Storage Optimization of Low-Level Manual Picking Warehouse Considering Vertical Travel

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Abstract. Order picking is considered the most critical operation in warehousing. In the low-level manual picking warehouse, in order to increase the warehouse capacity and make full use of the rack space, the overall height of the rack is increased by increasing the number of layer and reducing the height of each layer of the rack, or directly increasing the number of layers. These all lead to take more time to pick up the goods at the bottom or top of the rack. If the picking time of the vertical travel is not taken into account and the best-selling goods are placed on the bottom or top layer, the order picking efficiency will be greatly affected. Therefore, based on class-based storage strategy, we propose a picking time estimation model that considers the vertical travel. A heuristic algorithm is constructed to optimize the warehousing location and determine the optimal warehousing boundary of goods of different types on each shelf. The results show that the proposed algorithm can better optimize the storage location and reduce the picking time.

Keywords: Order picking · Low-level manual warehouse · Vertical travel · Storage optimization

1 Introduction

With the continuous development of economic globalization and e-commerce, consumption demand of consumers is becoming more and more personalized and diverse. As a result, enterprises receive more and more orders, and the types of goods are also more and more abundant. In this context, enterprise warehouses need more space to store goods, but as the soaring value of land, expanding the warehouse area requires significant costs. Therefore, for low-level manual picking warehouses, especially for e-commerce enterprise warehouses with a large variety of products, it is possible to expand the warehouse area and enhance the warehouse space utilization by increasing the number of shelves plies and reducing the height of each shelf layer, or directly increasing the number of shelves plies.

Order picking can be described as the process of picking up multiple items from a warehouse storage to complete a customer order. In low-level manual picking

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warehouses, order sorting operations are cumbersome and time-consuming. According to Topkkins et al. [1], the cost of sorting orders may account for 55% of all operating costs of the warehouse; and the labor of sorting orders accounts for about 60% of the total workload [2]. There are many factors to affect the efficiency of order picking, such as facility layout, order picking path, and storage allocation and so on. Among them, the optimization of storage places has been concerned by many scholars. Some scholars carried out a series of studies in the context of low-level manual picking warehouses without considering vertical travel. Le-Duc et al. [3] constructed an approximate model to estimate the picking distance in a dual-area warehouse, and proposed a heuristic algorithm to determine the optimal storage boundary problem with the goal of minimizing the distance of picking. Wu et al. [4] took a storage system as the research object, carried out a reasonable division of the storage area, and established an integer linear programming model with minimum picking time, then obtained the method of optimizing the location allocation. Pan et al. [5] considered the situation where multiple pickers work in the same area, and proposed a throughput model to optimize the order picking efficiency by considering a trade-off between the picking distance and the blocking-caused delay for the storage assignment. Bottani et al. [6] used genetic algorithm (GA) to optimize the storage location in the warehouse, which ultimately reduced the picking time and simplified the order picking operation. Glock et al. [7] created a U-shaped order picking system and proposed an alternative strategy for distributing products to shelves in it. Grosse et al. [8] considered the influence of human factors on the efficiency of the order picking system. Their results showed that learning and forgetting should be considered in order to properly plan the storage allocation strategy. Calzavara et al. [9] combined cost and ergonomics to conduct comprehensive planning of shelves with different layouts. Their results showed that the traditional pallet storage system could be replaced with half pallet on lower shelves to improve ergonomics and reduce costs. Li et al. [10] proposed an approximation algorithm to optimize the allocation of storage considering the turnover rate of goods and the correlation of demand. Zhang et al. [11] introduced the concept of demand-related models to describe the correlation between goods, and proposed a determining DCP method from historical data. On this basis, a new model was constructed to solve the problem of storage allocation. To solve this model, they developed heuristic and simulated annealing methods.

These scholars studied storage optimization from the perspective of models and algorithms. Some scholars carried out related research from the perspective of the combination of routing strategy and storage strategy, such as Dekker et al. [12] used computer simulation to combine storage and routing strategies, and then discussed the ways to improve the efficiency of warehouse operations. Dijkstra et al. [13] constructed the precise average path length mode of multi-lane and multi-cargo picking under the return, S-shaped, maximum gap and midpoint routing methods, and proposed a dynamic programming algorithm to allocate storage locations in the warehouse.

In addition, many scholars have done research on high-level shelf warehouses and AS/RS system warehouses with considering vertical travel. For example, Parikh et al. [14] pointed out that when vertical travel can be considered, models can be used to determine the optimal storage system configuration. Chan et al. [15] conducted research on storage allocation strategies and optimization orders, such as warehouse division, goods classification and division, to make the warehouse layout more reasonable and the sorting operation more efficient; Chen et al. [16] studied the optimization of shelf allocation and access path in automated warehouse based on mixed integer programming model, and proposed a two-stage decision-making algorithm based on graphics. Pan et al. [17] constructed a multi-layer "artificial" picking time estimation model based on the classification storage strategy and various routing strategies, with considering the shelf height, the number of layers and the vertical motion of the picking. Their experimental results showed that there is a sufficiently accurate practical use in the model. Kou et al. [18] used a random storage strategy in a parallel storage system based on belt conveyor to construct a travel time model for automatic retrieval and automatic storage systems, and used Matlab to verify the model and optimize the automatic storage system.

