Analytical Investigation of the Stiffness of Homogenous Isotropic Mechanical Materials with Different Cross-sections

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Abstract In order to achieve the required high level of mechanical material components as regards sustainability and safety, the stiffness of such materials has to be thoroughly investigated to avoid failure. This study investigates, analytically, the stiffness of some mechanical materials with different cross-sectional areas, whose mechanical properties are homogeneous and isotropic. The exerted force for steel material solids with different cross-sections was investigated. Comparisons were made and results are consistent with the ones in literature. A computer software, Maple, was used to plot the three-dimensional graphs of the relationship between the parameters. Specifically, it was observed that the steel hollow rectangular beam cross-section has a high axial stiffness compared to that of steel hollow circular beam cross-section. Also, the moment of inertia of the steel beams, considered, depend on their cross-sectional areas.

Keywords Stiffness · Homogeneous isotropic materials · Analytical investigation · Cross sections

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1 Introduction

Isotropic materials are materials with indistinguishable estimations of a property every direction. Glass and metals are good examples of isotropic materials [1]. A homogeneous material, on the other hand, is a material with similar properties at each point; it is uniform without inconsistencies. Materials can be both homogeneous and isotropic, in terms of properties; however, these two have diverse implications as regards their property of body and heading of the position [2]. The distinction between the two is that homogeneous material has a similar group of properties at each place, but an isotropic material has a similar-looking in the majority of the bearings at various purposes of the property. For example, steel demonstrates isotropic behaviour, although its microscopic structure is non-homogeneous [3–5].

Strain is "deformation of a solid due to stress"—change in dimension divided by the original value of the dimension. Stress is force per unit area. There are different types of stress [5–9]: T*ensile* stress, Compressive stress and Shearing stress. Stiffness is the rigidity of an object. Young's modulus, otherwise called the flexible modulus, is a measure of the solidness of strong material. It is a mechanical property of direct flexible strong materials. [10, 11]. The measure of the stiffness of solid material is known as elastic modulus or Young's modulus. A material with a very high Young's modulus can be approximated as rigid. Young's modulus is the ratio of stress to strain [12, 13]. In beams, the area moment of inertia can be used to predict deflection. In this study, the stiffness of such block of steel in the form of beam with circular hollow and rectangular hollow cross-sections was investigated. In particular, the axial stiffness, Bending stiffness and moment of inertial, for beam with circular hollow and rectangular hollow cross-sectional areas, were analyzed. Also, the second moment of area of these solids was investigated.

2 Formulation of Problem

The force F, exerted by the material when displaced by ΔL can be written as follows:

$$F = \frac{EA\Delta L}{L_0} \tag{1}$$

This is achieved by using Young's modulus.
The axial stiffness is given as

$$k = \frac{AE}{L} \tag{2}$$

where

A the cross-sectional area,

E Young's modulus,
L the length of the element.

The bending stiffness is given as:

$$K = \frac{F}{w} \tag{3}$$

where

F the applied force,
w the deflection.

Substituting Eqs. (1) into (3) gives

$$K = \frac{EA\Delta L}{\frac{L_0}{w}} = \frac{EA\Delta L}{wL_0} \tag{4}$$

3 Numerical Example

The moment of inertia of the steel hollow circular beam is obtained, converting inches to meters, as follows:

$$I_x = \frac{\pi}{64}\left[(d_2)^4 - (d_1)^4\right] \approx 0.0000008549\,\mathrm{kgm}^2 \tag{5}$$

where

$d_2 = 0.20055$ (the longer diameter)
$d_1 = 0.20000$ (the shorter diameter).

The axial stiffness of the steel hollow circular beam is obtained by adopting Eq. (5) and taken the young's modulus of the beam as 209 Mpa:

$$k = \frac{A_2E - A_1E}{L} = 0.98\left[\frac{22}{7}\left(\frac{0.127}{2}\right)^2 E - \frac{22}{7}\left(\frac{0.091}{2}\right)^2 E\right] \approx 1.27\,\mathrm{N/m} \tag{6}$$

The bending stiffness of the steel hollow circular beam is obtained by adopting Eq. (6) as follows:

$$K = \frac{EA_2\Delta L}{wL_0} - \frac{EA_1\Delta L}{wL_0} = \frac{\Delta LE}{wL_0}(A_2 - A_1) \tag{7}$$

