Assessment of Local Stresses and Strains Using NSSC Rules

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Abstract Practically, many machine components and structures contain irregularities in spite of careful and detailed design. Such stress concentration site drastically diminishes the properties of the material. The service cracks get initiated in the vicinity of stress raiser sites. For the reliable design of machine components, the knowledge of stress and strain near the stress raising site is essential. In particular, for high-strength and high-performance components, better insights of the behavior of these components are required for the reliable design. In this paper, the criteria commonly used to estimate the local stresses and strains, viz Neuber's rule and equivalent strain energy density (ESED) methods, are presented. An Excel-based program for the stress-strain response was developed which used a constitutive equations together with Neuber and the ESED method.

Keywords Neuber's rule · ESED · Inelastic · NSSC (Notch stress-strain conversion) rule · Local stress · Local strain

1 Introduction

Engineering structures and components when subjected to loading fail in the region of high stress concentration; therefore, it is necessary to know the inelastic stresses and strains. To assess the crack initiation and fatigue lives, notch stress/strain analysis

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and low-cycle fatigue concepts are often used. For such analysis, it is useful and essential to estimate the local inelastic notch stresses and strains [1–3]. Different methods, viz experimental methods [4, 5], inelastic FEA, and the NSSC rules, are available to estimate notch root stresses and strains. Experimental method is tedious and consumes lot of time. Among these, Neuber method [6] and equivalent strain energy method [7] are most popular and commonly used methods by designers. The objective of this paper is to review and assess the local stresses and strains in the vicinity of notches using NSSC rules, viz Neuber's rule and ESED method. Both the methods are programmed using Microsoft Excel to find the local plastic stresses and strains.

2 Johnson–Cook Elastic Plastic Material Model

Johnson–Cook elastic plastic material model [8] is used to define the material nonlinearity. In this paper, Aluminum 6063T7 material is used, even though in the present methodology any material can be used.

At normal strain rate and room temperature, the Johnson–Cook model is given by

$$\sigma = \left(a + b \in_p^n\right) \tag{1}$$

a is yield stress at low strains. b and n are strain-hardening constants.

Plastic strain, $\in_p = \in - \in_e$ and $\in_e = \frac{\sigma}{E}$

σ and $\in$ are true stresses and strains. E is Young's modulus of the material.

For Aluminum 6063T7, Johnson–Cook parameters are

$a = 90.26$ MPa, $b = 223.13$ MPa, $n = 0.374$, and ultimate stress $(\sigma_{\mathrm{m}}) = 175$ MPa.

The stress–strain curve is generated using Eq. (1) as shown in Fig. 1.

3 Notch Stress–Strain Conversion Rules

3.1 Neuber's Rule

The relation between local stress ($\sigma_{\max}$), local strain ($\in_{\max}$), theoretical or linear stress concentration factor (k_t), and nominal stress (σ_n) is given by Neuber's rule

$$\sigma_{\max} \in_{\max} = k_t^2 \frac{\sigma_n^2}{E} \tag{2}$$

The actual or local stresses and strains may be obtained by solving Eqs. (1) and (2) as shown in Fig. 2.

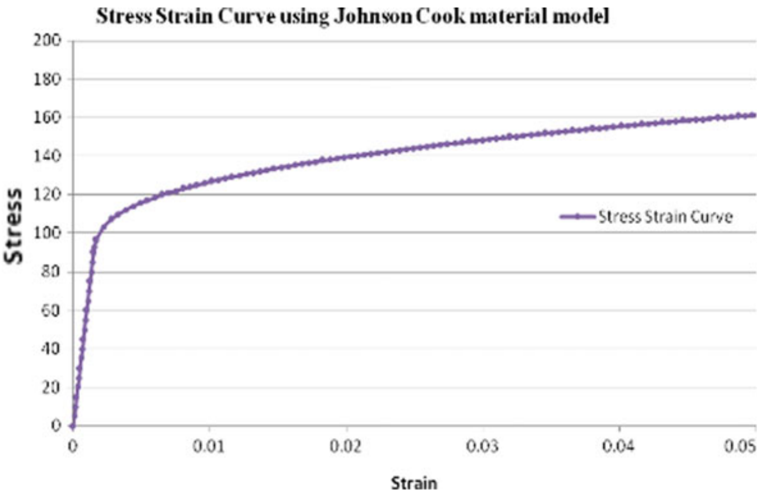
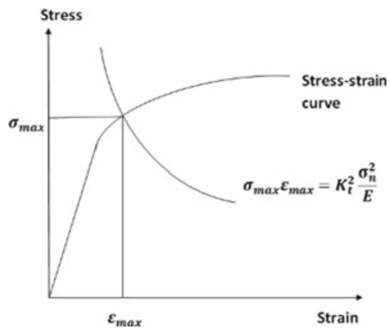