In summary, previous studies focused on low-level manual picking warehouses without considering vertical travel or high-rise warehouses and AS/RS system warehouses that consider vertical travel. However, in some low-level manual picking warehouses, in order to increase warehouse capacity, people tend to make full use of the shelf space by increasing the number of shelves and reducing the height of each layer of the shelf, or directly increasing the number of shelves to increase the overall height. This leads to the need to squat and other additional actions to pick up goods, and people need to step on the lower shelves to pick up the goods when picking up the top shelf goods. With the rapid development of e-commerce, the warehouse receives more and more orders every day. Under such circumstances, the order picking efficiency will be greatly affected if the goods with high order frequency are placed at the bottom or top layer, which takes more time for picking, without considering the vertical picking time. Therefore, this paper constructs an average picking time estimation model considering vertical travel, and proposes a heuristic algorithm to optimize the shelf storage.

2 Average Picking Time Estimation Model

2.1 Problem Description

The warehouse has a horizontal roadway (in the middle of the warehouse) and multiple working roadways. The layout of the warehouse is symmetric with respect to the lateral roadway, that is, i.e. the storage layout of the symmetric working roadway is completely consistent. The return path strategy is adopted, which is derived from Caron et al. [19]. The storage allocation uses a classification storage strategy. Figure 1 shows the warehouse layout of a horizontal roadway and six working roadways; each shelf consists of several columns and several layers, and the storage capacity on each shelf is equal to the number of columns. Figure 2 shows the layout of five shelves.

The order picking process of such warehouses can be described as follows: the pickers start picking the goods from the warehouse entrance/exit. First, they enter and the nearest working roadway where goods needed to be picked up. Then they travel to the position of the required goods, and pick up the goods from the shelf level until the farthest goods are picked up. Then they return to the middle of the horizontal roadway.

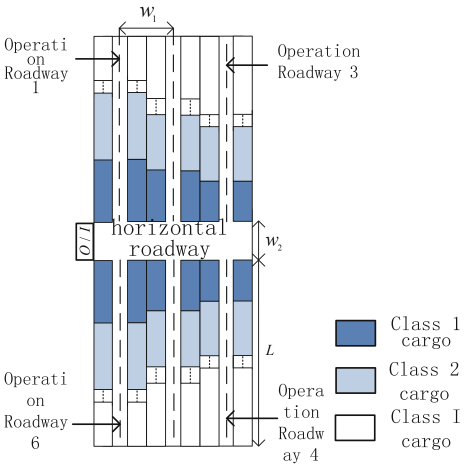

Fig. 1. Warehouse layout

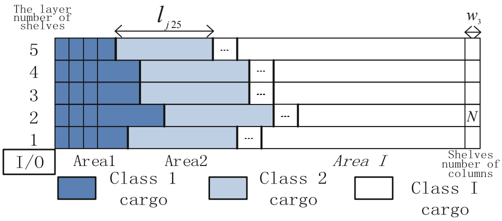

Fig. 2. Shelf layout of operation roadway

Next, the pickers enter the nearest working roadway where the goods needed to be picked up. Repeat the above process. After completing the farthest goods picking in the operation roadway, the pickers return to the warehouse entrance/exit. The picker first picks up the goods on one side of the warehouse and then picks the other side of the goods have the same total travel time with picking up the goods on both sides exactly.

2.2 Modeling Assumption, Parameter and Decision Variable

This paper proposes the following hypotheses:

(1) The storage shelves in the warehouse are identical, each storage space is of the same size, and each shelf can store multiple types of goods.
(2) The length, width and height of all operation roadway are equal.
(3) The working roadway is narrow enough that the picker can pick the goods on both shelves at the same time without changing the position.
(4) The picker can pick up the goods on the shelves regardless of the height of the shelves.

(5) Order frequency of the same category goods is equal. The order frequency of each category is defined as the number of goods needed for that category in a certain period of time.

The parameters in model as follows:

j	represents the sequence number of operation roadway $(j = 1, 2, \ldots, J)$;
i	represents the category of goods $(i = 1, 2, \ldots, I)$;
k	represents the layer number of shelves $(k = 1, 2, \ldots, K)$;
n	represents the layer number of columns $(n = 1, 2, \cdots, N)$;
(j, n, k)	represents the space coordinates of storing shelves;
w_1	represents the distance between two adjacent operation roadway center lines;
w_2	represents the width of horizontal roadway;
w_3	represents the length of each storage;
v	represents the speed of the picker;
t_k	represents the time of picking k layer numbers of the goods;
O_i	represents the order frequency of i goods.
S_i	represents the proportion of storage space of i goods, that is, the proportion of the total storage space;
q_{ji}	represents the numbers of i goods picked in j operation roadway;
p_{ji}	represents the probability of i goods picked in j operation roadway;
P_{ji}	represents the probability of picking the farthest goods in the category i goods area in the j operation roadway;
l_{jik}	represents the storage length of i goods on the k shelves in the j operation roadway;
HT_{ji}	represents the horizontal travel picking time in category i cargo area of j operation roadway;
VT_j	represents the vertical travel picking time in j operation roadway;
TT_j	represents the total picking time in j operation roadway

Decision variable:

n_{jik} represents the numbers of storage space of i goods stored in k shelves of j operation roadway.

2.3 Single Working Roadway Model

First, we consider the situation of single working roadway (all goods are in j th operation roadway), and the shelf layout of operation roadway is shown in Fig. 2, region 1, region 2,…, region I store separately 1 goods, 2 goods… I kinds of goods, the quantity of storage space for i goods in each layer is n_{jik}, within each class of cargo area, the probability of picking each stored cargo is equal.

