A Review on Vibration Suppression of Flexible Structures Using Piezoelectric Actuators

Aniruddha Mallick, Frank Crasta, and Vijaya Kumar N. Kottur

Abstract Piezoelectric Actuators are devices that convert electrical energy into a mechanical displacement or stress by a phenomenon known as Piezoelectric Effect. These crystals generate energy when mechanical stress is applied. This paper reviews the application of piezoelectric actuators in vibration suppression of flexible lightweight structures. Feedback control strategies are used for suppressing periodic and aperiodic vibrations. Negative velocity feedback and positive position feedback are two such common strategies employed for controller design. A finite element is defined for modelling of a uniform cantilever beam and a thin, flexible plate. The appropriate controller frequency is obtained by modal analysis, for monomodal vibration control.

Keywords Piezoelectric actuators · Piezoelectric effect · Vibration suppression · Feedback control strategies · Negative velocity feedback · Positive position feedback · Finite element · Modal analysis · Monomodal vibration control

1 Introduction

Lightweight flexible structures are used in aircraft, bridges, robot manipulators, large space structures, buildings, etc. Vibrations in any static structures are undesirable, as they induce stress in various structural elements. To reduce the stresses induced by high amplitude vibrations, damping, isolation or any other cancellation method is introduced in these structures. Damping will remove vibration energy, in the form

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of heat. Isolation protects a piece of equipment, by absorbing the mechanical energy from the system.

The two fields to look upon are the applications of smart materials and the adoption of various control techniques for active vibration suppression. Researchers have used smart materials such as Shape Memory Alloys (SMA) [1], Magnetorheological materials [2], Electrorheological materials [3], piezoelectric transducers, etc. Piezoelectric materials are suitable for use in structural vibration control because of (1) fast response, (2) it does not generate a magnetic field during conversion of electrical energy to mechanical motion. The most commonly used material as actuator is PZT (Lead-Zirconate-Titanate) material due to the high electromechanical coupling coefficient.

The piezoelectric materials can also be integrated with laminated composites. However, the challenge faced is the selection of elements for meshing, while accounting for the composite mechanical and electrical properties of the laminate plies. Based on the first-order shear theory (FSDT), a finite element is established to study the effect of stretching-bending coupling of piezoelectric actuator pairs on system stability of smart composite plates. A simple FSDT can be adopted which is similar to classic plate theory and uses a smaller number of unknowns [4]. Alternatively, the analysis can be performed by using a conventional beam or plane stress element, and the effects of the coupling terms can be translated as equivalent concentrated nodal loads.

Velocity Feedback and Positive Position Feedback (PPF) are popularly known control strategies in the classical control theory, due to their easy implementation. The PPF strategy is implemented by using an idealized modal model (analysis of dynamic properties of the system under the frequency domain) of an actuated structure. Lagrange's equations of motion are used to describe the behaviour of the system.

2 Need of Damping in Non-collocated Systems

Collocated control stands for the use of actuator and sensor on a structure when they are physically located at the same place and are energetically conjugated [8]. Studies conducted show how the zeroes migrate when the sensor moves away from the actuator. With the displacement of the sensor along beams with specific boundary conditions, pair of imaginary zeroes will reach infinity and then move towards origin along the real axis. This is known as non-minimum phase and it signifies non-collocated control systems.

Lead compensators produce sinusoidal output with phase lead and this compensates the undesirable phase lag caused by the poles, in the form of imaginary zeroes. However, they provide damping to all flexible modes. Notch filter is used along with lead compensator to introduce two zeroes alongside the flexible poles. It reduces the effect of flexible modes in non-collocated control. In case of systems with large uncertainties, notch filter is not useful as they are tuned to a single frequency and it is not subjected to parameter uncertainty. Therefore, damping is critical for non-collocated systems.