Substituting the deflection, w, and some manipulation gives:

$$K = \frac{3EI_x}{L^3} \tag{8}$$

Substituting the value of the parameters gives:

$$K = \frac{3(209)(0.000001)}{1.016^3} \approx 0.0006\,\mathrm{Nm}^2 \tag{9}$$

The moment of inertia of the steel hollow rectanguar beam cross-section is obtained as follows:

$$I_x = \frac{1}{12}BH^3 - \frac{1}{12}bh^3 \approx 0.0002222\,\mathrm{kg\,m}^2 \tag{10}$$

where

B the breadth of the outer part of the hollow rectangle
B the breadth of the inner part of the hollow rectangle
H the height of the outer part of the hollow rectangle
h the height of the inner part of the hollow rectangle.

The axial stiffness of the steel hollow rectangle beam was calculated to obtain:

$$k = \frac{A_2E - A_1E}{L} = \frac{E(1.83 - 1.02)}{0.0254(40)} \approx 166.62\,\mathrm{N/m} \tag{11}$$

The bending stiffness of the steel hollow rectangular beam was obtained:

$$K = \frac{3EI_x}{L^3} = \frac{3(209)(0.0002222)}{1.016^3} \approx 0.13284\,\mathrm{Nm}^2 \tag{12}$$

4 Results and Discussion

From the analysis, it can be gathered that the value of the moment of inertia and the bending stiffness of both steel hollow circular beam and steel hollow rectangular beam, under consideration in this paper, is less than one. However, the moment of inertia of the steel hollow rectangular beam is greater than that of steel hollow circular beam. This suggests that the stress in the steel hollow circular beam is less than that in the steel hollow rectangular beam, going by the formula for stress of a beam:

$$\partial = \frac{M_Y}{I} \tag{13}$$

where σ is the stress, M is the internal moment, y is the distance from the neutral axis and I is the area moment of inertia. Both the axial stiffness and bending stiffness of the steel hollow rectangular beam are greater than that of steel hollow circular beam. This implies that the former is more rigid than the later. It shows that the resistance of a member against bending deformation in higher in the steel hollow rectangular beam than in steel hollow circular beam. Also, more force is required to produce unit axial deformation in of steel rectangular circular beam than in of steel hollow circular beam. From Fig. 3, the moment of inertia of the steel hollow circular beam increases as the inner diameter decreases and outer diameters increase. This implies the smaller the hollow in the beam the higher the moment of inertia which suggests less stress on the beam (Figs. 1, 2, 4, 5 and 6).

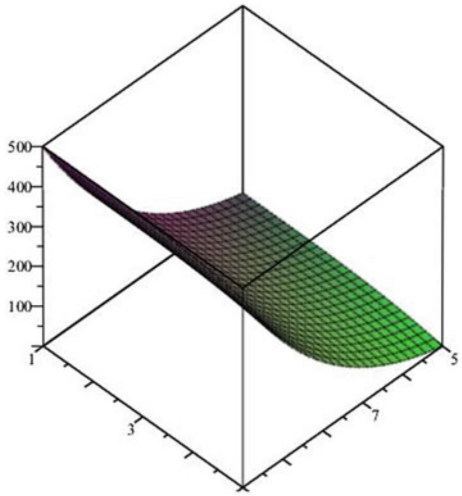


Fig. 1 Show 3D plotting of the relationship between *Ix*, *d*2 and *d*1 for the circular hollow beam

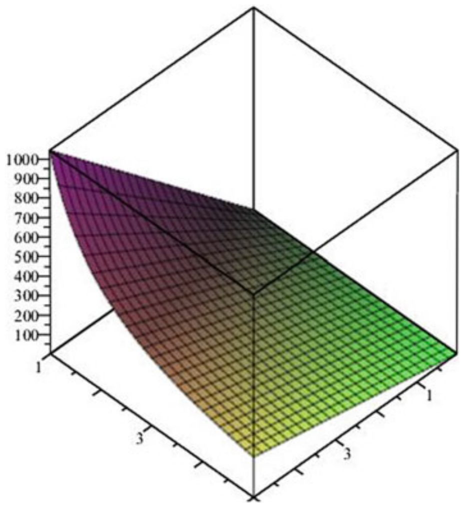


Fig. 2 Show 3D plotting of the relationship between *k*, *A* and *L* for the circular hollow beam

