Jennifer Way · Catherine Attard ·
Judy Anderson · Janette Bobis ·
Heather McMaster · Katherin Cartwright
Editors

Research in Mathematics Education in Australasia 2016–2019

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Chapter 1
Research in Mathematics Education in Australasia 2016–2019

Janette Bobis, Jennifer Way, Catherine Attard, Judy Anderson, Heather McMaster, and Katherin Cartwright

Abstract In this chapter we present an introduction to the tenth volume in the review series *Research in Mathematics Education in Australasia*. MERGA's four-yearly reviews present critical analyses of research in mathematics education in Australasia over the preceding periods. Moreover, they serve to highlight significant enduring and emerging trends and forecast possible directions for future research in mathematics education. In this chapter, we provide a historical overview of the four-yearly review series, describe the current review's production process and briefly introduce the review's overall structure.

Keywords Research in mathematics education in Australasia · RiMEA · MERGA review

1 Research in Mathematics Education in Australasia Review Series

The four-yearly reviews cover a wide cross-section of topics of research conducted in the Australasian region or by Australasian researchers abroad. Each review is

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published to coincide with the International Congress on Mathematical Education (ICME) conference. The first volume was published in 1984 (Briggs, 1984) to coincide with ICME-5 in Adelaide. The next volume is to be published in 2024 and, significantly, will coincide with ICME-15, which will be held in Sydney, Australia. More so than ever, the current and next review will spotlight Australasian research on the global stage of mathematics education.

The Mathematics Education Research Group of Australasia (MERGA) is a professional association whose members are interested in mathematics education research in Australasia. It provides a range of opportunities for members to raise important issues in mathematics education and share research findings that speak to how these issues might be effectively addressed. A major goal of MERGA is to encourage, promote and disseminate quality research in mathematics education. This goal is partly fulfilled by an annual conference and the association's two journals—*Mathematics Education Research Journal* and *Mathematics Teacher Education and Development*. Additionally, RiMEA also plays a major role as it serves to highlight significant enduring and emerging trends and forecasts possible directions for future research in mathematics education in Australasia and internationally. In accordance with the guidelines of previous volumes of RiMEA, only Australasian research published as readily accessible outputs in the review period 2016–2019 is included in RiMEA 10. Rather than attempting an exhaustive review of all research outputs in this period, chapter authors were requested to be selective, to highlight noteworthy findings or trends in the research and to provide a critical perspective.

The term 'Australasia' primarily refers to Australia and New Zealand. However, as was the case for RiMEA 2012–2015, chapter authors of the current RiMEA were provided with a slightly broader context than was historically the case. This broader context reflects the increasingly significant presence of Singaporean researchers in MERGA. Hence, the regional context was described to chapter authors to be inclusive of:

> … papers published in MERGA conference proceedings and articles published in MERGA journals by researchers from countries in the South Pacific and south-east Asian regions and with particular relevance to these regions should also be considered for inclusion in the review.

The current RiMEA was fashioned to be consistent with and maintain the high standard set by editors of previous volumes in the series. Previous RiMEAs and their editorial teams were:

- 2016—Makar, Dole, Visnovska, Goos, Bennison, and Fry
- 2012—Perry, Lowrie, Lodan, MacDonald, and Greenlees
- 2008—Forgasz, Barkatsas, Bishop, Clarke, Keast, Seah, and Sullivan
- 2004—Perry, Anthony, and Diezmann
- 2000—Owens and Mousley
- 1996—Atweh, Owens, and Sullivan
- 1992—Atweh and Watson
- 1988—Blane and Leder
- 1984—Briggs.

Although this review has been profoundly shaped by preceding volumes, it is important to note that the tone and substance of RiMEA is constantly evolving. This evolution is in part a reflection of events coinciding with each new review period as much as emerging trends in research foci. Notably, the current review period saw MERGA celebrate its 40th anniversary in 2017. Returning to Monash University, the site of our very first conference, the MERGA 40 conference organisers adopted the theme: *40 years on: We are still learning!* The theme was chosen to acknowledge the significant contributions of MERGA researchers over the past 40 years and highlighted "the impact and importance of our collective research for enabling new learning, innovation, and critique of mathematics education for those in our region and beyond" (Gervasoni & Forgasz, 2017, p. 3). 'New learning' is also an apt description for the collective work in this 10th volume of RiMEA. Like MERGA 40, RiMEA 10 is not only a means to disseminate research findings and reflect on the lessons of the past, it is a celebration of our new learning that allows a growing audience of researchers and practitioners to think forward and imagine mathematics education research of the future.

2 Editors and the Production Process

The current editorial team responded to a call for expressions of interest by the MERGA Executive to edit RiMEA 2016–2019 in the second half of 2017. The editors were then selected by the Executive from a pool of applicants. The current team comprises experienced and early career mathematics education researchers drawn from The University of Sydney and Western Sydney University. All editors are members of MERGA.

In February 2019 MERGA members were invited to submit an expression of interest to:

- lead a team of authors to write a chapter proposed by the editing team,
- suggest a chapter beyond those suggested by the editors, or,
- indicate an interest in joining a team of authors in one of the areas suggested by the editors.

In the call for chapter authors, it was requested that:

- Each chapter be written by a team of authors rather than an individual author,
- Author teams comprise experienced and early career researchers, and if possible, a blend of geographical diversity,
- All authors must be current MERGA members,
- Each individual author should only be involved in the writing of one chapter.

The result of this recruitment process was 11 author teams comprising 50 individual MERGA members. Final chapter topics eventually emerged from a blend of those originally proposed by the editorial team, those slightly modified by authoring teams and a few totally new topics proposed by authors to reflect emerging trends in mathematics education research. Added to this collection of chapters and keeping with the tradition of past RiMEAs, editors of the previous RiMEA were invited to compose a reflective chapter (Chap. 2) and a distinguished member of the MERGA community was invited to write a concluding reflective chapter. Combined with this introductory chapter composed by the current editorial team, RiMEA 2016–2019 comprises 14 chapters and involved 57 MERGA members in writing teams.

Except for the authors of the first two introductory chapters and the final reflective chapter, author teams submitted detailed outlines of their chapters to the editors by early March 2019 and full drafts by early July 2019. Each chapter was sent to at least two experienced reviewers in the relevant field of the chapter. Members of the editorial team consolidated comments from reviewers into a report that was returned to author teams by early September. Final chapter drafts were submitted by the end of December 2019 and reviewed by independent members of the editorial team to ensure reviewers' comments were addressed. Final chapters were formatted and copyedited by Bronwyn Lacken, to whom we are grateful for working across the holiday period. Final revisions were undertaken by chapter authors as needed throughout January 2020.

A distinguishing feature of RiMEA 10 is the lack of sections whereby chapters are clustered according to common themes, as was the case in the past few volumes of RiMEA. Instead, we chose to foreground this review with two chapters that reflect emerging areas of research interest and strength for MERGA members—STEM (Science, Technology, Engineering and Mathematics) and numeracy. Following these two chapters are chapters that focus mostly on topics of enduring concern to mathematics education researchers. However, within each of these enduring topics of interest, each team of authors have successfully identified new research trends and foreshadowed new directions for future research.

3 Concluding Comments and Acknowledgments

The editorial team consider it a great honour to have been bestowed the responsibility of creating the *Research in Mathematics Education in Australasia 2016–2019.* We hope that this volume, like so many RiMEAs before it, brings both pleasure and pride to all MERGA members as they read and reflect on the collective efforts of their colleagues. As editors we would like to express our gratitude to the MERGA Executive and members for their support throughout the making of this volume. Without the enormous support and generosity of time from all people involved—the researchers who forwarded copies of their work, author teams, reviewers, and the copy editor—this volume would not have been possible. We look forward to the next review of MERGA research, *Research in Mathematics Education in Australasia 2020–2023* that will be released to coincide with ICME-15 in Sydney, Australia.

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Chapter 2
Looking Back and Taking Stock: Reflections on the MERGA Research Review 2012–2015

Katie Makar, Shelley Dole, Jana Visnovska, Merrilyn Goos, Anne Bennison, and Kym Fry

Abstract Since 2004, each edition of the MERGA Research Review has invited the previous editorial team to write a chapter that reflects on issues that have occurred since the last Review. As the editorial team for Research in Mathematics Education in Australasia 2012–2015 (RiMEA-9), we have followed suit. The reflection chapters often compare the current MERGA Review with the previous one. Given that this is the tenth MERGA Review, we have taken the opportunity to look further back from RiMEA-1 in 1984 until now (RiMEA-10). Like the previous Review, we also comment on how mathematics education research in Australasia is affected by new reforms that have occurred in the past four years. In particular, we examine the implications of recent changes in initial teacher education, STEM teaching and learning, and the assessment of research impact and engagement. We use these three areas of reform to reflect on how related issues projected in the chapters of the last review played out and urge mathematics education researchers to focus their research on implications of these reforms for the field.

Keywords MERGA history · Research assessment · STEM · Teacher education reforms

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1 Introduction

As the editorial team of *Research in Mathematics Education in Australasia 2012–2015* (RiMEA-9, Makar, Dole, Visnovska, Goos, Bennison, & Fry, 2016), we continue the tradition of reflecting on the past four years in light of the chapters we edited in RiMEA-9. Former editors (RiMEA-8) Perry, MacDonald, Greenlees, Logan, and Lowrie (2016), saw the task of this chapter as being: "to reflect on the 4 years following the publication of this review, consider the directions the review foreshadowed and provide an overview of the context for the current review" (p. 14). We continued with their ethos in reflecting on the 17 chapters in RiMEA-9. Following the introductory chapter, Chaps. 2–16 were:

- Issues and Contexts for Mathematics Education
    - Reflections on the MERGA Research Review 2008–2011: Taking stock
    - A philosophical gaze on Australasian mathematics education research
    - Researching curriculum, policy and leadership in mathematics education
    - Mathematics education and the affective domain
    - Equity, social justice and ethics in mathematics education
    - Inclusive practices in mathematics education
    - Distribution, recognition and representation: Mathematics education and Indigenous students.
- Learning and Teaching
    - Mathematics education in the early years
    - Tertiary mathematics education
    - Innovative and powerful pedagogical practices in mathematics education
    - Assessment of mathematics learning: What are we doing?
    - Transformations of teaching and learning through digital technologies
    - Research into mathematical applications and modelling.
- Teacher Preparation and Development
    - Challenges, reforms, and learning in initial teacher education
    - The education and development of practising teachers.

The concluding Chap. 17, following tradition, was a reflection on the volume, and was written by an eminent MERGA scholar from a perspective of her choice. Lyn English (2016), winner of the 2012 MERGA Career Research Medal, authored the final chapter entitled *Advancing Mathematics Education Research within a STEM Environment.* English drew on the chapters in RiMEA-9 to forecast the future of STEM education, a significant initiative that deeply affects mathematics education in Australasia.

In this chapter, we first reflect on how MERGA Review chapters have highlighted research interests over time. The analysis of chapters and their authors in RiMEA-1 through RiMEA-10 provided insights into complexities of the drivers of our research community. We also look at three policy issues that have emerged since RiMEA-9 with respect to the future that its chapters foretold. In particular, we (1) examine recent policies in New Zealand and Australia and their implications for initial teacher education, (2) question if mathematics education is sufficiently present within the STEM Agenda and (3) summarise how mathematics education research fared in the recent impact and engagement assessment conducted by the Australian Research Council.

2 Looking Back at Previous Chapters in RiMEA

The current edition of *Research in Mathematics Education in Australasia 2016–2019* (RiMEA-10) is the tenth edition of the MERGA Research Review (as RiMEA is often referred to, or simply the "Review"). To have such an ongoing review of research in the field is rare. The Review series allows scholars to follow trends, trace patterns of research and their influences from and on educational policy in Australasia and gauge fruitful emerging areas of research. Authors of each chapter in the Review were asked to identify potential future directions of research based on their account of the research in the field in which they are reporting. Looking across editions of the Review, therefore, can give insight on where research momentum was moving, and if and how it meandered through time.

Authors of previous Reflection chapters have compared the chapters that they edited to the current Review. We take the opportunity in this tenth edition of the Review to look back a little further at the previous Reviews and comment on patterns we observed. A further gaze back is timely given that the impetus for the first Review was the opportunity of having the 1984 International Congress on Mathematical Education (ICME) held in Adelaide. Subsequent Reviews continue to coincide with ICME. Fittingly, the 2024 ICME will be held in Sydney when the 11th Review (RiMEA-11) will be launched forty years after MERGA's first Research Review.

The initial two volumes (1984 and 1988) did not yet have the spread of content chapters that is familiar to RiMEA readers today; but they provided annotated bibliographies and several topical review chapters to facilitate access to Australian research. RiMEA-3, published in 1992, was the first one with 'Australasia' in its title, and included references to mathematics education research from New Zealand. It was, however, not until RiMEA-5 that the first New Zealander appeared as a chapter co-author. This slow start to diversifying author teams is in considerable contrast to RiMEA practices of the recent Reviews, where half or more of the chapters are authored by international teams. The inclusion first of New Zealand, and later more international perspectives from the region, is a result of conscious commitments within the community. In 2008, RiMEA-7 editors explicitly defined Australasia and made a greater effort to include work of authors beyond Australia and New Zealand. Similar efforts have been since reflected in increasingly international composition of chapter author teams. Table 1 lists the number of chapters with at least one co-author

Table 1 Authorship participation in Reviews across Australasia

	Year	Number of chapters	Number of chapters with at least one co-author from	
			New Zealand	Australasia (Non-AUS, Non-NZ)
RiMEA-1	1984	5		
RiMEA-2	1988	5		
RiMEA-3	1992	13		
RiMEA-4	1996	16		
RiMEA-5	2000	12	1	
RiMEA-6	2004	16	6	
RiMEA-7	2008	16	7	
RiMEA-8	2012	16	10	
RiMEA-9	2016	17	8	3
RiMEA-10	2020	14	7	2

from listed location, in comparison with the total number of chapters. (Authors affiliated only with a non-Australasian institution were not included in the counts, there were several from RiMEA-8 on.)

Returning to the structure of RiMEAs, reflections, trends, and/or future-oriented chapters became a stable presence in most of the volumes. Besides front and back matters chapters, we categorised chapters into several themes to gain a sense of changes in foci through different periods. The themes are presented in Fig. 1 and include:

- Focus on learners by age;
- Theoretical underpinnings of research;
- Educational issues;
- Teacher learning, pedagogies, practices;
- Systemic issues; and
- Mathematics domains and proficiencies.

In Fig. 1, each opaque coloured box represents a chapter, while semi-transparent coloured boxes illustrate a theme that was a substantial part in another chapter. This was most useful in capturing the structure of early volumes. For example, the 1988 annotated bibliography on *Psychology in Mathematics Education* included sections devoted to research on affect and exceptional students; the annotated bibliography on *Problem Solving* included substantial sections on research in algebra, geometry, and early arithmetic. In 2004, a chapter on *Learning to Teach Mathematics* predominantly discussed teacher education, but also included significant content related to in-service teacher professional development. Similarly, a chapter on *Social Justice and Sociocultural Perspectives in Mathematics Education* (2004) presented extensive

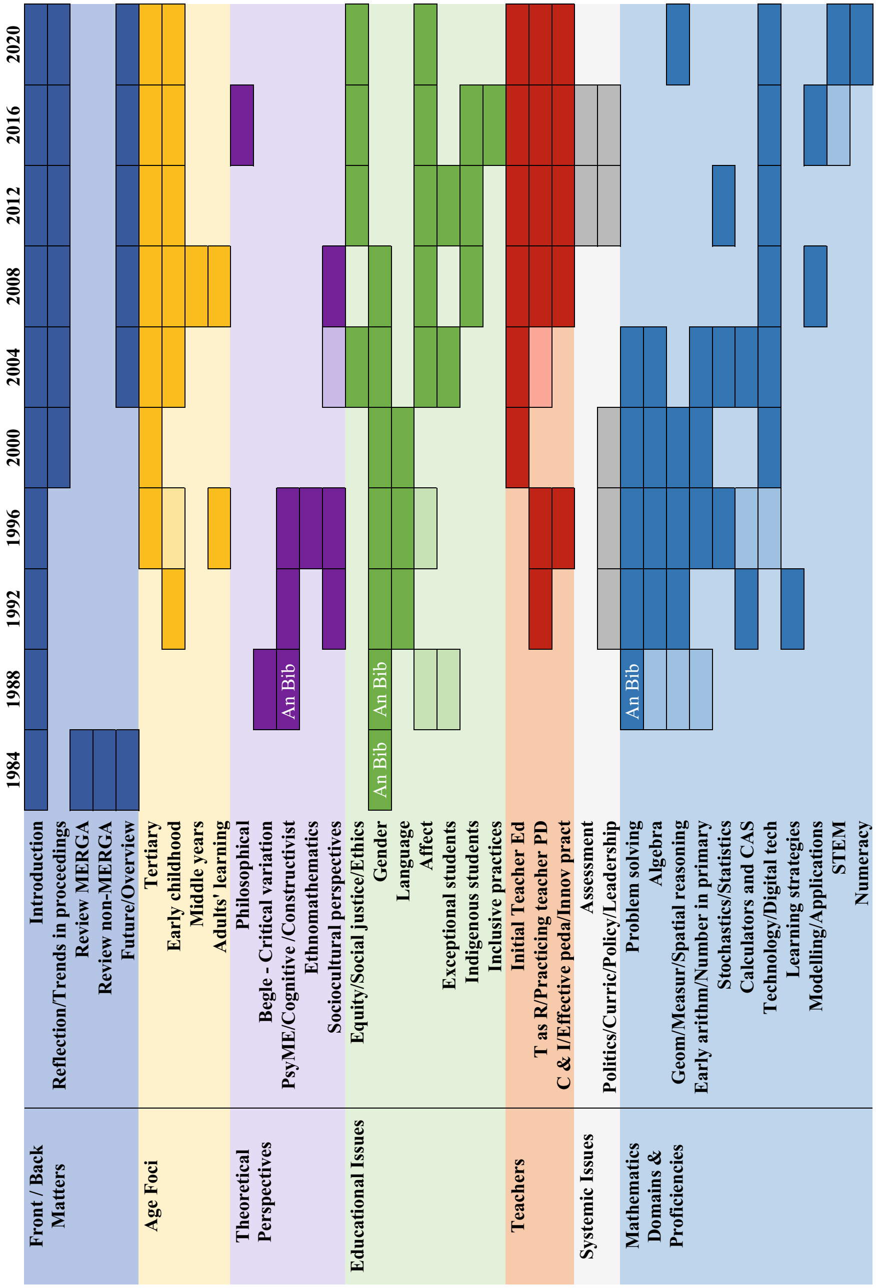

Fig. 1 Themes of MERGA Review chapters 1984–2020

discussion of various themes within social justice agenda, and then somewhat more briefly addressed the underpinning theoretical matters within sociocultural theories.

It was not always obvious where a specific chapter fits best, or which sub-section deserved to be represented independently. In making the categorising and highlighting decisions, we were guided by the aim of illustrating developments that took place over time. For example, a chapter on sociocultural theories was included in 1992, was absent in 2000 (with chapters on gender and language carrying on some of the theoretical debates), then re-appeared in 2004 and more strongly in 2008, and was subsequently subsumed within specific educational issues of equity, social justice, and ethics (with independent chapters on gender, indigenous students, exceptional students, or inclusive practices, as the distinct debates were highlighted by editors).

A changing research climate is evident in other patterns. While learning theories production and justification remains important for research in educational settings, it is less commonly the distinguishing element in a study. As theoretical research results faced limitations when attempting to guide practice, in-depth explorations of those practices became research-worthy in their own right. Student learning was initially the primary target of research; yet over time, an increasing emphasis was evident on explorations of teaching. This transition of focus from learning to teaching appears to be well illustrated in how the blue block of mathematical domain chapters ceased dominance around the same time that the red block of teachers' work chapters became firmly established. In this transition, not only did new theoretical paradigms enter the scene, but pragmatic considerations gained legitimacy for researchers to study and work to improve practices in classrooms and schools. Early signs of this transition are evident in chapters like *Teachers as Researchers* (1992). While learning theories continue to provide tools for disciplined inquiry, the drive for advances within specific educational issues might have provided new organising principles along which research work is conducted and collated. As the Philosophical chapter (2016) reminds us, even when our theoretical underpinnings are specified, philosophical assumptions are more and more likely to remain unexplored and implicit.

Tightening of accountability measures in Australia and New Zealand swung funding priorities from basic (pure theoretical) towards more practice-oriented research (see Clarke et al., 2012; Clements, 2008). The shifts in the Review chapter themes from *learning theories* towards *specific educational issues* that emerged in early 2000s, and those from *learning* to *teaching* that followed closely, appear to follow these funding changes as well as being mirrored in recent policy commitments to research with demonstrable engagement and impact that we discuss in a later section. This historical overview of RiMEA chapters demonstrates that—perhaps with an advantage to other fields of research—the mathematics education community has long carried an appreciation for the pragmatic alongside the theoretical.

Another shift, discernible in the chapters on 'Mathematics Domains and Proficiencies', is the transition from mathematical domains (e.g., number, algebra, stochastics, geometry) to mathematics as activity (e.g., statistical and spatial reasoning, modelling, STEM, numeracy). Here the transition from conceptualising the key problems as being those of how mathematics is created by the human mind, to recognising the need for addressing the problems of *how and why* mathematics might be created and sustained by collectives of problem-solvers appears highly relevant. Similar shifts can be imagined within the broader context of STEM disciplines. Approaches to teaching and learning of mathematics, science, engineering, and technology as both products of human inventiveness, and purposeful creations capable of addressing problems that humanity faces are highly consistent with the directions apparent across the Reviews. We later discuss the extent to which this is the direction taken up in recent policies regarding STEM.

The four chapters—two on assessment (2012, 2016) and two on curriculum/policy issues (2012, 2016)—are examples of how chapters in RiMEA can be prompted by policy changes. On the one hand, both assessment chapters deal with issues around NAPLAN and international tests (how high stakes accountability testing should not be the only/main focus, or how could NAPLAN become more meaningful in face of curricular goals) at the time when these became influential, and NAPLAN started shaping teaching practices. The two curriculum chapters, on the other hand, are both very strongly shaped by introduction of new curricula in Australia and New Zealand. Here, the 2012 chapter is about how the curricula were developed and 2016 more about how to think about, and research, the relationships among curriculum, policy, and leadership, especially in the space of 'bringing' the official curriculum to shape the operational curriculum effectively.

In interpreting the table, it is useful to note that it does not speak to the individual research foci taken up by the researchers in Australasia. Instead, it is useful to view the chapters as those topics where the critical mass of researchers existed at the time and was overt enough, or organised enough, that (1) a group of chapter authors was formed to propose or undertake the chapter and (2) the editorial team recognised the theme as both a significant contribution and as sufficiently different from other proposals to be accepted. The cases of statistical reasoning (2012) and spatial reasoning (2020) chapters are instructive in that regard. In spite of the seeming 'disappearance' of mathematical-domain-focused chapters, the chapters were formed around a mathematical content theme when a sizeable productive *community* of researchers existed, on which the chapter would draw. For instance, Australian and New Zealand researchers were instrumental in the work of the International Collaboration for Research on Statistical Reasoning, Thinking, and Literacy, but also in the Modelling and Applications community, each of which organise bi-annual domain-specialised international conferences. Similarly, a recent completion of a large Australian project (e.g., ELSA) on spatial reasoning generated the core of the chapter contributions.

We do not take the change in chapter foci as indicating that there is no longer interest in researching content domain learning. But where do the researchers with strong focus on a specific mathematical content domain send their work for consideration when the chapter themes are announced? It is very likely that they are oriented by the number of issues, including the pedagogical approaches used, modes of delivery, or a specific policy framing that was relevant to how the research study was framed (e.g., equity, numeracy, STEM). In this way, the shifts in the Review chapter themes might illustrate that drawing on existing insights into mathematical learning allows researchers to attend to additional problems of practice.

