Chapter 13
Teaching and Learning Mathematics with Digital Technologies

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Abstract This chapter provides a critical synthesis of research on technology-related classroom practice from the early years of schooling through to tertiary and initial teacher mathematics education. The synthesis considers ways in which research has explored the use of digital technologies through the three dimensions identified by Pierce and Stacey's map of pedagogical opportunity: mathematics content (subject), in classroom interactions (classroom), and through task design (task) (2010). The chapter also provides a synthesis of the research methodologies undertaken within the included studies. The review concludes with a discussion of emerging themes and a range of future directions for research into technology-related mathematics education.

Keywords Digital technologies · Digital tools · ICT · Mathematics · Tablets

1 Introduction

The use of digital technologies in the teaching and learning of mathematics continues to evolve, along with the increasing number and range of devices being used

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in mathematics education contexts and beyond formal education institutions in students' lives. Developments in digital technologies have led to a blurring of the boundaries between school and home learning. Digital technologies and blended learning approaches in schools and tertiary institutions have the potential to transform learning, and Australasian research in this field continues to emerge. Research investigating the influence of digital technology on learning, teaching, educational outcomes, and the delivery of curriculum content continues to be critical to ensure educators gain the maximum benefit of any potential affordances of digital technology.

In the previous Research in Mathematics Education in Australasia (RiMEA) chapter on digital technologies Geiger, et al., (2016) future directions for research were suggested, including a need for:

- Principles to assist teachers in discerning the quality of applications/software;
- Provision of insight in participation and management of interactions and relationships in online environments;
- Exploration of the potential of virtual worlds and technologies;
- Investigation of new forms of instruction such as blended and flipped learning approaches;
- Documentation of the role of digital technologies in pre- and in-service teacher education;
- Exploration of the notion of *futures* in terms of new digital tools; and
- Investigation of teaching approaches that leverage off the affordances of digital tools.

This chapter is a critical review of Australasian research conducted between 2016 and 2019 in relation to the use of digital technologies in mathematics education from early childhood though to tertiary education, including initial teacher education (ITE). Although some of the research directions suggested in the previous review have been undertaken during the current period, others have not. For example, there is only one study in this review that provides evidence of research exploring the potential of virtual worlds (Marshman, Woolcott, & Doyle, 2017) and there has also been little focus on technologies or investigations into participation and management of interactions and relationships in online environments. However, much of the research cited in the previous review has been built upon and expanded, such as the use of screencasting technologies to gain insight into students' mathematical reasoning and understanding (e.g., Murphy & Calder, 2017; Prescott & Maher, 2018), implying a move away from simply using digital technologies to build understandings of mathematical content as well as a shift away from students consuming content authored by others to students authoring their own content.

This review also sees the continued development of theoretical frameworks. Work by Lowrie and Larkin (2019) provides a heuristic for early years STEM learning that provides early years educators with guidance for using digital technologies. Research by Larkin (2016a) resulted in a proposed framework to assist teachers in determining the pedagogical potential of concrete and virtual manipulatives.

To build on the 2012–2015 RiMEA digital technology chapter Geiger, et al., (2016), we again consider the literature through elements of Pierce and Stacey's map of pedagogical opportunities (2010). However, in this review, we separate each stage of education into early years (Ages 0–8), school (primary and secondary—Year 3–12), tertiary, and ITE to allow us to compare and contrast the research being conducted within each stage of school and identify any gaps in the current research. Within each stage of school and ITE education we specifically explore the dimensions of *Tasks*, *Classroom Interactions*, and *Subject.* Through each of these dimensions we then interrogate recent research to seek themes relating to learners and learning, teachers and teaching, and classrooms. In our analysis of research conducted in tertiary mathematics education we take a slightly different approach in recognition of the differences between school and tertiary level education. To do this we briefly summarise research reviews found in our search. We then consider a range of studies directed at either improving pedagogical approaches or the creation and use of student resources. We then provide a critical synthesis of the research methodologies utilised within the studies cited in this chapter. The chapter concludes with an analysis of emerging themes and suggestions for future research.

2 Technology in Early Years Education

In this section we critique research that utilises digital technologies to develop particular mathematical content areas. According to Pierce and Stacey's pedagogical map, digital technologies can be useful in three sub-domains; however, the research conducted in this sub-strand in relation to early years mathematics only focusses on one i.e., rebalance emphasis on skill, concepts, and applications.

2.1 Subject

A strong emphasis of research involving mathematical subject areas in this period has concerned, as an umbrella term, spatial reasoning. Dindyal (2015) provides an important overview for the framing of research into spatial reasoning and makes the observation that, given children's exposure to a large number of technological devices, it is now normal and appropriate for teachers to use technology in teaching young children. This technology can be leveraged to "develop subtle ways of dealing with geometrical concepts and spatial reasoning at large" (p. 524). Dindyal also sounds two areas for caution: firstly, equity of access for various groups of young learners; and secondly to remind educators of the importance of effective planning in relation to the use of technology to support geometry learning.

The intersection of spatial reasoning and digital technologies in early years STEM is critiqued by Lowrie, Logan, and Larkin (2017), in a symposia publication. The three papers focussed respectively on: a) developing a learning program to promote children's engagement in STEM (Logan, Lowrie, & Bateup, 2017); b) the place of spatial reasoning in the early years (Lowrie, Logan, & Larkin, 2017); and c) using a design based approach to develop a mapping app (Larkin & Kinny-Lewis, 2017). In each of the papers the role of digital technology, in this case using custom designed apps for tablets, was prominent.

The symposia papers indicate that a focus on STEM practices, that include STEM ideas, methods and values, rather than developing integrated content-based learning experiences derived from the respective disciplines, is one that enhances student engagement with STEM, supported by digital technologies. Their pedagogical approach suggests that it is more important that young children actively use the iPads to develop mathematical understanding rather than passively using the iPads to achieve goals set by app developers.

As opposed to a stronger focus on the use of Interactive White Board (IWB) as a preferred technology reported in Geiger et al. (2016), and as an exception from the focus on spatial reasoning, Muir, Callingham, and Beswick (2016) investigate the impact of the use of an IWB on the learning of mental computation by students in Year One, and also on the pedagogy used by their teacher to assist students in their learning. Their findings indicate that the teacher demonstrated a high level of TPACK. This TPACK understanding resulted in learning experiences for students that were purposeful, individualised, and conceptually focussed. The learning was supported via the technology because it supported the teacher's content knowledge (via online resources) and subsequent levels of personal confidence, as well as encouraging a sequential set of instructions that were flexible and responsive to student learning needs.