Expected picking time TT_j in operation roadway includes the vertical travel expected picking time VT_j and the horizontal travel expected picking time. We can determine expected picking time by adjusting the farthest reach area, that is, the probability of farthest pick position in region i multiply the corresponding desired horizontal expected picking time, multiply 2 is the time of entry and exit in the operation roadway. Expected picking time in operation roadway can be obtained as follow:

$$TT_j = 2\sum\nolimits_{i=1}^{I} P_{ji} \times HT_{ji} + VT_j \qquad (2-1)$$

(1) Vertical travel expected picking time

When picking one good, we can calculate the probability that the goods on each shelf in the category are picked up by $l_{jik}\big/\sum\nolimits_{k=1}^{K} l_{jik}$, and then multiply the picking time required to pick each shelf t_k to get the horizontal expected picking time of the goods on each shelf. Then add the time spent on all shelves and multiply the probability of each category of goods will be picked so as to get the vertical travel desired picking time. When the expected quantity of picking goods is q_j, the vertical travel expected picking time is:

$$VT_j = q_j \sum_{i=1}^{I} p_{ji} \sum_{k=1}^{K} \frac{l_{jik}}{\sum\nolimits_{k=1}^{K} l_{jik}} t_k \qquad (2-2)$$
$$l_{jik} = n_{jik} \times w_3$$

(2) Horizontal travel expected picking time

We consider the horizontal travel expected picking time in the following situation:

(1) If the farthest picking goods are in region 1, this means that all goods to be picked are in region 1, i.e. $\sum\nolimits_{i=1}^{I} q_{ji} = q_{j1} = q_j$, its probability is:

$$P_{j1} = p_{j1}^{q_j} \qquad (2-3)$$

According to the ratio of the storage length l_{jik} of each shelf to the total storage length of the cargo in the operation lane, to obtain the probability of picking each shelf cargo, and then multiply the horizontal expected picking time of each shelf, adding them together to get the total horizontal expected picking time HT_{ji}

$$HT_{j1} = \sum_{k=1}^{K} \left[\left(\frac{l_{j1k}}{\sum\nolimits_{k=1}^{K} l_{jik}} \right) \left(\frac{l_{j1k}}{v} \times \frac{q_{j1}}{q_{j1}+1} \right) \right] \qquad (2-4)$$

Remark: q_{j1}random variables x are uniformly distributed in the interval $[0, 1]$. Its maximum expected value is $q_{j1}\big/(q_{j1}+1)$ [3]. The maximum expected value (the time to get to the furthest position) is $\left(l_{j1k}/v\right) \times \left(q_{ji}/(q_{ji}+1)\right)$ when pick q_j goods in region 1. Therefore, Eq. (2-4) is valid when the situation is generalized to each shelf.

(2) If the farthest picking goods are in region 2, this means that all goods to be picked are in region 1 or region 2, but not only in region 1, i.e. $\sum_{i=1}^{I} q_{ji} = q_{j1} + q_{j2} = q_j$, its probability is:

$$P_{j2} = (p_{j1} + p_{j2})^{q_j} - (p_{j1})^{q_j} \qquad (2-5)$$

Calculating the horizontal stroke picking time HT_{ji} is basically the same as the calculation method of HT_{j2}:

$$HT_{j2} = \sum_{k=1}^{K} \left[\left(\frac{l_{j2k}}{\sum_{k=1}^{K} l_{j2k}} \right) \left(\frac{l_{j1k}}{v} + \frac{l_{j2k}}{v} \times \frac{q_{j2}}{q_{j2}+1} \right) \right] \qquad (2-6)$$

q_{j2} is the number of goods to be picked in the area 2. It is difficult to calculate HT_{J2} by formula (2-6) directly, so we can calculate HT_{J2} by estimating. First, we calculate the expectation of picking quantity $E(q_{j2})$ as formula (2-7):

$$\begin{aligned} E(q_{j2}) &= \sum\nolimits_{q_{j2}=1}^{q_j} q_{j2} P(B/A) \\ &= \sum\nolimits_{q_{j2}=1}^{q_j} q_{j2} \frac{P(AB)}{P(A)} \\ &= \sum\nolimits_{q_{j2}=1}^{q_j} q_{j2} \frac{P(AB)}{P_{j2}} \\ &= \frac{\sum\nolimits_{q_{j2}=1}^{q_j} q_{j2} \begin{pmatrix} q_{j2} \\ q_j \end{pmatrix} \left(((p_{j2})/(p_{j1}+p_{j2}))\right)^{q_{j2}} \left(((p_{j1})/(p_{j1}+p_{j2}))\right)^{q_j - q_{j2}}}{\left((p_{j1}+p_{j2})^{q_j} - p_{j1}^{q_j}\right)} \\ &= \frac{(q_j p_{j2})/(p_{j1}+p_{j2})}{(p_{j1}+p_{j2})^{q_j} - p_{j1}^{q_j}} \end{aligned} \qquad (2-7)$$

As shown in the Eq. (2-7), event B refers to the quantity of goods picked in region 2, event A refers to all of the goods are in region 1 or region 2, but not only in region 1; so event AB refers to the quantity of goods picked in region 2 is q_{j2}, and the quantity of goods picked in region 1 is $q_j - q_{j2}$. The final step of the formula is derived from the expectation of the binomial distribution.