3 Reflection on Three Policy Developments

Since the previous review, new policy developments have occurred that impact the research landscape in mathematics education. We focus on three of these—initial teacher education, STEM teaching and learning, and recent assessment of research impact and engagement—in relation to the previous MERGA Research Review. Initial teacher education reforms were also highlighted in the Perry et al. (2016) reflection chapter, highlighting how this area of work has been under constant scrutiny by policymakers. We encourage readers to compare policy developments in initial teacher education from their chapter and how they have continued to change in our reflection below. We recognise that the Closing the Gap initiative, which Perry et al. included in their reflection chapter, is missing from ours. Perry and his colleagues commented on the slow progress made on this initiative in Australia. We found developments slowed even further, leaving little to report. Better progress on similar initiatives in New Zealand is discussed below within initial teacher education. Researchers in mathematics education have been actively urging governments to take action, as outlined in the chapters in this volume.

3.1 Initial Teacher Education

Australia and New Zealand have each had new policies affecting initial teacher education (ITE). We briefly reflect on these policies, referencing the 2016 MERGA Review (2012–2015). Implications of teacher accreditation were also addressed as an issue within the political landscape in Australia and New Zealand in the last reflection on the MERGA Research Review (Perry et al., 2016).

In a review of the existing ITE system, the New Zealand Education Council (2016) made a number of recommendations to guide the ongoing development of ITE. The recommendations spanned entry requirements, program design and accreditation, and system-wide management issues. The Education Council described the change: "[the] area of specification and assessment of outcomes from ITE is the most important long-term step it can take to strengthen the ITE system" (p. 11). From 1 July 2019, new requirements governing the approval, monitoring and review of ITE programs were introduced (Teaching Council, 2019). Standards for graduating teachers have been aligned with the *Standards for the Teaching Profession* (Education Council, 2017) and ITE providers must demonstrate how programs enable pre-service teachers to meet each standard *in a supported environment*. Interpretation of the standards extends beyond the elaborations of the Standards and includes consideration of Tātaiako cultural competencies (Ministry of Education, 2011) and Tapasā cultural "compass" (Ministry of Education, 2018) to ensure graduating teachers develop culturally responsive teaching practices that address the learning needs of students from Māori and Pasifika backgrounds, respectively. There is also a strong emphasis on demonstrating how programs enable graduates to develop inclusive teaching practices that cater for the diverse learning needs of all students, especially those from disadvantaged backgrounds and those with additional learning needs.

In their chapter in the previous MERGA review, Vale, Atweh, Averill, and Skourdoumbis (2016) noted that in many ITE programs issues of equity, social justice and ethics are dealt with in general education courses and suggest that these issues should be an integral part of all mathematics education courses. The limited research in Australasia during the previous review period on how ITE programs can support teachers to develop mathematics teaching practices that cater for diversity (Anthony, Cooke, & Muir, 2016) may reflect the lack of attention to such issues in mathematics education courses. Research is needed to assist ITE providers to design courses that enable pre-service teachers to develop inclusive practices for teaching mathematics.

A recent review was extended to the provision of compulsory schooling in New Zealand (Tomorrow's Schools Independent Taskforce, 2018) and identified a number of system-wide challenges that the current system of self-governing schools is struggling to address. In the report, authors made recommendations on eight key issues they believe need to be addressed: governance, schooling provision, competition and choice, disability and learning support, teaching, school leadership, school resourcing and central education agencies. It is unclear at this stage which of the recommendations will be adopted. However, the findings and recommendations of this review are likely to influence the content of New Zealand's Initial Teacher Education programs. It is promising to see that inclusion is a vital part of the vision for education in New Zealand. The Australasian mathematics education community looks forward to learning about this progress in the coming years.

The advent of the Australian Professional Standards for Teachers occurred in 2012 as Australia progressed towards nationally consistent accreditation of initial teacher education programs aligned with the standards. The Teacher Education Ministerial Advisory Group's (2014) position paper, entitled *Action Now: Classroom Ready Teachers—Report of the Teacher Education Ministerial Advisory Group* (now commonly referred to as the TEMAG Review), was addressed in relation to new entry requirements into initial teacher education programs associated with personal levels of literacy and numeracy. A test to measure literacy and numeracy of aspiring teachers was developed by the Australian Council for Educational Research (ACER).

The TEMAG Review has had a major impact in the last four years on initial teacher education programs in Australia, and it is predicted that this will continue. A rationale for TEMAG was to raise the profile of the teaching profession in the eyes of the public (Bahr, 2016). It appeared to level blame for Australian students' falling literacy and numeracy standards, as measured on international assessments such as TIMSS and PISA, on the poor preparation of graduate teachers, and by default, the institutions that prepare teachers. The release of the latest PISA 2018 results, and Australia's slippage in mathematics in particular, will continue to put pressure on improving the preparation and development of mathematics teachers. The media continue to highlight low tertiary admission scores for entry to initial teacher education (ITE), publishing league tables of the cut-off scores of post-year 12 students who are offered places into education by higher education providers (HEPs) across the country. The test for measuring literacy and numeracy (referred to as LANTITE—Literacy and Numeracy Test for Initial Teacher Education) is now fully implemented in all Australian initial teacher education programs. HEPs can elect to use this test as an entry requirement into ITE programs, or as a requirement for graduation. The ACER website states that "the test standard is literacy and numeracy achievement equivalent to the top 30% of the Australian adult population" (ACER, 2019). As with annual results of Australia's National Assessment Plan Literacy and Numeracy (NAPLAN) test results, the Australian media continues to sensationalise results of LANTITE and the literacy and numeracy of teacher graduates. Interestingly, there is no equivalent to Australia's national literacy and numeracy testing policy for ITE providers in New Zealand. Rather, institutions must have entry testing which reflects university entrance of literacy and numeracy requirements and HEPs must provide their assessment tools as part of the ITE programme approval process (Teaching Council, 2019).

We highlight Anthony and her colleagues' (2016) chapter in the previous Review to remind the mathematics education community of the value of examining the ongoing impact of policy associated with initial teacher education and on the field. For example, Anthony et al. reported that changes in New Zealand had resulted in two HEPs offering only postgraduate ITE programs. Pre-service teachers' numeracy, preparation for teaching both numeracy and mathematics, as well as pedagogical content knowledge has been the focus of research, as reported in the previous review period. Anthony et al.'s caveat was associated with the complexity of measuring pre-service teachers' preparedness for teaching numeracy and mathematics, and the issue of pre-service teacher confidence and enjoyment of mathematics, which was the focus of work by Young-Loveridge, Bicknell, and Mills (2012) for New Zealand pre-service primary teachers. Their exploration of initial teacher education from a mathematics education perspective spanned teacher preparation and accountability, effectiveness and policies; teacher preparation for the knowledge society, and included studies associated with curriculum, opportunities to learn within coursework, designing opportunities to learn in school settings, the continuum of teacher learning; and teacher preparation for social justice. Anthony et al. offered valuable insights into what the mathematics education community can offer and where it needs to continue to build strength:

> The politicised attention to teacher preparation and the press to institute reforms will not abate in the near future…we must build on the existing large-scale studies concerning mathematics teacher entry and graduating knowledge/testing to address concerns around accountability, equity, and access for teacher candidates. (p. 321)

They end their review by highlighting the large-scale national Australian project *Inspiring Mathematics and Science in Teacher Education* (see Goos & Bennison, 2018), which they state

> provides an example of collaboration between academics from different communities of practice … [that] bodes well for the opportunity for mathematics teacher educators to open up their practice, to share their practice, and learn in, from and for practice. Only then will mathematics teacher educators be able to experience the benefits of a learning community of practice that we so readily advocate for teacher and student learning. (p. 321)

The mathematics education community has a strong history of research into initial teacher education as attested through the richness of Anthony et al.'s (2016) chapter in the previous RiMEA. The impact of national Australian policy associated with literacy and numeracy standards, and specifically the LANTITE, is a potential focus of research for the mathematics education community to improve the policy landscape in ITE. With respect to recent developments and future RiMEA chapters, we find it possible that attention to measures introduced to assess pre-service teachers would lead to return of the chapter on assessment in 2024 in this context. Would these policies impact our teaching lives and push us to develop stronger research-based arguments and advocacy in this space? The implications of measures on the upcoming generations of teachers of mathematics would be worth documenting.

3.2 STEM Agenda 2016–2026

The *National STEM Agenda 2016–2026* (Education Council, 2015) endorsed by the Australian Ministers of Education set out the priorities for the next decade in building Australian students' STEM (Science, Technology, Engineering and Mathematics) capabilities. The report argued that "STEM literacy is increasingly becoming part of the core capabilities that Australian employers need" (p. 4) as a critical driver of the future national economy. The Agenda set out goals, actions and principles to guide future initiatives undertaken by the Commonwealth. Activity in STEM has been present for a number of years, but recent momentum has accelerated. For example, the ELSA project (Lowrie & Logan, 2019) addresses STEM learning in early years. At the policy level, Toh, Kaur, and Tay (2019) outlined Singapore's response to STEM in updating the national mathematics curriculum and Anthony (2018) suggested that recent funding of research in mathematics education in Australia, New Zealand and Singapore are likely linked to government interests in STEM. Despite challenges (Timms, Moyle, Weldon, & Mitchell, 2018), the STEM Agenda is an opportunity for the mathematics education community to do more to connect to STEM initiatives in schools.

In the final chapter of the previous Review, English (2016) expressed the critical importance in mathematics education for greater connection to the STEM movement at both school and university level. Members of the MERGA community have indeed published on STEM policy and practice since the last Review and a new chapter that reviews research on STEM is given in the current Review (see Chap. 3, this Volume). Murphy, Macdonald, Danaia, and Wang (2019) examined state-level versions of the STEM agenda across the areas of STEM capabilities—STEM dispositions, STEM educational practices, Equity, Trajectories, and Educator capacities. It is concerning that they found little focus in the research on STEM dispositions and improving equity access to STEM (see Prieto & Dugar, 2017; Wilkie & Tan, 2019). Focused on the "rising premium on skills in STEM" (p. 1), Prinsley and Johnston (2015) from Australia's Office of the Chief Scientist outlined a position paper on STEM teaching in the primary schools. In its short report, the paper lists dozens of STEM initiatives and case studies that progress STEM teaching in primary schools. Within these, one key project in mathematics education has been *reSolve: Maths by Inquiry* (2015–2018, www.resolve.edu.au).

The *reSolve* project was a $7.4M partnership between the Australian Academy of Science and the Australian Association of Mathematics Teachers, funded by the Australian Government to develop a large set of classroom resources, online professional development and to organise 300+ "Champions" in schools to build capacity and scale the use of inquiry-based learning in mathematics. *reSolve* is driven by a protocol that emphasises mathematics as purposeful, with inclusive and challenging tasks, and a productive classroom culture that embraces higher-order thinking, collaborative inquiry and dispositions that support productive struggle and confidence to take intellectual risks (Thornton, 2017). The *reSolve* project ensured that its products were significantly linked to the Australian Curriculum: Mathematics for Years F-10 (particularly Mathematical Proficiencies, which are often overlooked) and to the AITSL Australian Professional Standards for Teachers. These connections were intended to ensure that the curriculum and professional development were explicitly linked to teachers' professional work. The project has ended, but an additional $1M in funding to 2020 was provided to continue promoting and updating the resources. *reSolve* is a significant initiative that the mathematics education community can capitalise on to further mathematics education reform, and highlight the potential contribution of mathematics education within the STEM agenda.

If policymakers, like the public, see mathematics as no more than fluency in number facts, there is little opportunity for them to see how mathematics connects to the STEM agenda. As national governments across the world see jobs of the future coming from STEM areas, there is an emphatic rise in funding initiatives focused on STEM. A recent report on 69 STEM initiatives being funded in 2018 by the Australian and state governments confirms the millions of dollars being spent on these projects, with only nine of them focused specifically on mathematics (Education Council, 2019). Panizzon and Corrigan (2017) analysed the 2016 STEM Program Index (SPI) listing published by Australia's Chief Scientist of 250 active STEM programs catering to schools and students. Only 36 of these included explicit mention of mathematics. Because innovation and entrepreneurship had been identified as recognised drivers of the economy, the authors sought to investigate the extent to which STEM promoted these and other valued characteristics. In their summary table, the contrast was striking in comparing particular characteristics to their appearance in the SPI for STEM, science and mathematics (a few of which are listed in Table 2). The table speaks to a number of areas highly valued by STEM that have been adopted in science to some extent but not in mathematics. The entries that did include mathematics were almost exclusively listed under "STEM content" (characterised by 33 of 36 programs), with "Motivation" as the second most common characteristics listed in the 36 mathematics programs (12 programs). Because SPI are funded projects, it emphasises that in some areas, innovations in mathematics teaching and learning are not being included in the national STEM conversation.

Much of the work we do in mathematics education contributes to the development of STEM, including the applications of particular curriculum content, pedagogy and

Table 2 Selection of characteristics in the 2016 STEM Program Index (Panizzon & Corrigan, 2017)

Characteristic	STEM	Mathematics	Science
Communication	4	0	10
Creativity	12	2	6
Critical thinking	3	0	3
Curiosity	0	0	1
Entrepreneurship	2	0	0
Independent thinking	2	0	1
Innovation	12	0	1
Inquiry	13	2	9

assessment. However, English (2016) argued that there is a danger within STEM that mathematics is being overlooked or sidelined, with most of the emphasis placed on science, digital technologies (including coding) and engineering, possibly because policymakers cannot envisage how mathematics could fit into their futures agenda. She advocated for more work in statistics, problem solving and modelling as places where mathematics can continue to raise its profile. These three areas foster generic, as well as mathematics-specific skills and processes that are significantly needed in STEM, yet under-developed in many mathematics classrooms. English concluded her chapter with four recommendations that are critical for the mathematics education community to engage meaningfully with the STEM agenda:

- seeking to raise the profile of mathematics in STEM through statistics, modelling and problem-solving
- capitalising on and extending national assessment items that build on rich mathematical experiences
- emphasising twenty-first century skills in mathematics: creativity and innovation, critical thinking and problem-solving, and communication and collaboration
- connecting to and engaging with mathematics related to computational thinking and coding (pp. 366–368).

These recommendations create useful avenues for the mathematics education research community to continue to connect with the international push for STEM education. We anticipate that they will orient both international research agendas broadly, and Australasian research agendas within the period leading to RiMEA-11.

3.3 Assessment of Research Impact and Engagement

In December 2015, as part of its *National Innovation and Science Agenda*, the Australian government announced the development of a national assessment of research engagement and impact. The Australian Research Council (ARC) and the Department of Education and Training (DET) released an *Engagement and Impact Assessment Consultation Paper* in May 2016 to seek feedback from stakeholders on how this assessment should be undertaken (ARC & DET, 2016). Subsequently a pilot study of research engagement and impact was conducted in 2017, and in 2018 the inaugural Engagement and Impact Assessment (EI2018) was implemented as a companion exercise to Excellence in Research for Australian (ERA).

The ARC *Consultation Paper* drew on the definition used by the Academy of Technological Sciences and Engineering to develop metrics for Australian universities' research engagement. Engagement was defined as:

> the interaction between researchers and research organisations and their larger communities/industries for the mutually beneficial exchange of knowledge, understanding and resources in a context of partnership and reciprocity. (ATSE, 2015)

However, the *Consultation Paper* noted that metrics, which are largely based on research commercialisation income and patents, may not capture the complexity of some forms of research engagement.

The Australian Research Council defines research impact as:

> the demonstrable contribution that research makes to the economy, society, culture, national security, public policy or services, health, the environment, or quality of life, beyond contributions to academia. (ARC, 2012)

While noting that there were no clearly defined indicators for research impact, the Consultation Paper referred to peer reviewed case studies—similar to those reported in the UK REF exercise—as being an appropriate means of assessment. Nevertheless, it was acknowledged that full case studies are expensive to produce. As a compromise, a template was developed for impact case studies that requested a short summary of the impact; a list of beneficiaries and countries in which the impact occurred; a narrative that clearly outlined the research impact, especially the impact made beyond academia with specific reference to appropriate evidence; and a description of the research that led to the impact. They also required an extended explanation of the "approach to impact", demonstrating how the university putting up the case study had facilitated the research to seek and attain its impact.

Altogether 38 of Australia's 40 universities submitted engagement and impact case studies in Education (FoR13; *EI 2018 Institution Report*). Of these, 21 were rated as demonstrating "high" for engagement (12 of 38), impact (17 of 38) and/or approach to impact (5 of 38). According to the rating scale used in the assessment (see https://dataportal.arc.gov.au/EI/NationalReport/2018/pages/introduction/index.html?id=ei-rating-scales), those so rated were characterised by:

- **highly effective interactions** between researchers and research end-users outside of academia for the mutually beneficial transfer of knowledge, technologies, methods and resources and research engagement that is **well integrated** into the development and ongoing conduct of research within the unit of assessment;
- having made a **highly significant contribution** beyond academia, with a clear link between the associated research and the impact was demonstrated.

Four of the high-impact case studies were in mathematics education: *Transforming mathematics education in preschool and primary school contexts* (Macquarie

University); *Improving mathematics teaching to integrate numeracy learning across the curriculum* (The University of Queensland); *Addressing the needs of at-risk learners in Numeracy and Literacy* via *the QuickSmart program* (University of New England); and *YuMi Deadly Centre (YDC) programs to improve mathematics teaching and learning in schools with a high Indigenous student population* (Queensland University of Technology). Summaries of all case studies rated as having high impact are available on the ARC website (see https://dataportal.arc.gov.au/EI/Web/Impact/ImpactStudies).

In a research symposium presented at the MERGA 40 conference, Goos, Geiger, Bennison, Dole, and Forgasz (2017) raised some key issues in undertaking this national assessment of research engagement and impact, and proposed some approaches to evidencing engagement and impact in the context of mathematics education research. In her symposium paper, Goos (2017) argued:

> …there is surely value for mathematics educators in retrospectively analysing our own research to illuminate the opportunities taken, decisions made, and relationships built in pursuing research that makes a difference. Such an analysis might help us not only to learn "where to look" for evidence of past impact, but also to plan future research projects with an eye to demonstrating potential benefits for educational policy and practice. (p. 637)

We suggest that there may be benefits for mathematics education researchers not only in learning from the results of ARC Engagement and Impact assessments over future years, but also in learning both "where" and "how" to look at their own research to plan for and evidence its impact. In New Zealand, this more prospective approach to planning for impact is illustrated by the Teaching and Learning Research Initiative (see http://www.tlri.org.nz/home), which seeks to enhance the links between educational research and teaching practices to improve outcomes for learners. Mathematics education is well represented amongst the research projects funded by this initiative (Teaching and Learning Research Initiative, 2019). For example, one project funded in 2019/2020 investigates the use of home languages as a resource for multilingual students in learning statistical probability in two multicultural classrooms, while another aims to uncover Pasifika learners' mathematical funds of knowledge. Both these projects involve partnerships with schools that are expected to improve outcomes for learners.

A number of new research projects in Australia and New Zealand funded since the last Review provide assurance that mathematics education research will continue to contribute to the policy issues that we raise in this chapter. Over a recent 3 year period (2016–2018), over 20% of ARC Discovery projects funded in education (FoR13) were in mathematics education. In 2017, a full 35% of education projects funded were in mathematics education. The *Mathematics Education Research Journal* Special Issue (Anthony, 2018) discussed these projects, the issues they sought to address, and aimed to highlight the types and methodologies of projects that are being funded in this era of research reform. The relatively strong presence of mathematics education research has significant potential to impact the issues raised above in teacher education, STEM and research impact.

4 Conclusion

We began this chapter with an analysis of patterns we have observed in the first ten MERGA Research Reviews (RiMEA-1 through RiMEA-10) to encourage researchers in our field to consider how research in mathematics education has changed in the past 40 years. We know of no other group that has documented their research history in this way. From our long look back, we next reflected on recent changes in the landscape that we think are important for us to consider as an organisation.

Indeed, since 2015 a number of policy reforms have affected mathematics education research. The persistent reforms around initial teacher education will likely continue. This is an important area for mathematics education researchers in Australasia to examine the impacts of these reforms, including on the dwindling number of pre-service teacher education students entering with significant qualifications in mathematics. Mathematics education researchers in Australasia have already been reporting on this issue in the following chapters of this Review. We were happy to see a new chapter on STEM, perhaps following the chapter written by English (2016) at the end of the last Review. We see this as an essential area of research in our field given the poor representation of mathematics in this reform (as predicted by English). The STEM (2016, 2020) and Numeracy (2020) chapters allow us to speculate that future research may increase its emphasis on inter-disciplinary research. The new Engagement and Impact Assessment in Australia, following a similar reform in the United Kingdom, reflects the increasing pressures that policymakers place on researchers to demonstrate that their research has a direct public or commercial benefit. Given the high number of impact case studies rated as "high" in education that were from mathematics education research, we are perhaps fortunate to be in a field with close ties to schools and classrooms; but it is also undeniable that attention to research engagement and impact had been cultivated in this field over several past decades. The timing of the high number of 2017 Australian Research Council Discovery Projects funded in mathematics education was perhaps serendipitous, or possibly a reflection of policymakers' interest in the potential of mathematics education research to positively impact society (Anthony, 2018).

We believe that each of the reforms that we have outlined in this chapter will affect the future research agendas of mathematics education researchers. We also believe that we have both an opportunity and a responsibility in mathematics education research to act and seek to influence the future set of policy reforms. We look forward to reading this volume and the reflection of the current editors in the next Review: *Research in Mathematics Education in Australasia 2020–2023*.

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Chapter 3
The Contribution of Mathematics Education Researchers to the Current STEM Education Agenda

Judy Anderson, Lyn English, Noleine Fitzallen, and Duncan Symons

Abstract STEM education is relatively new in the Australasian education landscape but is beginning to forge a place in school curricula. Driving the attention to STEM education are calls for the implementation of learning experiences that prepare students for a future that relies on them being innovative problem solvers. This has posed many challenges for teachers who wrestle with the most appropriate way to engage students in STEM learning while still attending to student development of mathematics discipline knowledge. The research reported in this chapter situates STEM education within current education policies and curricula, explores the various ways in which integrated STEM education is conceptualised and implemented, illustrates the role teacher education plays in preparing teachers to implement STEM learning, and showcases the student learning possible when an integrated approach to learning is adopted. This review draws attention to the diverse nature of the research undertaken in STEM education in the last four years and suggests that future research is needed to explore curriculum reforms that ensure mathematics learning is developmental, to investigate the way in which mathematical understanding supports development of understanding of other STEM disciplines, and to examine professional learning programs that assist teachers and pre-service teachers to develop pedagogies that make STEM learning effective and sustainable.

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1 Introduction

STEM (science, technology, engineering and mathematics) education is a recent field of research for mathematics education researchers in the Australasian context and as such is undertheorized and disparate. This chapter aims to review research that focuses on the role mathematics plays in STEM education (Maass, Geiger, Ariza, & Goos, 2019), thus raising the profile of mathematics (English, 2016a) and providing researchers with advice about developing a stronger case for mathematics as central to quality STEM education (Fitzallen, 2015). While policy reforms are embryonic, several Australian state and territory jurisdictions have supported schools developing STEM education initiatives and programs (e.g., Australian Curriculum, Assessment and Reporting Authority [ACARA] STEM Connections Project, 2016; NSW Department of Education Integrated STEM Projects—http://stem-nsw.com.au) although these initiatives rarely have a research base or use rigorous evaluation mechanisms to test the efficacy of their findings. The chapter also aims to critique such agendas and to provide timely advice to facilitate new ways of integrating mathematics into STEM education.

Two challenges in reviewing research in this area involve the interpretation of the terms "STEM" and "STEM education" Fraser, Earle, & Fitzallen, (2019), and the perceived potential of STEM to solve a myriad of identified issues. STEM has been used as an umbrella term for all four disciplines, to describe connections between the disciplines, or to include an even broader collection of related fields such as health, agriculture, and the environment. In addition to a lack of clarity of definition, STEM education has been proposed as the solution to many concerns in mathematics education including:

- poor student performance on international tests (Thomson, De Bortoli, Underwood, & Shmid, 2019);
- fewer enrolments in advanced mathematics subjects in senior secondary schooling (Coupland et al., 2017);
- disengagement in school mathematics (Murphy, MacDonald, Wang, & Danaia, 2019);
- underqualified mathematics teachers requiring enhanced content knowledge and 21st Century competencies (Beswick & Fraser, 2019); and
- shortages of well-qualified workers in some STEM fields (Holmes, Gore, Smith, & Lloyd, 2018).

