^\circ)^2} = 12.932$ kPa and
$q_4 = 57 + 12.932 = 69.932$ kPa.
Settlement of the fourth layer, $s_4 = \frac{69.932}{25000} \times 1 = 2.8 \times 10^{-3}$ m.

$$s_s = s_1 + s_2 + s_3 + s_4 = \left(2.58 \times 10^{-3} + 2.98 \times 10^{-3} + 2.46 \times 10^{-3} + 2.8 \times 10^{-3}\right) \text{ m} = 0.00938 \text{ m} = 9.38 \text{ mm}$$

So, the total settlement becomes $s = s_\text{p} + s_\text{s} = 29.5 + 10.82 = 40.32$ mm.

(iv) By Greenwood and Thompson [15]:
Area of stone column, $A_\text{c} = \frac{\pi}{4} \times 0.5^2 = 0.1963$ m^2.
Area supported by column, $A = 0.866 \times (1.25)^2 = 1.353 - \frac{\pi}{4} \times 0.5^2 = 1.157$.
Undrained cohesion of clay, $c_\text{u} = 25$ kPa.
$\frac{A}{A_\text{c}} = \frac{1.157}{0.1963} = 5.85$.
By using Bowel's chart for $c_\text{u} = 25$kPa, $\frac{\Delta H}{\Delta H'} = R = 1.95$
Considering settlement of untreated soil, $\Delta H = 230.1$ mm,
settlement of stone column treated soil, $\Delta H' = \frac{230.1}{1.95} = 118$ mm.
The summary of all the results is presented in Table 6.

Table 6 Summary of all results

Method	Bearing capacity in kPa	Settlement in mm
For untreated soil		
	167.05	230.1
For soil treated with stone column		
IS 15284, PART-1, 2003	208.85	102.43
Etezad et al. [11]	424.4	
Afshar and Ghazavi [10]	448.592	
Castro [23]		60.87
Zhang et al. [26]		40.32
Greenwood and Thompson [15]		118

7 Conclusions

Out of many published research papers, only a few papers have been referred to in this review paper. Selection of the referred papers was based on the availability of bearing capacity and settlement formulae. It is noticed from the present study that there is a wide variation in the results of bearing capacity and settlement suggested by different researchers. The following conclusions are drawn from this review paper:

(i) Bearing capacity of the untreated soil is improved by 1.5–3 times with installation of the stone column in a triangular pattern at a spacing of 2.5 times its diameter.
(ii) The Indian Standard code [4] gives a conservative result on the improvement of bearing capacity, hence a detailed study and upgradation of the codal provision are required.
(iii) The expected consolidation settlement of untreated soil is reduced by 2–4 times with installation of stone columns.
(iv) The most conservative result was obtained by Greenwood and Thompson [15].
(v) The codal provision in IS code [4] suggests a possible reduction of 0.4–0.5 times the expected settlement of untreated column.
(vi) The overall settlement is to be reduced to an effective use of stone column below building foundation.

References

1. Hughes JMO, Withers NJ (1974) Reinforcing of soft cohesive soils with stone columns. Ground Eng 7(3):42–49
2. Barksdale, R. D., Bachus, R. C., (1983) “Design and Construction of Stone Columns Volume II, Appendixes.”Federal Highway Administration Office of Engineering and Highway Operations Research and Development, Washington
3. Lo SR, Zhang R, Mak J (2010) Geosynthetic-encased stone columns in soft clay: A numerical study. Geotext Geomembr 28(2010):292–302. https://doi.org/10.1016/j.geotexmem.2009.09.015
4. IS 15284-1 (2003) “Design and construction for ground improvement-Guidelines.” Part-1: Stone columns [CED 43: Soil and foundation engineering], ICS 93.020
5. Black, J. A., Sivakumar, V. Madhav, M. R., and Hamill, G. A. (2007). “Reinforced stone columns in weak deposits: laboratoryModel study.”Journal of geotechnical and geoenvironmental engineering, ASCE, Vol. 133, No. 9, https://doi.org/10.1061/(asce)1090-0241(2007)133:9(1154)
6. Maheshwari P, Khatri S (2010) Nonlinear response of footings on granular bed-stone column-reinforced poor soil. Int J Geotech Eng 4(4):435–443. https://doi.org/10.3328/IJGE.2010.04.04.435-443
7. Saroglou H, Antoniou A, Pateras S (2009) Ground improvement of clayey soil formations using stone columns: A case study from Greece. Int J Geotech Eng 3(4):493–498. https://doi.org/10.3328/IJGE.2009.03.04.493-498
8. Rajesh S, Jain P (2015) Influence of permeability of soft clay on the efficiency of stone columns and geosynthetic-encased stone columns – a numerical study. Int J Geotech Eng 9(5):483–493. https://doi.org/10.1179/1939787914Y.0000000088