We can get the HT_{j2} by replacing q_{j2} with $E(q_{j2})$ [3]:

$$\begin{aligned} HT_{j2} &\approx \sum_{k=1}^{K}\left[\left(\frac{l_{j2k}}{\sum_{k=1}^{K} l_{j2k}}\right)\left(\frac{l_{j1k}}{v}+\frac{l_{j2k}}{v}\times\frac{E(q_{j2})}{E(q_{j2})+1)}\right)\right] \\ &= \sum_{k=1}^{K}\left[\left(\frac{l_{j2k}}{\sum_{k=1}^{K} l_{j2k}}\right)\left(\frac{l_{j1k}}{v}+\frac{l_{j2k}}{v}\times\frac{(q_jp_{j2}/(p_{j1}+p_{j2}))/\left((p_{j1}+p_{j2})^{q_j}-p_{j1}^{q_j}\right)}{(q_jp_{j2}/(p_{j1}+p_{j2}))/\left((p_{j1}+p_{j2})^{q_j}-p_{j1}^{q_j}\right)+1}\right)\right] \\ &= \sum_{k=1}^{K}\left[\left(\frac{l_{j2k}}{\sum_{k=1}^{K} l_{j2k}}\right)\left(\frac{l_{j1k}}{v}+\frac{l_{j2k}}{v}\times\frac{q_jp_{j2}/(p_{j1}+p_{j2})}{(q_jp_{j2}/(p_{j1}+p_{j2}))+(p_{j1}+p_{j2})^{q_j}-p_{j1}^{q_j}}\right)\right] \\ &= \sum_{k=1}^{K}\left[\left(\frac{l_{j2k}}{\sum_{k=1}^{K} l_{j2k}}\right)\left(\frac{l_{j1k}}{v}+\frac{l_{j2k}}{v}\times\frac{q_jp_{j2}}{q_jp_{j2}+(p_{j1}+p_{j2})\left((p_{j1}+p_{j2})^{q_j}-p_{j2}^{q_j}\right)}\right)\right] \end{aligned} \quad (2-8)$$

(3) If the farthest picking goods are in region i ($i = 3, 4, \ldots I$)

If the farthest picking goods are in region i, this means that all goods to be picked are in region 1 to region i, but not only in region 1 to region i-1, i.e. $\sum_{a=1}^{I} q_{ja} = \sum_{a=1}^{i} q_{ja} = q_j$, its probability is:

$$\begin{aligned} P_{\substack{ji \\ i\geq 3}} &= (p_{j1}+p_{j2}+\cdots+p_{ji})^{q_j} \\ &\quad -(p_{j1}+p_{j2}+\cdots+p_{j,i-1})^{q_j} \\ &= \left(\sum_{a=1}^{i} p_{ja}\right)^{q_j} - \left(\sum_{a=1}^{i-1} p_{ja}\right)^{q_j} \end{aligned} \quad (2-9)$$

Let $H(p_{ja}, q_j) = (\sum_{a=1}^{i} p_{ja})^{q_j} - \left(\sum_{a=1}^{i-1} p_{ja}\right)^{q_j}$. The expectation picking time of horizontal stroke $HT_{ji,i\geq 3}$ is calculated in a similar way HT_{J2} is calculated:

$$HT_{\substack{ji \\ i\geq 3}} \approx \sum_{k=1}^{K}\left[\left(\frac{l_{jik}}{\sum_{k=1}^{K} l_{jik}}\right)\left(\sum_{a=1}^{i-1}\frac{l_{jak}}{v}+\frac{l_{jik}}{v}\times\frac{q_jp_{ji}}{q_jp_{ji}+(\sum_{a=1}^{i} p_{ja})H(p_{ja}, q_j)}\right)\right] \quad (2-10)$$

Finally, we can get the total expected picking time in the operation roadway TT_J:

$$\begin{aligned} TT_j &= 2\sum_{i=1}^{I} P_{ji}\times HT_{ji}+VT_j \\ &= 2\times p_{j1}^{q_j}\times HT_{j1}+2\sum_{i=2}^{I}\left[H(p_{ja}, q_j)\times HT_{ji}\right]+VT_j \end{aligned} \quad (2-11)$$

2.4 Multiple Operation Roadway Model

The average picking time estimation model for multi-job roadway includes two parts: the expected picking time of all operation roadway and the expected picking time of the horizontal roadway.

First, the probability of i goods being picked in j th operation roadway p_{ji} can be obtained by the ratio of the storage length of i goods stored in j operation roadways to the storage length of i goods stored in all operation roadways, multiplied by the probability of i goods be picked O_i:

$$p_{ji} = O_i \frac{\sum_{k=1}^{K} l_{jik}}{\sum_{j=1}^{J} \sum_{k=1}^{K} l_{jik}}, \forall j \in J, i \in I$$

Then, we calculate the probability of entering each operation roadway. If a cargo is picked, the probability of entering the working roadway j is the sum of the probability of picking all the goods picked in the roadway $\sum_{i=1}^{I} p_{ji}$. If q items cargo is picked, the probability of entering the working roadway can j be obtained by the probability that all the selected items are not in the working roadway j is:

$$b_j = 1 - \left(1 - \sum_{i=1}^{I} p_{ji}\right)^q$$

Next, calculate the expected quantity q_j of goods picked up in the working roadway j based on entering the working roadway j:

$$q_j = \frac{q \sum_{i=1}^{I} p_{ji}}{b_j}$$

TT_j is the time taking to pick up q_j goods from the center-line of the horizontal roadway to the farthest picking location in the operation roadway j. In formula (2-12), $p'_{ji} = p_{ji} / \sum_{i=1}^{I} p_{ji} (\forall j \in J)$; $w_2/2v$ is the time taking from the center line of the horizontal roadway to the entrance of the working roadway.