STEM education is also viewed as essential for students to connect knowledge across and between subjects (Chalmers, Carter, Cooper, & Nason, 2017), and to develop students' capacity to solve real-world, complex problems (English, King, & Smeed, 2017).

While the interest in STEM education continues, teachers and school leaders are under pressure to implement strategies addressing such issues and to design STEM curriculum to meet the needs of their students. This pressure comes without a great deal of support and guidance, particularly without a sound evidence base of what works and what role mathematics might play (Maass et al., 2019). Some researchers argue we consider STEM education as integrated curriculum with a focus on inquiry-based learning (e.g., Bybee, 2013). This raises challenges for teachers when the curriculum is written as separate STEM subjects, recommended STEM pedagogies are ill defined, and some teachers may be teaching out-of-field without the requisite deep content knowledge of mathematics or indeed, of any of the STEM subjects (Beswick & Fraser, 2019). However, given the intense international interest in STEM education in recent years, it is critical we examine the STEM landscape (Tan, 2018), particularly from the mathematics education perspective (Maass et al., 2019; Tytler, Williams, Hobbs, & Anderson, 2019), and consider current policy advice and research initiatives in the Australasian context.

Our literature search has revealed many Australasian research papers refer to the term "STEM education" to justify their research within the current socio-political agenda and yet frequently the papers refer to just one of the STEM subjects. We have decided to focus our review on those studies which have considered an integrated approach to STEM education (where mathematics is connected to at least one other of the STEM subjects), allowing students to choose to use knowledge from more than one subject to solve problems. This distinguishes our review from the "numeracy across the curriculum agenda" that promotes students being able "to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life" (Department of Employment, Education, Training and Youth Affairs, 1997, p. 15). Like Goos, Geiger, Dole, Forgasz, and Bennison (2019, p. 214), we "consider the two agendas as complementary" but seek to distinguish the research reviewed here from that in Chap. 4.

The review of research for this chapter was informed by the descriptive framework for integrated STEM education proposed by Honey, Pearson, and Schweingruber (2014). Their original framework had four features, each with specific subcomponents but we have adapted these to fit the purpose of reviewing the contribution of mathematics education researchers to the current STEM education agenda—see Fig. 1.

This framework guides the structure of the chapter. We begin by discussing the goals of STEM education as presented in policy documents and reports. Next, we consider the nature and scope of integrated STEM in the school curriculum. This is followed by STEM implementation as reviewed through research into teacher education, and finally, evidence of outcomes for students through the development of STEM capabilities is presented. Our conclusion identifies issues from the Australasian research and suggests potential new research opportunities to explore the role of mathematics in integrated STEM education.

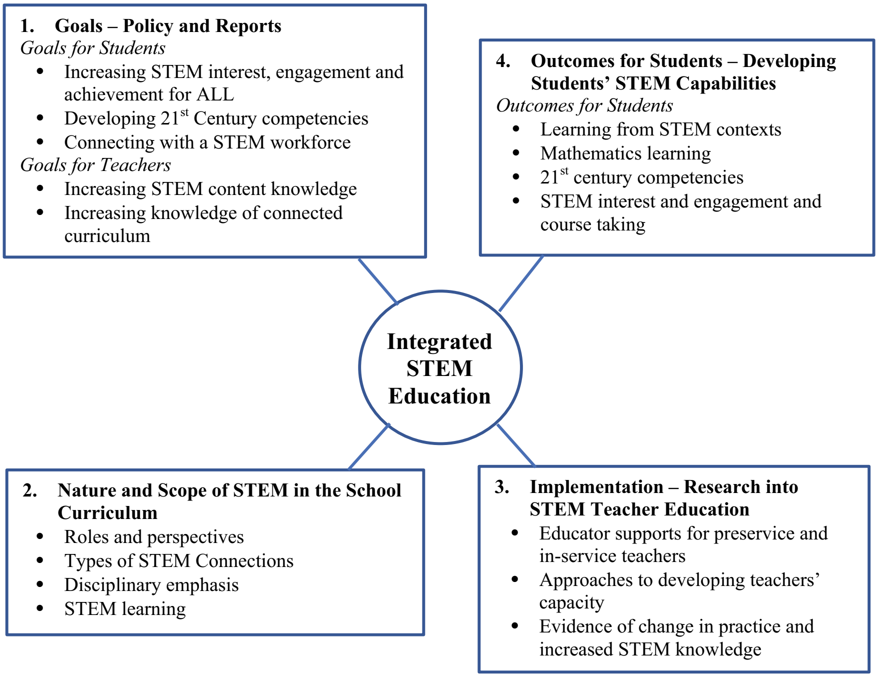

Fig. 1 Descriptive framework of integrated STEM education showing general features and subcomponents (adapted from Honey et al., 2014, p. 32)

2 Goals of STEM—Policies and Reports

The current STEM education agenda has been informed by international trends, particularly from the United States of America [US], but also by a desire for governments to promote calls for greater innovation in the STEM fields, and to develop a competitive edge in research and innovation (Commonwealth of Australia, 2017). The speed with which new technologies including artificial intelligence is impacting workplaces has left governments concerned about being "left behind" in the race to compete on an international scale and to have sufficient well qualified STEM researchers, designers and innovators to meet workplace demands (Office of the Chief Scientist [OCS], 2017). It is therefore unsurprising that efforts are being made at all levels of education to promote STEM and to encourage more students to consider careers in STEM related fields (Prieto & Dugar, 2017). Since this is the first chapter to review STEM education research for *Research in Mathematics Education in Australasia* (RIMEA), this section of the chapter summarises the rise of STEM education in Australasia, considers STEM policies and reports about school education, and provides a brief account of inputs from governments, professional organisations, and other business and industry groups. This section of the chapter seeks to identify the goals for students and teachers as represented through policies and reports in Australasian contexts.

Focused interest in STEM education in Australia began when the Federal Government appointed Professor Ian Chubb to the role of Chief Scientist in 2011. While some publications about STEM education appeared before his appointment (e.g., Tytler, Osborne, Williams, Tytler, & Clark, 2008), the launch of a range of influential reports (e.g., OCS, 2014) accompanied by funding for school- and university-based STEM initiatives helped to set the scene for what has followed. The OCS reports argued for building a stronger, more competitive economy through education and training, research and international engagement stating "science and innovation are recognised internationally as key for boosting productivity, creating more and better jobs, enhancing competitiveness and growing an economy" (OCS, 2014, p. 7). These aims have been strengthened as government departments and other organisations have called for reform in school education (Murphy, MacDonald, Danaia, & Wang, 2019).

2.1 STEM Education in Australasian Schools

Reports calling for a greater focus on inspired teaching, inquiry-based learning and increased attention in schools to the preparation of students for careers in the mathematical sciences (e.g., OCS, 2016a) led to a rapid rise of STEM-based initiatives in Australia (OCS, 2016b). Given the diversity of approaches to the STEM education agenda, the Australian Federal Government called for a united approach with common goals and actions (Education Council, 2015). The strategy promotes greater attention to the separate STEM subjects as well as the use of integrated STEM approaches in school education to address two clearly stated goals:

> Goal 1: Ensure all students finish school with strong foundational knowledge in STEM and related skills.
>
> Goal 2: Ensure that students are inspired to take on more challenging STEM subjects. (p. 5)

2.1.1 Improving STEM Interest, Engagement and Achievements for All Students

We should acknowledge that much of the STEM policy, programs and practices agenda in schools is driven by concerns about students' declining performances on international tests, students' lack of engagement and falling enrolments in STEM subjects, and the disparities which exist between sections of the student population in Australian and New Zealand schools. Unlike other countries, it is not compulsory to study mathematics or science in the senior secondary school (grades 11 and 12) in most Australian states and territories or in New Zealand. From 2001 to 2015 enrolments in the final year of schooling, for the higher, more challenging levels of mathematics, dropped for both genders with only 11.5% of males and 6% of females choosing to participate (Mack & Walsh, 2014). While there has been an increase in the number of students completing grade 12 studies, a growing proportion of them leave school with no mathematics or science, most of whom are female (Li & Koch, 2017). Coupled with these concerns, Australian students' performances on the international assessments of Trends in International Mathematics and Science Study [TIMSS] and Program for International Student Assessment [PISA], have declined with fewer students meeting the highest benchmarks, and significant gaps between the performances of male and female students, between metropolitan and rural/remote students, and between non-Indigenous and Indigenous students (Thomson et al., 2019; Thomson, Wernert, O'Grady, & Rodrigues, 2016).

Both Australia and New Zealand have policies and programs designed to address the "gap" in STEM performance and engagement of Indigenous students. In Australia, school completion rates and transition to university STEM programs are much lower for Indigenous students than for their non-Indigenous peers (Paige, Hattam, Rigney, Osborne, & Morrison, 2016). However, some programs have had success. Through his work with the Aboriginal and Torres Strait Islander Mathematics Alliance [ATSIMA], Matthews (2015) has led several initiatives with results indicating 48% of South Australian Indigenous grade 12 students now complete at least one STEM subject. Paige et al. describe six case study programs with potential to address transition into the higher education sector for Indigenous students and call for further reform and policy initiatives to stabilise, grow and strengthen such programs given fluctuating funding arrangements. They call for government commitment to "develop a national policy and strategy on Indigenous STEM" (p. ix) and agree with Morris (2017, p. 2) that the recent Australian Government STEM agenda offers "little to address the disparities in learning outcomes" for Indigenous learners. It appears that policy rhetoric rarely results in real outcomes; so much is still to be done to improve STEM interest, engagement and achievements for Indigenous learners in Australian schools.

The situation is similar in New Zealand regarding results on international tests, level of interest and enrolment patterns in STEM subjects, and disparities between student groups, although there have been policy and program recommendations to address these issues. Regarding inequity for Māori students, the New Zealand government developed a national strategy to improve education outcomes and launched *Ka Hikitia—Managing for Success: The Draft Māori Education Strategy 2008–2012* (Ministry of Education, 2009). One recommended approach to enhance engagement and help students understand the usefulness of STEM knowledge to solve real-world problems, is to "connect" the curriculum. The New Zealand Ministry of Education also encourages connections between schools and their wider community including connections with scientists and local business and industry (McKinley, Gan, Bunting, & Jones, 2015). Such connections are reliant upon teachers and school leaders having the capacity and commitment to enact curriculum redesign and community engagement.

2.1.2 Developing 21st Century Competencies

In Singapore, the calls to incorporate STEM education into the curriculum are not driven by students' poor performance on TIMSS and PISA, but rather from a desire to focus on 21st Century Competencies (Ministry of Education, n.d.), and to provide hands-on, authentic learning experiences for students (Teo, 2019). In her review of major changes in the education system in Singapore, Kaur (2019, p. 29) refers to the current phase as "values-based, student-centric", so that every student has access to a new economic future, and the system focuses on students' aspirations and interests to build values and skills. Besides a raft of informal STEM competitions and challenges, the STEM agenda has been largely realised in schools through the Applied Learning Programme (ALP), implemented by the Ministry of Education in 2013. ALP is aimed at cultivating a joy of learning by providing school-based, hands-on experiences encouraging students to explore new skills such as coding. Schools can design their own ALP curriculum or work collaboratively with industry, community-based organisations, higher education institutes, and/or professional bodies to deliver curriculum. Initially voluntary, the Ministry announced all primary schools will have an ALP by 2023 and while it could involve STEM, languages, humanities, business, entrepreneurship, and aesthetics, STEM is the most popular program (Teo, 2019). As further evidence of the impact of STEM, a Multi-Centric Education Research and Industry STEM Centre was established at the National Institute of Education in Singapore in 2018 aimed at enhancing STEM literacy "through cross-disciplinary partnerships in research, teaching, and outreach" (Teo, 2019, p. 3).

2.1.3 Supporting Well-Qualified STEM Teachers

Most jurisdictions acknowledge that for STEM education to be successful, teachers require new knowledge and understanding and support to design integrated approaches to STEM and to use inquiry-based pedagogies in classrooms (Tytler et al., 2019). Promoting inspired teaching and the need for further professional development for STEM teachers was accompanied by reports indicating a shortage of STEM teachers in secondary schools (Productivity Commission, 2012) and a concern with the quality of teacher preparation programs (OCS, 2014). In both Australia and New Zealand, governments have supported teacher education programs to recruit more high performing STEM graduates such as the *Teach for Australia* program and the *Teach First* and *Manaiakalani Digital Teachers Academy* programs in New Zealand. To further support teachers in New Zealand, a national strategic plan, *A Nation of Curious Mind*s, is a government initiative with a ten-year goal to promote the importance of science and technology and since 2015 it has funded over 175 projects for more than $NZD 6 million (https://www.curiousminds.nz). In Australia, $64 million was committed to support a raft of programs including principals' capacities to lead STEM initiatives in their schools (e.g., Hatisaru, Beswick, & Fraser, 2019). While developing students' STEM capabilities and teachers' STEM knowledge and understandings are key goals for the STEM agenda, promoting STEM aspirations beyond school is still dominant in the STEM policy discourse.

2.2 STEM Workforce Engagement and Preparation

To further promote a STEM agenda of filling projected STEM career requirements and "work ready" citizens, key professional organisations seized the opportunity to put their case for national economic and political priorities. For example, the Australian Industry Group (2015) advocated for developing STEM skills and promoting STEM careers in schools to address projected shortfalls in the workforce, while Engineers Australia (2017, p. 37) called for schools to promote engineering since "participation in high school STEM subjects is a means to an end: mathematics and science are the tools used by engineers to solve real world problems." Both organisations recommended greater industry connections with schools and shared responsibility to engage students in understanding workplace opportunities and skills requirements. Further adding to the focus on skills, the Foundation for Young Australians (2018), a not-for-profit organisation, described the challenges for young people to enter meaningful employment and indicated the need for schools to provide "courses that teach enterprise skills (such as problem-solving, communication and teamwork) [which] can increase the speed of attaining full-time work by 17 months" (p. 9). It seems, schools must take on the responsibility to fill the shortfall of the STEM workforce, promote STEM careers, and ensure students develop necessary employability skills, as well as address gender inequities (Barkatsas, Cooper, & McLaughlin, 2019).

From this brief review of policy documents and recent STEM reports, several goals have been identified. Integrated STEM education is recommended to develop students' engagement and participation in STEM subjects and careers, to develop students' STEM subject knowledge as well as skills and capabilities to improve performance, to address diversity and equity issues in STEM performance and participation, and to help teachers develop capacity to design and effectively deliver integrated STEM curriculum through inquiry-based pedagogies. These are ambitious goals that are not necessarily informed by a sound evidence base, and have potential to impact mathematics teaching and learning. The following sections of this chapter aim to report Australasian research conducted largely by mathematics educators, which either challenges or affirms the potential of realising these goals.

3 STEM Education in the School Curriculum: Roles, Perspectives, and Discipline Integration

Australasian researchers in mathematics education have focused primarily on the discipline itself, as evident in our journals, *Mathematics Education Research Journal, Mathematics Teacher Education and Development, Australian Primary Mathematics Classroom*, and the new *The Australian Mathematics Education Journal*. With the global focus on STEM education and in particular, integrated STEM education, our researchers have branched into linking mathematics with the other disciplines (e.g., Chalmers & Nason, 2017; English, 2019; English & King, 2017, 2019; Loong & Herbert, 2018). Although the importance and benefits of integrated STEM programs have been well documented (e.g., Timms, Moyle, Weldon, & Mitchell, 2018), concerns for maintaining the integrity of the respective disciplines, especially mathematics, have been expressed frequently by Australian researchers (English, 2016b, 2017; Fitzallen, 2015; Hobbs, 2019; Little, 2019).

The role of STEM education in the school curriculum has been examined by several Australian researchers. Two main perspectives appear in the publications of mathematics education researchers, namely, (a) a general overview of the role of STEM education in the curriculum (e.g., English, 2016b, 2017, 2018a; Timms et al., 2018) and (b) specific studies of integrated STEM education in Australian schools (e.g., English & King, 2017; English, King, & Smeed, 2017; Fitzallen, Wright, & Watson, 2019; Holmes, Prieto-Rodrieguez, Hickmott, & Berger, 2018; Loong & Herbert, 2018). We examine the role of STEM education in the school curriculum from these two perspectives, although sometimes both perspectives are presented within the one publication.

3.1 STEM Education in the Curriculum: Roles and Perspectives

Studies examining the roles of STEM education in national and international curricula have been undertaken by a number of Australian researchers including Chalmers et al. (2017), English (e.g., 2016a, 2016b, 2017, 2018a or 2018b), Watson, Fitzallen, English, and Wright (2019); Hobbs (2019), MacDonald, Hunter, Wise, and Fraser (2019), and Timms et al. (2018). Several authors have claimed that STEM education is undergoing an "identity crisis", with numerous concepts and terminology being associated with the acronym (e.g., Hobbs, 2019). Hobbs highlighted the pressing need to map out a future for STEM education, at least in Australia, to ensure quality STEM learning across the full spectrum of education systems. Moving beyond the rhetoric of what STEM education needs to do to actually doing it, that is, implementing effective STEM programs was emphasised by Hobbs as well as Timms et al. (2018). Importantly, Timms et al. indicated the need for a new agreed-upon definition of STEM education including a rationale for why the disciplines belong together. It is questionable how much interdisciplinary education is occurring across Australian states, despite recommendations by state education departments. MacDonald et al. (2019) indicated how many states are drawing on the *National STEM Education Strategy* (2015) in developing their programs, such as Tasmania. Nevertheless, as MacDonald et al. rightly indicated, the teaching on integrated STEM requires "a change in teacher pedagogies to encompass real world problem solving or design-based approaches" (p. 85), as noted by Fraser et al. (2019).

A year-long professional program conducted by Anderson, Wilson, Tully, and Way (2019) displayed significant improvements in students' STEM attitudes and aspirations. Once again, the challenges teachers and school systems face in creating and implementing integrated STEM projects were highlighted by the researchers. Issues pertaining to program sustainability and scalability in school systems where demands on teachers' time appear to be escalating were also raised. Further discussion on these issues appears in the next section.

Concerns regarding adequate coverage of core disciplinary content have been raised by several researchers (e.g., Chalmers et al., 2017; English, 2016a, 2016b). Issues highlighted include the weakening or even neglect of disciplinary content within some integrated STEM programs. Chalmers et al. further noted how technology and engineering frequently appear as "add-ons" to standard mathematics and science curricula rather than being integrated in meaningful and essential ways. One solution offered by Chalmers et al. is to focus on the "big ideas" across the STEM disciplines, including representations, systems, relationships. This use of shared disciplinary "big ideas" appears not to have received the attention warranted in studies of STEM integration.

Other concerns expressed in the literature include the lack of focus on integrated STEM literacy (Fraser et al., 2019), with such literacy frequently addressed in terms of the individual disciplines rather than collectively. The Tasmanian Government, however, recognises the importance of integrated STEM literacy as enabling "learners to identify, apply and integrate key ideas and processes from science, technology, engineering and mathematics to understand complex problems and offer innovative solutions" (https://stem.education.tas.gov.au/whatisstem/).

In response to problematic issues regarding STEM integration, Rennie, Venville, and Wallace (2018) pointed out how integrated STEM education serves to help students explore and experience the types of connectedness that occur in the real world. As these authors rightly indicated, many of the current policies and practices favour siloed, disciplinary approaches that focus primarily on what can be measured in achievement testing. Such approaches do not mirror the multidisciplinary nature of students' lives outside of school or the myriad problems faced by the world today. Rennie et al. emphasised the contributions of an integrated approach to STEM education including increasing the opportunities for students to engage in "contextual, multidisciplinary issue-based learning" (p. 91). The authors present rich examples and sound justification for integrating at least some components of the STEM subjects in developing a curriculum that students find relevant, motivating, and meaningful. As such, Rennie et al.'s chapter provides worthwhile recommendations for addressing the continuing concerns regarding a lack of integrated STEM education in school curricula. Studies such as the one conducted by Watson et al. (2019) serves as an illustration of how mathematics, statistics in particular, can be integrated into STEM curricula (see also English, 2018c).

An appealing and informative model for exploring the nature of STEM with students was presented by Lyons (2018). His Russian nesting doll analogy can assist students in appreciating the nature of STEM and the intrinsic connections among the disciplines. The analogy conveys the disciplines as related and integral to the whole even though the four dolls (representing the four disciplines) are separate. The largest doll represents technology, as Lyons maintained that this discipline is usually the "exterior, public face of STEM" (p. 38). Technology is dependent on engineering, which in turn is reliant on both science and mathematics. Lyons regarded mathematics as the core of STEM, that is, it is foundational to the other disciplinary areas; each of the larger dolls contributes a further layer of applicability.

As numerous authors have documented, teachers' lack of experience and pedagogical knowledge in designing and implementing integrated STEM programs can be a major stumbling block in progressing the field. For example, Simoncini and Lasen (2018) reported on Australian early childhood teachers' beliefs on the importance of STEM education. Their findings revealed that the teachers considered STEM education to be important but ranking behind young children's social–emotional development. STEM education was perceived in terms of its individual disciplines and was seen as providing opportunities for play-based and/or hands-on learning. Developing children's "habits of mind" was also considered an important feature of STEM education, with teachers referring to specific habits such as "experimenters", "inquirers", and "predictors". Establishing foundations for subsequent formal STEM learning was further cited by some of the teachers.

Although beyond the publication period for this chapter, it would be remiss not to acknowledge the detailed and informative study by Marginson, Tytler, Freeman, and Roberts (2013), namely, *STEM: Country Comparisons*. As the title implies, their study provided a detailed global account of STEM initiatives across a number of nations including Australia. It would be informative to compare their findings of 6-7 years ago with the state of STEM education in these countries today. It could be conjectured that substantial progress has been made in the intervening years, with more integrated STEM projects being implemented in schools. In Australia, for example, several state education departments now feature sections on integrated STEM projects within their curriculum. The NSW education system incorporates an "Integrated STEM Project" unit, with rich opportunities for connecting the disciplines and developing integrated learning. The major focus of their integrated units is on "aligning syllabus outcomes, [and] promoting higher order thinking through authentic project-based tasks." The unit provides a guide for "integrated teaching and learning, inquiry learning and design thinking" (http://stem-nsw.com.au/leading-stem/stage-4-integrated-stem-project).

3.2 Research on Integrated STEM Learning in the Curriculum

In this section, we consider studies that have integrated two or more of the STEM disciplines. While mathematics has been highlighted in several of these integrations, in some studies the discipline has not played as prominent a role.

3.2.1 Mathematics and Technology

The use of technology in mathematics learning has received considerable attention over the years, but more as a tool ("add-on") rather than as an integrated, essential component of an activity. A good example of STEM integration involving mathematics and technology appeared in a study by Chalmers and Nason (2017), where they focused on "Big Ideas" in the meaningful learning of STEM knowledge in robotics contexts. The authors connected robotics learning and mathematics learning, in particular proportional reasoning, which they cited as central to ways in which the motion of a robot can be controlled through programming. They indicated how the relationships between the building of the robot, the values used to program it, and the movements of the robot are frequently proportional in nature.

3.2.2 Mathematics and Science

A frequently cited article by Fitzallen (2015) emphasises the importance of mathematics in STEM education and how its contributions have been overlooked in many integrated STEM programs. For example, Fitzallen illustrates the contributions of mathematics to science, contributions that have been frequently neglected in the rush to develop and implement STEM within the curriculum. Similar sentiments regarding a weakening of mathematics in some integrated STEM experiences were expressed by English (e.g., 2016a, 2016b, 2017). She argued that there has been an inequitable discipline representation in many STEM programs, with STEM being viewed primarily in terms of science.

A valuable project linking mathematics and science was implemented by Coupland et al. (2017). The "Maths Inside" project was funded by the Australian Mathematics and Science Partnership Program [AMSPP], and was designed to help teachers and students appreciate how mathematics is applied "inside" science and other areas. The overall aim of the program was to improve uptake and participation of students in mathematics and science courses at the secondary and tertiary levels, as well as develop the confidence, knowledge, and skills of the classroom teachers. The project supported mathematics teachers through providing rich, investigative learning resources, including video case studies of CSIRO scientists and mathematicians.