2.2 Classroom Interaction

This section addresses research related to the impact of digital technologies on Early Years learning environments. In terms of the two sub-dimensions of the Pierce and Stacey pedagogical map i.e., social dynamics and didactic contract, the emphasis of these articles leans more heavily towards the social dynamics element; however, there are some implications in terms of how tablets also become tools that change the didactic contract. Pierce and Stacey provide a full description of social dynamics and didactic contract, for our purposes, social dynamics refers to changes in the classroom environment –e.g., paired work on an iPad or group work on a digital table and didactic contract refers to the role of technology in becoming, alongside the teacher, a new authority for learning and teaching –e.g., tutor type software.

A body of research investigated the impact of the use of digital technologies, in this case iPads, in impacting classroom interactions to enhance student learning. Calder (2017) suggests a number of consistent outcomes from research conducted across a number of projects regarding student led, data collection and interpretation activities. Firstly, the iPads could collect data in formats not possible via pen and paper—visual, audio etc. Secondly, the mobility of the devices allowed for immediate, in situ data collection and early interpretation. Thirdly, the devices facilitated the communication of findings to peers and teachers. Calder (2017) also reports of improved attitudes towards mathematics more broadly as a consequence of using the tablets. Furthermore, iPad screencasting was found to enable students to represent their solutions to a problem involving division with remainder (Murphy & Calder, 2017). The authors report that the screen casting app enabled the recording of multiple modes of communication (drawings, downloaded images, mathematical symbols, spoken and written language), and thus assisted the students to clarify their thinking and allow teachers to gain further insight into students' thinking and identification of misconceptions.

In a largely conceptual article, a heuristic—Experience, Represent and Apply (ERA) was used to propose a new way of using digital technology in preschool to support the learning of STEM (Lowrie & Larkin, 2019). This includes early number and spatial reasoning experiences including sorting, patterning, position and location language, perspective taking, encoding and decoding, debugging, and classifying. They suggest that Experience [E] is what children already know about a STEM concept, based on their existing social and language experiences; Represent [R] occurs when children use apps to engage with, and then represent, various STEM concepts. These representations include creating images, interpreting pictures, visualising and using symbols; Apply [A] activities occur where children build on their learning through a range of off-app activities, guided by their educators and their families.

The conceptual work of this paper forms the underpinning of a 2017–2019 project where the ERA framework was deployed in the context of STEM practices and provide data from approximately 400 educators and 4000 children, in over 100 centres Australia wide (Lowrie, Logan, & Larkin, 2019). The data indicates that the ERA heuristic was instrumental in assisting educators to embed STEM in play-based learning environments (Lowrie et al., 2019).

2.3 Tasks

The use of coding to support the development of digital representations of spatial concepts was the focus of research by Miller and Larkin (2017). Somewhat novelly for school based research, both intervention and control groups were established in a six-week coding and robotics teaching experiment with Year 2 students. Their aim was to explore how students developed mathematical knowledge and thinking as they participated in lessons using Scratch Jnr on desktop computers. The authors were seeking to determine how coding in primary school classrooms could support, or provide opportunity for, the learning of mathematics and in particular develop the proficiencies of problem solving and reasoning. Miller and Larkin indicate that limited research has been conducted on whether coding and robotics provides opportunities to develop early algebraic thinking. Following a small-scale intervention, their findings indicated that some students demonstrated higher levels of mathematical thinking than "required" in Year 2 by: working with 90 degree turns; demonstrating perspective taking abilities; and deducing a repeating pattern to provide a generalised code for making a square. Miller and Larkin claim that these findings are an early indication that coding and robotics may support children to identify and deduce patterns, an established precursor for more sophisticated algebraic thinking.

3 Primary and Secondary Education

This section reviews the research literature related to technology use in the primary and secondary school years. Most literature included in this section focussed on the primary school setting, revealing a distinct lack of research attentive to the specific needs of the secondary mathematics classroom. In terms of mathematical content, the majority of studies interrogated technology as it is used to promote geometric understanding however some dealt with other mathematics topics. Pedagogically, the key role of the teacher as an informed decision maker was emphasised, with greater emphasis placed on *how* teachers decided to use technological tools, rather than *which* tools they chose. The power of technology to make students' thinking observable was also noted, along with its potential to foster collaborative practices among students. In terms of how classroom tasks were impacted by technology, the notion that technology adds 'fun' elements to learning was critiqued and the need for teachers to have guidance in choosing technological applications was examined.

3.1 Subject

The technology used for teaching and learning in mathematics can vary according to the mathematical content under consideration. In line with previous reviews Geiger, et al., (2016); Geiger, Forgasz, Tan, Calder, & Hill, (2012) we examine research focussed on technology used to promote learning across a range of mathematical content areas. Although we find examples of research studies on a range of mathematics topics typically covered in the primary and secondary school years, there is a greater proportion of studies examining the role of technology for promoting geometric or spatial understanding.

A number of studies focussed on the pedagogical value of iPads in the primary classroom, particularly in relation to the teaching of geometry. Larkin, Kortenkamp, Ladel, and Etzold (2019) draw on Artifact Centric Activity Theory (ACAT) to describe an evaluative technique which enables teachers to evaluate the pedagogical potential of apps. The evaluative method described is applicable to both primary and secondary teachers and considers teaching as a network involving students, mathematical content, apps, the mathematical content within the apps, and the classroom context. The pedagogical value of any particular app is therefore dependent on a range of factors, not least the specific mathematical content under consideration. Given the large number of geometry apps available, Larkin (2016b) proposes a process for evaluating their educational potential for the primary classroom based on Dick's (2008) three measures of fidelity (cognitive, mathematical and pedagogical) and in relation to the *Australian Curriculum: Mathematics* finding that app quality is highly variable, with many apps only enabling student to 'trace' geometric shapes, rather than consider their geometrical properties.

Although most studies focussed on technology use for the teaching of geometry, Gorman and Way's (2018) study of six Year 4 students investigated the potential of technology to illuminate concepts related to number, and specifically decimal fractions. In this study a 'zoomable' number line was employed as the basis for student exploration of decimal fractions. The students were interviewed post-exploration and encouraged to share their thinking as they undertook the lesson. The students revealed knowledge of decimal density, whole number thinking, and the role of place value in understanding decimals. Their interviews exposed the cognitive conflict experienced during the investigation with the dynamic number line and the resulting 'self-correction' in reasoning which ultimately occurred. The authors point out that many 'teachable' moments occurred during the investigation highlighting the potential of this interactive tool for whole class instruction.