3 Direct Velocity Feedback

If a load is to be moved by means of a motor, the common control method used is Proportional (P) control. It will produce system output proportional to the displacement required and will oscillate about the steady-state value for some time. By using Proportional-Derivative (PD) control, faster response with fewer oscillations is obtained, as compared to P control. The alternative to this method is to use a second feedback loop which returns the rate at which displacement is changing. This is known as velocity feedback.

The governing equation of motion is obtained by using Hamilton's variational principle

$$M\ddot{x} + C\dot{x} + Kx = f. \tag{1}$$

where M, C and K denote the mass, damping and stiffness matrices, respectively, and f stands for the perturbation. Let $\dot{y}$ be a set of velocity measurements

$$\dot{y} = B^T\dot{x}. \tag{2}$$

where B is the influence matrix of dimensions $(n \times m)$. The damping matrix $C = BGB^T$. The velocity distributions which belong to the null space of B^T will remain undamped.

4 Positive Position Feedback (PPF)

This control strategy is appropriate for structures equipped with strain actuators and sensors. It uses a second-order filter. The position response received from the second-order auxiliary system is passed through a second-order filter and force feedback is given to the structure.

For designing the PPF controller, an idealized modal model of a flexible structure is considered. Either Lagrange's [3] or Hamilton's [4, 7] equation can be used to formulate the equation of motion for the structure. The Lagrange equation is

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial(T-V)}{\partial\dot{q}}\right) + \frac{\partial(T-V)}{\partial q} = Q. \tag{3}$$

where ψ is the displacement of the structure; ζ_s and ω_s are the damping ratio and the natural frequency of the structure; η is the displacement of the compensator, ζ_c and ω_c are the damping ratio and frequency of the compensator; K_{PPF} is the feedback gain. Shan et al. [5] described the state-space form of the equations describing the PPF technique.

When a general case of Multiple Input Multiple Output (MIMO) system with m collocated actuator/sensor pairs and an array of l second-order filters.

$$M\ddot{x} + Kx = Bu \tag{4}$$

$$y = B^T x \tag{5}$$

$$\ddot{v} + \beta_f \dot{v} + \Omega_f^2 v = Ey \tag{6}$$

The structure, sensor and controller equations are (4), (5) and (6) respectively, where $\beta_f = \text{diag}(2\xi_f\omega_f)$, $\Omega_f^2 = \text{diag}(\omega_f^2)$ and $G = \text{diag}(g_i)$, $(g_i) > 0$. G is the diagonal positive gain matrix. E is a rectangular matrix ($l \times m$) which allows the number of filters to be greater than actuators (l modes are dampened by m actuators).

There are three situations regarding the comparison of structural frequency and compensator natural frequency [5],

Active Flexibility: $\zeta_s < \zeta_c$ (due to decrease in stiffness term).
Active Damping: $\zeta_s = \zeta_c$ (due to increase in damping term).
Active Stiffness: $\zeta_s > \zeta_c$ (due to increase in stiffness term).

The robustness of the controller increases by increasing ζ_c as this ensures larger region of active damping.

5 Structure Modelling

5.1 Uniform Cantilever Beam

For system modelling, the flexible structure (for example, robot-arm manipulator [5]) bonded with piezoelectric (PZT) actuators is modelled as a uniform cantilever beam. It is assumed as a Euler-Bernoulli beam. The Euler-Bernoulli equation gives a relation between the deflection of the beam and the applied load

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(EI\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}\right) = q. \tag{7}$$

where $w(x)$ is the curve that defines the deflection of the beam. q is the force per unit length. E is the elastic modulus and I is the area moment of inertia.

Let $w(x, t)$ be the deflection of the beam in the z-direction at point x and at time t. The deflection can be represented as

$$w(x,t) = \sum_{k=1}^{m} \phi_k(x) q_k(t) = \phi(x) q(t). \tag{8}$$

where $\phi_k(x)$ is the shape function, $q_k(t)$ is generalized modal coordinate and k represents the mode number ($k = 1, 2, 3, \ldots, m$). A shape function is a function that is used in finite element simulations to combine the solutions obtained at the mesh nodes. Generalized modal coordinates are obtained by a coordinate transformation,

$$\{q(t)\} = [\psi]\{p(t)\}. \tag{9}$$

where $[\psi]$ is the modal transformation matrix and $p(t)$ denotes the principal coordinates. Such a transformation decouples the coupled set of equations of motion (of higher DOF) so that they can be solved using the single DOF approach.