One solution to raising the profile of mathematics in secondary education was proposed by Little (2019), namely, by forming connections between content, skills, and practices through real-world applications. Little's detailed analysis of the literature highlighted the challenges teachers face in linking mathematics with science. Such challenges have been widely reported (e.g., Tytler et al., 2019), but it is questionable how these might all be resolved without systematic changes. A word of caution was proposed by Little regarding possible negative impacts on student achievement in mathematics. That is, does connecting mathematics and science remove the "rigour, abstractness, and sequential approach to mathematics teaching and learning"? (p. 458).

3.2.3 Mathematics, Science, and Engineering

Numerous longitudinal studies by English and colleagues (e.g., English, 2016c; English & King, 2017, 2019; King & English, 2016) involve the implementation of engineering education in primary and middle schools. Students' application of mathematics and science concepts were key components of these engineering studies. In King and English (2017), fifth-grade students were given the problem of designing and building an optical instrument that enabled them to see around corners. In sketching their design and building their instrument, students were observed to apply scientific concepts of light, mathematical understandings pertaining to geometry, angles and measurement, and technology concepts related to how light travels through a system. Students' final constructions worked successfully, allowing light to travel through their instrument and be reflected from mirrors to their eyes as they viewed the object. The outcomes of the study indicated that the planning of STEM activities needs careful consideration to ensure that the desired STEM concepts and associated disciplinary learning are central to activity success. Such planning is not easy, and again, indicates the importance of adequate professional development.

A book on early engineering learning, edited by English and Moore (2018), focused on engineering design as linking the STEM disciplines. Engineering education has received limited attention in Australian schools, especially in the primary school, despite being one of the most practical and real-world domains in which all children can engage. The chapters report on several intervention programs, theoretical developments, and assessment frameworks all of which are applicable to early STEM learning. The role of mathematics, in particular, spatial skills, in early engineering learning is an important feature of the book, as are "habits of mind", the more generic skills required across the STEM fields. Such skills include systems thinking, innovative problem finding and solving, visualizing, and collaborating and communicating.

The topic of habits of mind in early childhood STEM education was also examined in Simoncini and Lasen's (2018) article cited earlier. Applying their *Early Childhood STEM Habits of Mind Framework*, these researchers illustrated how the framework could assist in the planning of integrated STEM learning experiences in the early learning years. Simoncini and Lasen considered the potential of their framework to facilitate "more holistic understandings and a shared language, among early childhood educators, parents and children, concerning STEM education" (p. 353).

3.2.4 Mathematics, Science, and Technology

Campbell, Speldewinde, Howitt, and MacDonald (2018) observed four pre-school centres in an effort to determine how science, mathematics, and technology arose in outdoor learning centres. Informed by play-based education pedagogies, the researchers explored approaches to STEM education in the early years. Their findings included how integrated STEM, particularly science and mathematics, arises through children's play and the various themes arising from their interests. Their study indicated how different practices and educational pedagogies can be applied to support STEM learning in the early years.

3.2.5 Mathematics, Science, Engineering, and Technology

In another study by English (2019) involving fourth-grade classes, the four STEM disciplines were linked through a focus on design. In the activity, "Fancy Feet," students first investigated their feet and shoe measurements, their shoe materials, the popular shoes and shoe features of their peers, and the roles of engineers in shoe manufacture. They then designed and constructed their own pairs of shoes from supplied materials.

A conceptual framework, "Towards Informed Design" (adapted from Crismond & Adams, 2012), was advanced for exploring students' learning while designing. English (2019) applied this framework to analysing students' use of design strategies, including posing their own problems and shoe design aims, their shoe design sketches, and their testing and reflecting on their products. Redesigning and reconstructing their shoes followed. The activity was able to elicit multiple concepts that together contributed to students' success and to their integrated STEM learning. Such learning included concepts of material properties and their functions, notions of aesthetics and style, 2-D and 3-D perspectives, shape properties, spatial visualisation, measurements, and design sketches. Overall, students' responses indicated not only rich integrated STEM learning but also progress towards becoming an informed designer, an increasingly important skill in today's world.

This section of the chapter has outlined the diversity of integrated STEM approaches currently being investigated in school settings by Australasian mathematics educators. Many of these approaches have been driven by researchers' interests in examining the ways integrated STEM curriculum can support and develop students' mathematical understandings as well as their general capabilities. Without substantial support for teachers, such initiatives may never become embedded in curriculum. The next section examines studies into developing pre-service and in-service teachers' knowledge and understanding of integrated STEM curriculum and pedagogies.

4 Implementation—Research into STEM Teacher Education

Given the context of a national agenda encouraging a move towards developing students' 21st Century skills and dispositions, and sustained engagement with STEM subjects, it is necessary that teachers (both pre-service and in-service) receive high quality professional learning opportunities, particularly about integrating STEM curriculum. While there has been some attention to this field of research in the US (e.g., Honey et al., 2014), and to a lesser extent in Europe (e.g., Radloff & Guzey, 2016), within the Australasian region this is still an emerging field of research. Several concerns arise when examining the Australasian professional learning landscape and the potential to support implementation in the teaching and learning of integrated STEM. One concern is current policy reforms in Australia directed at the need to "standardise" the learning of pre-service teachers (PSTs) through mandated professional standards and accreditation of teacher education programs. Another concern are the differences between the STEM subject matter knowledge of primary and secondary teachers with significant implications for preparing them to develop integrated STEM tasks, lessons and units of work. These concerns raise important issues about effective integrated STEM professional learning that have been considered in some of the studies reviewed in this section.

Several Australasian mathematics educators, typically in collaboration with science or technology educators, have examined aspects of integrated STEM PST education (e.g., Jorgensen & Alden, 2018; Symons & Blannin, 2019). There have also been several programs aimed at developing practicing teachers' capacity to design and implement integrated STEM curriculum (e.g., Anderson et al., 2019; Hobbs, Doig, & Plant, 2019; Lowrie & Larkin, 2019). A review of these studies follows and concludes with suggestions for potential research in STEM professional learning.

4.1 Research About Pre-service STEM Education

Several higher education institutions in Australia have been exploring how to embed integrated STEM curriculum into primary PST education programs, although it is clearly a "work in progress". In research at the University of Melbourne, Symons and Blannin (2019) and Symons, Redman, and Blannin (2017) investigated the way PSTs negotiated the transition between the various component STEM subjects when teaching integrated STEM during professional experience in schools. PSTs formed professional learning communities to observe each other teach, recording short excerpts of feedback in an online collaborative environment, or Padlet. The excerpts were coded for the type of feedback provided by peers, with most about general pedagogy, typically subject specific, and equally about either science or mathematics. It was not evident whether the STEM methods subjects were integrated or taught separately. While the STEM subjects are still taught separately at the University of Wollongong, educators are finding ways to provide a connected experience for primary PSTs (Nielsen, Georgiou, Howard, & Forrester, 2019). The educators provide experiences to support knowledge integration through projects like the *Case of the Mystery Bone—A Unit of Work on Measurement for Grades 5 to 8* (Clarke, 1996) where students become forensic scientists, solving a mystery murder. It is challenging for institutions to connect subjects in pre-service education and still meet accreditation requirements, but the intentions are evident in these studies.

In the secondary pre-service education context, a one-year, end-on program was developed in partnership with schools, governments, and Griffith University to establish a STEM Centre of Excellence where PSTs from the STEM subjects undertook their education program in parallel in both school and university settings (Jorgensen & Alden, 2018). With various backgrounds in STEM professions, the program enlisted quality candidates and supported their development as STEM teachers through practice-based immersion into schools to ameliorate the theory-praxis nexus. While not integrating the STEM subjects within the program and regardless of their undergraduate qualifications, all PSTs were encouraged to enrol in mathematics methods courses because of the shortage of qualified mathematics teachers and to be able to teach junior secondary STEM classes. Data collected in the first two years of the program from the pre-service teachers, their mentors and school principals indicated a substantial increase in the readiness of these graduates to plan, teach and reflect on STEM practice.

If PSTs are to be prepared to teach integrated STEM education in schools, it is clear tertiary institutions will need to find ways to support them in developing subject matter knowledge across the disciplines, and provide opportunities for them to develop integrated tasks, lessons and units of work before they enter the profession (e.g., Smith, Fitzallen, Watson, & Wright, 2019). To support such endeavours, the University of South Australia designed a more integrated, place-based experience for their STEM PSTs who spent up to 20 hours in a STEM industry placement during their program (White, MacGregor, & Panizzon, 2018). In integrated STEM teams of science, mathematics and design and technology, PST participants learnt about the skills required and capabilities needed in the workplace, the types of career opportunities available, and worked together to develop units of work that demonstrated applications and connections to industry. Evidence of connecting with industry partners to support teacher knowledge and understanding of STEM has also occurred in in-service education programs.

4.2 Research About In-service STEM Education

Several researchers have been developing and delivering in-service STEM education, particularly at the secondary school level, with studies identifying a range of challenges related to secondary school contexts and the constraints on working across the STEM subject boundaries. One action research study was launched by ACARA in 13 Australian secondary schools to investigate whether teachers could use curriculum documents to make connections between the STEM subjects, and design integrated STEM projects for their students (ACARA, 2016). The STEM projects aimed to improve student confidence and capacity for transfer of knowledge across disciplines, encourage girls to remain engaged in STEM, and to identify connections between classroom learning and future employment opportunities. While not all aims were met fully, the study demonstrated that an integrated STEM approach developed students' ability to collaborate with others, to transfer knowledge, and to understand the usefulness of STEM in the workplace. However, implementation issues arose with teachers' knowledge and capacity, the inflexibility of timetables and staffing in schools, and "inconsistent content coverage of some learning areas within a single project" (ACARA, 2016, p. 20). Coverage of content has been an ongoing issue in other integrated STEM approaches, particularly for mathematics (e.g., English, 2016b) with some suggesting "much STEM work does little to emphasise mathematics" (Doig & Williams, 2019, p. 1). Attention to these issues in teacher education is needed.

While many professional learning opportunities are over short periods of time, one approach to supporting STEM teachers has demonstrated substantial impact in schools throughout its three-year duration (Hobbs et al., 2019). Working with grades 7 and 8 science, technology and mathematics teachers from ten schools in the Geelong region of Victoria, *Successful Students—STEM Program*, researchers developed a STEM Vision Framework to guide school teams in developing a STEM vision for their school by providing a common language of STEM curriculum and pedagogies. Since there is no one way to implement integrated STEM education, as illustrated in the section on STEM education in the school curriculum in this chapter, schools were tasked with developing their own approach to meet the needs of their students. After only six months in the program, 79% of teacher participants reported improved knowledge and understanding, after 12 months, 100% reported improved capability, and after 24 months in the program, 75% reported evidence of "embedded classroom innovations" (p. 211). Case studies of schools revealed a variety of approaches to the integration of the STEM subjects depending on school needs and expertise, but overall, there was increased use of problem solving, investigations, and design challenges. The factors identified as ensuring success included teacher release time, STEM expert support via a project officer, and STEM academic leadership from the Deakin University team. Challenges included designing appropriate assessment, maintaining integrity of the subjects, and engaging all teachers and school leaders.

Similar supports and challenges were identified by Anderson and her colleagues at the University of Sydney (Anderson et al., 2017, 2019) in their year-long program—*STEM Teacher Enrichment Academy*. The initial STEM academy program began in 2014 with each secondary school sending a team of 6 teachers (typically 2 mathematics, 2 science and 2 technology) to work collaboratively to design integrated STEM tasks, lessons and units of work. Initially data were collected from teachers to determine potential changes in practice with evidence of greater use of problem solving and inquiry-based learning approaches, particularly for mathematics teachers (Anderson et al., 2017). Within a social cognitive theory perspective, the 108 matched pre- and post-surveys for the secondary teachers revealed gains in *individual self-efficacy* was highly significant with a large effect size ($Z = -7.75$, $p = .000$, $r = .53$). Additionally, secondary teachers' increase in measures of *outcome expectancy* was also statistically significant ($z = -3.21$, $p = .001$, $r = .22$). Data collection from a small number of case study schools enabled a more detailed investigation with observations in classrooms and interviews with students (Tytler et al., 2019). Results suggest redesigning the curriculum to build integrated STEM projects into programs is possible although time-consuming, and it also requires changes to school structures such as more flexible timetabling, regular meeting times, and careful selection of teachers to ensure success. Overall, data from the secondary STEM Academy reported successful schools have: a clear vision and purpose; support from school leadership; an energetic and resourceful STEM team leader; intention to bring others with them (teachers, parents, community); and capacity to find solutions to challenges (Anderson, 2019).

In 2017, a new STEM Academy program was developed for primary school teachers (Anderson et al., 2019). Like the secondary program, teams of teachers from participating schools met three times over a one-year period to develop school-based plans for integrated STEM curriculum. Supported by an experienced mentor, schools created innovative STEM projects with students frequently driving the investigations with their own inquiry questions. Data from 29 participating schools in 2018 revealed the 102 teacher participants developed new understandings of STEM-related pedagogies and STEM careers. Students in each of the teachers' classes completed pre- and post-surveys with evidence of improved STEM attitudes and aspirations in some schools but not all. These data raise questions about situational learning of teachers and students in such projects. One case study school was investigated in Anderson et al. to determine the specific factors leading to positive STEM outcomes. Initial data analyses reveal teacher capacity and leadership support as key to driving effective integrated STEM curriculum and pedagogical change. Critical to effective integrated STEM curriculum is the design of effective tasks (Widjaja, Hubber, & Aranda, 2019) and/or the selection of appropriate tasks from the myriad now available online.

Investigating how to support less qualified or inexperienced STEM teachers' selection and use of suitable STEM resources formed the focus of the STEMCrAfT project implemented at the University of Tasmania (Beswick & Fraser, 2019). It "was designed cognisant of the relative inexperience and relatively high likelihood of teaching being out-of-field or working with colleagues who are out-of-field, in rural and remote locations" (p. 959), a situation not uncommon in many parts of Australia. Based on participating teachers' responses to a series of questions, the researchers developed a STEMCrAfT framework to guide and support teacher selection of resources, but the framework was also useful to provide impetus for additional professional learning and heightened community engagement (Fraser, Beswick, & Crowley, 2019).

As is frequently the case, research into teacher education typically involves small cohorts of teachers, and rarely investigates the impact on student learning outcomes—this is still the case in the studies reviewed here. Scaling up many of these models of teacher learning would be expensive and time consuming, and sustainability remains an issue after the program or intervention has ceased. Future directions for research within the area of in-service and pre-service teacher education within Australasia could focus on the development of robust and empirically tested frameworks to support the professional learning of teachers tasked with developing integrated STEM curriculum. Likewise, pre-service teacher education must take seriously the ever more pressing wave of pressure being applied to ensure that future teachers have the skills and dispositions to support students' critical and creative thinking. We need pre-service teachers who feel confident transitioning into schools knowing that they have the preparation to develop students' STEM capabilities. Also needed are continuing efforts to support in-service teachers (e.g., Hatisaru et al., 2019). This is necessary so that the pre-service teachers get the support they need to implement STEM learning experiences as beginning teachers.

5 Outcomes for Students—Developing Students' STEM Capabilities

Conceptualised in the late 1960s, Realistic Mathematics Education called for mathematics classrooms to link learning activities to real-world contexts, involve problem solving, and include inquiry-based learning (Treffers, 1993). These suggestions are echoed in the more recent calls for reforms to promote STEM education (Rennie et al., 2018). This implies that although strong cases were put forward for the RME reforms, they have not become regular practice in the majority of mathematics classrooms. Exploring the reasons for not adopting the RME reforms broadly is beyond the scope of this chapter but the issue is raised to highlight the need to make STEM education sustainable (Anderson et al., 2019). Hence, this section addresses issues related to the question, "What evidence is there that STEM education addresses the demands of discipline specific mathematics education and adds value to established practice?"

5.1 Utilising STEM Contexts to Engage Students in Learning

Promoting engagement in learning STEM disciplines is paramount. In the earlier years of schooling, engagement and enthusiasm for exploring different topics is high but often wanes as students progress through the later years, particularly in the secondary years of schooling. Although mostly only covering the early and primary years of schooling, recent STEM research makes innovative use of a diverse range of contexts to excite students (e.g., Fitzallen et al., 2019), pre-service teachers (e.g., Smith et al., 2019) and teachers (e.g., Anderson et al., 2019) about STEM ideas and concepts, and to stimulate their learning. Table 1 provides a snapshot of the contexts and the main topics of activities in the research reported. Integration of and

Table 1 Contexts and topics of STEM learning activities

Context	Topic	Reference
Science		
Performance of catapults	Stored energy and forces	Fitzallen, Watson, Wright, & Duncan (2018)
Changes in the temperature of hot and iced water	Transfer of heat	Fitzallen, Wright, Watson & Duncan (2016)
Seed dispersal devices	Structural features of living things	Smith et al. (2019)
Weather Watch	Observable changes in the sky	Campbell et al. (2018)
Technology		
Hand-made and machine-made licorice	Judge the suitability of processes	Watson, Fitzallen, English, & Wright (2019)
Design of shoes	Create design ideas using models and drawings	English (2018c)
Safety features of vehicles	Create and adapt design solutions	Ward, Lyden, & Fitzallen (2016)
Wind turbines	Create and adapt design solutions	Ward, Lyden, Fitzallen, & Panton (2018)
Engineering		
Optical instruments	Effects of structural changes	King & English (2016)
The stability of towers	Effects of forces on materials and systems	English & King (2017)
Earthquake resistant buildings	Effects of forces on materials and systems	English, King, & Smeed (2017)
Paper bridges	Effects of forces on materials and systems	English & King (2019)
Wind powered cars	Effects of structural changes	Anderson et al. (2019)
Robotic prosthetic devices	Refinement of products and processes	Ward, Lyden, Fitzallen, & Leon de la Barra (2016)
Mathematics		
Using the Square Kilometre Array telescope to explore spirals.	Geometric shapes and patterns	Coupland et al. (2017)
Conducting a survey	Graphical representations and decision making	English et al. (2017)
Spinners and counters	Outcomes of chance experiments	English, 2018c; Wright, Watson, & Fitzallen (2019)
Bees with backpacks	Properties of geometric shapes	Coupland et al. (2017);
Photo story	Chronological order of events	Lowrie & Larkin (2019)

connections among the multiple disciplines within the contexts that are the foundation of the STEM learning activities are not noted and the list is not exhaustive.

5.2 Utilising STEM Contexts to Develop Students' Problem Solving and Critical Thinking Skills

Prominent in the STEM activities is the application of the engineering design process. It was used to support learning in design and technologies (e.g., English, 2019; Ward et al., 2018) and science (e.g., Fitzallen et al., 2018; Smith et al., 2019). The engineering design process supports the implementation of structured, yet flexible learning environments. It includes multiple phases such as identifying a problem, generating solution ideas, designing and constructing a solution, evaluating the performance of the solution, and potentially redesigning or modifying the solution, and evaluating the changes made.

During the phases of the engineering design process that involve identifying a problem and constructing a solution through the construction of a device or model, students engage in problem solving. The students who constructed a wind-powered seed dispersal device were required to select one design from three to improve before conducting a second round of trials (Fitzallen et al., 2019). They then had to determine which features of the device should be modified and which materials would assist in improving its performance. Ward et al. (2018) required their students to work out which parts of a system needed adjustment, and how much adjustment was needed, to optimise the output from a wind turbine model. Similarly, students had to problem solve to get a prosthetic limb and hand to grip and pick up items (Ward et al., 2016).

The cyclical nature of the engineering design process at the construct, evaluate, and redesign phases assists students to employ critical thinking skills. This was evidenced in the report on students launching ping pong balls from a catapult (Fitzallen et al., 2018). The students evaluated the data collected from the initial trials of the catapult to suggest potential improvements that would make the balls travel further. Similarly, students made changes to a wind-powered seed dispersal device based on data collected from trials (Fitzallen et al., 2019). A different approach to students thinking critically about the design of shoes was implemented by English (2019). The students in that study evaluated their designs and redesigns, to which they assigned a score of 1-5. The opportunity to make an assessment based on established criteria and backed with a justification supported the students to make knowledge-driven decisions and reflect on their knowledge and skills.

5.3 Utilising STEM Contexts to Develop Students' Understanding of Mathematics

Research on STEM education rarely documents the development of understanding of mathematical ideas and concepts. One exception is the research conducted as part of a 4-year longitudinal study entitled, *Modelling with Data*: *Advancing STEM in the Primary Curriculum* (English, Watson, & Fitzallen, 2017). That study followed students from year 3 to year 6 and focused on the implementation of statistics education activities, more specifically, the big ideas of variation and expectation. For many of the activities, learning about the statistical ideas was conceptualised and implemented as the Practice of Statistics (Watson, Fitzallen, Fielding-Wells, & Madden, 2018). The practice of statistics is a simplified four-stage framework that simulates the steps undertaken by statisticians in a statistical investigation—Pose Questions (English et al., 2017), Collect Data (e.g., Watson et al., 2019), Analyse Data (e.g., Fitzallen, Watson, & Wright, 2017), and Make Decisions from Data (e.g., Fitzallen, Wright, & Watson, 2019). Framing several of the STEM learning experiences on the framework across the four years of the project enabled the students' capabilities in managing and recording data, creating graphical representations from the data, using graphing software to analyse data, and making sense of their data when answering research questions to be developed and sustained.

The *Modelling with Data* project illustrates the way in which mathematical ideas can be developed within STEM contexts from both student learning and pedagogical practice perspectives. For instance, the activities—Licorice, Catapults, The Heat is On!, Viscosity, Plant Growth, Seed Dispersal—included students measuring distances, temperature, and time, which were used as data for the statistical analyses undertaken (Fitzallen & Watson, 2019). In many cases, the application of other mathematical concepts and skills, such as drawing 3-D representations in 2-D form (English, 2019) and weighing the mass of objects (Watson et al., 2019) were reported but the level of achievement of those mathematical ideas at the year levels targeted or their development over time were not reported. It would be beneficial for mathematics education for the development of mathematical content embedded within STEM learning activities to be the focus of STEM education research.

The issue of not taking advantage of the mathematics learning opportunities embedded within STEM activities arises often. In the delivery of a STEM outreach program that implemented engineering-focused learning activities, Ward et al. (2016) acknowledged the mathematical ideas associated with designing, constructing, and trialling a vehicle to protect an egg during a collision. These included drawing 2D representations of a model, measuring speed and rates of change, and measuring angles and force. Their report, however, only described the application of the mathematics. The same issue arises in their report on students designing a prosthetic limb and constructing a model of an ear drum (Ward et al., 2016), and optimising the performance of a wind turbine (Ward et al., 2018). Ward and her colleagues contend this is a limitation of STEM outreach programs. Although valuable for delivering informal teacher professional learning and for supplementing the school curriculum, the short, intermittent nature of outreach programs is not conducive to supporting sustained learning for any of the STEM disciplines. Although not set within a STEM outreach program, in their research that explored students' construction of an optical device, King and English (2016) suggested the students would have benefited from additional time to develop and draw designs, to reflect on progress made, and to develop understanding of the mathematics and science concepts that applied.

Developing student experiences to take advantage of the learning afforded by multiple disciplines in individual activities is complex. There is, however, evidence that digital devices and software support that process and make student learning hands-on and potentially more engaging. For example, Lowrie and Larkin (2019) illustrate the benefits of using digital tablets in the early years to deliver learning experiences based on stories and familiar contexts. The activities developed include creating patterns from photos students take, determining the chronological order of events from story segments, and building arguments by recording ideas and listening to the ideas of others. At the primary level of schooling, the exploratory data analysis software, *TinkerPlots*, was pivotal in supporting students to organise data, determine trends in the data collected, compare the performance of devices, predict future performance from the data, and explore the probabilities embedded in spinners (e.g., Fitzallen et al., 2019). Adding to these practical examples is a scoping review written by Hickmott, Prieto-Rodriguez, and Holmes (2018) that supports linking explicitly computational thinking facilitated by new technologies to the learning of mathematics in multi-discipline learning contexts. Further developments in the use of digital technologies in STEM education are anticipated with the recent emergence of research on student coding (e.g., Miller, 2019).