$$\begin{aligned} TT_j &= 2 \times \frac{w_2}{2v} + 2 \times \left(p'_{j1}\right)^{q_j} \times HT_{j1} + 2 \sum_{i=2}^{I} \left[H\left(p'_{ja}, q_j\right) \times HT_{ji}\right] + VT_j \\ &= \frac{w_2}{v} + 2 \times \left(p'_{j1}\right)^{q_j} \left(\sum_{k=1}^{K} \left(\frac{l_{j1k}}{\sum_{k=1}^{K} l_{j1k}} \times \frac{l_{j1k}}{v} \times \frac{q_j}{q_j + 1} \right) \right) + 2 \times \\ & \sum_{i=2}^{I} \left\{ \begin{array}{l} H\left(p'_{ja}, q_j\right) \times \\ \left[\sum_{k=1}^{K} \left(\frac{l_{jik}}{\sum_{k=1}^{K} l_{jik}} \left(\sum_{a=1}^{i-1} \frac{l_{jak}}{v} + \frac{l_{jik}}{v} \times \frac{q_j p'_{ji}}{q_j p'_{ji} + H\left(p'_{ja}, q_j\right) \sum_{a=1}^{i} p'_{ja}} \right) \right) \right] \end{array} \right\} \\ & + q_j \sum_{i=1}^{I} p'_{ji} \sum_{k=1}^{K} \frac{l_{jIk} t_k}{\sum_{k=1}^{K} l_{jIk}} \end{aligned} \quad (2-12)$$

We can get the total expected picking time in each working roadway by adding the expected picking time in each working roadway:

$$TT^a = \sum_{j=1}^{J} TT_j b_j$$
$$= \sum_{j=1}^{J} \left[TT_j \left(1 - \left(1 - \sum_{i=1}^{I} p_{ji} \right)^q \right) \right]$$
$$= \sum_{j=1}^{J} \left\{ \left[\frac{w_2}{v} + 2 \times \left(p'_{j1} \right)^{q_j} \times HT_{j1} + 2 \times \sum_{i=2}^{I} \left[H\left(p'_{ja}, q_j \right) \times HT_{ji} \right] + VT_j \right] \times b_j \right\} \tag{2 − 13}$$

Next, we calculate the expected picking time in the horizontal roadway. Due to the symmetry of the warehouse layout, we assume that the probability of at least one of the working roadways j or the working roadways $(J - j + 1)$ being accessed is n_j, which can be obtained by calculating the probability that neither of the working lanes are accessed (m_j and m_{J-j+1}):

$$n_j = 1 - (1 - m_j)(1 - m_{J-j+1}) = 1 - (1 - m_j)^2 = 1 - (1 - \sum_{i=1}^{I} p_{ji})^{2q}.$$

The expected picking time TT^c in the horizontal roadway is obtained by multiplying the probability of the furthest picking goods in the working roadway j with the time spent on arriving working roadway j on the horizontal roadway:

$$TT^c = 2\sum_{j=1}^{J/2} \left[(2j-1)(w_1/2v) \left[\left(\sum_{e=1}^{j} n'_e \right)^q - \left(\sum_{e}^{j-1} n'_e \right)^q \right] \right]$$
$$= \sum_{j=1}^{J/2} \left[\frac{(2j-1)w_1}{v} \left[\left(\sum_{e=1}^{j} n'_e \right)^q - \left(\sum_{e=1}^{j-1} n'_e \right)^q \right] \right] \tag{2 − 14}$$

And $n'_j = n_j \Big/ \sum_{j=1}^{J/2} n_j (\forall j \in J/2)$. Finally, we can estimate the average picking time TT according to TT^a and TT^c.

3 Storage Location Optimization

The average picking time can be estimated using a 2-section model for a given layout $(L, J, I, K, N, w_1, w_2, w_3)$, order frequency O_i, storage allocation scheme n_{jik}, pick quantity q, and picking time in every layer t_k. In this section, if a storage space ratio S_i is given, the formulas are used to optimize the storage location.

3.1 Model Building

In many cases, the problem of determining the optimal storage space for each category of goods on each shelf floor in each working roadway often occurs, for example, when the warehouse first starts to be used, and the goods classification or order mode change. This section defines the problem as follows: based on various given parameters of the warehouse, we determine the optimal storage boundary of the types of goods on each shelf in each working roadway, so as to get minimize the total average picking time, based on the classification storage strategy (probability of picking each category of goods and the proportion of total storage required). The objective function is:

$$\min TT = TT^a + TT^c \tag{3 − 1}$$

The constraint conditions are:

$$\sum\nolimits_{i=1}^{I} n_{jik} = N, \forall j \in J, k \in K \tag{3 − 2}$$

$$\sum\nolimits_{J=1}^{J} \sum\nolimits_{k=1}^{K} n_{jik} = s_i KNJ, \forall i \in I \tag{3 − 3}$$

$$p_{ji} = O_i \sum\nolimits_{k=1}^{K} n_{jik} \Big/ s_i KNJ, \forall j \in J, i \in I \tag{3 − 4}$$

$$n_{jik} = n_{J-j+1,ik}, \forall j \in J, i \in I, k \in K \tag{3 − 5}$$

$$n_{jik} \geq 0, \forall j \in J, i \in I, k \in K \tag{3 − 6}$$

The objective function is to pick up q goods with the minimum picking time. The picking time consists of two parts: the operation roadway and the horizontal roadway picking time. Use the formula (2-13) to calculate the picking time of the operation roadway, and use formula (2-14) to calculate the picking time of the horizontal roadway.