Fig. 1 Stress-strain response using Eq. (1) [9, 10]

Fig. 2 Assessment of local stresses and strains using Neuber's rule



3.2 Neuber's Rule to Calculate Local Stresses in Plastic Region

An Excel-based program is prepared to calculate the local stresses for given nominal stress (σ_n). Johnson–Cook material model for Aluminum 6063T7 and Neuber's hyperbola is plotted in Excel sheet. For given values of nominal stress (σ_n), local stress ($\sigma_{\max}$) can be calculated graphically using the Excel-based program.

3.2.1 Input to the Program

In the worksheet, only yellow colored cells as given in Table 1 are to be given as input to find local stress and strain.

Table 1 Input to the program

Only Enter the yellow Colored Data to Find Local Stresses and Strains at Notch by using Neuber's Rule			
Modulus of Elasticity	E	60400	MPa
Theoretical Stress Concentration factor based on Geometry	k_t	2.42	-
Nominal Stress at which Local Stress and Strains to be calculated	σ_n	65	MPa

3.2.2 Excel-Based Program

To find local stress and strain by Neuber's rule, two equations need to be solved, stress-strain equation (Eq. 1) and Neuber's hyperbola (Eq. 2). The screen shot of Excel sheet shown in Fig. 3 represents stress and corresponding strain values using Johnson–Cook material model for Aluminum 6063T7 with yield stress (a) = 90.26 MPa, hardening parameter (b) = 223.13 MPa, hardening exponent (n) = 0.374618, and Young's modulus (E) = 60,400 MPa. The Excel sheet shown in Fig. 4 presents data

Johnson Cook Material Model Data

$$\sigma = \left(a + b\varepsilon_p^n\right)$$

Modulus of Elasticity (MPa)	Yield Stress (MPa)	Hardening Modulus (MPa)	Hardening Exponent	Elastic strain	Plastic Strain	Strain	Stress (MPa)
E	a	b	n	ε_e	ε_p	ε	σ
60400	90.26	223.13	0.374618	0	0	0	0
60400	90.26	223.13	0.374618	8.27815E-05	0	8.27815E-05	5
60400	90.26	223.13	0.374618	0.000165563	0	0.000165563	10
60400	90.26	223.13	0.374618	0.000248344	0	0.000248344	15
60400	90.26	223.13	0.374618	0.000331126	0	0.000331126	20
60400	90.26	223.13	0.374618	0.000413907	0	0.000413907	25
60400	90.26	223.13	0.374618	0.000496689	0	0.000496689	30
60400	90.26	223.13	0.374618	0.00057947	0	0.00057947	35
60400	90.26	223.13	0.374618	0.000662252	0	0.000662252	40
60400	90.26	223.13	0.374618	0.000745033	0	0.000745033	45
60400	90.26	223.13	0.374618	0.000827815	0	0.000827815	50
60400	90.26	223.13	0.374618	0.000910596	0	0.000910596	55
60400	90.26	223.13	0.374618	0.000993377	0	0.000993377	60
60400	90.26	223.13	0.374618	0.001076159	0	0.001076159	65
60400	90.26	223.13	0.374618	0.00115894	0	0.00115894	70
60400	90.26	223.13	0.374618	0.001241722	0	0.001241722	75
60400	90.26	223.13	0.374618	0.001324503	0	0.001324503	80
60400	90.26	223.13	0.374618	0.001407285	0	0.001407285	85
60400	90.26	223.13	0.374618	0.001490066	0	0.001490066	90
60400	90.26	223.13	0.374618	0.001494371	0	0.001494371	90.26
60400	90.26	223.13	0.374618	0.001532535	0.000005	0.001537535	92.56513683
60400	90.26	223.13	0.374618	0.001584794	0.00005	0.001634794	95.72153487
60400	90.26	223.13	0.374618	0.001602202	0.00008	0.001682202	96.77300978
60400	90.26	223.13	0.374618	0.001708608	0.0005	0.002208608	103.1999533
60400	90.26	223.13	0.374618	0.001772129	0.001	0.002772129	107.0366006
60400	90.26	223.13	0.374618	0.001817692	0.0015	0.003317692	109.7885915
60400	90.26	223.13	0.374618	0.001854483	0.002	0.003854483	112.0107993
60400	90.26	223.13	0.374618	0.001885881	0.0025	0.004385881	113.9071854