6 Conclusions and Implications for Further Research

In their commentary on "inter-disciplinary" mathematics, Doig and Jobling (2019) caution it is not a new approach but in the current STEM landscape they suggest "there is always the possibilities that some integrations may not contribute to successful mathematical learning for all students" (p. 250). It is, therefore, incumbent on mathematics education researchers with interests in integrated STEM learning to continue to champion for the focus to be on the mathematics that is the foundation of STEM learning activities to ensure students' mathematical learning is maximised.

Over the last four years, several Australasian mathematics education researchers have focused on aspects of STEM education which helped to frame sections of the chapter. These include: critiquing the role of STEM education in the school curriculum (e.g., English, 2016b; Smith & Watson, 2018, 2019); emerging school and community approaches to designing STEM programs (e.g., Tytler, Williams, Hobbs, & Anderson, 2019); developing teachers' knowledge and understanding of STEM education (e.g., Anderson et al., 2017); nurturing pre-service teachers' knowledge of STEM (e.g., Fitzallen & Brown, 2017; Jorgensen & Alden, 2018; Symons et al., 2017); developing students' STEM capabilities (e.g., English & Watson, 2018; Watson et al., 2019); and promoting students' STEM career aspirations (e.g., Holmes, Gore, et al., 2018). In concluding, we offer some key issues that we consider warrant further attention in STEM education research.

1. Policies have tended to provide advice on STEM education in schools, without sufficient evidence on how integrated STEM curriculum supports students' learning in mathematics, particularly in the secondary school context. In claiming that policy is fragmented and disjointed, Timms et al. (2018) recommended a focus on three challenges—strategies to improve student outcomes; strategies for building the STEM teacher workforce; and strategies for rethinking the STEM curriculum including a shift to focus on STEM practices and an integrated STEM curriculum. Another worthwhile perspective was offered by Chalmers et al. (2017), namely, developing "big ideas" of STEM both within and across the STEM disciplines.
2. Of major concern is the issue of whether mathematics will be incidental (e.g., applied learning) or foundational in integrated STEM education. In either case, how do we support teachers to know the difference between the two approaches and thus design learning activities accordingly? Siemon, Banks, and Prasad (2019) argued for both approaches to STEM education—"a coherent, well-planned sequence of mathematics instruction focused on developing the key ideas and strategies and the connections between them, and the opportunity to apply this knowledge in rich, integrated settings that require collaborative endeavour and exercise process skills" (p. 79). Clearly, attention to this issue is critical in ensuring mathematics does not get left behind in integrated STEM education.
3. Furthermore, there is little research illustrating how mathematics supports the development of the other STEM subjects. This is a critical area for future research. Although multiple classroom activities abound, more research is needed on the mathematics outcomes and how a developmental approach to mathematics learning within STEM can be implemented.
4. Associated with the above points are urgently needed curriculum reforms to guide teachers work. Lowrie, Downes, and Leonard (2017) argued for the inclusion of STEM education in the *Australian National Curriculum* as a general capability. They further called for a national framework to identify STEM practices including ideas developed, methods applied, and outcomes achieved. Such a framework could encourage teachers to engage more productively with STEM education Lowrie, Logan, & Larkin, (2017). Although professional learning is a key component here, it needs to be strongly supported by school leadership team/s.

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Chapter 4
Facets of Numeracy: Teaching, Learning and Practices

Vince Geiger, Keiko Yasukawa, Anne Bennison, Jill Fielding Wells, and Carly Sawatzki

Abstract The purpose of this chapter is to develop an inclusive and coherent discussion about research developments within numeracy while, at the same time, highlighting the contributions of its different facets. These facets include two broad contexts in which numeracy development and practices take place, schooling/initial teacher education and the workplace, and two centred on specific areas of mathematical content, statistical and financial literacy. Research in this review is analysed through the dimensions of the Model of Numeracy for the 21st Century—contexts, mathematical knowledge, tools, dispositions and critical orientation. The chapter concludes with a discussion of potential new directions for numeracy research.

Keywords Adult numeracy · Critical orientation · Financial literacy · Mathematical literacy, numeracy · Statistical literacy

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1 Introduction

Numeracy is a term used to identify the mathematical knowledge and capabilities needed to accommodate the demands of informed, contributory and critical engagement with private, civic and work life. While the term numeracy is prevalent in countries such as Australia, the UK, Canada and South Africa, other expressions (e.g., quantitative literacy, critical mathematical literacy, mathemacy, matheracy) are used internationally to identify related constructs, nuanced by differences in nature and intention, that revolve around the notion that mathematics and practices associated with its use are fundamental to navigating personal, economic and socio-political worlds. Thus, discussions about numeracy can be complex as some authors use the terms such as numeracy, quantitative literacy and mathematical literacy synonymously, while others draw distinctions between these constructs (Niss & Jablonka, 2014). The situation is further complicated by the lack of an equivalent term in some languages (Frejd & Geiger, 2017). For example, in Sweden, while words exist for literate (litterat) and illiterate (illitterat) they are not used in relation to mathematics. In this chapter, we will use the term numeracy to cover all related notions while also identifying different aspects that are established lines of research enquiry.

The origin of numeracy as a field of research is generally linked to the *Crowther Report 15–18*, where it was defined as a type of quantitative thinking—the mirror image of literacy (Ministry of Education, 1959). Over time, the definition of numeracy has evolved to encompass different and more sophisticated capabilities, consistent with the demands of an ever-changing world. A widely accepted broad definition of numeracy was developed by the OECD (referred to as mathematical literacy) through its PISA (Programme for International Student Assessment) initiative. This has been revised on a regular basis since 1999, currently as below:

> an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well founded judgements and decisions needed by constructive, engaged and reflective citizens. (OECD, 2017a, p. 65)

Another broad definition of numeracy, also developed for the OECD, anchors the assessment content and parameters of PIAAC (Programme for International Assessment of Adult Competencies).

> PIAAC defines numeracy as the ability to access, use, interpret and communicate mathematical information and ideas, in order to engage in and manage the mathematical demands of a range of situations in adult life. (PIAAC Numeracy Expert Group, 2009, p. 21)

As in the case of PISA, the definition of numeracy for PIAAC is subject to systematic review. A recent review (Tout et al., 2017) identified four areas that require attention in the development of a revised definition: (1) disposition to use mathematics; (2) seeing mathematics in a situation; (3) critical reflection and action; and (4) degree of accuracy. The third of these areas refers to critical capacities that are increasingly seen as an essential component of being numerate—needed to accommodate the demands of complex problems encountered when negotiating a world characterised by rapid change (e.g., Geiger, Goos, & Forgasz, 2015; Maass, Geiger, Ariza, & Goos, 2019). Thus, being numerate requires more than facility with basic mathematics as it also involves the critical use of mathematics to: solve problems in the real world; make decisions and judgements; and provide evidence in support of, or to discredit, arguments or stated positions.

While becoming numerate is often portrayed as the acquisition of mathematics-based cognitive behaviours and skills, it can also be viewed as specific practices related to the use of mathematics within particular contexts. When numeracy is viewed as a practice, how mathematics is employed is related to the physical and social context in which a task is situated. The notion of situatedness is tied to ways of thinking, modes of reasoning and means of knowledge generation within communities that are defined by distinct social or cultural types of activity. As Yasukawa, Jackson, Kane, and Coben (2018, p. 9) explain:

> An NSP [numeracy as a social practice] perspective focuses on what people do with numeracy through social interactions in particular contexts, rather than on people's performance of mathematical skills in isolation of context…Moreover, a focus on practice entails viewing numeracy activity as culturally, historically and politically situated.

The notion of numerac y as a social practice represents a significant departure from traditional views of numeracy that emerged from the need to identify the mathematics needed to prepare students for the demands of higher education, employment and adult life (e.g., Cockcroft, 1982). This perspective demonstrates how the scope of numeracy has broadened over time to include a wider range of learning environments and practices. Different perspectives have, in turn, fostered the development of research communities with specific foci within numeracy. In this chapter we present a review of research conducted within four such foci, which we term facets. These include two facets devoted to the contexts in which numeracy development and practices take place—schooling/initial teacher education and the workplace—and two that centre on particular mathematical topics—statistics and financial literacy. While it may be argued that there are other emerging areas that might receive attention, for example the role of digital literacy in numeracy, we have selected the four facets canvased here because they represent concentrations of research effort over the period of review.

In previous issues of RiMEA, research into different facets of numeracy was reviewed under a variety of headings in separate chapters. The purpose of this chapter is to develop an inclusive and coherent discussion about research developments within numeracy while, at the same time, highlighting the contributions of its different facets. To develop this discussion, we will first describe the theoretical lens used to frame the review of research in the field. Second, research conducted across each of the identified facets of numeracy will be outlined. Then, a synthesis of research developments will be presented followed by a discussion of future directions for research in the field.

1.1 Theoretical Lens

To develop a synthesis of the contributions that studies of different facets of numeracy have made to research in the field, we draw upon the Model of Numeracy for the 21st Century (Goos, Geiger, & Dole, 2014). The model was initially developed as a synthesis of relevant literature but has been extended and validated through a series of research projects (e.g., Goos, Geiger, Dole, Forgasz, & Bennison, 2019; Geiger, 2019). The model goes beyond broad definitions of numeracy by outlining four key dimensions, *contexts*, *mathematical knowledge*, *tools*, and *dispositions*, which are activated through an analytical and evaluative capability, a *critical orientation*—represented in Fig. 1 and described in Table 1. While initially conceived as a tool for teachers' planning for and reflection on their teaching and learning practice in numeracy, the model has also been used as a scaffolding instrument for the design of numeracy and interdisciplinary STEM tasks (e.g., Geiger, 2016, 2018; Geiger et al., 2018); informing initial teacher education instruction in numeracy (Goos et al., 2019) and as an embedder-of-numeracy identity (e.g., Bennison, 2016a). Additional detail about this model can be found in Goos et al. (2014).

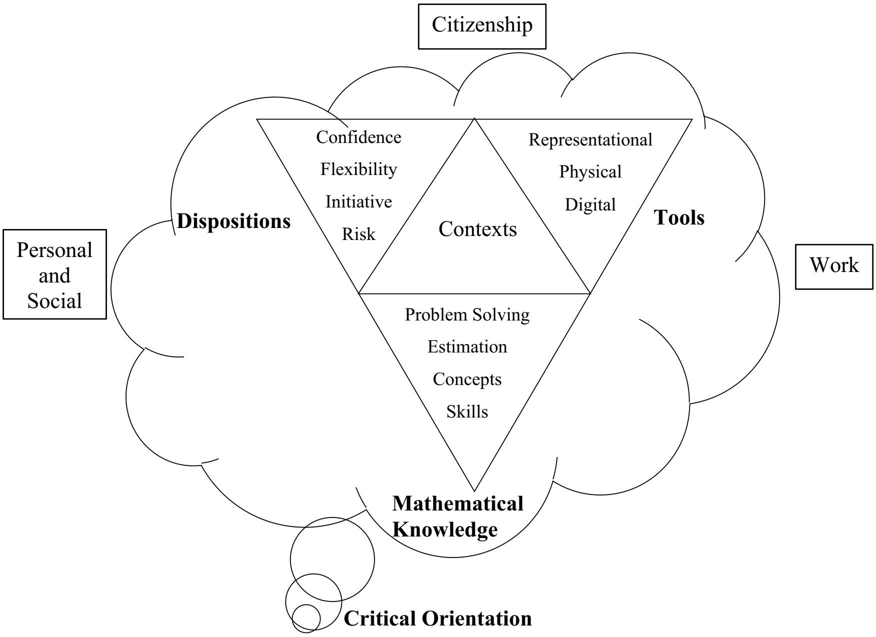

Fig. 1 A model for numeracy in the 21st century (Goos et al., 2010, 2014)

Table 1 Descriptions of the dimensions of the Model of Numeracy for the 21st Century (Goos, Dole & Geiger, 2012)

Mathematical knowledge	Mathematical concepts and skills; problem solving strategies; estimation capacities
Contexts	Capacity to use mathematical knowledge in a range of contexts, both within schools and beyond school settings
Dispositions	Confidence and willingness to use mathematical approaches to engage with life-related tasks; preparedness to make flexible and adaptive use of mathematical knowledge
Tools	Use of material (models, measuring instruments), representational (symbol systems, graphs, maps, diagrams, drawings, tables, ready reckoners) and digital (computers, software, calculators, internet) tools to mediate and shape thinking
Critical orientation	Use of mathematical information to: make decisions and judgements; add support to arguments; challenge an argument or position

2 Numeracy in Schooling and Initial Teacher Education

Schooling plays an important part in developing the numeracy capabilities of children and adolescents. Across the review period, there is significant research literature devoted to the development of students' numeracy capability and teachers' effective numeracy practices within the schooling sector. By association, this research also involves Initial Teacher Education (ITE) students. Policy and practice in both sectors in Australia have been influenced by national (e.g., National Assessment Plan—Literacy and Numeracy [NAPLAN]) and international numeracy assessments (e.g., PISA, PIAAC) that have raised concern over the quality of numeracy teaching and learning practice. This concern has led, in Australia, to the Literacy and Numeracy Test for Initial Teacher Education (LANTITE) initiative.

2.1 Numeracy Research in School Settings

2.1.1 Teachers and Teaching

If developing numerate citizens is a goal of school education in Australia, then it is imperative that teachers have the capacity to promote numeracy learning in the subjects they teach. In support of this endeavour, Bennison (2016a, 2016b) developed a framework for *identity as an embedder-of-numeracy* that includes cognitive and affective attributes that impact on teachers' capacity to address numeracy across the range of subjects. In addressing this issue further, Forgasz and Leder (2016) explored the numeracy competence and confidence of practicing teachers through an online survey based on tasks drawn from the 2010 Year 9 NAPLAN test. They found that only 60% of teachers performed at a level expected of people aged 17 or older on a task requiring critical evaluation, indicating that the numeracy proficiencies of practising teachers require further investigation.

While developing students' numeracy capability is seen as important, few studies have investigated effective pedagogies in this area. Against this trend, Geiger (e.g., 2016, 2018, 2019) investigated how primary and secondary teachers design and implement numeracy tasks across the curriculum. This work drew on the model for Numeracy in the 21st Century (Goos et al., 2014) and the literature of task design in mathematics to generate evidence that teachers use two primary approaches to designing numeracy tasks—utilising the curriculum as a lens for identifying promising ideas and archiving potential starting points for development when planning lesson sequences.

In one of the few studies related to numeracy in early childhood settings, Chigeza and Sorin (2016) reported on an arts-based project that found children's numeracy capabilities were enhanced through attention to spatial orientation (e.g., placement of objects in space), quantifying objects (e.g., some, many, few) and expressing the attributes of objects.

2.1.2 Assessment

Research related to assessment of numeracy in Australian schooling has primarily focused on NAPLAN and PISA. This includes studies into the impact of NAPLAN testing and the results of teachers' mathematics pedagogy and curriculum planning (e.g., Carter, Klenowski, & Chalmers, 2016), student engagement with mathematics and attitudes towards numeracy (e.g., Carmichael, Muir, & Callingham, 2017; Parnis & Petocz, 2016) and school level practices that have contributed to improved numeracy outcomes (Muir, Livy, Herbert, & Callingham, 2018). Numeracy assessments have also been used extensively to highlight inequities in student achievement (e.g., Thomson, De Bortoli, & Underwood, 2016), especially in the case of Australian Indigenous students (e.g., Chua, Khan, Humphry, & Hassell, 2017) and those from disadvantaged backgrounds (e.g., Goss, Sonnemann, Chisholm, & Nelson, 2016).

Gender has also been a focus in the secondary analysis of NAPLAN data. In a study that analysed data from a sample of independent schools in Western Australia participating in National Partnerships aimed at improving literacy and numeracy, Chua et al. (2017) found that males performed better than females and that the gap increased as students moved through schooling. Logan and Lowrie (2017) also found that males performed better on two-stage orientation questions and suggested that girls need explicit practice in developing processing skills for this type of question. Rather than analysing student performance, Exley and Trimble-Roles (2017) mapped the language used in the Year 3 NAPLAN example test against the Year 3 Australian Curriculum: English (ACARA, 2017). They found that the use of language in the test was more complex than what students were expected to understand at that year level, implying that teachers need to emphasise the mathematical meaning embedded in word problems through classroom discourse.

2.2 Numeracy Research in ITE Settings

Research related to numeracy in ITE settings has focused primarily on the content of pre-service programs and, in Australia, on LANTITE. LANTITE was introduced in 2017 to ensure that ITE students are in the top 30% of the population for numeracy upon graduation (TEMAG, 2014). Consistent with this demand, Furness, Cowie, and Cooper (2017) argued that ITE programs needed to provide opportunities for students to develop "knowledge of mathematics, the disposition to use this knowledge in an ethical manner for social/political action and the capacity to recognise when it is useful and/or being used" (p. 721). In addition to addressing mathematical knowledge, they recommended ITE programs provide opportunities to understand the politics of mathematical knowledge and how they could support others to develop agency through the use of critical mathematical thinking.

Using a whole of program approach in a one-year graduate ITE course for prospective primary teachers in New Zealand, the Mathematical Thinking and Reasoning in ITE (MARKITE) project investigated the impact on ITE students' confidence, competence and critical awareness of mathematics encountered in teachers' work (Cooper, Cowie, Furness, Peter, & Bailey, 2017). Findings indicate positive outcomes for the ITE students in response to explicit attention to the mathematical thinking embedded in the program. In another study conducted within the ITE context, Forgasz and Hall (2019) evaluated a required numeracy course introduced into primary and secondary Master of Teaching programs. Overall findings indicated that participating ITE students developed increased confidence with incorporating numeracy into their teaching, greater awareness of the differences between numeracy and mathematics, and were more aware of the numeracy demands on teachers. Similar findings were noted in a study conducted by Bennison (2019) related to course outcomes for undergraduate secondary ITE students.

O'Keeffe (2016) and O'Keeffe, O'Halloran, Wignell, and Tan (2017) have contributed to the conversation about the lack of clarity surrounding numeracy tasks in LANTITE through a linguistic analysis of the ten sample items published by the Australian Council for Educational Research. Their findings revealed the high linguistic demands within the test items, leading the researchers to question whether LANTITE is achieving its purpose.

3 Adult Numeracy

3.1 Assessment of Adult Numeracy Capability—Insights from Policy Research

Research studies on the assessment of adult numeracy capabilities reflect the debates and discussions between those who conduct research *for* policy and those who conduct research *of* policy (Lingard, 2013, cited in Black & Yasukawa, 2016, p. 166).

Exemplars of the former are the Survey of Adult Skills (SAS) and PIAAC (http://www.oecd.org/skills/piaac/), sponsored by the OECD. SAS measures adults' proficiency in key information-processing skills—literacy, numeracy and problem solving in technology-rich environments—and how adults (16–65 years in age) use their skills in personal, civic and work life. The survey has, to date, been conducted in over 40 OECD member and partner countries, including Australia, New Zealand and Singapore.

The translation of findings of international large-scale assessments (ILSAs) into local policies and practices has provoked dialogue and contestations between research undertaken by transnational policy advisory organisations, multinational corporations and peak bodies such as the OECD, Pearsons, the Australian Industry Group and 'traditional' academic researchers (Farrell, 2014). Black and Yasukawa (2016), for example, argue that there is an increasing dominance of a neoliberal discourse surrounding adult learning, viewing literacy and numeracy almost exclusively in terms of the human capital necessary for economic productivity. They suggest that this is a consequence of particular groups of policy actors, who include employer and industry peak bodies, and policy advisory arms of government, exerting an unbalanced influence, for example, on the development of Australia's National Foundation Skills Strategy for Adults (NFSS). In Black and Yasukawa's view, a rebalancing of policy would require an understanding of the meaning and nature of literacy and numeracy in adults' lives, including in the workplace, and attention to in situ studies that provide insights that cannot be gained from surveys.

Osmond (2016) expressed similar concern in a historical analysis of the evolution of a strong community of practice among adult basic education teachers in New South Wales in the 1970s. In their view, the convergence between the public discourses and the professional discourses has been lost in recent times due to a narrowing of policy interests in literacy and numeracy to economic outcomes. Similarly, Hunter (2016), writing from the New Zealand context, describes the impact of the OECD's standardised assessment framework for measuring adult learners' progress and the associated accountability regime. She is critical that these initiatives effectively discount the extensive research available on literacy and numeracy as situated practices that are highly contingent on the socio-cultural and political contexts in which people engage in these practices.

ILSAs in education have themselves become a subject of interest for an increasing number of international researchers. For example, Gorur (2019) studied the phenomena of ILSAs such as PIAAC and PISA by using theoretical resources from Actor Network Theory (ANT) to suggest that their effectiveness could only be improved by focusing more carefully on "the description of the assemblages that make up ILSAs. …focusing on the practices that link actors together and sustain ILSAs as believable and relevant" (p. 223). In a further use of ANT, researchers have analysed media reports about Australia's results on ILSAs, for example, ALL and SAS (Yasukawa, 2019; Yasukawa & Black, 2016). In addition to this work, Yasukawa, Hamilton, and Evans (2017) undertook a comparative study of national media responses to SAS in Japan, England and France. A salient observation from these studies was that a simple and 'catchy' message was needed for stories to be newsworthy. This often meant that important background detail or expert commentary, other than that of the OECD, was absent.

In a contrasting view, Coben and McCartney (2016) are less pessimistic about the disempowering potential of ILSAs. While acknowledging the hegemony of the state and the tensions that new compliance and accountability regimes present for practitioners, they urge critics to engage in debate rather than to settle for a reductive binary of compliance/non-compliance. In the same spirit Coben and Alkema (2018) proposed a conceptual framework for measuring adults' numeracy both as social practices and as technical skills, thereby rejecting what they describe as the "prevailing polarized positions in the academic and policy literature" (p. 75).

3.2 From a Focus on the Individual to Their Environment

Evans, Yasukawa, Mallows, and Creese (2017) have observed that while many countries participating in the SAS have adults who are assessed as having very low levels of proficiencies in numeracy (and literacy), they are not enrolled in adult education programs that may help them to develop their numeracy. This brings into question how a group of people who are assessed as low proficiency manage numeracy-embedded aspects of their lives. As a way of gaining insight into this paradox, Evans et al. (2017) propose the notion of the *numerate environment* to examine how an adult's circumstances support (or hinder) numerate practices suggesting that the following need to be considered—The: (1) demands that practices may make on an adult; (2) opportunities practices may offer to an adult; and (3) supports/resources and barriers that exist or develop within these practices that impede an adult's numeracy development. They argue that this perspective re-focuses adults' meaningful numeracy development and practices on the demands and opportunities of their environment rather than simply on an individual's cognitive abilities.

The importance of the environment in promoting numeracy development has been further explored by Morris, Hanckel, Yasukawa, and Gamage (2017) in a study based on semi-structured interviews with 18 adults who were homeless or who were at risk of homelessness. In this study, several interviewees expressed a wish to develop their numeracy to be more effective in everyday tasks such as shopping and managing their health. These responses highlighted the need for material supports, such as provision of reading glasses and food at the venues for learning, in addition to social and affective supports such as non-judgemental tutors. Findings of the study suggest that further research is needed into ecological approaches as these may provide greater insight into the role of socio-material environments in developing adult numeracy capability.

3.3 Numeracy Learning and Teaching—Sites of Adult Numeracy Provision

3.3.1 Numeracy in Remote Australian Aboriginal and Torres Strait Islander, Maori and Pacific Communities

Research about adult numeracy teaching and learning in remote Australian Aboriginal and Torres Strait Islander communities and Maori and Pacific communities in New Zealand highlight the need to take an ecological approach not only in researching numeracy but also in designing numeracy provision, and indeed educational provision more broadly.

In a study based on five training programs in remote Indigenous Australian communities, Guenther et al. (2017) found that the development of foundational skills, including numeracy, were crucial to retention in training and that family and community support, solidarity amongst the learner group and local ownership of the programs were vital factors for success. Unanticipated findings included a range of potential intergenerational benefits as an outcome of the program, for example, gaining skills needed to help children and grandchildren with their schoolwork. Local ownership of the programs also enabled the creation of a safe and supportive learning environment that in turn helped to create community cohesion as well as "community healing" (Guenther et al., 2017, p. 20). Similar personal and social gains have been reported (Bauer, 2018; Disbray & Bauer, 2016) in programs at a community learning centre in Yuendumu, a remote Aboriginal community in Central Australia.