In the constraint conditions, formula (3-2) indicates the number of storage spaces each shelf is N; formula (3-3) indicates the storage space required of each category goods; formula (3-4) indicates relationship between p_{ji} and n_{jik}, formula (3-5) represents the symmetry of the warehouse layout; the non-negative property of n_{jik} is represented by Eq. (3-6).

With the increase of the quantity of picking goods, working roadways, goods categories and shelf layers, the objective function will become very complex. Therefore, this paper tends to propose the following heuristic algorithm to solve this problem.

3.2 Heuristic Algorithm

First, define the following terms: the same roadway layout means that in all the operation roadway, the quantities n_{jik} of goods of the same category on all shelves are the same. We defined that a and i are adjacent categories if $|a - i| = 1$.

The heuristic algorithm proposed in this paper is a 2-opt switching technology, which belongs to the neighbor search heuristic algorithm. In the algorithm, the picking time of each shelf is sorted in descending order. If it takes the same time, they will be sorted in ascending order according to the number of shelves. The highest ranking of shelves in the longest picking time is rank as 1, the lowest ranking of shelves in the shortest picking time is rank as K, and the corresponding numbers are $1, 2..., K$. The heuristic algorithm in this paper is divided into two parts. The first part is horizontal optimization of storage location. The initial layout is the same roadway layout. Considering the picking time of horizontal travel, the adjacent goods are exchanged between the operation roadways step by step. The goods of high order frequency far from the starting point of the warehouse and the goods of low order frequency near the starting point of the warehouse are exchanged. This part refers to the heuristic algorithm of Le-Duc et al. [3]. The second part is storage vertical optimization, as shown in Fig. 3. On the basis of the first part optimization, considering the time spent on vertical travel, goods with long picking time and high shelf order frequency are exchanged with goods with short picking time and low shelf order frequency between adjacent categories in each operation roadway, so as to determine the optimal storage boundary of each category of goods on each shelf in each operation roadway and minimize picking time. Since the warehouse layout is symmetrical, only half of the warehouse needs to be considered.

To illustrate this method, assume there is a semi-layout of three-layer shelves in three operation roadways (numbered as operation lanes 1 to 3 and shelf layers 1 to 3, in which shelf layer 3 takes the most picking time, shelf layer 2 takes the least picking time, and shelf layer 1 takes the middle picking time) and three categories of goods (category A, B, and C). First of all, only consider the picking time in the horizontal direction, let start from the farthest working lane 3, the class A goods in the working lane 3 and the class B goods in the working lane 1 are exchanged, then the class B goods in the working lane 3 and the class C goods in the working lane 1 are exchanged, and then the exchange is carried out between the working lane 3 and the working lane 2. Finally, consider the exchange between operation lane 2 and operation lane 1. After completing the previous steps, start to consider the vertical travel picking time. Between adjacent categories in each operation roadway, first exchange the storage locations of class A goods on third shelf and class B goods on second shelf one by one, then exchange the storage locations of class A goods on third shelf and class B goods on first shelf one by one, finally exchange the storage locations of class A goods on first shelf and class B goods on second shelf, and so on until the exchange between all adjacent categories and all operation lanes is completed.