5.2 Plate

A smart isotropic finite element [7] is used for vibration control of composite plate with bonded piezoelectric PZT actuators. Piezoelectric materials have two constitutive laws, one which is used for sensing and the other for actuation.

$$\{\sigma\}_{3\times1} = [C]^{(E)}_{3\times3}\{\varepsilon\}_{3\times1} - [e]_{3\times2}\{E\}_{2\times1}. \tag{10}$$

$$\{D\}_{2\times1} = [e]^{T}_{2\times3}\{\varepsilon\}_{3\times1} + [\mu]^{(\sigma)}_{2\times2}\{E\}_{2\times1}. \tag{11}$$

where $[C]$ is the mechanical constitutive matrix measure at constant electric field. The above equation can be combined into a single equation

$$\{\varepsilon\} = [S]\{\sigma\} + [d]\{E\}. \tag{12}$$

where $[S]$ is the compliance matrix and $[d]$ is the electromechanical coupling matrix. This law is extensively used for vibrational control, noise control and shape control.

PPF strategy and strain actuators/sensor pairs are relied on for active damping. Nearly collocated configuration of actuator and sensor is preferred as they guarantee alternating poles and zeroes at low frequencies. Precise tuning of the controller natural frequency on the targeted mode is necessary.

Taking an example, a 4-noded isoparametric FE element can be formulated, assuming 2-D plane stress.

At each node, there are three DOFs: (1) $u(x, y, t)$, (2) $w(x, y, t)$ (Displacement components), (3) $E_z(x, y, t)$ (Electrical degree of freedom in z-direction).

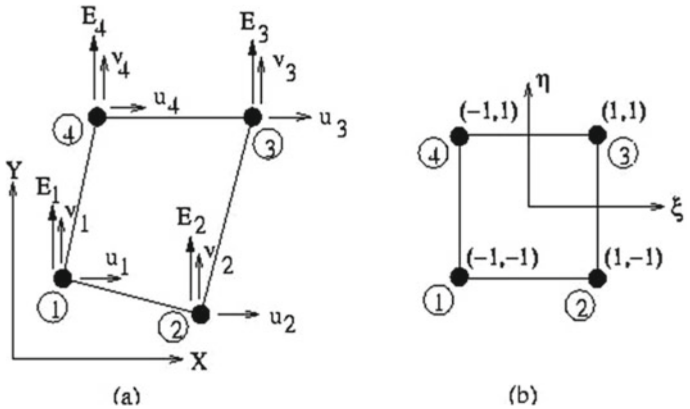

Fig. 1 **a** 4-noded isoparametric element. **b** Coordinate transformation into square of order 2

$$u(x, y, t) = \sum_{i=1}^{4} N_i(\zeta, \eta) u_i(t). \tag{13}$$

$$w(x, y, t) = \sum_{i=1}^{4} N_i(\zeta, \eta) w_i(t). \tag{14}$$

$$E_z(x, y, t) = \sum_{i=1}^{4} N_i(\zeta, \eta) E_{zi}(t). \tag{15}$$

where N_i stands for shape function at the node; ζ and η are the isoparametric coordinates and are the nodal mechanical DOFs. Through a Jacobian transformation, the actual geometry is mapped to a square of order 2 defined in the generalized coordinate system (Fig. 1b). First-order Shear Theory is used to obtain the following linear strain displacement relations

$$u(x, y, z, t) = u_0(x, y, t) + z\beta_x(x, y, t). \tag{16}$$

$$v(x, y, z, t) = v_0(x, y, t) + z\beta_y(x, y, t). \tag{17}$$

$$w(x, y, z, t) = w_0(x, y, t). \tag{18}$$

The nodal displacement vector is used with shape function to obtain reduced strain vector. The governing equation is thus found.