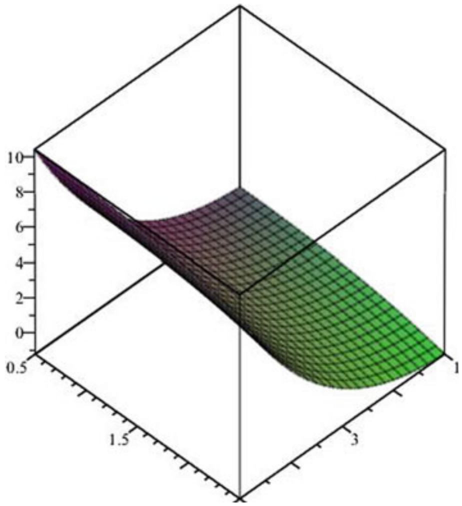


Fig. 3 Showing 3D plotting of the relationship between *K*, *Ix* and *L* for the circular hollow beam

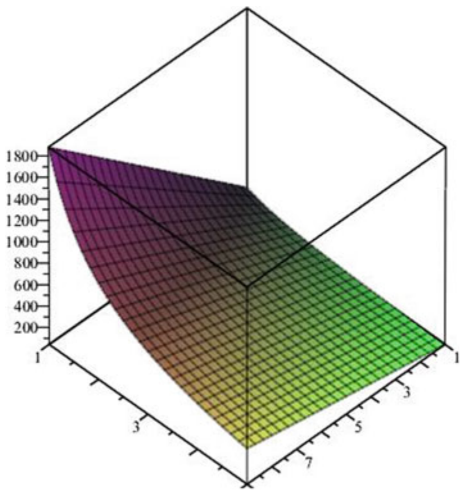


Fig. 4 Shoing 3D plotting of the relationship between I_x, H, and h for the rectanglar hollow beam

5 Conclusion

The study set out to investigate analytically the stiffness of homogenous isotropic mechanical materials with different cross-sections. Steel circular hollow beam and steel rectangular hollow beam were used as case studies. The moment of inertia, the strain, the stress, the stiffness, the deflection, the Young modulus and the relationship between these were investigated and analyzed mathematically. It was observed that the steel hollow rectangular beam is more rigid than the steel hollow circular beam,

Fig. 5 Showing 3D plotting of the relationship between k, A, and I for the rectangular hollow beam

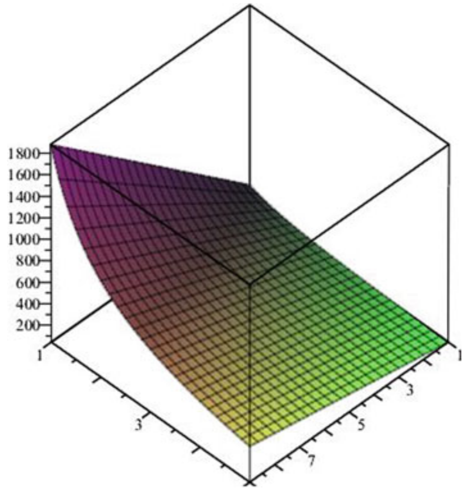
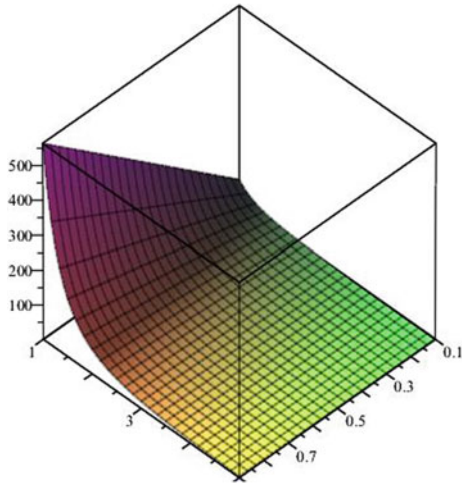


Fig. 6 Showing 3D plotting of the relationship between K, I_x, and L for the rectangular hollow beam



going by the numerical values used to calculate their axial stiffness and bending stiffness.

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