In New Zealand, Furness, Robertson, Hunter, Hodgetts, and Nikora (2017) study confirmed the criticality of an ecological approach in adult literacy and numeracy programs in the development of broad personal, social and human capital outcomes that can make a real difference to the lives of Maori and Pacific people. This approach also led to a re-interpretation of *wellbeing* as something broader than the physical health of an individual to include the spiritual and emotional, as well as a harmonious relationship between individuals and their (socio-material) environment.

3.3.2 Numeracy in Vocational Education and Training

In both Australia and New Zealand, vocational education and training (VET) is an important site for numeracy development. One of the distinctive features of numeracy teaching and learning practice in VET programs is the embeddedness of mathematical concepts and methods in industry specific practices, posing challenges to teachers who have regarded mathematics as a purely academic discipline. For example, Flynn, Pillay, and Watters (2016) reported on a study of two industry-school partnerships (ISPs) in Queensland in which schools and the minerals and energy industry co-developed a curriculum contextualised to support students commencing an industry apprenticeship. The notion of *boundary-crossing* (e.g., Akkerman & Bakker, 2011) was used as an analytical lens to examine the movement across organisational boundaries of schools and companies operating within the industry. Findings from the study by Flynn et al. (2016) indicated that the partnerships led to the development of a curriculum that "mirror[ed] authentic workplaces practices and address[ed] problems that apprentices were experiencing in the workplace" (p. 322).

Most VET programs are delivered outside the K-12 school systems by providers who employ industry experts as teachers, however, these individuals may not be equipped with the pedagogical knowledge necessary to address the needs of their learners. Two action research studies, one in Australia (Livock, 2016) and one in New Zealand (Schwenger, 2018) involved researchers working with vocational teachers to address the numeracy needs of their students. Livock's study (2016) focused on nursing students, while Schwenger's study (2018) addressed the numeracy requirements of electrotechnology students. Both studies indicated that action research was an effective mechanism to develop vocational teachers' skills in embedding vocationally relevant numeracy into their teaching.

Research conducted in New Zealand investigated the occupational discourses of carpentry and automotive technology (Parkinson & Mackay, 2016; Parkinson, Mackay, & Demecheleer, 2018). Parkinson and Mackay (2016) analysed the Builder's Diary, "a daily account of the work done on the building site over a year-long period" (p. 290). They found that the Diaries included records of students' use of numeracy skills such as measurements and calculations through both verbal and visual texts. This illustrated that numeracy, for these apprentices, was highly purposeful and contextualised, and highlighted the degree of their participation in the discourse community of their industry. Thus, in vocational contexts, numeracy is related to competence with particular mathematical concepts and skills as well as a making meaning capacity, a mode of discourse for communication and a means to represent work practices in modes that are accepted in an industry.

3.3.3 Numeracy in Vocation Aligned Higher Education

Numeracy is also an area of research in enabling programs supporting university courses leading to specific vocations. For example, Galligan et al. (2017) investigated university nursing students' perceptions of their preparedness for the numeracy demands of their course. They found students expressed both over-confidence (graphing) and under-confidence (algebra) in comparison to their actual performance in a numeracy quiz and concluded that students should be given very clear guidelines about the numeracy expectations in courses. The importance of providing clear guidelines about the numeracy demands of courses has been highlighted by other researchers (e.g., Lisciandro, Jones, & Geerlings, 2018; Miller-Reilly & O'Brien, 2018), who recommend that university preparation courses provide affective support to address attitudes, aspirations and anxiety related to mathematics learning.

3.3.4 Numeracy in Adult Basic Education Programs

While there are limited studies related to basic numeracy programs, Muscat et al. (2016) and Morony et al. (2017) report on the successful delivery and evaluation of a health literacy course to adults identified with low literacy and numeracy proficiency within New South Wales Technical and Further Education TAFE colleges. The course made use of PIAAC data to examine the relationship between literacy and numeracy proficiencies and health outcomes and to understand decision-making around risks of available treatments.

3.4 Numeracy Practices

There is a strong tradition of research in numeracy as a social practice within adult education. For instance, Yasukawa et al. (2018) mapped the *terrain* of numeracy as social practice research to identify distinct themes in a sub-field dominated by ethnographic research approaches that focused on what people *do* with mathematics in particular social (work, community, home) contexts. This research raised questions about: the transferability of formal school maths to everyday contexts; the politics of knowledge; *academic* versus *everyday* numeracies; and experts vs lay knowledges of problem solving in different contexts. The invisibility of maths in many everyday practices also poses challenges to researching numeracy practices outside of school contexts (FitzSimons & Boistrup, 2017). These numeracy practices are multimodal, drawing on symbolic, visual and material resources, as well as other sensory perceptions including touch and hearing.

School mathematics has often been criticised for being divorced from numeracy in people's everyday lives. Northcote and Marshall (2016) investigated the topics, frequency, amount, type, difficulty level and methods used in adults' everyday calculations (outside of their paid workplace) through analysis of interview data and participants' log of calculation activities to find:

> Over 80% of all calculations were related to number and algebra and just over 60% were related to measurement and geometry. Very few calculations (less than 1%) related to statistics and probability. (pp. 11–12)

Other workplace studies show that arithmetic calculations are not always the most strongly featured mathematical skill in numeracy practices. For example, Alangui's (2018) work on building stone walls that hold in terraced rice paddies in the northern Philippines found that there were complex processes involved in the walls' construction that required mathematical thinking influenced by historical beliefs and customs including: classifying and defining the stones, soil and land; explaining the causes of fracturing of stones and erosion of the walls; estimating the height of walls, areas of land and number of stones needed; and decision-making about positioning of stones and shapes of stones to use in relation to different kinds of spaces. In a similar fashion, Kane (2018) focused on the numeracy practices of urban waste collectors and orchard managers, finding estimation played a significant role. These workers were always conscious of the consequences of the degree of accuracy or precision with which they made decisions.

In a study that examined the development of onsite critical numeracy practices in manufacturing companies and higher education, Yasukawa (2018) found that workers were increasingly subjected to new performance targets aimed at productivity increases. While many workers were aware that these targets were not delivering any personal benefit, they did not challenge their employers' demands. In the higher education sector, casually employed academics likewise felt they were being exploited by institutional demands and developed, in collaboration with their trade union, a response. This took the form of collective learning about casual pay calculations and the collection of data related to unpaid labour, leading to the lodgement of an industrial dispute that eventually led to backpay. The outcome led Yasukawa to conclude that a mediator to facilitate collective learning and action, rather than just individual learning, was crucial for enabling workers to develop critical numeracy practices.

4 Statistical Literacy

The increased availability and accessibility of data demands that citizens be statistically literate. Relevant research is reviewed in the sections that follow with particular focus on the role of mathematical knowledge, context, representational skills and critical reasoning capabilities in becoming statistically literate.

4.1 Development of Statistical and Mathematical Knowledge

Many statistical concepts were previously thought too complex to be grasped during formal schooling, however, the prevalence of data usage in society and the resultant need for a statistically literate population has necessitated the development of essential statistical understandings in all citizens. Consequently, significant research effort has been devoted to identifying key statistical ideas and investigating the capacity for school-age students to develop informal understanding of these concepts. For example, English (2018) proposed that early statistical literacy is underpinned by the foundational concept of chance and the constructs of variation, expectation, prediction, distribution and informal measures of centre.

4.1.1 Variation and Expectation

Watson (2018) demonstrated that 6-year-olds can recognise and discuss variation in data before being able to express data-based expectation. Other research provides further evidence of primary-aged children's capacity to: identify variation (e.g., English, 2018; Watson, 2018); explain why variation occurs (e.g., English, 2018); and make comparisons and draw conclusions about variations between groups, within groups and over time (e.g., Chick, Watson, & Fitzallen, 2018). It has also been noted that older primary children can: identify variation; consider the means of reducing error-based variation before comparing pre- and post-test representations (hat plots); and conjecture about the success of their actions (Fielding-Wells, 2018a).

4.1.2 Distribution

Research into an understanding of distribution has provided insight into students' appreciation for centre, shape and spread. For example, informal conceptions of distribution have been seen to develop through the use of informal language with primary students describing the shape of a distribution as *lumpy* and *with humps* (English, 2018) or *spread out*, *squished* or *bunched* (Chick et al., 2018). Students have also been reported as using terminology such as *clumps* and *outliers* when describing range (Fielding-Wells, 2018a). In the secondary years, Arnold and Pfannkuch (2016) have carried out research into Year 10 students' development of distribution as a conceptual understanding to provide a framework for describing distributions.

4.1.3 Informal Inference

To be statistically literate, citizens must develop an appreciation for the foundations of inference and be able to apply critical reasoning to statistical claims. This can include using an inference as an approximation of a feature of a population subject to formal limitations. Research by Makar, for example, investigated how Foundation year children (4–5 years in age) develop the underpinning structures of inference (Makar, 2016; McPhee & Makar, 2018), while English and Watson (e.g., English, 2018; English & Watson, 2018) and Fielding-Wells (2018a) have explored the capacity for students in later primary years to make informal inferences when working with data in familiar contexts.

4.1.4 Sampling

Understanding the difference between working with populations and samples is crucial when working with data, however, students are often not provided with the source of data they are asked to analyse. Work by Watson and English (2016) has shown that children at the upper primary level can appreciate the difference between sample and population, understand the nature of samples as predictors of population characteristics and develop the capacity to draw on data structures to support predictions.

4.2 Processes for Developing Statistical Literacy

Research related to the development of statistical literacy has tended to focus on the promotion of conceptual knowledge though investigations/inquiries and/or modelling approaches. While multiple frameworks for the implementation of statistical investigations have been developed, Watson et al.'s (2018) synthesis of statistical practice suggests these frameworks exhibit the following common principals: *problem posing* (asking, understanding and refining statistical questions); *planning for and collecting data* (including decision making about sample sizes and methods); *data analysis* (cleaning, organising and representing data, summarizing and reducing data); and *drawing conclusions* (decision making, inferring and responding to the problem posed). Other studies have also stressed the importance of engaging learners in complete investigations so that they: experience authentic statistical practice; become aware of the decision-making involved; and develop an understanding of the need for statistically relevant questions (Makar, 2018a; Watson, 2018).

Studies into the implementation of investigations/inquiries have addressed specific aspects of associated processes including: facilitating students' capacity to pose and critique problems (e.g., Arnold & Pfannkuch, 2019; Watson & English, 2017a); documenting the scaffolds and supports experienced teachers use in their classrooms (e.g., Allmond & Makar, 2018; Fielding-Wells, 2018b); and using investigations to facilitate the development of key conceptual statistical understanding (e.g., Makar, 2018b).

Recently, Lehrer and English (2018) proposed a framework for data modelling that draws on aspects of investigation/inquiry but also incorporates the need to generate statistical models to draw conclusions and informal inferences. This approach is supported by other research into the application of data modelling that suggests school students have the capacity to develop and use statistical and probability models to draw inferences and conclusions (e.g., English, 2018; English & Watson, 2016, 2018; Fielding-Wells, 2018a). In other work related to statistical thinking, Callingham, Watson, and Oates (2019) have proposed a learning trajectory for statistical reasoning based on the "big ideas" of statistics from Callingham and Watson's (2017) previous work on realistic expectations of middle school students.

4.3 Knowledge of the Context in Which the Data Is Embedded

The context from which data is collected is crucial. Thus, students' learning needs to be embedded in familiar contexts (Budgett & Rose, 2017; Makar, 2018a; Watson & English, 2018) or contexts that are developed with the students during the statistical investigation process (English, 2018; Watson & English, 2017b). At the same time, topics for statistical investigations are typically designed to coincide with curriculum areas (e.g., Fitzallen, Watson, Wright, & Duncan, 2018; Watson, Fitzallen, English, Wright, 2019). Watson (2017) also addresses the use of existing, open data sets for student explorations and notes their potential for use with middle school students, suggesting this may be a way of preparing students for working with *big data* in their future lives.

4.4 Written and Representational Literacy Skills

Representing and visualising data are crucial skills in statistics. Accordingly, research has shifted towards how graphs and other representations are developed as tools. For example, in the early years, children have been encouraged to formulate their own representations so that they: convey meaning to the child; demonstrate that data can be represented; and reinforce the need for data to support conjectures (e.g., Makar, 2018a). Further work in this area with primary aged students (English & Watson, 2018) investigated the use of representations to make comparisons between data sets, to make conjectures about variation and to ascertain changes to distribution and variation.

Increasing attention is now being paid to the use of digital tools (e.g., software, apps, spreadsheets) in relation to data visualisation (e.g., Prodromou & Dunne, 2017; Watson, 2017). Virtually all the previously mentioned statistics research has addressed and discussed the ways in which students explore, develop, design, interpret, manipulate and/or critique graphical representations. Some of these representations, especially in the earlier primary years, were child generated (e.g., Makar, 2018a), however the introduction of software to facilitate visualisation of larger data sets, for example, the use of TinkerPlots™ has provided opportunity for data comparison, re-representation, and pattern identification in the primary classrooms (e.g., Watson & Fitzallen, 2016). TinkerPlots™ has also been used to facilitate children's representation of data, as well as to enable the generation of data samples when working with open data, such as that available via *CensusAtSchool* (Watson 2017).

4.5 Capacity and Disposition to Adopt a Critical Stance

There has been an additional research focus on developing the capacity to critique evidence-based claims. In this work, learners make assessments on factors that strengthen or weaken an argument or inference. In this vein, Prodromou and Dunne (2017) stress the need for statistically literate citizens to be aware of, and recognise, the potential for data to be intentionally misrepresented through representations that demonstrate and/or obfuscate information. This research includes instances where students evaluate media claims (e.g., Budgett & Rose, 2017) or generate topics or questions for investigation (Watson & English, 2016).

While disposition has received limited attention in the development of statistical literacy, English (2018) incorporates aspects of disposition into her foundations for early statistics and probability including critical awareness, appreciation of uncertainty, flexibility and seeking connections.

5 Financial Literacy

Financial literacy education research is an emerging field of study that is currently characterised by three main features. First, it is promoted by governments and policymakers out of concern for the level of financial literacy needed to navigate the growing complexity of the financial landscape. Second, it is shaped by surveys, education programs and program evaluations typically funded and branded by the finance industry. Third, it draws interest from scholars with diverse expertise—in behavioural economics, education, finance, psychology and sociology.

Recently, financial literacy has gained international attention through its inclusion as a component of PISA. In this assessment, financial literacy is defined as:

> …knowledge and understanding of financial concepts and risks, and the skills, motivation and confidence to apply such knowledge and understanding in order to make effective decisions across a range of financial contexts, to improve the financial well-being of individuals and society, and to enable participation in economic life. (OECD, 2017a, p. 87)

Together, the mathematical and financial literacy assessment results of PISA provide an overall picture of 15-year-olds' capacity to apply their accumulated knowledge and skills to real-life problems and decisions. Across the 10 OECD countries that participated in the financial literacy component of PISA 2015 (New Zealand and Singapore did not participate), 45% of the top performers in mathematical literacy were also top performers in financial literacy with a correlation of 0.74 (OECD, 2017b). It was also noted that students in Australian metropolitan schools achieved more highly than those in provincial and remote schools, and that non-Indigenous students significantly out-performed their Indigenous counterparts (Thomson & de Bortoli, 2017). An example of an everyday financial task that Australian students found challenging would be interpreting information presented in payslips and invoices.

In addition to what can be understood from international testing regimes such as PISA, mathematics education researchers have begun to investigate effective approaches to financial literacy teaching and learning at school. This recent body of work reveals four key insights. First, finance is an example of a real-world context within which numeracy and mathematics teaching and learning can be meaningfully situated. Second, financial problem contexts can productively develop sophisticated mathematical knowledge and skills. Third, classroom tasks and pedagogical practices can be designed to promote a perspective on financial problem-solving. Fourth, teacher professional learning that promotes socially just, culturally responsive pedagogical practices is needed. Each of these insights is discussed below.

5.1 The Role of Context in Teaching Numeracy and Financial Literacy

A series of design-based research studies in Australia and New Zealand positioned finance as a real-world context within which mathematics can be meaningfully situated (e.g., Hunter & Sawatzki, 2019; Sawatzki, 2017; Sawatzki, Downton, & Cheeseman, 2019; Sawatzki & Goos, 2018; Sawatzki & Sullivan, 2017a). A noteworthy finding from this work was that students value learning about unfamiliar, novel and imaginable financial problem contexts that they deem useful in their lives beyond school (Sawatzki, 2017). While acknowledging such tasks can be pedagogically demanding for teachers, Sawatzki (2017) argued that learning opportunities of this nature have the potential to broaden students' horizons and better prepare them for economic participation.

Others have argued (e.g., Sawatzki & Goos, 2018; Sawatzki & Sullivan, 2017a) that there are social and mathematical dimensions to student financial problem-solving which pose both opportunity for and a threat to learning. While financial problem contexts and the associated social considerations can excite and engage students, they can also prove distracting. An effective financial problem context creates a need to do mathematics while encouraging students to contribute social and cultural insights about money matters to class discussion. For example, in a study by Sawatzki and Goos (2018) upper primary school students were asked to price lolly bags for sale as part of a fundraising [enterprise] activity. Differences were found between students who gave loss-making and break-even responses that were sensitive to the affordability of items and those who provided profit-making answers that seemed more concerned with generating a profit. Thus, being aware of students' personal beliefs, attitudes, values and dispositions is central to creating and/or selecting financial problem contexts that fit local circumstances (Hunter & Sawatzki, 2019; Sawatzki & Goos, 2018). This is particularly true for those living and learning in disadvantaged communities where finance industry sponsored teaching resources can be, at the least, disconnected from students' financial realities, and, at the worst, marginalising (Blue & Pinto, 2017).

5.2 Developing Sophisticated Mathematical Knowledge and Skills

A number of studies have explored the potential for financial literacy lessons to engage students in mathematical processes and develop sophisticated mathematical knowledge and skills. For example, Sawatzki and Sullivan (2017a) used tasks requiring students to make sense of a situation where a shoe sale offer is being shared by two characters, both of whom would like to save some money. Student responses to this task revealed that it is not always appropriate to retain/reuse familiar mathematical models to solve financial problems and that it is important to check how the social and mathematical thinking was associated with a financial problem context.

Building on the work of Sawatzki and Sullivan (2017a), Sawatzki et al. (2019) examined how 10–12-year-old students solved a financial problem where two characters negotiate to share the cost of a taxi ride. They found that while the vast majority of the student participants reported having caught a taxi before, few were familiar with the cost structure for doing so—i.e., flagfall plus cost calculated by distance travelled—and few had considered that a taxi ride and the associated costs might be shared for mutual benefit. A finer grained analysis showed that multiplicative reasoning and rate thinking was evident in slightly over half (54%) of the student participants' work samples—capabilities that are essential for students' development in financial literacy capability.

5.3 Developing a Critical Orientation to Financial Problem-Solving

Since financial decision-making involves choice, Sawatzki and Sullivan (2017a) argue that it is important that financial tasks and pedagogical practices be designed to promote critical capabilities. These should involve: accepting and confronting challenge; identifying, comparing, and contrasting multiple options; and developing solution arguments that weave together mathematical and social aspects of reasoning. They further argue that students should also learn to make judgements about the reasonableness of their social and mathematical thinking against a financial problem context before committing to a decision. This position is in alignment with that of Blue, O'Brien, and Makar (2018), who suggest that inquiry mathematics lessons can promote the social interaction and collaboration needed to support discussion of the socio-mathematical tensions inherent in financial decisions that have the potential to affect others.

5.4 Implications for Teacher Education

The studies discussed in this section indicate a need to support teachers in understanding students' unique and dynamically changing financial literacy learning needs in order to enact socially just and culturally responsive pedagogical practices. For example, Sawatzki and Sullivan (2017b) argued that teachers would benefit from professional learning related to reading and interpreting the possibilities for financial literacy teaching and learning across curriculum documentation, noting that Mathematics and Humanities and Social Sciences (HaSS) are disciplines that might form natural partnerships. Sawatzki and Goos (2018) and Hunter and Sawatzki (2019) go further by suggesting that it is only through exposing the social and cultural factors that influence student thinking that teachers can understand and begin to address tensions between neoliberal and social justice ideologies in ways that are sensitive to students' beliefs, values, interests, local conditions and needs.

6 Conclusion and Future Directions

In this chapter we have reviewed studies from established areas of numeracy research—schooling/ITE, adult and workplace learning, and statistical and financial literacy. Each of these areas represent distinct facets of the broader notion of numeracy. While distinct, research carried out within these facets has a primary focus on identifying and understanding the conditions and learning and teaching practices that lead to the empowerment of young people and adults with the capability and disposition to use mathematics when solving problems in the real world. In this final section, we provide a synthesis of research outlined in the chapter, using the dimensions of the Model for Numeracy in the 21st century (Goos et al., 2014) as an analytical lens—context, mathematical knowledge, dispositions, tools and critical orientation.

6.1 Context

Within each facet of numeracy, context was identified as crucial. In the case of schooling, investigations have been conducted into how teachers design effective numeracy tasks (e.g., Geiger, 2016, 2019) that are embedded in contexts students find accessible and that accommodate their interests. Context has also been important in the development of ITE programs that can best support future teachers' numeracy capability development. In this space, a type of future school-as-a-workplace approach was documented in which mathematics rich contexts are used as the basis for problematized school-based scenarios, for example, curriculum planning based on the interpretation of NAPLAN data (e.g., Forgasz & Hall, 2019).

While much discussion on the role of mathematics in the workplace has centred on large scale assessments such as PIAAC, Yasukawa et al. (2018) and others (e.g., FitzSimons & Boistrup, 2017; Kane, 2018) point to the importance of numeracy practices that are defined by situational and social contexts, arguing that this brings into question the possible transfer of numeracy capabilities. The situated nature of numeracy practices also means that research into adult numeracy must include ecological approaches as well as large scale surveys.

Context is inseparable from both financial and statistical literacy because relevant data and information itself is generated within real world-contexts. Research within statistical literacy indicates that learning needs to be developed within contexts familiar to the learner or developed by students during a process of inquiry (e.g., Makar

2018a; Watson & English, 2018), although Watson (2017) also notes the potential for using existing, open data sets in developing students' statistical literacy. By way of contrast, Sawatzki's (2017) work in financial literacy found students value learning about unfamiliar, novel, though imaginable, financial problems as long as these are seen as relevant to their lives beyond school.

6.2 Mathematical Knowledge

Research within each facet of numeracy presented here emphasised the importance of mathematical knowledge. Bennison (2016a, 2016b), for example, noted mathematical knowledge as a cognitive attribute in a teachers' identity as an embedder-of-numeracy framework. The introduction of LANTITE in ITE programs is largely aimed at guaranteeing the standard of future teachers' mathematical knowledge before they enter the profession.

The mathematics embedded in the workplace is not always visible (FitzSimons & Boistrup, 2017) and is typically intertwined with role specific practices. Accordingly, research related to mathematics in the workplace raises questions not only about the transferability of mathematical knowledge across contexts but also the value of different types of such knowledge—academic versus "everyday" and expert versus lay (Yasukawa et al., 2018).

Statistical literacy and financial literacy share the common challenge of promoting students' mathematical knowledge at the same time as providing the conditions under which they learn to use mathematics in context. In the case of statistical literacy, this includes research into: variation and expectation (e.g., Chick et al., 2018; Watson, 2018); distribution (e.g., Arnold & Pfannkuch 2016; Fielding-Wells, 2018a); informal inference (e.g., Makar, 2016; McPhee & Makar, 2018); and sampling (e.g., Watson & English, 2016). The work of Sawatzki et al. (2019) provides specific examples of how the mathematical knowledge that underpins financial literacy is closely intertwined with the context of a task as well as students' beliefs about and attitudes towards issues such as fairness and ethical practices.

6.3 Dispositions

Research that has a direct focus on dispositions towards applying mathematics in the real world appears to have received limited attention across the period of review. That said, English (2018) has observed that appreciation of uncertainty and flexible thinking are essential attributes for data-based inquiry aimed at promoting statistical literacy. With a note of concern, however, Miller-Reilly and O'Brien (2018) have identified the need for affective support in order to address the negative attitudes and anxiety that can be related to the learning of mathematics within the workplace. Documenting a different perspective on dispositions, Sawatzki and Sullivan (2017a) observed, in a study related to financial literacy, students' attempts to reconcile the tension between making pragmatic judgements about financial propositions and the social/ethical consequences of their decisions.