Fig. 3 Screen shot image of stress and strain values for Al 6063T7

Only Enter the Coloured Data to Find Local Stresses and Strains at Notch by using Neuber's Rule

Modulus of Elasticity	E	60400	N/mm2
Theoretical Stress Concentration factor based on Geometry	Kt	2.42	--
Nominal Stress at which Local Stress and Strains to be calculated	σ_n	65	N/mm2

Local Stress Strain Equation based on Neuber's Rule

$$\varepsilon_{\max} \cdot \sigma_{\max} = K_t^2 \cdot \frac{\sigma_n^2}{E}$$

Young's Modulus	Linear Stress Concentration factor	Nominal Stress at which Local Stress,Strains to be calculated	Local Stress by formula	Local Stress for plotting graph	Local Strain	Constant
E	K_t	ε	σ_{max}	σ_{max}	ε_{max}	$\sigma_{max} \cdot \varepsilon_{max}$
60400	2.42	65	0	0	0	0
60400	2.42	65	819.3142384	819.3142384	0.0005	0.409657119
60400	2.42	65	409.6571192	409.6571192	0.001	0.409657119
60400	2.42	65	273.1047461	273.1047461	0.0015	0.409657119
60400	2.42	65	204.8285596	204.8285596	0.002	0.409657119
60400	2.42	65	163.8628477	163.8628477	0.0025	0.409657119
60400	2.42	65	136.5523731	136.5523731	0.003	0.409657119
60400	2.42	65	117.0448912	117.0448912	0.0035	0.409657119
60400	2.42	65	102.4142798	102.4142798	0.004	0.409657119
60400	2.42	65	91.03491538	91.03491538	0.0045	0.409657119
60400	2.42	65	81.93142384	81.93142384	0.005	0.409657119
60400	2.42	65	74.48311258	74.48311258	0.0055	0.409657119
60400	2.42	65	68.27618653	68.27618653	0.006	0.409657119
60400	2.42	65	63.02417219	63.02417219	0.0065	0.409657119

Fig. 4 Screen shot image of stress-strain values for Neuber's hyperbola

for Neuber's hyperbola. Figure 4 also shows screen shot image of Excel sheet to generate stress–strain curve for Neuber's hyperbola.

3.2.3 Output of the Program

The data in the two sheets (Figs. 3 and 4) is used to plot a graph of stress-strain curve and Neuber's hyperbola. As explained in Sect. 3.1, the intersection of these two curves gives local stress and strain at the notch and is shown in Fig. 5.

3.3 Equivalent Strain Energy Density (ESED) Method

One more popular method to estimate local stress is ESED approach given by Molski and Glinka [7]. The strain energy per unit volume is called as strain energy density, and mathematically, it is expressed as $W = \int_0^{\in} \sigma(\in) . d \in$, and graphically, it is considered as area under stress-strain curve (Fig. 6). From stress-strain constitutive relation, within an elastic limit $\sigma = E . \in$. From the above equation, $w = \frac{\sigma^2}{2E}$.