Finally, the links between computational thinking and mathematics have been explored. Calder (2018) examined the ways in which 10 year-olds engaged with mathematical ideas using *Scratch* for coding. There was evidence that the students improved in their spatial awareness, understanding of angles and positioning of coordinates. On a broader level, the links between computational thinking and mathematics were explored by Hickmott, Prieto-Rodriguez, and Holmes (2018). In a comprehensive scoping review they examined the literature base in relation to how computer programing can foster mathematics learning. They found that the research in this space was generally conducted by computer science academics rather than researchers with an education background, and that it was generally small in scale, focussing more on programming than the mathematics concepts involved. They also reported a relative lack of studies involving statistics, probability, functions and measurement, in comparison to number and algebra.

Although there is a clear focus on geometrical and spatial content in the research examined for this review, it is also clear that much emphasis is placed on the centrality of the teacher as the key decision maker in relation to the choice of technology, mathematical content and pedagogical approach. These decisions were the focus of one study (Loong & Herbert, 2018) which examined how two teachers made such decisions in their primary school classrooms. In this study, the authors examined the teachers' decisions using the SAMR (Puentedura, 2006) and TPACK (Mishra & Koehler, 2006) frameworks. They examined the teaching activities chosen by the teachers through the SAMR lens, leading to conclusions about the teachers' TPACK developmental stage. They surmise that teachers need advanced levels of TPACK to move to the enhanced or transformative levels described by the SAMR framework.

3.2 Classroom Interaction

The research reviewed in this section focuses on how the use of digital technologies plays out in primary and secondary classroom learning environments. As with research presented in the previous section, most of the research focussing on classroom practices involving technology was conducted in primary schools, signalling a need for more studies in secondary schools. Several studies conducted in primary school investigated the potential for digital technologies to enhance engagement with mathematics through a variety of mechanisms.

Across a series of papers Calder and Murphy (2017, 2018a, 2018b) explore how apps can be used by teachers in the primary classroom to promote mathematical understanding and engagement. They highlight that the affordances of particular apps are less important than the pedagogical decisions that the teacher makes when employing the app in the classroom (Calder & Murphy, 2018a), however, they do find that some app affordances are effective for promoting engagement and mathematical understanding. Apps that facilitate simultaneous screencasting and voice-recording created by the students themselves were seen to provide a new dynamic learning environment increasing student engagement (Calder & Murphy, 2018b). This work was further extended through the introduction of the idea of an assemblage as a means to understand the interplay between social and technical entities as apps are employed in the classroom (Calder & Murphy, 2018c). In this study the use of screencasting was viewed as a means of encouraging collaborative ways of working in the mathematics classroom, stimulating the contestation and validation of mathematical ideas and processes. Prescott and Maher (2018) also examined the use of screen casting, in particular focussing on *Explain Everything* and *Educreations*, as a means of allowing students to think mathematically. The study revealed the capacity for screen casting to facilitate teachers' formative assessment strategies by making students' thinking visible and by allowing students the opportunity to critique each other's work. This type of collaborative work within the mathematics classroom was found to increase student engagement. Similarly, in a study of 11 primary and secondary mathematics teachers, Ingram, Williamson-Leadley, and Pratt (2016) found high student engagement when using a 'Show and tell' app. The authors surmise that the act of making 'mathematical thinking visible' resulted in rich mathematical discussions about problem solving.

The concept of student engagement in mathematics through technology use in primary schools was directly addressed in three publications (Attard, 2018; Hilton, 2018; Orlando & Attard, 2016). Attard (2018) synthesises the results from three qualitative studies examining mobile technologies employing classroom observations, student focus groups and teacher interviews. Using the Framework for Engagement in Mathematics (FEM) (Attard, 2014) as a lens, she concluded that mobile technologies such as iPads do have the potential to improve student engagement in mathematics. However, this is not universally the case. The degree to which engagement is improved depends heavily on the pedagogical practices that embed their use, rather than the technological devices themselves. In a larger study in Queensland, Hilton

(2018) examined the impact of iPad use in Years 2–6 including the use of control group classes without iPad*s*. The study, which also employed Attard's (2014) FEM, found evidence that the use of iPads in mathematics had a positive influence on student engagement, enjoyment and self-perceptions, particularly for boys.

In addition to studies focussing on student engagement, Orlando and Attard (2016) conducted a study focussed on early career teachers and their use of mobile technologies for teaching mathematics. This paper reported on three related case studies of teachers in both primary and secondary school settings. The study questioned the untested but widely accepted myth that early career teachers should be well-versed in emerging technologies and may even be able to act as technology leaders in schools. The study also concluded that the tendency for early career teachers to be viewed in this way can be detrimental to their development as teachers.

Two studies (Willacy & Calder, 2017; Willacy, West, Murphy, & Calder, 2017) focussed on students in primary and lower secondary schools. Using a case study approach with four students age 11–13 years, the researchers found that three of the four students did experience increased engagement when apps were introduced into their mathematics classroom (Willacy & Calder, 2017). Taking a broad view of the factors influencing technology use, the study revealed three inter-related themes, which need to be considered to enable positive engagement through the use of technology: individual student aspects; pedagogical aspects; and societal aspects. The potential for mobile technologies to enable personalised approaches to learning was examined in the second study with upper primary school students (Willacy et al., 2017). As with other studies examined in this chapter, the key role of the teacher, rather than the technology itself, was emphasised. The degree to which personalised learning was achieved depended heavily on teacher decisions with regard to the extent of teacher direction, customisation features, work places and student-led learning.

3.3 Tasks

In this section we examine how technology can impact on the tasks that teachers use for teaching and learning. Kawka and Larkin (2018) question the notion of 'edutainment' in relation to apps developed to engage young students in mathematics. They examine the integration of popular culture, fantasy and 'fun' elements into apps designed to educate children about mathematics. Interestingly, they reveal that many of the fictional contexts within the apps have little meaningful connection to the mathematics that the children are meant to learn. The authors question the notion of 'fun' as a useful construct for learning mathematics and caution against choosing apps for their 'fun' value, which can be construed as a significant distraction from the mathematics content.

Ratnayake, Oates, and Thomas (2016) investigated how 12 teachers, working in groups of three, used digital technology to develop and implement algebra tasks for secondary mathematics classrooms. They determined that a range of factors assisted the teachers as they worked collaboratively to design suitable tasks, including having a clear focus on the mathematics content, proficiency with digital technologies, clearly set goals for the lessons and considered student thinking and potential difficulties.