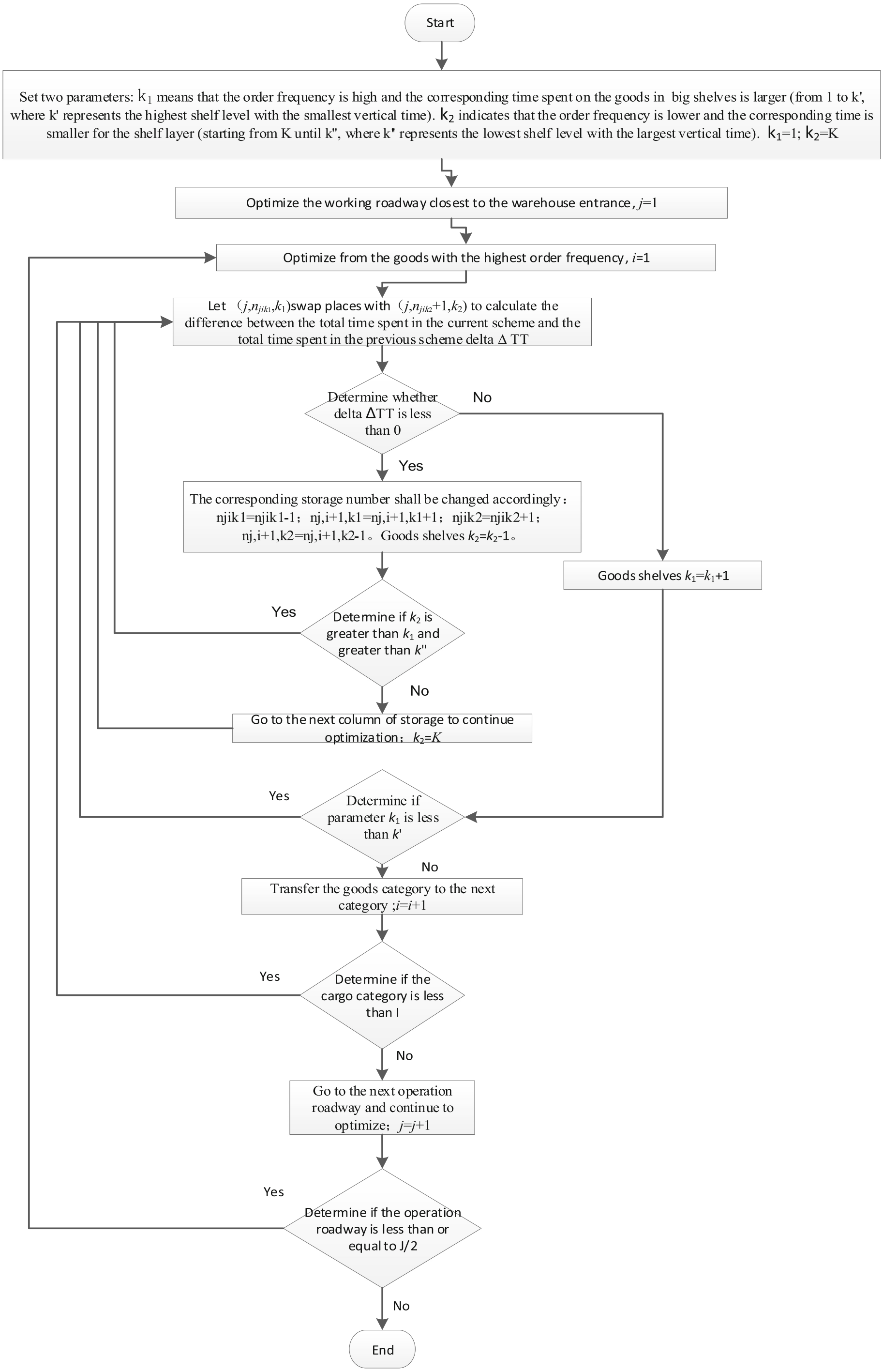

Fig. 3. Flow chart of storage vertical optimization

3.3 An Illustrative Example

In order to evaluate the performance of heuristic algorithm and determine the best storage area in the warehouse, assume that an average of 100,000 pieces of goods are picked up in the warehouse A every day, and the picking time required for each shelf is estimated (in reality, according to the actual situation the time required for each shelf can be measured in the warehouse). Refer to the parameters in literature [3], various numerical experiments are carried out to compare with the heuristic algorithm in literature [3], which did not optimize the storage location in the vertical direction. In these experiments, this paper considers the classified storage strategy of three layouts (with 6, 12 and 24 work lanes) and different picking quantities (6, 12, 24 and 48 goods). The warehouse layout parameters are as follows:

$$N = 90, K = 6, w_1 = 15, w_2 = 10,$$
$$w_3 = 1, v = 2, t_k = (2, 1, 1, 1, 1, 4)$$

Goods are divided into three categories according the order frequency and ratio of storage space, namely ABC storage strategy, as shown in Table 1.

Table 1. Classified storage strategy (order frequency/storage space ratio)

Classified storage strategy	Class A(%)	Class B(%)	Class C(%)
	80/20	15/30	5/50

According to the above parameters, the comparison results of the average picking time are shown in Table 2:

Table 2. The comparison results of the average picking time

Numbers of operation roadway	Quantity to be picked each time	Total quantity to be picked	Average picking time		The time saved by the algorithm in this paper
			Le-Duc' heuristic algorithm	This paper heuristic algorithm	
6	4	100000	2797531.11	2794564.96	2966.15
	12	100000	1782121.96	1776317.91	5804.05
	24	100000	1254183.73	1237251.41	16932.32
	48	100000	864585.33	818266.00	46319.33
12	4	100000	3730103.36	3727897.67	2205.69
	12	100000	2449103.45	2445371.48	3731.97
	24	100000	1803769.53	1797844.51	5925.02
	48	100000	1268285.28	1251245.43	17039.85
24	4	100000	5168603.00	5167128.97	1474.03
	12	100000	3316805.86	3314125.63	2680.23
	24	100000	2475759.60	2471967.24	3792.36
	48	100000	1814884.50	1808899.98	5984.52

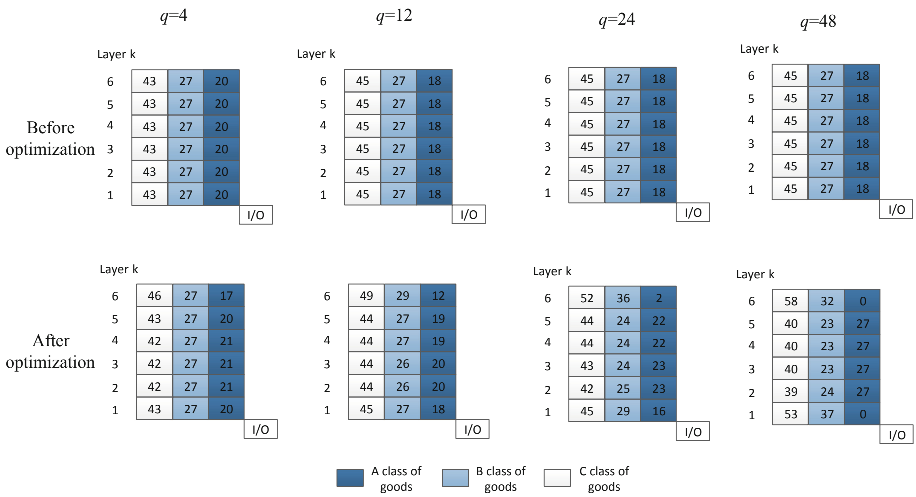

Fig. 4. Operation roadway layout before and after optimization

As we can see from Table 2, compared with the heuristic algorithm from literature [3], the heuristic algorithm proposed by us can reduce the picking time by optimizing the storage location in the vertical direction, and can save more time as the picking quantity increases. Therefore, considering the picking time on the vertical travel in the low-level manual picking warehouse can better optimize the storage, reduce the picking time and improve the picking efficiency.