Let, w_s = Strain energy density due to nominal stress and
w_σ = Strain energy density due to notch stress

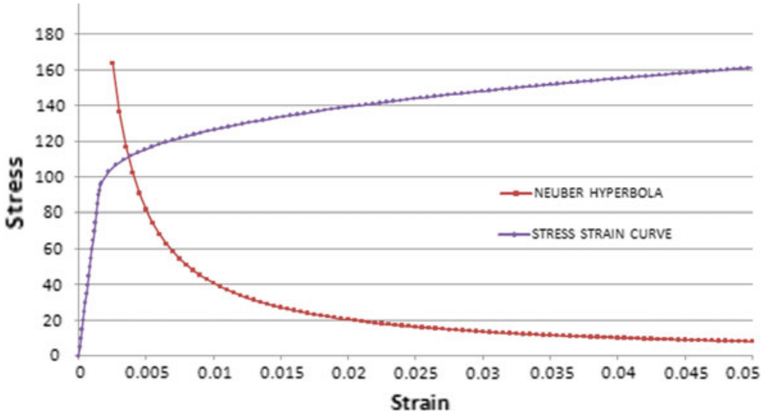
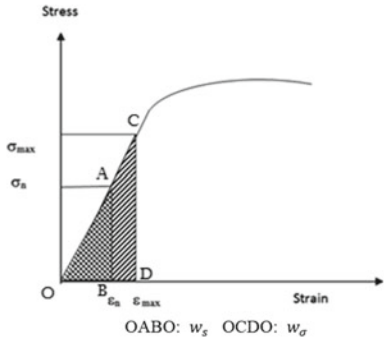


Fig. 5 Determination of notch stress using Neuber

Fig. 6 Energy density due to nominal and notch stress [7]



$$w_s = \frac{\sigma_n^2}{2E} \tag{3}$$

$$w_\sigma = \frac{\sigma_{\max}^2}{2E} \tag{4}$$

$$k_t = \frac{\sigma_{\max}}{\sigma_n} \tag{5}$$

From Eqs. (3), (4), and (5), the relation between k_t, w_s, and w_σ is

$$w_\sigma = k_t^2 w_s \tag{6}$$

Graphical interpretation of w_σ and w_s is shown in Fig. 6. As per Glinka's approach [7], k_t, w_s, and w_σ, can be correlated as $k_t^2 = \frac{W_\sigma}{W_s}$.

For finding maximum stress $\sigma_{\max}$ in the plastic region, ESED method [7] states that "it is reasonable to assume that the energy distribution does not change significantly if

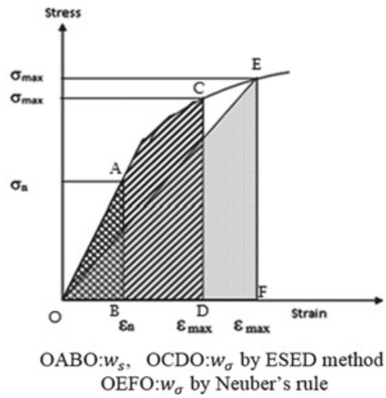


Fig. 7 Comparison between Glinka's and Neuber's method [7]

localized plasticity is surrounded by predominantly elastic material," i.e., the energy ratio given in Eq. (6) should be almost the same for elastic region and local yielding due to the effect of relatively large volume of the elastically strained material. The volume of the material that is strained elastically is more near the material that has undergone plastic deformation. Therefore, the assumption made in the above statement is reasonable. The relation can also be used if the material undergoes plastic deformation at the notch root. Equation (6) is used to find local stress in plastic region.

The comparison between ESED method and Neuber's method is shown in Fig. 7. The strain energy w_σ is represented by the triangle as shown in Fig. 7 by Neuber's rule which can be expressed as follows.

From Eqs. (2) and (6)

$$\frac{1}{2}\sigma_{\max} \in_{\max} = w_\sigma \tag{7}$$

Left hand side of Eq. (7) is area of a triangle under stress-strain curve as shown in Fig. 7, and RHS of the Eqn. is strain energy density at notch root. According to ESED method [7], the same energy w_σ is represented by area as shown in Fig. 7. From Fig. 7, it can be seen that the stresses and strains calculated on the basis of Neuber's rule are always more than or equal to those obtained using ESED method, i.e., Neuber's rule overestimates the results.