In summary, the research on digital technology use for mathematics teaching and learning in primary and secondary classrooms is heavily skewed towards primary school settings, indicating a gap in ongoing knowledge development in secondary schools. This is potentially concerning as there is a significant cost for parents and schools in providing access to digital technologies for learning in secondary schools and yet the educational benefits have not yet been widely evaluated by researchers. Also, there is evidence that many mathematics topic areas are not being taught with technology, and that there is a clear preference for geometrical and spatial content. While this is not necessarily of concern, it does highlight that digital technologies are possibly under-utilised in the mathematics classroom and that their potential is yet to be fully realised. In general, there was little research on 'task' development and use. While pre-developed apps were popular in primary settings, there was only one study demonstrating how secondary teachers might collaborate to develop digital tasks for secondary students. Interestingly this study highlighted the importance of teachers having well-developed technology skills, emphasising the role of TPACK as an enabler for effective use of digital technologies for teaching and learning mathematics.

4 Initial Teacher Education

As was the case in Geiger, et al. (2016), there continues to be a lack of research on particular subject areas of mathematics in ITE. Although a number of the research articles discussed here include mathematical content e.g., Fractions and Division (Handal, Campbell, Cavanagh, & Petocz, 2016); data collection and interpretation, (Geiger, et al., 2016); or Geometry (Larkin 2016b)—this content is the vehicle supporting changes to either Classroom Interactions or Tasks in mathematics education—rather than being the focus of the research.

Almost all of the research conducted in this review period has focussed on the Classroom Interaction aspect of the pedagogical map. As in Geiger, et al. (2016), we take a broad perspective of what constitutes a "classroom". Given the rise of blended and fully online mathematics education in tertiary institutions, this is appropriate and timely; indeed, each of the research projects discussed in this section incorporate online learning experiences of one kind or another. The articles in this section are classified as to whether the primary focus was on how digital technologies change the didactic contract or the changed social dynamics that resulted from their use.

4.1 Classroom—Didactic Contract

A major focus within this theme concerns the pedagogical opportunities that arise when digital technology changes the didactic contract between lecturer and students.

The use of Mathcasts, defined here as screen recordings of explanations of mathematics concepts, was trialled by Galligan, Hobohm, and Peake (2017) to determine their usefulness in supporting pre-service teachers (PSTs) to learn mathematics. In this project, using a framework designed by the authors, the PSTs created their own mathcast demonstrating how they intended to teach a mathematics concept to primary school students. Galligan et al. (2017) report a number of interesting findings. Firstly, the PSTs found the framework useful as it helped them focus on purpose and context, structural elements such as visual quality, clarity and fluency of delivery, and their own developing PCK. Secondly, the creation of a mathematics artefact engaged students in the learning process and consequently improved their personal understanding of mathematics (CK). Finally, the mathcasts become a resource that PSTs could use in their future mathematics classroom practice.

Although using a different technology, in this case online modules, an argument is made by Geiger, et al. (2016) that mathematics should be taught as it is practised, i.e., as a dynamic inquiry into the nature of real-world phenomena. The conference paper is part of a broader project that sought to enable undergraduate PSTs to experience mathematics and science in a similar manner to how mathematicians and scientists practise it beyond the confines of the classroom. Here the authors provide insight into the process of developing one online module—*Modelling the present: Predicting the future*—to better understand how collaboration between mathematicians, scientists, mathematics educators and science educators can be utilised in designing an online learning module with a focus on mathematical modelling. The authors report that the creation of online modules provided opportunities for PSTs to contextualise mathematics in real world contexts, where mathematics is central to both understanding and solving the problem scenario. The use of digital tools (spreadsheets, online tools and resources and short explanatory videos prepared by mathematicians) was critical in this endeavour. As students were engaged in authentic mathematics activities using real contexts and real data (collected online), an important by product of this creative approach was enhanced positive dispositions towards mathematics.

(2016) Although still using iPads, and still concerning changes to the didactic contract, a different approach to technology use is critiqued by Galligan, Hobohm, and Peake (2017). Handal, Campbell, Cavanagh, and Petocz (2016) examine the role of mathematics apps in providing a "technology that is ubiquitous in nature, highly portable and endowed with multimedia capabilities offering a new dimension to curriculum making learning accessible 'anywhere, anytime'" (p. 200). However, the content validity of the apps is not always clear. These authors created a tool, which was used by PSTs, to measure the content validity of mathematics apps. The findings of this research were mixed: on one hand, students found the apps useful for their future teaching and could see that, once the instructional role for the apps was clarified, their use could create rich learning experiences; on the other hand, most

PSTs could not determine differences between four TPACK constructs—TPACK, PCK, TPK and TCK in relation to the apps and thus more work needs to be done in assisting PSTs to clearly understand the different pedagogical uses for apps.

4.2 Classroom—Social Dynamics

The next section of the review examines a series of articles (Larkin, 2016a, 2017a, 2017b) outlining ongoing development of two large, cross campus undergraduate mathematics courses offered in blended and face to face modes. Although each publication takes a slightly different perspective on the use of digital technologies to support mathematics learning, each focuses on classroom dynamics (in this case the classroom includes online interactions and face to face lectures, workshops and tutorials), and how these dynamics can be changed via the use of digital technologies. Thus, they will be considered here as one, extended research project.

The overarching conceptual framework for the series of research publication is Transactional Distance Theory (TDT). According to Larkin, TDT is a theory with its origins in the work of Moore (1993) in distance education, that suggests modifications to three core classroom elements (structure, dialogue and student autonomy) are critical in student learning, especially when this learning includes asynchronous interactions. In moving from a face to face to blended course, Larkin (2016a) made changes to the allowable 'three hours contact per week' using a (1 + 1 + 1) model which consisted of a one hour online pre-recorded lecture with a focus on theories of mathematics learning; a one hour weekly workshop with a focus on demonstrations of appropriate language, materials and symbolic representations; and a one hour tutorial that specifically enacted MPK in various teaching scenarios. Larkin (2016a) reports that feedback from students indicated that they appreciated the flexibility of the online components that complemented, but did not replace, some of the face-to-face components of the course.