9. Poorooshasb, H. B., Meyerhof, G. G (1996) "Consolidation settlement of rafts supported by stone columns", Journal of Southeast Asian Geotechnical Society, 1996, Vol. 27, No. 2, pp. 84-97, 1996
10. Afshar J. N., Ghazavi, M., (2012) "A simple analytical method for calculation of bearing capacity of stonecolumn." International Journal of Civil Engineering, Vol. 12, No. 1, Transaction B: Geotechnical Engineering, January 2014
11. Etezad, M., Hanna, A. M., Ayadat, T. (2015). "Bearing capacity of a group of stone columns in soft soil."InternationalJournal of Geomechanics,ASCE, ISSN 1532-3641/04014043(15)
12. Ambily., A. P., Gandhi, S.R., (2004) "Experimental and theoretical evaluation of stonecolumn in soft clay."ICGGE
13. Ambily, A. P., Gandhi, S. R., (2007) "Behavior of stone columns based on experimental and fem analysis." Journal of geotechnical and geoenvironmental engineering, ASCE, 2007.133:405-415, DOI:10.1061/(ASCE)1090-0241(2007)133:4(405)
14. Niroumand, H., Kassim, K. A., Yah, C. S., (2011) "Soil improvement by reinforced stone columns based on experimental work." EJGE, Vol. 16 [2011], Bund. L
15. Bowles, J. E.,(1988) "Foundation analysis and design."4th edition,McGraw-Hill, Singapore
16. Guetif et al (2007) Improved soft clay characteristics due to stone column installation. Comput Geotech 34(2007):104–111. https://doi.org/10.1016/j.compgeo.2006.09.008
17. Taube, G., Martin, P.E., P.G. (2002) "Stone Columns for Industrial Fills", Nicholson Construction Company, Cuddy, Pennsylvania
18. Bachus, R. C. and Barksdale, R. D. (1984), "Vertical and lateral behaviour of model stone columns.", Proceedings of the International Conference on In-situ Soil and Rock Reinforcement, Paris, 99–104
19. Hu, W. (1995) "Physical modeling of group behavior of stone column foundations", Ph.D. Thesis, University of Glasgow
20. Rao NS, Reddy KM, Rumar PH (1997) Studies on groups of stone columns in soft clays. Geotechnical Engineering-Bangkok 28(2):165–182
21. Bae, W. S., Bang, W. S., Byung, C. A. (2002) "Behavior of foundation system improved with stone columns", Proc. 12th Int. Offshore and Polar Engineering Conf., Kitakyushu, Japan, 675–678
22. Ghazavi M, Afshar JN (2013) Bearing capacity of geosynthetic encased stone columns. Geotext Geomembr 38(2013):26–36. https://doi.org/10.1016/j.geotexmem.2013.04.003
23. Castro J (2016) An analytical solution for the settlement of stone columns beneath rigid footings. Acta Geotech 11:309–324. https://doi.org/10.1007/s11440-014-0358-4
24. Sexton, B. G., McCabe, B. A., Castro, J., (2014) "Appraising stone column settlement prediction methods using finite element analyses." ActaGeotechnica (2014) 9:993–1011, https://doi.org/10.1007/s11440-013-0260-5
25. Zahmatkesh, A., Choobbasti, A. J., (2010) "Settlement evaluation of soft clay reinforced by stone columns, considering the effect of soil compaction." IJRRAS 3 (2), May 2010
26. Zhang, L., Zhao, M., Shi, C., Zhao, H., (2013) "Settlement calculation of composite foundation reinforced with stone columns."International journal of geomechanics, ASCE, 13(3): 248–256, https://doi.org/10.1061/(asce)gm.1943-5622.0000212
27. Christian JT, Carrier WD (1978) Janbu, Bjerrum and Kjaernsli's chart reinterpreted. Can Geotech J 15:123–128