6.4 Tools

Explicit attention to tools was only noted in this review within research associated with statistical literacy. Digital tools within this facet of numeracy are considered to be essential for data visualization and analysis (e.g., Prodromou & Dunne, 2017; Watson, 2017). These tools, including software, apps and spreadsheets, provide the means for data comparison, re-representation, and pattern identification (e.g., Watson & Fitzallen, 2016) and also facilitate the generation of data samples from large open data sets such as *CensusAtSchool* (Watson, 2017).

6.5 Critical Orientation

The capacity to adopt a critical orientation or critical stance is a consistent theme in research across all facets of numeracy. Furness, Cowie, and Cooper (2017), for example, point out the need for ITE programs to address the politics of mathematical knowledge and its role in shaping society. This is a salient point given the finding of Forgasz and Leder (2016) that only 60% of practicing teachers performed at or above the expected level of 17-year olds on a task requiring critical evaluation. At the same time, the work of Geiger (2019) has shown that it is possible to design tasks and implement pedagogies that embed a critical orientation to using mathematics to solve real world problems within school contexts.

Black and Yasukawa (2016) argue for a more critical stance within the field of adult numeracy because of an increasing neoliberal discourse. Consistent with this perspective, Osmond (2016) takes note of how public discourse has shaped the work of practitioners working in adult literacy and numeracy, for example, the standardising of assessment frameworks and accountability regimes in New Zealand (Hunter, 2016). The danger of such standardisation is that it takes little account of the types of knowledge practices inherent in different occupations and in participatory citizenship. In a different study involving higher education workers, Yasukawa (2018) documented how workers took a critical stance in relation to their industrial conditions by taking part in collective numeracy learning in order to build a case that was eventually lodged by their union as part of an industrial dispute.

The capacity to critique evidence-based claims is central to becoming statistically literate. Prodromou and Dunne (2017), for example, see sensitivity to the potential misrepresentation of data as an important attribute of informed citizenship. Similarly, Budgett and Rose (2017) argue a critical orientation is essential when evaluating media claims.

Sawatzki and Sullivan (2017a) suggest that effective tasks and pedagogical practices are fundamental if students are to develop a critical orientation to financial problem-solving and decision-making. They also point out the need to promote the type of socially-responsible thinking that must accompany financial decision-making, a position supported by Blue et al. (2018) who argue that students should learn to evaluate how the financial decisions they generate within a real-world scenario may affect the lifestyles or livelihoods of others.

7 Future Directions

This review points the way to significant future research opportunities in the field of numeracy.

From the perspective of schooling, there are many opportunities to develop students' numeracy capabilities in the early years, however, there appears to have been limited research in this area during the period of review. Within ITE settings, there is evidence that numeracy focused courses and programs have positive impact on ITE students' numeracy capability. However, there appears to be a gap in the literature related to how successfully courses embedded within ITE programs prepare students for the numeracy related practices they encounter in the workplace.

There are ongoing debates, within adult numeracy, between those focused on research *for* policy and those who are investigators *of* policy. These discussions in themselves are fostering new research agendas, particularly in relation to large-scale assessments such as PISA and PIAAC. Given the international prominence of these assessments, there will be continuing opportunity for further policy related research in this area. The place of numeracy/mathematics in adult education and the workplace is often less visible. Thus, it is not surprising that many studies pay less attention to particular types of mathematical knowledge and skills, focusing, rather, on the sociocultural contexts of teaching and learning. The diversity of contexts within which mathematics is embedded in adult education and the workplace, however, means that there are significant challenges associated with developing a coherent and cohesive theory in this space. There is still much work to be done before ecological approaches and large-scale survey programs can be leveraged in concert to generate new insights into the field.

Many of the challenges for conducting research in statistical literacy are associated with monumental shifts in the accessibility, type and quantity of statistics used in society, which have been catalysed by rapid technological change. The notions of 'big data', 'open data', and 'metadata' are relatively new but increasingly prevalent. Access to large data sets and the new opportunities for the visualisation and management of data provided by emerging technologies offer great potential for future research. Given the context of a world in which data is now freely available and the means of its analysis readily accessible, research into the skills and capabilities now needed by citizens to be statistically literate is a matter of some urgency.

There are promising opportunities for research in financial literacy related to socially responsible decision-making. Given changes in public attitude towards, and confidence in, large established financial institutions, the way in which financially sound but ethical and socially just decisions are made is an important area for ongoing research.

Given the ubiquitous nature of digital technologies and their influence on all aspects of the economy, environment and society, the role of digital tools in promoting numeracy capability appears to be an under-researched area of enquiry. Understanding the capability needed with digital tools as part of modern citizenship is an area requiring urgent attention given the rapidly changing nature of life in the 21st century.

Finally, there are now large-scale data sets available from national and international studies (e.g., the Longitudinal Surveys of Australian Youth [LSAY], PIAAC) that provide opportunity for secondary analysis in relation to different facets of numeracy. While this direction in research has great potential, we should bear in mind one of the corner-stones of numerate citizenship that was a recurring theme throughout this review—that a critical stance should be adopted when analysing such data, in the sense of critique and also from the perspective of socially just and responsible decision-making.

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Chapter 5
Advancing Our Understanding of Initial Teacher Education Through Research

Jennifer Way, Michael Cavanagh, Fiona Ell, Sharyn Livy, and Heather McMaster

Abstract The purpose of this chapter is to explore the influences on, and tensions within, initial teacher education in mathematics, and to identify the research outcomes that might advance our understanding of learning and practices in this context. We have used a socio-spatial approach to organise our review into the three 'spaces' of policy, teacher educators and pre-service teachers, and to reveal tensions that exist between these spaces. Our analysis of the research highlighted the multiple roles of teacher educators—as researchers, teachers, collaborators and curriculum designers—as they mediate the tensions that occur between the spaces.

Keywords Initial teacher education · Teacher educators · Pre-service teachers

1 Introduction

This chapter reflects upon Australasian research in the period 2016 to 2019 pertaining to the education of pre-service teachers in primary and secondary programs in Australasia, with particular emphasis on their preparation to teach mathematics in school settings. Our purpose is to explore the influences on, and tensions within, initial teacher education, and to identify the research outcomes that might advance our

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understanding of learning and practices in this context. Minimal attention has been given to the areas of STEM Education, Numeracy development, Early Childhood Education, or the role of technologies, as these are more deeply explored in other chapters of this book (Chaps. 3, 4, 9, 13 respectively). Since around 2000 we have seen a steady increase in research regarding pre-service teacher mathematics education in Australasia, perhaps indicating the maturation of teacher educators' capacity for professional 'introspection' and inquiry into their own practices, and into the impact of those practices on emerging teachers of mathematics. The increased influence of political agendas and national policies on teacher education in Australia and New Zealand has given rise to a range of pressures on the curriculum, and to issues around student access to courses and progression in their studies of mathematics education. In contrast, the more stable, highly centralised teacher education scene in Singapore has generated much less research focused on pre-service teachers. As highlighted in the relevant chapter of the previous edition of this book, we need a strong evidence-base for practice in initial teacher education to inform debate around current issues (Anthony, Cooke, & Muir, 2016).

The research on initial teacher education (hereafter referred to as ITE) is diverse and complex in its motivations, stakeholder perspectives, approaches, theoretical drivers and impact. As the conceptual organiser for this chapter we have selected a *socio-spatial approach*, which is a theoretical frame that has recently proven useful in comprehensive studies of teacher education (Rowan, Mayer, Kline, Kostogriz, & Walker-Gibbs, 2015). The *socio-spatial approach* is the representation of teacher education through three imagined 'spaces'—the *Conceived Space* of policy makers, the *Perceived Space* of teacher educators, and the *Lived Space* of pre-service teachers in their courses and school placements. Such an approach enables us to capture the perspectives of the range of stakeholders in ITE. As suggested in our visual representation (see Fig. 1), the three spaces have distinct characteristics, yet are overlapping and interrelated. For example, the government may launch a new policy that must be interpreted by teacher educators and translated into changes in ITE programs, which in turn affects the experiences of the pre-service teachers. A research study might be situated within a particular space or be strongly from the perspective of

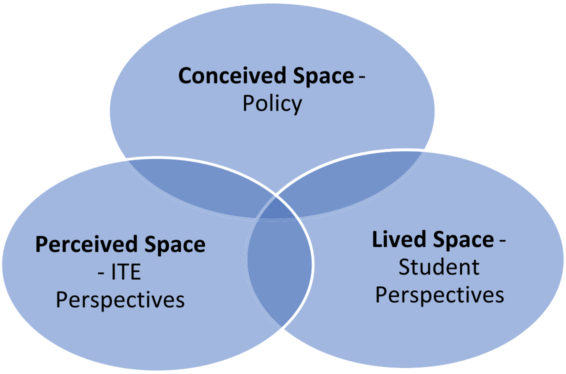

Fig. 1 A diagrammatic presentation of the socio-spatial approach representing teacher education

a particular space (e.g., the teacher educator). Alternatively, the study might explore more than one space and perspective (e.g., The students' experiences of the practices used by the ITE educators).

In reviewing the mathematics education research relevant to ITE in Australasia, we began by sorting the literature according to the tri-spatial categories of policy, teacher educators and the student perspective. Although mathematics education researchers rarely study the *Conceived Space* of Policy itself, it sets the context for much of their research and provides background for international colleagues to interpret their findings. As expected, we found overlap in much of the research for the *Perceived Space* of the teacher educators' work and the *Lived Space* of the pre-service teachers. However, examining research studies from these two perspectives highlighted aspects of the research contexts and methodologies that we may otherwise have overlooked. The chapter has been organised into the three spaces and concludes with a critical discussion that draws together our review by reflecting on how the research appears to be advancing our understanding of initial teacher education.

2 Policy—The Conceived Space

The Conceived Space of teacher education is the domain of political agendas and policy makers that define the ideal characteristics and processes required to produce teachers that will meet the educational and economic goals of the government. In Australia and New Zealand, as with various other countries, current agendas value "standards-based reforms, accountability and efficiency" (Rowan et al., 2015, p. 281).

2.1 Australia Context

In Australia, the majority of teacher education takes place in its 35 universities, which are subject to the Australian Government's higher education policies and funding arrangements, yet are highly influenced by the state systems in which they are located. At the school level, the curriculum and the work of teachers is constitutionally the jurisdiction of the states and territories, yet has increasingly been influenced by the centralised agendas and policies of the Australian Government (Mayer, 2016). Such agendas largely focus on accountability, quality assurance and compliance of both schools and ITE institutions in their curriculum design and delivery of programs.

At the end of 2014, the report of the national Teacher Education Ministerial Advisory Group (TEMAG), titled *Action Now: Classroom Ready Teachers*, set out eight principles for 'improving' ITE programs. The implementation of these principles, mainly in the form of standards and procedures for accreditation of ITE programs, and professional learning of individual teachers, continues to influence the teacher education landscape. Interestingly, the 8th principle is "Research: accreditation generates and relies upon a strong research base that informs program design and delivery, and informs the continual improvement of teacher education programs by providers" (TEMAG, 2014, p. xii)—a statement that implies a dual responsibility between policy makers and teacher educators for what constitutes quality ITE.

However, a number of government-imposed measures have been introduced, with the strength of the research base being questionable, marking unprecedented interference of national policies in the qualification requirements of ITE programs. From July 2016 all primary and secondary pre-service teachers have been required to pass an online *Literacy and Numeracy Test for Initial Teacher Education*, known as LANTITE, before they are 'allowed' to graduate from their degree (Australian Government, Department of Education and Training Education, 2015; Services Australia/AITSL, 2015/2018). Also, the introduction of the *Teaching Performance Assessment* (TPA) now requires ITE providers to build into their programs and final Professional Experience placements, supports for pre-service teachers to appropriately complete this externally imposed assessment. Although TPA is a new 'teacher registration requirement', it becomes, like passing LANTITE, a quasi-graduation-requirement for the teaching degree. While some policy-driven changes may be viewed as impositions by some ITE institutions, other institutions have seized particular directives as opportunities for innovation and research. For example, one recommendation arising from the TEMAG (2014) report was creation of primary curriculum specialisations. This has allowed mathematics educators to forefront the quality of pre-service teachers (as opposed to lack of quality) through the creation of a *Primary Mathematics Specialisation* in their programs and opened a new area for research.

2.2 New Zealand Context

New Zealand has seen a more recent wave of policy directions impacting tertiary institutions, and more specifically, ITE providers. Notably, *The Education (Tertiary Education and Other Matters) Amendment Act 2018* is intended to "… increase provider accountability and strengthen monitoring and compliance" (NZ Government, 2018). In December 2018 the Teaching Council of Aotearoa New Zealand, and New Zealand Qualifications Authority (NZQA) released the *ITE Programme Approval, Monitoring and Review Requirements*, for initial implementation mid 2019. The program requirements are accompanied by *Graduating Teacher Standards* (Teaching Council, 2018), developed through consultation with the education community. Interestingly, at around the same time, the report on *Pedagogy and Student Performance in Primary School Mathematics* was released (Education Review Office, 2018). So, in New Zealand we see multi-layered centralised reforms and accountability beginning to flow through the ITE space, perhaps forecasting a wave of associated research in the next few years.

2.3 Singapore Context

For the last decade Singapore has ranked highly in a range of international benchmark student assessments (TIMMS, PISA, International Baccalaureate). The success has been attributed to high teacher quality, together with factors such as a centralised system, education reform and high levels of funding (Deng & Gopinathan, 2016; Loh & Hu, 2019). The Singaporean education system, along with other high performing systems has attracted the attention of governments and policy-makers from other countries seeking to increase education performance. However, such success can also bring tensions within the system. Recently concerns have been raised about the decline in ITE intakes (apparently due to a stable workforce), yet with an increase in attrition rate of early-career teachers; with a call for teacher education programs to equip pre-service teachers "…with the knowledge of and strategies for circumventing, or better still resisting, the politics of performativity in an environment of educational neoliberalism" (Loh & Hu, 2019, p. 14). Some researchers have cautioned against hasty 'policy borrowing' by other countries, as a closer look at pedagogy at the school classroom level in Singapore tends to reveal direct instruction approaches that may not be compatible with contemporary directions in education (Deng & Gopinathan, 2016). Such research directs our attention to the international Conceived Space, where the Organisation for Economic Co-operation and Development (OECD) has become a powerful player in influencing education policy through its far-reaching data collection and publication of international test and survey results.

2.4 The Conceived Space as a Research Context

Very few mathematics education researchers operate inside the Conceived Space, and there have been calls for educators to make greater efforts to gather the type of evidence needed to more strongly inform the policy that impacts their work, particularly in Australia (Rowan et al., 2015; Sleeter, 2014). The following sections of Perceived Space and Lived Space present research that is inevitably influenced, directly and indirectly, by the policies generated in the Conceived Space. While only a portion of research responds explicitly to policy-driven activities (such as the Australian assessment, LANTITE), the context in which much of the research takes place is nevertheless shaped by such policies.

3 ITE Educators—The Perceived Space

Research in the Perceived Space addresses arrangements, actions and innovations designed to enhance pre-service teachers' experiences of learning to teach mathematics. While there is some overlap with literature on the Lived Space of the pre-service teachers themselves, the Perceived Space is focused on the perspectives of tertiary educators and the ways in which they interpret the policies and ‘messages’ from the Conceived Space and translate them into curriculum and teaching practices for their pre-service teachers. In both their teaching and research, teacher educators seek to reconcile established theories and contemporary research in the mathematics education field with the directives, constraints (and opportunities) imposed by the policies from the Conceived Space, while also attempting to address the needs of their cohorts of pre-service teachers.

Our critical review of the literature in the Perceived Space yielded three key themes: (1) teacher educators’ knowledge, learning and the nature of their work; (2) teacher education pedagogies; (3) the curriculum of teacher education. The Perceived Space section has been organised under these three themes. The studies reviewed largely report on teacher education experiences and programs within a single setting. Many involve research that has been designed to enhance the capabilities of pre-service teachers in response to TEMAG, (2014). More studies have focused on primary pre-service teachers than secondary teacher education.

3.1 Teacher Educators: Their Knowledge, Learning and the Nature of Their Work

Teacher educators themselves are less often the focus of research than the pedagogies they use or the curriculum they deliver. However, in recent years such research has increased, so in this section we explore the implications of research that reveals the complexity of knowledge and skills required by mathematics teacher educators to be effective practitioners in their ongoing curriculum development and teaching work.

Partly in response to the TEMAG (2014) calls for increased accountability of teacher educators for the quality of their graduates, Chick and Beswick (2018) and Muir, Wells, and Chick (2017) explored the concept of pedagogical content knowledge for mathematics teacher educators (MTEPCK). MTEPCK describes the knowledge of content and pedagogy (both pedagogy for teachers, and pedagogy for preparing teachers) that teacher educators employ when teaching preservice teachers. Chick and Beswick (2018) provided an extensive table that shows the relationship between pedagogical content knowledge (PCK) for school mathematics teachers (SMTPCK) and MTEPCK, highlighting the meta-level ‘teaching about teaching’ that teacher educators do as they are simultaneously teaching the pre-service teachers in front of them. Muir et al. used Rowland, Huckstep, and Thwaite’s (2005) ‘knowledge quartet’ to explore episodes from mathematics teacher education classes, to see if similar types of knowledge were used in teacher education classes to those used in primary or secondary classrooms. The knowledge quartet framework proved to be useful for understanding the PCK of mathematics teacher educators, and allowed the researchers to identify strategies (such as talk-aloud techniques) and additional content (such as knowing how to teach PCK as well as how to teach mathematics content) that were evident in the work of mathematics teacher educators.

The following cluster of studies investigated the professional learning of mathematics educators and the evolving nature of their work. The studies are further connected by the common thread of teacher educators collaborating with others, across institutions, discipline boundaries, specialist areas of teacher education or sectors of education. The findings of these studies carry implications for the nature of teacher educators' work, teacher educator identity and the skills and knowledge they need to be effective.

Some large-scale government funded projects encouraging interdisciplinary collaborations have provided opportunities for mathematics educators to engage in 'boundary-crossing' research and to learn from their colleagues as they do the work of curriculum design. Such projects serve to expand the scope of the Perceived Space to encompass academics from beyond teacher education departments, and in some cases, cross over into the Lived Space of the pre-service teachers. As part of the multi-institutional *Inspiring Mathematics and Science in Teacher Education* (IMSITE) project, Goos and Bennison (2018) explored the role of teacher educators as boundary crossers and brokers, as they worked with mathematicians to develop mathematics education courses to better integrate content taught in the university's mathematics department with mathematics pedagogy in the education department. An important finding of this study was that, "Theorising interdisciplinary collaboration in terms of communities and boundary practices makes it possible to conceptualise the boundaries between disciplines as sociocultural differences that are generative of new practices—and, therefore, new learning" (Goos & Bennison, 2018, p. 273). In a different institution, the researchers similarly concluded that interdisciplinary collaboration allowed "…teams to preclude, manage, and respond to challenges in ways that can ultimately enhance pre-service teacher education." (Butler et al., 2019, p. 87). At a third university, the formation of a community of practice involving mathematicians, mathematics educators, pre-service teachers and in-service teachers, led to the suggestion that such a community may support the retention of pre-service teachers and contribute to their growth of teacher identity (Tully, Poladian, & Anderson, 2017).

Within a different large project, *Enhancing the Training of Mathematics and Science Teachers*, the *Opening Real Science* project utilised interdisciplinary collaboration among mathematics and science educators, mathematicians and scientists to design a substantial online learning module focused on mathematical modelling for pre-service teachers (Geiger et al., 2018). The project team successfully utilised a collaborative and emergent design process that incorporated the Numeracy Model for the 21st Century (Geiger, Forgasz, & Goos, 2015) and the 5Es science instructional model (Bybee, 2009) to produce a series of contextualised cases, under the theme of *Modelling the present: Predicting the future*.

The participants in the IMSITE and *Opening Real Science* studies outlined in the previous paragraphs, were taking the role of 'curriculum workers', an aspect of teacher educators' work explored by Bragg and Lang (2018). These researchers used self-study in a 'collaborative peer learning team' of two, to iteratively develop an assessment task for mathematics pre-service teachers. This was also a collaboration that crossed boundaries, as Lang provided expertise in assessment while Bragg was the mathematics teacher educator.

In the New Zealand project *Learning the Work of Ambitious Mathematics Teaching*, researchers focused on practice-based strategies to promote inquiry practices in their pre-service teachers, using a reflective 'rehearsal' approach and collaborations with schools (Anthony, 2018). Under the same project, Anthony, Averill, and Drake (2018), focused specifically on the teacher-educator learning that occurred as a group of teacher educators across two institutions worked together to implement pedagogies of practice with their pre-service teachers. (Further discussion of these pedagogies occurs in the next section of this chapter). Their design-based study gave a rare in-depth account of teacher educator professional learning, as they unfolded the act of teaching using rehearsals and discussions of practice as a way to open practice-based spaces in ITE settings. A key finding from this work was that learning from and with pre-service teachers through exploring practice-based pedagogies opened up new possibilities for being a teacher educator.

In a different form of collaboration, Downton, Muir, and Livy (2018) discussed how co-teaching pre-service teachers with a colleague from a school resulted in teacher educator professional learning, giving further evidence that new ways of working with others can lead to development and learning. Sellings and Brandenburg (2018) took a different approach to educators' learning by outlining an initiative that collected and used data about preservice teachers' mathematics and confidence to shape mathematics teacher education courses. They recommended that teacher educators use 'data praxis' to inform their work, implying that mathematics teacher educators will need skills in data analysis and data use.

3.2 Teacher Education Pedagogies

This section reports a range of studies of the Perceived Space of ITE that present, analyse and discuss innovative teacher education pedagogies. The studies addressed ways to prepare pre-service teachers to be effective teachers of mathematics, providing descriptions of key features, and, in most cases, data about the impact of the approaches on pre-service teacher learning. Work on teacher education pedagogies in the review period can be clustered into three categories: (a) creating approximations of practice or authentic experiences in ITE settings, (b) building pre-service teachers' identities as teachers of mathematics, (c) using digital technologies to support pre-service teacher learning.

In recent years there has been an increase in practice-based pedagogies that support pre-service teachers learning to respond to the complexity and unpredictability of teaching (Anthony, Averill, & Drake, 2018). Typically, practice-based approaches include a representation of teaching (e.g., a video or scenario), a decomposition of practice, and an approximation of practice such as a 'rehearsal' with either peers or children; and are often designed to promote an inquiry stance to teaching practice (Anthony, 2018; Cochran-Smith & Lytle, 2009, Grossman, Hammerness, & McDonald, 2009).

An example of approximation of practice is using rehearsals—where pre-service teachers teach their colleagues, and the teacher educator orchestrates a meta-level discussion of their teaching (Anthony, Averill, & Drake, 2018). Work within the *Learning the Work of Ambitious Mathematics Teaching* project in New Zealand also used rehearsals of practice by pre-service teachers with small groups of 9–11 year-olds, with the intention of developing an ongoing inquiry stance towards practice (Anthony, 2018). Anthony, Averill, et al. (2018) used rehearsals with pre-service teachers to develop skills in questioning, conducting discussion and noticing critical features of school students' contributions. In particular, this research team have focused on the teacher educator's use of questions in rehearsals to co-construct understandings with the pre-service teachers (Averill, Drake, Anderson, & Anthony, 2016). Focusing on productive discourse techniques, Wright (2017) explored the benefits and difficulties of using defined *Talk Moves* in tutorial classes with pre-services teachers, as a means to simultaneously develop their knowledge for teaching and rehearse their pedagogy for classroom teaching. Although reporting generally positive outcomes, Wright (2017) highlights the need for the pre-service teachers to have sufficient classroom experience to be able to productively role-play young students in mathematical discussions. A New Zealand study explored first-year teachers' uptake of problem-solving approaches and investigated which elements of pre-service teachers' experiences contributed to their persisting (or not) with teaching in this way (Bailey, 2018). It was found that experiencing problem solving as learners in ITE course contributed to the pre-service teachers' use of problem solving in their subsequent teaching.