The 1 + 1 + 1 was, by and large, a structural change but did not account in detail with the student experience of the model in terms of student engagement, supported by digital technology, in blended, online environments. In a subsequent article Larkin (2017a) argues that the integration of digital technologies into the existing university digital architecture is important for the uptake of these technologies as PSTs resist changing platforms to access content. The primary research contribution of this project was the choice of digital tool (i.e., Desktop Capture) for the delivery of content. Whilst studio-recording labs were available and encouraged by the university, Larkin (2017a), based on the research of Hibbert (2014) and Popova, Kirschner, and Joiner (2014) made the pedagogical decision to record the lectures using Desktop Capture in his office. Feedback from PSTs (Larkin, 2017a) indicated that they valued the sense of familiarity and the relaxed tone of the office-based desktop captured lectures, and this encouraged their engagement with them. In addition, the delivery of much of the theory in the online lectures had the added benefit of "freeing up" the face to face components to be much more interactive with small group activities and teaching demonstrations being the normal pattern of lecture delivery (Larkin, 2017a). The final component of the project was a study that sought to determine the impact of video or no video of the lecturer in the delivery of online lectures on the experience of PSTs. In this study (Larkin 2017b), the PSTs engaged with online lectures that were modified to include accompanying video of the lecturer for the entire lecture, only the start of the lecture, or no video at all. The small pilot study found that the PSTs overwhelming preferred to see the lecturer during the entire online lecture and that this deepened their engagement with the course.

4.3 Tasks

We conclude this section with a discussion regarding three research projects investigating the impact of modifying tasks to support ITE learning of mathematics. Two of the three relate to the task dimension of simulating real situations; however, one is from the perspective of teachers and their pedagogy and the other from the perspective of learners and their learning. The third relates to the use of online challenging tasks.

Digital Learning Objects (DLOs) are becoming more common as many university mathematics education courses are expanding their footprint in the online space. The reasoning of PSTs in relation to how and why they selected digital learning objects (DLOs) when planning to teach mathematics was investigated by Hawera, Sharma, and Wright (2017). These authors indicate that PSTs were positive in their intentions to use DLOs in their teaching of measurement as they found them likely to provide opportunities for children to access, construct, review and consolidate mathematical thinking and also help children to understand measurement concepts and/or formula. Hawera et al. (2017) suggest that one implication from their research is that tertiary mathematics education courses should provide PSTs with ample time to explore the use of the DLOs in their pedagogical practice, thereby supporting the development of student TPACK expertise.

A second technology beginning to be more broadly utilised in Higher Education is the use of simulations. Similar to the Hawera et al. (2017) paper, the research by Marshman, Woolcott, and Doyle (2017) focussesd on a task that simulates real experiences; however, the focus for these researchers was much more closely aligned to the perspective of learners and their learning (albeit the learners are PSTs). The paper investigated whether immersive technology (in this case CAVE2TM—a 3D, full body experience) could support their developing spatial thinking. In this study, learning experiences for PSTs that centred on spatial reasoning were provided; including an examination of both learning (as understandings) and perceptions. The findings from Marshman et al. (2017) were mixed. On the positive side, the immersive experience was an engaging one for PSTs and it encouraged increased collaboration with peers. Whilst the 3D spatial environment was initially confronting, PSTs generally found ways to utilise this unique resource and think about their personal spatial reasoning competence. On the negative side, some PSTs continued to have difficulty reconciling the 3D reality of objects and the various perspectives from which these objects could be viewed, perhaps because their previous experiences of shapes and objects were largely (un)developed based on 2D experience with school-based geometry. In addition, some PSTs expressed confusion regarding their spatial understanding of parallel lines and perspective—given that in the 3D representations the parallel lines appeared to meet. Overall, as spatial reasoning is malleable, Marshman et al. (2017) argue that it is vital PSTs are given opportunities to improve their spatial thinking and reasoning skills as part of their ITE and are encouraged to continue developing these skills.

The third of the research papers in the task component investigated, using Expectancy-Value Theory (EVT), how challenging tasks could be used to improve ITE students mathematics capability (Fielding-Wells et al., 2019). The authors used EVT as it provided them insight into the engagement potential of a task by identifying individual's motivational influences, according to two constructs—expectancy of success and valuing of task. Their findings suggest, in order for ITE to be competent and confident in teaching mathematics using challenging tasks, they need to overcome the notion that mathematics is only procedural, they need to have their own learning about challenging tasks scaffolded, and the value and utility of challenging tasks needs to be made more explicit.

5 Use of Digital Technologies in Tertiary Mathematics

In this section we review the research literature related to the use of digital technology at the tertiary level in non-ITE contexts. There is an established and growing body of literature that contrasts the different experiences, both in teaching and learning, of school versus tertiary level mathematics education (see Clark & Lovric, 2009). In partial recognition of these differences, we depart from the structure set out in the previous sections of this chapter. We instead divide the research into two broad sections: First, as a distinguishing characteristic of the current iteration of the quadrennial review, we briefly summarise five research reviews. Second, we consider a range of studies, most involving the use of video, directed at either improving pedagogical approaches or the creation and use of student resources. Overall, we found 19 studies involving the use of digital technologies in tertiary mathematics education, a significant drop from the last review where 35 were found. Most (13) of these studies were published in peer-reviewed journals with the remaining found in conference proceedings (4) and book chapters (2). Once again, the subject context for these studies reflect an ongoing focus on first-year undergraduate mathematics teaching and learning.

5.1 Research Reviews

Perhaps most noteworthy in this quadrennial review are the number of "stock takes", to use Oates' (2016) expression, related to the use of digital technology in tertiary mathematics. Just over one quarter of the papers examined the state of the field or a subfield, most systematically. Compared to past quadrennial reviews, this appeared to be somewhat of an anomaly which suggested to us a level of maturity in the field as several researchers reflect on past studies, gauging the state of various subfields of digital technology usage at the tertiary level. Compared to the last review, many of these studies continue to suggest, implicitly and explicitly, that a gulf continues to exist between the promise and actual benefits of using digital technologies (see Laborde & Sträßer, 2010).

There is a common theme concerning interventions proposed to bridge this gulf. Lake et al. (2017) and Thomas, Hong, and Oates (2017), for example, focus on 'innovations' enabled using technology. One theme Thomas et al. (2017) emphasise is teacher agency: teachers' choices are critical to the successful implementation of digital technologies. Similarly, regarding the use of Computer Algebra Systems (CAS), Tobin and Weiss (2016) argue a new curriculum is needed to take advantage of CAS, rather than simply adding CAS to the current curriculum. An example is provided by Ponce Campuzano et al. (2019) in this review relating to teaching Vector Calculus with GeoGebra. Finally, Trenholm, Peschke, and Chinnappan (2019), investigating the state of fully online mathematics instruction through the lens of large-scale research, found this modality of instruction is not working well compared to either face-to-face instruction in mathematics or to fully online instruction occurring in other disciplines. They suggest more pedagogical (not just technological) innovations are needed, which they argue both face-to-face and fully online teaching may benefit from. Overall, these reviews draw attention to the human element associated with effective use of digital technologies as a tool for teaching and learning.