3.3.1 ESED (Molski and Glinka) Method to Calculate Local Stresses in Plastic Region

To find local stress and strains in plastic region knowing E, σ_n and k_t, first $W_s = \frac{\sigma_n^2}{2E}$ is determined, then $W_\sigma = k_t^2 W_s$, and finally, $\sigma_{\max}$ is located on stress-strain curve such that area under the curve is W_σ. Excel sheet is prepared to calculate W_s and W_σ for

Table 2 Data for ESED (Molski and Glinka's) method

Modulus of elasticity E (MPa)	Nominal stress σ_n (MPa)	Theoretical stress concentration factor k_t	W_s	W_σ	Local stress from area under curve using hypergraph 2D σ_{max} (MPa)	Local strain from area under curve using hypergraph 2D ε_{max}
60,400	0	2.3042	0	0	0	0
60,400	10	2.3042	0.000827815	0.004395147	23	0.00038
60,400	20	2.3042	0.003311258	0.017580588	46	0.00076
60,400	30	2.3042	0.007450331	0.039556323	69	0.00114
60,400	40	2.3042	0.013245033	0.070322353	92.11	0.00153
60,400	45	2.3042	0.016763245	0.089001728	97.6	0.00173
60,400	50	2.3042	0.020695364	0.109878676	100.5	0.00193
60,400	55	2.3042	0.025041391	0.132953198	102.8	0.00216
60,400	60	2.3042	0.029801325	0.158225294	104.7	0.00240
60,400	65	2.3042	0.034975166	0.185694963	106.4	0.00266
60,400	70	2.3042	0.040562914	0.215362206	107.9	0.00294
60,400	75	2.3042	0.04656457	0.247227022	109.4	0.00323
60,400	80	2.3042	0.052980132	0.281289411	110.8	0.00355
60,400	85	2.3042	0.059809603	0.317549375	112.1	0.00388
60,400	90	2.3042	0.06705298	0.356006911	113.3	0.00421

known values of E, σ_n, and k_t as shown in Table 2. Locating σ_{max} for given σ_n is done using hypergraph [11]. Stress-strain curve is imported to hypergraph. Hypergraph has facility to calculate area under curve between two limits. First point is set to (0, 0), and second point (σ_{max}, ε_{max}) is manipulated using curser in order to get area under curve equal to W_σ as shown in Fig. 8. The procedure is repeated for all values of σ_n and (σ_{max}, ε_{max}) values are tabulated in Table 2.

4 Conclusion

From the reported work on nonlinear analysis of assessment of local stresses and strains, it was observed that the past researchers used ESED method proposed by Glinka and Neuber for comparing their methodology. Therefore in this paper, the procedure to evaluate local stresses and strains using NSSC rules (Neuber and ESED) is explained. An Excel-based program was developed to assess inelastic stresses and strains by using Neuer's rule. The systematic approach was also explained to assess inelastic stresses and strains by area under stress-strain curve using hypergraph-2D

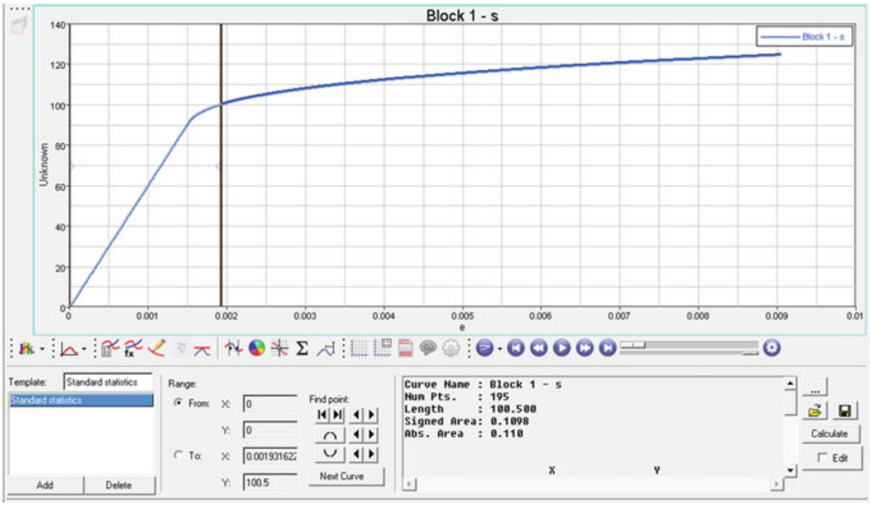


Fig. 8 Calculation of local stress and strains by area under stress-strain curve using ESED method (Tool used: Hypergraph-2D)

for ESED method. The method used in this paper would help design engineer to assess notch stresses and strains by NSSC rules without tedious numerical calculations.

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