Figure 4 shows the comparison of 4, 12, 24, 48 goods in the 6 operation roadways before and after vertical optimization of storage location of operation roadway 1. Between adjacent categories, the optimized operation roadway layout will exchange the higher order frequency goods with longer picking time in a shelf with lower order frequency goods with shorter picking time in a shelf, we place more A goods in 2–5th layer with the least time spending, and place fewer A goods in 1st and 6th layers. We should place more B type goods on the 6th and 1st floors than the middle layer, this is because Class A goods occupy more storage locations of 1st and 6th layers. More Class C cargo is placed on the 6th and 1st floors, while on the 2–5th floor there are fewer Class C cargoes. With the number of picks increasing, this phenomenon becomes more apparent as the number of picks increases, the picker needs to take a complete operation roadway to pick up all the goods. At this time, reducing the vertical pick picking can reduce the total picking time, so more and more goods with high order frequency are placed on the shelf layer with less picking time.

4 Conclusion

Based on the existing research, this paper optimizes the storage position in the vertical direction for the low-level manual picking warehouse that takes a long time to pick up goods at the bottom or top of the shelf. The goods with high order frequency and the longer picking time on the shelf are exchanged with those with low order frequency and the shorter picking time. The results show that the optimization of the storage position can reduce the picking time. And, with the number of picking increasing, the picking time can be reduced more significantly.

In this paper, only one warehouse layout is considered, and the influence of vertical travel on picking time can be further studied later in other low-level manual picking warehouse layouts.

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References

1. Tompkins, J.A., White, J.A., Bozer, Y.A., et al.: Facilities Planning. Wiley, Hoboken (2010)
2. Petersen, C.G.: An evaluation of order picking routeing policies. Int. J. Oper. Prod. Manag. **17**(11), 1098–1111 (1997)
3. Le-Duc, T., De Koster, R.M.B.M.: Travel distance estimation and storage zone optimization in a 2-block class-based storage strategy warehouse. Int. J. Prod. Res. **43**(17), 3561–3581 (2005)
4. Wu, L., Zhou, Z., Zheng, J.: The analysis of storage zone and storage location of a specific corporation. Ind. Eng. Manag. **4**, 24 (2006). (in Chinese)
5. Pan, J.C.H., Shih, P.H.: Evaluation of the throughput of a multiple-picker order picking system with congestion consideration. Comput. Ind. Eng. **55**(2), 379–389 (2008)
6. Bottani, E., Cecconi, M., Vignali, G., et al.: Optimisation of storage allocation in order picking operations through a genetic algorithm. Int. J. Logist. Res. Appl. **15**(2), 127–146 (2012)
7. Glock, C.H., Grosse, E.H.: Storage policies and order picking strategies in U-shaped order-picking systems with a movable base. Int. J. Prod. Res. **50**(16), 4344–4357 (2012)
8. Grosse, E.H., Glock, C.H., Jaber, M.Y.: The effect of worker learning and forgetting on storage reassignment decisions in order picking systems. Comput. Ind. Eng. **66**(4), 653–662 (2013)
9. Calzavara, M., Glock, C.H., Grosse, E.H., et al.: Analysis of economic and ergonomic performance measures of different rack layouts in an order picking warehouse. Comput. Ind. Eng. **111**, 527–536 (2017)
10. Li, M., Jiang, X.: A method for storage location assignment considering COI and demand correlations. Oper. Manag. **27**(09), 22–23 (2018). (in Chinese)
11. Zhang, R.Q., Wang, M., Pan, X.: New model of the storage location assignment problem considering demand correlation pattern. Comput. Ind. Eng. **129**, 210–219 (2019)
12. Dekker, R., De Koster, M.B.M., Roodbergen, K.J., et al.: Improving order-picking response time at Ankor's warehouse. Interfaces **34**(4), 303–313 (2004)
13. Dijkstra, A.S., Roodbergen, K.J.: Exact route-length formulas and a storage location assignment heuristic for picker-to-parts warehouses. Transp. Res. Part E: Logist. Transp. Rev. **102**, 38–59 (2017)
14. Parikh, P.J., Meller, R.D.: A travel-time model for a person-onboard order picking system. Eur. J. Oper. Res. **200**(2), 385–394 (2010)

15. Chan, F.T.S., Chan, H.K.: Improving the productivity of order picking of a manual-pick and multi-level rack distribution warehouse through the implementation of class-based storage. Expert Syst. Appl. **38**(3), 2686–2700 (2011)
16. Chen, C., Lu, Z.: Optimization for storage location assignments and interleaving problems in an automated storage/retrieval system. J. Manag. Eng. **26**(01), 42–47 (2012). (in Chinese)
17. Pan, J.C.H., Wu, M.H., Chang, W.L.: A travel time estimation model for a high-level picker-to-part system with class-based storage policies. Eur. J. Oper. Res. **237**(3), 1054–1066 (2014)
18. Kou, X., Xu, G., Yi, C.: Belt-conveyor based efficient parallel storage system design and travel time model analysis. Int. J. Prod. Res. **56**(23), 7142–7159 (2018)
19. Caron, F., Marchet, G., Perego, A.: Routing policies and COI-based storage policies in picker-to-part systems. Int. J. Prod. Res. **36**(3), 713–732 (1998)