Related to the role of the teacher, a recurring issue identified in the research, which is challenging successful innovation, concerns how to communicate mathematically in digital technology-enabled mediums. Several researchers have raised concerns about the constraints these technologies place on the effective communication of mathematical language, syntax and symbolism, critical to interactions and, ultimately, successful task completion in mathematics (Maclaren, Wilson & Klymchuk, 2017; Tobin & Weiss, 2016; Trenholm, Alcock, & Robinson, 2016). For example, as input devices, the qwerty and mouse are used differently with different software packages and tool pallets. This increased cognitive load may add to an already challenging subject to learn and does not compare with the familiarity of freehand writing of mathematics on paper or chalk/whiteboards. Such challenges appear indicative of ongoing struggles to reorient the nature of learning mathematics (see Abrahamson & Sánchez-García, 2016).

5.2 *Teacher Pedagogy and Student Resources*

The one technological tool that continues to challenge the current boundaries of educational practice in tertiary mathematics (and more broadly) is video. The growing accessibility and ubiquity of video technology have provided teachers and students with the means of producing and editing quality videos for use in teaching and learning. Not counting reviews, just over half (8) of the studies focussed on some aspect of video use. Of these studies, three focussed on tasks using student-created videos, suggesting a growing area of pedagogical innovation and a need for further research (Dunn, Loch, & Scott, 2018; Galligan, Hobohm, & Peake, 2017; Loch & Lamborn, 2016).

Most studies of video use relate to the delivery of direct instruction. Studies on the use of screencasts (Dunn, Loch, & Scott, 2018; McLoughlin & Loch, 2016) and recorded lecture videos (Tisdell & Loch, 2017; Trenholm, Hajek, Robinson, Chinnappan, Albrecht, and Ashman, 2019) reflect a continuing interest in exploiting this technology for the delivery of tertiary mathematics instruction. Although most research suggests students are generally satisfied with this form of instruction (either as a supplemental resource or a replacement for live lectures; e.g., Trenholm, Alcock, & Robinson, 2012), Trenholm, et al. (2019) found 'regular' recorded lecture video use associated with increased measures of surface approaches to learning. This quasi-experimental pre- and post-test study design, using validated scale measures, provided some needed insight into learning processes around the use of video in teaching and learning tertiary mathematics. Further directions for research include measuring the effect of interactive activities placed at specified points in recorded lecture videos.

As identified in the last quadrennial review, more insight is needed into these processes as they relate to the use of digital technologies. Currently dominant are qualitative methodological approaches using, generally, thematic analysis, with surveys by far the favoured research instrument. Like the last review, most questions interrogated students' perceptions of their learning experience. Without diminishing the important contribution of these efforts (or the use of qualitative research approaches), future research might consider more quantitative approaches.

At the tertiary level, some of this research may be done using log and administrative data, such as "click" data which may be culled from Learning Management Systems. These approaches, now commonly falling under the umbrella of learning analytics research, were evident in our review. Of all the quantitative research approaches we found, most (4) used administrative and/or log data in combination with other data (Johnston, 2017; Quinn, Hajek, & Aarão, 2017; Tisdell & Loch, 2017; Trenholm, et al., 2019). The relatively small number of related studies suggests this research approach is still in its infancy in this field. Notwithstanding current challenges around consent and ethics (e.g., Slade & Prinsloo, 2013), this is an area for potential further development, not least given the capacity for data production and collection associated with the use of digital technologies. Alongside current dominant qualitative approaches, this work may help to clarify some of the complexities associated with the use of various digital technology tools in tertiary education.

Our review of literature related to tertiary mathematics has highlighted a number of strengths and gaps in the research. For example, research into the use of video continues to be an area of strength, though many questions remain. The first-year experience, particularly related to engineering mathematics, remains a steady focus, though little research appears to be targeting mathematics subjects taught in later years or even at the graduate level. Of particular note, more than one quarter of the studies we found were reviews, which suggested to us a level of maturity in this area of research in Australasia.

6 Current Methodological Approaches in Technology Research

In a move away from previous chapters reviewing digital technologies, we now shift our focus to explore the methodologies undertaken in the research reviewed above. An understanding of methodological approaches will provide further insight into future directions for research and the ways in which the research might be conducted.

Although digital technologies and their corresponding use in mathematics education are evolving, some degree of constancy and rigour is beginning to settle over the methodologies utilised to examine the field. The range of lenses used is still eclectic with each distinctive in nature, yet within each of these distinctive methodologies, most are consolidating their approach, enhancing validity as more studies use them. Some are hinged to elements that might centre predominantly on a curriculum area or application. For instance, instrumental orchestration (Trouche, 2004) is frequently used in CAS and dynamic geometry research; however, only one of the recent Australasian studies have used it, i.e., Thomas, Hong, and Oates (2017) in their examination of first year mathematics students' use of digital technologies. Other methodologies relatively prevalent in the field of using digital technologies in mathematics education are: design-based; socio-cultural, such as semiotic mediation; interpretative phenomenology and action research.

This range of methodologies enables us to examine the field in a more critical way. If comparable processes, affective aspects, and/or conceptual thinking are situated in similar contexts, but examined through the varying lenses of differing methodologies, they might open up varying perspectives and insights. Having this range of perspectives and insights allows critical comparison of consistencies or tensions between the studies. Likewise, critical analysis can be applied to studies that are situated in different contexts but use the same methodology to examine the research questions.

Importantly, it is the research questions themselves that predominantly drive the selection of the methodology and research design. Although usually considered as research designs, strong arguments have been made for mixed methods (e.g., Creswell & Creswell, 2018) and case studies (e.g., Yin, 2014) to also be considered as methodologies. Hence, these are included in this synthesis. As well, the on-going development of digital technologies has opened up opportunities for new ways to generate data to answer research questions in the field, and new methodologies have accompanied these emerging approaches. The consideration of each methodology will be illustrated with examples from the associated literature already discussed in this chapter.