6 Modal Analysis

Modes are inherent properties of a structure. They are characterized by their natural frequency and modal shape. Modal analysis is needed to find out the modal parameters of the structure [6]. This method is an important tool for vibration diagnosis, analysis and control.

Modal analysis is carried out in Ansys workbench. The cross-section of a cantilever beam is created and then extruded up to a given depth. The parameters given in Table 1 are taken into account. The beam is automatically meshed and fixed support constraint is applied to one end of the beam. The displacement should be zero at the fixed support.

From the above observations (Fig. 2; Table 2), it can be concluded that the convenient mode shape to control is the first mode. Thus, a PPF control system can be developed using the first mode frequency, 9.8491 Hz as the controller frequency [6]. An accurate value of the controller is obtained, by reducing the mesh size and defining the mesh by commands.

Table 1 Aluminum alloy beam parameters

Density	2700 kg/m^3
Young's modulus	68.9×10^9 (Pa)
Size	$0.5 \times 0.04 \times 0.003$ m^3

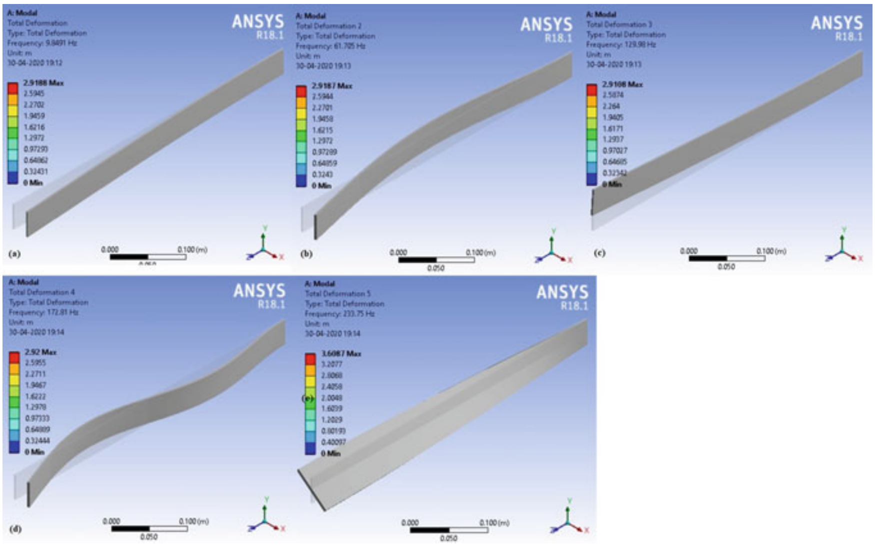

Fig. 2 Mode shapes: **a** first mode, **b** second mode, **c** third mode, **d** fourth mode, **e** fifth mode

Table 2 List of mode frequencies

Tabular Data

	Mode	Frequency [Hz]
1	1.	9.8491
2	2.	61.705
3	3.	129.98
4	4.	172.81
5	5.	233.75

7 Conclusion

Control laws have guaranteed stability due to the interlacing of the poles and zeroes of the structure along the imaginary axis. Stability is ensured for collocated pairs of sensors and actuators. If they are not collocated or the location of the actuator-sensor pair is such that controllability and observability are weak, the system becomes unstable [8]. It has been studied that two such control methods viz., negative velocity feedback and positive position feedback have been developed in the field of vibration suppression.

8 Scope for Future Work

The challenge faced by each of these control methods is that they do not account for unknown disturbances. Unknown or random disturbances are not accounted for since controller design revolves around the suppression of periodic vibrations. A general model has been proposed for disturbance rejection [9]. This model considers unknown disturbances and responds excellently in adverse conditions. Until more research is conducted, the proposed control method works the best.

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