A case study is an in-depth examination of a particular case, with its focus often a contemporary phenomenon within a real-life context (Yin, 2014). Case studies can be employed to investigate phenomena that are collaboratively designed by multidisciplinary teams. For instance, the processes utilised between mathematicians, scientists, mathematics educators and science educators in designing an online learning module for mathematical modelling (Geiger et al., 2016). Their process included three phases: initial case study development; case study review; and the linking of case studies. Reporting on a case study from within a larger action research project, Muir, Callingham, and Beswick (2016) examined a case where the aim was for teachers to enhance their teaching practice through the use of IWBs. The single lesson case gave particular nuanced understanding of how to make effective use of the features of the technology to engage students and maintain their interest, encourage participation, and demonstrate particular mathematical strategies and skills.

In a case study of a Regional Health Schools outpatients' engagement in mathematics learning, Willacy and Calder (2017) reported on four teenage students' use of apps in their learning, with the case study methodology revealing insights into ways to keep students engaged when working in situ at home. Comparative case studies can reveal more fine-grained insights as the differences and similarities between the cases are compared. An example of such a comparison highlighted the complexities of primary-school teachers' use of digital technology through two case studies (Loong & Herbert, 2018). This allowed them to investigate their research questions with the rich and varied generation of data around particular situations. As well, comparative case studies were undertaken with: Pre-service teachers' (PSTs') using a blended-learning approach (Larkin, 2016a, 2017a) in which he reported the impact changes to delivery mode (online, blended, and face-to-face) had on experiences of the course; the comparison of three studies to answer questions related to student engagement when using digital technologies in mathematics teaching and learning (Attard, 2018); the examination of whether using apps motivated reluctant learners in three different locations (Calder & Campbell, 2016); and the analysis of early-career teachers' experiences of using digital technology to teach mathematics (Orlando & Attard, 2016) which indicated across varying contexts, that teaching with technology is different to using technology.

While examining the case of a primary school that used screencasting apps on mobile technologies to produce "create-alouds" Prescott and Maher (2018) indicated that the approach provided school-wide insights into the opportunities for the teachers to explore collaborative tasks and formative assessments. In a similar way, Galligan and Hobohm (2018) used a case study to examine tertiary students' use of an evaluative tool to develop effective maths-casts. Meanwhile, a sequential case study was used to consider a flipped classroom approach with a numerical methods course (Johnston, 2017). As well as the depth of insight that the data revealed, the sequential approach enabled an ongoing development of instruments, each informed by the previous.

Many of the studies in this chapter used variations of sociocultural research methodologies. Underpinned by Vygotsky's (1978) participatory theories, the sociocultural lens is concerned with perceptions and interpretations that are imbued with the cultural and historical discourses from which they emerge. The concern for participants' connections and perceptions of their lived-in worlds, e.g., classrooms, has led to their manifestation in various forms of educational research. Several studies reported research viewed through a sociocultural lens: in a study of the ways primary-school teachers use "show-and-tell" apps in their mathematics teaching (Ingram, Williamson-Leadley, & Pratt, 2016); with the examination of six primary children using decimals on an interactive number line (Gorman & Way, 2018); and with examining the video-recordings of seven-year-olds using a screencasting app to explain their solutions of a simple division problem (Murphy & Calder, 2017). An interpretative methodology was also used to derive a version of socio-technical assemblage from the collaborative analysis of teachers using a range of creative apps, materials and the associated social elements (Calder & Murphy, 2017); and with Willacy, West, Murphy, and Calder (2017) in their investigation of personalisation and differentiation when using MT. Using a VR "e-cave" Marshman, Woolcott, and Dole (2017) investigated pre-service teachers' experiences with immersive technology and whether their reflections on their 3D thinking and reasoning abilities supported spatial thinking when developing learning activities. These, and others such as Oates' (2016) personal reflection, were able to analyze participants' reasons and motivations and gain fine-grained insights within the specificity of a particular situation.

The aim of mixed methods methodology is to use multiple methods, data sources, and analytic approaches to better capture the breadth and depth of complex phenomena and enhance understanding, with one data strand intersecting with, adding to, and making meaning for the other (Creswell & Plano Clark, 2011). In the examination of the influence of teaching and learning mathematics with iPads on students' engagement and attitudes to mathematics, Hilton (2018) combined surveys of over 400 participants in the first year, pre and post-intervention surveys in the second year, and semi-structured interviews with small focus groups, in the third year. The weaving of these methods enabled in-depth analysis of the complex phenomena, with each method opening up space to enhance the understandings of the other. Survey and focus groups were undertaken in a similar approach to examine Stage II mathematics students' perspectives of a flipped-classroom approach to lectures (Novak, Kensington-Miller, & Evans, 2017). Likewise, research such as Hawera, Sharma, and Wright's (2017) study of how PST's how can best be supported to use digital technologies for mathematics teaching also used a mixed methods approach, employing pre and post-intervention testing and video-recorded observation to explore how students developed mathematical knowledge and thinking as they participated in coding and robotics lessons. In another mixed methods study undertaken in engineering mathematics tertiary classes, McLoughlin and Loch (2016) explored the role of screencasting in scaffolding flexible learning and engagement, while in order to analyse complex phenomena or settings Maclaren, Wilson, and Klymchuk (2017) used mixed methods to examine the place of gesture and annotation in teaching STEM subjects using pen-enabled Tablet PCs. This also included some lecturers of pure mathematics and statistics classes.

Other studies adapted or cultivated methodological approaches to best explore their research, such as the development of an instrument that integrated the semantic items of three related scales aimed at characterising the perceived worth of mathematics-education apps (Handal, Campbell, Cavanagh, & Petocz, 2016). While this collected a mixture of numerical and written data, the analysis was qualitative and they considered it effective in establishing content worth. In another novel approach, Kawka and Larkin (2018) aimed to disrupt the notion of using apps to project mathematics learning as a fun experience. They created a digital artwork, *Arithmomania* that challenges how users interact with education apps by employing the aesthetic of the glitch, characterised by reifying disorder and malfunction. Glitching enabled the divergence of the educational component from the fun component, with the mathematical element portrayed as varying layers of colour and sound.

Several studies undertook systematic literature reviews to gain insight and analysis of particular phenomena e.g., Lake et al. (2017); Tobin and Weiss (2016); and Trenholm et al. (2019b). There were also a number of papers that engaged a contemporary hermeneutic methodology, where the data were analyzed through iterations of interpretation, shifts in researcher perspective and then re-engagement from fresh perspectives. This methodology enabled the layering of interpretations of data which when done collaboratively with teacher co-researchers allowed rich insights and understandings to evolve. This methodology was used to examine a number of questions related to primary children using apps for learning (Calder & Murphy, 2018a, 2018b, 2018c) and in research where primary children used the coding app *Scratch* to design number games for their younger buddy class (Calder, 2018).

Design-based research involves iterations of the review and design process, with the intervention practice followed by the review and modify stages of the cycle. Through this process, the intention is to incrementally improve an artifact or process within its situated context, with the aim to enhance practice (Anderson & Shattuck, 2012). While employing a design-based methodology to develop a mapping app for developing spatial reasoning in early years learners, Larkin and Kinny-Lewis (2017) used the four-stage agile design principles: Discovery, Alpha, Beta, and Live, as they underwent iterations of design and feedback. They incorporated user feedback from the early years' learners through observations of the children's behaviours (e.g., smiling, looking confused) as key indicators of their level of engagement with the app. Also utilizing design iterations of interactions between the user (Subject), app (Artifact) and mathematics content (Object), Larkin et al. (2019) further developed the ACAT framework to analyze the ways that two apps were used with classes (Group). They also considered how they behave when used (Rules) and contend that

ACAT is highly useful for evaluating any mathematics app. A quasi-experimental design study was utilised to investigate the use of recorded lecture videos (RLVs) in undergraduate mathematics instruction (Trenholm et al., 2019a).

Also, utilizing iterative design elements in the development of an instrument to investigate the quality of mathematical apps, Larkin and Milford (2018a, 2018b) initially used an integrated framework evolved from three existing measures: the Haugland Scale, Productive Pedagogies, and Gee's Principles. The framework went through several design and trial iterations, before being coupled with a statistical tool, cluster analysis, to revisit the earlier evaluation. This combination of design cycles, that included measures of perception and review hinged to mechanistic statistical analysis, is perhaps indicative of potential methodologies or research designs that integrate both socio-cultural and machine-driven lenses through which to generate and analyze data, with the consequent unpacking revealing better understanding of a range of digitally-enhanced realities.

Action research follows similar principles and an iterative process as does design-based research, but has a deliberately situated reflection stage in the cycle, rather than the reflection being ongoing, while also not having a distinctive collaborative design stage (Anderson & Shattuck, 2012). Also creating a purpose-designed instrument, Galligan, Hobohm, and Peake's (2017) paper describes an action research process of developing and refining a tool for the creation and evaluation of quality student-produced mathscasts. The study then analyses its effectiveness in relation to pedagogy and mathematical understanding. Other studies used action-research iterations to initiate change; Loch and Lamborn (2016) when aiming to make mathematics relevant to first-year engineering students, and Quinn, Hajek, and Aarão (2017) with their intention to optimize the blending of online and face-to-face teaching and learning for first-year engineering students.

7 Conclusions and Future Directions for Research

This chapter presented an analysis of Australasian research conducted between 2016 and 2019 pertaining to the use of digital technologies across all stages of education. In doing so we interrogated the research to understand the current foci of research, and if, and what, changes have occurred since the last review (Geiger, et al., 2016). As stated in the introduction, areas of research have been built upon and expanded, and as in the last review, we found a broad research agenda and a broad range of methodologies employed in research related to the use of digital technologies.

The authors of the last review made several suggestions for future research into technology enhanced mathematics education. This current review has revealed many of the suggestions have not yet been realized, particularly in relation to the management of interactions and relationships that occur in online environments. Given the increasing use of such environments, this is of some concern, as is the lack of research into the influence of social media on mathematics teaching and learning. Related to this is the limited exploration of virtual worlds and technologies that promote student design within online environments. However, this use of technology is not yet common in schools and this may account for the lack of emerging research in this area.

Several themes emerged from our review that address suggestions made in the previous review. Research on the use of screencasting across the school and tertiary sectors, including ITE, appears to have increased and is a dominant feature in this review. This increase implies some developments in task design and a shift away from the use of consumer focussed applications such as drill and practice apps and apps that are buried in 'fun' contexts such as those described in the paper by Kawka and Larkin (2018). Task design has been suggested by Tsai and Chai (2012) as a possible third-order barrier to effective technology integration. They posit that such knowledge "lies in the dynamic creation of knowledge and practice by teachers when they are confronted with the advancement of ICT and its associated pedagogical affordances" (p. 1058), believing the capacity for 'design thinking' is the new barrier to technology use in education.

Although the increased use of screencasting provides us with some evidence of technology redefining pedagogical practices, there is little research reporting on other and more innovative task design using emerging technologies such as virtual reality and aritifical intelligence. We also found evidence the third order barrier may be limiting the ways in which students interact with digital technologies due to the level of teacher decision-making and its influence on how task design and technology is used. Throughout the research reviewed it is evident that teachers remain in control of how technology is used and in some cases, where learning is 'flipped' and live lectures or lessons are replaced by video-recordings, it appears that rather than transforming learning, practices have reverted to teacher-centred approaches. This leads us to question how 'anywhere, anytime' learning influences the ways teachers interact with their students as discussed in the work by Trenholm, et al. (2019).

This current review period revealed a narrow emphasis of research on the use of digital technology to teach mathematical content. Although studies that include, what could be termed Geometry, appear to dominate in the early childhood, school, and tertiary sectors, there is no evidence of digital technology being used in ITE to develop content knowledge. This is of some concern in early childhood and primary ITE given the current concerns in some countries about the mathematical content knowledge held by generalist early childhood and primary teachers. However, of note with regard to mathematics content is an increased level of research investigating mathematics teaching and learning within the context of STEM education in the early and primary years.

A final theme that has emerged more strongly in this review is that of student engagement. Although there were several studies that specifically focussed on digital technology and its influence on engagement, others reported on student engagement as a by-product of technology use across all levels of education. Given that the Attard study (2018) indicated engagement is largely dependent on the teacher's technology-related pedagogical practices, we find strong links across each of the themes emerging from this review and make the following recommendations for future research directions:

- Investigation of how digital technologies are being used to develop mathematical content knowledge in pre-service early childhood and primary teachers, including those undertaking specialisations in mathematics;
- Develop deeper understandings of how technology can be used to position students to have more voice and control in mathematics classrooms and promote rich, two-way interaction;
- Exploration of the relationship between teacher technology-related decision-making and teacher engagement in the planning of technology-related practice;
- Focus on innovative task-design with current and 'over the horizon' technologies
- Further investigation into the use of digital technologies to teach the breadth of content knowledge across all levels of education; and
- Investigation into the effects of learning management tools on the ways students access teaching resources (including videos) beyond the classroom and the ways these tool provide teachers access to evidence of student learning.

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