Other researchers have investigated different approaches to 'bringing the classroom' into ITE coursework. Working across three universities, researcher/educators used a videoed Year 6 mathematics lesson delivered by an experienced teacher as an authentic learning stimulus for their pre-service teachers (Watters, Diezmann, & Dao, 2018). The university students observed, discussed and debated practices that were evident in the video, with the research data suggesting that such practices afford insights "into how theory and practice are intertwined" (Watters, Diezmann, & Dao, 2018, p. 249). Co-teaching with a practicing primary teacher on campus was shown to facilitate meaningful mathematical discourse and assist in connecting theory and practice (Downton, Muir, & Livy, 2018). Another example of approximation of practice was the use of authentic samples of school-student work to activate and develop pre-service teachers' knowledge of early multiplicative thinking (Livy, Downton, & Muir, 2017).

While approximations of practice can serve to prepare pre-service teachers for the teaching roles, educators often employed pedagogies more specifically designed to support the 'identity' formation of their students—based on the premise that pre-service teachers' teaching of mathematics rests on their identity as teachers of mathematics. Several studies addressed how teacher education can build strong identity through various approaches. For example, Axelsen, Galligan, and Woolcott (2017) offered nine secondary pre-service teachers an additional learning opportunity with in-school mentors to enhance their confidence, based on the Enhancement, Learning, Reflection (ELR) process (developed through the collaborative efforts of six universities). The results of the study reveal the potential of the ELR process to be used as a teaching method that can build the confidence and self-efficacy of pre-service teachers. Other research has focused more specifically on affective responses and mathematics anxiety as factors in identity development (Marshman, Galligan, Axelsen, Woolcott, & Whannell, 2018; Perkins, 2016; Wilson, 2016b). These studies utilised a variety of structured experiences and reflective techniques to allow both educators and pre-service teachers to better understand the relationships between emotional responses to critical moments and subsequent performances in university classrooms or while teaching in schools.

Another group of studies pertaining to teacher educators' pedagogies are those that focused on the use of digital technologies to support pre-service teacher learning. These studies, like much of the research situated in the Perceived Space, tend to have a dual purpose—one directed at developing pre-service teachers' knowledge, and the other at translating the knowledge into teaching practices. Such studies tend to be based on the premise that providing authentic, personalised and collaborative learning experiences using mobile technologies within their course, would not only assist the pre-service teachers in engaging with their own learning, but also in realising how mobile technologies can open up mathematics learning opportunities in primary school classrooms (Schuck, 2016). An example of such a pedagogical choice is the immersion of pre-service teachers in a cylindrical 3D virtual environment which developed their personal competence in spatial reasoning and also impacted on their ability to develop questions that would lead to rich teaching activities having spatial reasoning components (Marshman, Woolcott, & Dole, 2017). In another study, the digital application, *Slowmation,* was used as a learning and teaching tool, with pre-service teachers required to construct "… a three-to-four minute video animation using a series of digital still images to demonstrate an understanding of a scientific or mathematical concept" (Paige, Bentley, & Dobson, 2016, p. 1).

3.3 Curriculum of Mathematics Teacher Education

Teacher educators continue to be challenged to design a curriculum that meaningfully integrates content and pedagogical approaches that foster pre-service teachers' mathematical knowledge for the twenty-first century. In designing, implementing and evaluating their curriculum, teacher educators respond to political agendas and policies (often linked to funding opportunities for innovations and research), established and emerging research findings, and the needs of their student cohorts.

In a response to the TEMAG report (2014) some teacher education programs in Australia now offer subjects that prepare pre-service teachers to specialise in primary mathematics teaching, while maintaining a generalist teaching qualification. Questions have been raised about the contrasting expectations of school principals and leaders and policy standards for these graduates—and about how to build the pre-service teachers' skills to meet these expectations (McMaster, Way, Bobis, & Beswick, 2018).

In 2015, the Australian government decided that all graduating teachers would need to pass a national literacy and numeracy test, LANTITE. About 95% of pre-service teachers pass these tests, but little is known about the specific difficulties faced by the remaining 5% in coping with the nature and content of the tests. ITE providers have responded to the situation in various ways, depending on the needs of their own pre-service teachers. One university reported success in building the confidence and self-efficacy of a group of pre-service teachers who self-nominated to work together to improve their numeracy (Brown & O'Keeffe, 2016). The researchers' role was to provide appropriate material and supportive feedback and facilitate knowledge-sharing between group members. Another university used the announcement of the national numeracy test and the incorporation of numeracy as a 'general capability' within the Australian Curriculum, as impetus to introduce a new compulsory unit of study into their Master of Teaching program for all primary and secondary school teachers (Forgasz & Hall, 2019; Forgasz, Leder, & Hall, 2017). Pre- and post-unit questionnaires and interviews showed an increase in participants' confidence in their abilities to incorporate numeracy into their various areas of teaching. In addition, the pre-service teachers became more aware of the differences between numeracy and mathematics, the numeracy demands placed on teachers outside the classroom, and the potential for cross-curricular learning experiences to stimulate engagement in challenging mathematics.

The mathematical knowledge of pre-service teachers has been a focus of research over many years. During the period of 2016–2019 researchers have attended to features of curriculum and course delivery that may influence the development of mathematical content knowledge (MCK) and pedagogical content knowledge (PCK) and other aspects of pre-service teachers' development as teachers of mathematics. In contrast to the Australian situation of an imposed *Numeracy* test, a *Mathematics* mastery test is embedded within the Singaporean secondary ITE program (Tay, Ho, Cheng, & Shutler, 2019). The inclusion of the test was a response to a need to raise the MCK levels of pre-service teachers, who have three opportunities to reach mastery level, each time triggering self-study expectations or additional course components to 'upgrade' their MCK. The majority of students passed within two attempts and very few required the additional study triggered by a third attempt (Tay, Ho, Cheng, & Shutler, 2019). The researchers emphasised that the process is intended to be supportive of teacher development rather than punitive.

In a study comparing the mathematical content knowledge of pre-service primary school teachers in Australia and China, Norton and Zhang (2018b) found that the Australian pre-service teachers commonly struggled with the mathematical content they were required to teach. They documented that Chinese teachers spend a larger portion of their 4-year undergraduate degree learning to teach mathematics, whereas in Australia there is a greater emphasis on generic skills than on a mathematics-specific pedagogy. One conclusion was that Australian teachers may not have sufficient time to deepen their MCK and connect it with PCK prior to commencing as a teacher

(Norton & Zhang, 2018a). Curriculum for secondary mathematics teacher education often includes mathematics courses taught by mathematicians and pedagogy courses taught by mathematics educators. The findings of several studies have suggested that that greater coherence or integration of the curricula of these two types of course would benefit pre-service teachers (Marshman & Goos, 2018; Norton, 2018; Seen, Fraser, Beswick, Penson, & Whannell, 2016). Livy, Herbert, and Vale (2019) conducted a longitudinal study that identified how pre-service teachers' program structure (rather than brief interventions), including course work and practicum, can influence development of pre-service teachers' learner identity, build teacher identity and develop their mathematical content knowledge. A clear message from the studies reported here is that ITE educators should critically review the larger structures of curriculum to maximise the productive influence on pre-service teachers' development of knowledge and teacher identity.

Some of the most challenging curriculum work of mathematics educators arises in online or blended-learning (mixture of face-to-face and online) environments. Transactional Distance Theory (Moore, 2007) was used as an epistemological framework for designing a new blended learning primary mathematics course across three campuses of the same university (Larkin, 2016). Through providing high levels of structure and high levels of dialogue, the course engaged pre-service teachers and developed their MCK and PCK, and self-confidence (Larkin, 2016; see also Chap. 13, this Volume).

The need for educators to look more deeply into the apparent knowledge of their students and consider how to address their needs in coursework design appears to be a widespread issue. Toh (2017) was concerned about the MCK of pre-service secondary school teachers in Singapore. Although they perform well in International comparative studies and are highly efficient in algebraic manipulation, he identified topics such as decimal representation in which they did not perform so well, and found that they generally lacked a deep understanding of mathematical proofs. A comparative study across the USA, Australia and South Korea investigated how pre-service secondary mathematics teachers (PSMT) conceived of proof and proof teaching (Lesseig, Hine, Na, & Boardman, 2019). One of their key findings was that teacher educators should "… design content and activities that build on PSMTs' strengths and enable PSMTs to productively coordinate disciplinary components of proof (most often learned in mathematics content courses) with pedagogical knowledge of proof, such as developmentally appropriate representations" (p. 414).

3.4 *The Perceived Space as a Research Space*

In the analysis of research pertaining to the work of ITE educators we see the positive outcomes that can flow from the funding for large inter-institutional and interdisciplinary projects that often accompanies new government policies in Australia coming from the Conceived Space. Numerous smaller studies have probed deeply into local contexts and innovative practices. The collection of research provides insight into the evolving roles of the mathematics educators as collaborators and ‘boundary-crossers’, pedagogical innovators, curriculum designers, and above all, learners. A number of studies have sought to understand and learn from the student’s experiences, thereby firmly bridging the Perceived Space of mathematics educator and the Lived Space of their pre-service teachers.

4 The Student Perspective—Lived Space

The Lived Space refers to the experiences of pre-service teachers in their initial teacher education programs, and concerns their perspectives on their professional experience activities, their university studies, and their experience of learning mathematics. An interesting tension arises in the ITE research within and about the Lived Space of pre-service teachers. As the research is usually conducted by the mathematics educators themselves, it is strongly influenced by their own perspectives and motivations. The findings of the research on student experiences are often used to reflect on aspects of the educators’ work in the Perceived Space. While this overlap cannot be discounted, in our selection of literature for the Lived Space section of this chapter, we have tried to focus on studies that have used methods that substantively seek the pre-service teacher’s voice. The focus of the selected studies (or sometimes parts of studies) is to better understand the lived experiences of pre-service teachers in, (a) their coursework, (b) their professional experience placements in schools, and, (c) their learning of mathematics in general.

4.1 Coursework Experiences

Research has investigated the impact of course design on pre-service teacher learning. ITE programs often include a blend of online and face-to-face components. Strang and Larkin (2018) conducted a five-year study in primary teacher education programs at two university campuses to investigate how the mode of delivery influenced student success. Results suggest that delivery mode was not a critical factor. Instead, it was more important that academics focused on how their units were structured to promote development of pre-service teachers’ MCK and PCK, and on building rapport with students. Research on the development of pre-service teachers’ MCK and PCK shows that these two types of knowledge are related and can be developed simultaneously. A deep, connected knowledge of mathematics is necessary for teachers to be able to facilitate mathematically constructive classroom discourse and more advanced problem-solving and reasoning (Norton & Zhang, 2018a, 2018b).

The challenges of developing pre-service teachers’ pedagogical practices within the university setting was investigated by Livy et al. (2017). They reported on the experiences of two cohorts of pre-service teachers as they participated in tutorial classes designed to assist them to notice and discuss Year 2 students’ responses to an array task. The authors analysed how the pre-service teachers selected and sequenced work samples from five children. Results indicated that the use of children's work samples supported pre-service teachers to learn about teaching, but that the pre-service teachers' own foundation knowledge of mathematics was crucial in enabling them to analyse students' understandings and sequence student work samples to support a key mathematical idea (Livy et al. 2017).

Several studies have investigated the difficulties that pre-service teachers can encounter when attempting to adopt a problem-solving approach in their teaching practice. In teaching problem-solving lessons, pre-service teachers' own experiences of problem-solving impact on their skill in creating instructional contexts that provide their students with engaging, challenging tasks, and with their ability to emphasise mathematical thinking above the application of procedures (Sullivan, 2011). Berenger (2018) investigated the problem-solving strategies of 179 first year pre-service teachers and found that their solution processes relied largely on numerical procedures. He recommends that more attention be given to providing pre-service teachers with instructional approaches to improve their own knowledge in problem solving and to support their learning of heuristics. In Singapore, Kaur (2017) reported on the impact of such a course on the preparation of pre-service teachers in Singapore to teach problem solving to grades 7 and 8. Interviews with five pre-service teachers at the end of the course showed that it had deepened their own understanding through the problems they solved and they welcomed Polya's framework as a tool to help their future students. The course also included six micro-teaching sessions in which they were expected to teach using the strategies they had learnt in class, in particular, how to communicate mathematical knowledge.

Microteaching, and forms of practice-based learning, continue to be integral part of many initial teacher education programs, and some researchers have sought to better understand the perspective of pre-service teachers in these experiences. Murphy (2016) reported on a microteaching experience focused on teaching through collaborative group work for participants in a specialist primary mathematics program. Twenty-one pre-service teachers responded to an online survey and wrote personal reflections on their experiences. Murphy analysed the written accounts to identify the pre-service teachers' awareness 'in-the-moment' and how they noticed student learning. She found tensions between pre-service teachers' need to cover the content of the lesson and their desire to include more collaborative student talk. Murphy concluded that while it was important for pre-service teachers to know the content, they also needed to consider how their planned teaching strategies, or those which they recognised later in reflecting on the lesson, enabled them to elicit students' talk about mathematical concepts.

Seeking the perspectives of pre-service teachers who have been involved in new pedagogical practices as part of their coursework can provide valuable feedback for ITE educators, which can lead to enhancement of learning in further iterations of those practices. As part of the *Learning the Work of Ambitious Mathematics Teaching* project (see Sect. 3.2 of this chapter), the researchers collected reflective information from pre-service teachers through survey items, open-response questions and semi-structured interviews to gain an understanding of the benefits and difficulties of 'rehearsals and coaching' from the pre-service teachers' viewpoint, together with video analysis of the micro-teaching episodes (Averill et al., 2016). The majority of responses were positive and identified the aspects of the experiences that were most meaningful and beneficial for their learning. However, the pre-service teachers also highlighted variability in experiences across the different class groups and reinforced the "…importance for the coaching of rehearsals of respect, reciprocity, and sound relationships across the class—as well as between the coach and the presenter." (Averill et al., 2016, p. 498).

Other research has focused on assisting pre-service teachers to become more critical, knowledgeable, skilled and confident in using digital technologies. These studies typically employ the technological pedagogical content knowledge (TPACK) framework. For example, Handal, Campbell, Cavanagh, and Petocz (2016) used the TPACK model to investigate the perceptions of 373 primary pre-service teachers from three universities of apps designed for mathematics learning and teaching. Participants critiqued the educational value of three apps and their responses indicated a great deal of variability in how they rated them. The researchers concluded this was most likely because pre-service teachers were unfamiliar with basic principles of multimedia instructional design. Hāwera, Sharma, and Wright (2017) examined how 40 New Zealand pre-service teachers made decisions about which digital learning objects they would use for mathematics learning and teaching. Like Handal et al. (2016), Hāwera and colleagues reported that allowing pre-service teachers opportunities to explore and evaluate technology resources could support the development of TPACK expertise.

4.2 In School Experiences

Studies situated in professional experience contexts include research on how partnerships can bridge the gap between theoretical approaches presented in initial teacher education programs and the reality of classroom practice. One group of studies investigated how the experiences of the pre-service teachers relates to the further development of their mathematical content knowledge (MCK) (Hine & Thai, 2019; Livy, Vale, & Herbert, 2016; Wilson, 2016a), with some focused more strongly on factors influencing the enactment of existing MCK and PCK (Daniel, 2017; Gronow, Cavanagh, & Mulligan, 2019; Little & Anderson, 2016). The development of MCK and PCK was discussed earlier in this chapter in relation to curriculum structure and coursework. A notable difference in the research reviewed here is that the pre-service teachers are enacting their knowledge for teaching inside the world of schools rather than tertiary institutions. For example, Gronow et al. (2019) investigated how pre-service teachers made use of a framework for noticing mathematical structure (introduced in coursework) in their teaching during practicum. A second group of studies investigated the affective domain and the ways in which relationships between participants (and visitors) in the Lived Space could promote development as a teacher

(Axelsen et al., 2018; Cavanagh & McMaster, 2017; Hāwera & Taylor, 2017; Marshman et al., 2018; McMaster & Cavanagh, 2016). While all the studies highlighted the critical importance of in-school learning for pre-service teachers and shared a common goal of better understanding ITE through research, each study presented unique findings.

Livy et al. (2016) explored primary pre-service teachers' MCK during a practicum and found that the nature of the professional experience placement was an important factor in developing their MCK. In particular, professional experiences with upper-primary classes alongside concurrent primary mathematics units at the university enabled participants to learn more mathematics. The role of the mentor teacher was also crucial in enhancing the quality of pre-service teachers' learning experiences by providing guidance on lesson planning, offering constructive feedback, and being a good role model when teaching lessons of their own.

Wilson (2016a) investigated how one secondary pre-service teacher's MCK developed during a four-week professional experience placement. Data comprised three of the pre-service teacher's mathematics lesson plans, her lesson notes about the content, and an interview that took place five weeks after the completion of the placement. Wilson (2016a) noted the importance of MCK for teaching mathematics and concluded that lesson planning could provide a useful way for pre-service teachers to learn mathematical content prior to teaching it. This 'preparation phase' included detailed notes and diagrams which helped to improve her content knowledge and became a useful resource for her future teaching. Daniel (2017) explored how the learning goals of six secondary mathematics pre-service teachers influenced their MCK by observing ten algebra lessons and interviewing the participants. She found that procedural knowledge dominated teacher talk and written work, limiting pre-service teachers' ability to identify the underlying algebraic concepts and support their students' understanding. Rather than analysing aspects of planning and enactment of MCK, Hine and Thai (2019) investigated secondary pre-service teachers' perceptions of their own readiness to teach mathematics, before and after their practicum. A key finding was that participants' confidence in their MCK for teaching lower-secondary mathematics was greater than their confidence for teaching senior mathematics content.

The factors which influence how secondary mathematics pre-service teachers implement problem solving were examined by Little and Anderson (2016). They reported that while most participants expressed a willingness to use problem-solving tasks in their lessons, their ability to do so was constrained by their perceptions of the students' mathematical abilities and the additional preparation time to plan these tasks. Another crucial factor was the degree to which the mentor teacher understood the reform-oriented goals of the teacher education program and whether mentors possessed the knowledge and skills to support the implementation of problem-solving tasks in mathematics lessons. A different perspective of the implementation of problem-solving was provided by the research of McMaster and Cavanagh (2016) and Cavanagh and McMaster (2017). They examined a professional experience learning community where four primary pre-service teachers engaged in co-planning and co-teaching mathematics lessons, as well as peer observation and shared reflection.

The learning community was effective in building trust among the pre-service teachers and the mentor teacher's supportive role enabled them to successfully implement a series of problem-solving lessons. The need for a 'supportive bridge' between course work and classroom teaching was also a finding of the Hāwera and Taylor (2017) study, that investigated six Māori pre-service teachers' views in a focus group discussion about factors that helped them link their university and practicum experiences. Participants valued working with a lecturer who was proficient in the indigenous language and who could offer support for teaching in Māori medium contexts. Navigating the tensions in their relations with their mentor teachers was highlighted by participants as a means of strengthening school-university partnerships.

Another important issue related to in-school experiences is how pre-service teachers develop the capacity to become reflective practitioners. Axelsen and colleagues (2018) and Marshman et al. (2018) used critical incidents as a catalyst for pre-service teachers to learn about teaching. The authors used an Enhancement-Learning-Reflection (ELR) process to guide pre-service teachers' reflections on their teaching experiences and help them consider the types of emotions they experienced as they chose critical moments from their own lessons to consider. An analysis of these critical moments found participants' often expressed anxiety when reviewing their teaching of mathematics, but that peer and other observers were less likely to detect it. The authors concluded that the ELR intervention helped pre-service teachers gain a more realistic impression of their teaching and realise that it was often better than they supposed it to be.

4.3 Experiences of Learning Mathematics

Much of the research on affective aspects of pre-service teachers' development has investigated their attitudes to mathematics and how this can be improved (See also Chap. 7, this Volume). Mathematics anxiety has often been reported, particularly for prospective primary teachers. Itter and Meyers (2017) analysed 150 pre-service teachers' written reflections about their attitudes to mathematics and found that nearly three-quarters of the participants acknowledged negative or ambivalent attitudes to mathematics. The authors advocate for learning activities that allow pre-service teachers to engage in mathematical investigations designed to promote conceptual understanding. Sanders, Nielsen, Sandison, and Forrester (2019) explored mathematics-anxious pre-service primary teachers' perspectives of collaboratively solving mathematical tasks in a whiteboard room. The public arrangement of whiteboards in the classroom and the participatory nature of solving problems on these whiteboards can reduce the anxious students' typical avoidance patterns and encourage them to engage more in mathematics learning.

Marshman et al. (2018) examined how pre-service teachers identified their emotions as they viewed video of critical moments from their lessons. The analysis of affect was an important means of increasing pre-service teachers' pedagogical confidence. Yeigh et al. (2016) also suggest issues related to affect become more prominent in reflections on teaching and found that pre-service teachers' emotional awareness can support their pedagogical confidence. Tran and Javed (2017) considered the impact of two first-year, undergraduate numeracy units in improving non-traditional pathway pre-service teachers' attitudes towards mathematics. The authors reported a growth mindset among the majority of their sample. These students believed that they could improve their mathematical understanding if they worked hard. This was attributed to explicit teaching about the development of a growth mindset and supporting structures enabling them to do well in their first mathematics unit.

Pre-service teachers can develop more positive attitudes to mathematics when limitations in their content knowledge are addressed and when explicit reference to the importance of affect is made throughout teacher education programs (Ingram, Linsell, & Offen, 2018). Ingram et al. examined the MCK of 83 primary pre-service teachers throughout a three-year initial teacher education program. At the start of the program, many of the participants had MCK levels below the desired standard and held negative attitudes towards mathematics. The pre-service teachers became more positive about mathematics as their teaching identity grew, along with their confidence to teach mathematics.

4.4 The Lived Space as a Research Space

Very few studies have been specifically designed to deeply explore the experiences of pre-service teachers from their own perspectives, without the additional goals of evaluating the practices of teacher educators. The synergy between the practice of the mathematics educators and the experiences of their preservice-teacher students is a natural and necessary part of teaching and learning. However, in our attempt to make some distinction between research located in the Perceived Space and research located in the Lived Space, we have highlighted the difference between collecting data to 'measure' characteristics of pre-service teachers, and using approaches that seek to understand pre-service teachers' experiences from their own perspective. Numerous studies have provided in-depth examples of the experiences of small groups of pre-service in unique contexts, and these offer ITE valuable insights into a range of characteristics of specific groups of pre-service teachers. However, to significantly advance our understanding of ITE perhaps more longitudinal studies with rich data drawn directly from pre-service-teachers are needed across a range of institutions.

5 Conclusions

In the opening of this chapter we stated our goal of exploring the influences on, and tensions within, initial teacher education, and to identify the research outcomes that might advance our understanding of learning and practices in this context. We also acknowledged the call of previous authors for mathematics educators to build a strong evidence-base for practice in initial teacher education, to inform debate around current issues and policy directions. Applying the lens of the tri-socio-spatial approach to the diverse body of studies about ITE allowed us to see how the intersection between spaces of policy, ITE programs and pre-service teachers' experiences provide rich research and learning contexts. The mathematics educators in their multiple roles as researchers, teachers, collaborators and curriculum designers mediate the tensions that occur between the spaces. One such tension that has long existed between the Perceived Space and Lived Space, is the relationship between the theory and practice of teaching. Building on the emerging body of research prior to 2016, mathematics educators have continued to explore a range of practice-based pedagogies that engage pre-service teachers in quasi-authentic teaching experiences that allow both educators and students to enact, analysis and reflect on reform-based teaching practices, such as challenging tasks and problem-solving, or productive mathematical discourse. However, there are very few studies that move into the Lived Space of the pre-service teacher in their professional experience placements in schools, to investigate the tensions they experience in this 'high stakes' performance situation as they attempt to enact contemporary pedagogies in real mathematics classrooms. This boundary is a difficult one to cross.

It is notable that research that examines the changing roles of the mathematics educator as an innovative and reflective practitioner has continued to build. However, it is interesting to note that such research tended to be in relation to the interface between the educator and the pre-service teacher, rather than about more holistic identity formation that encompasses the multiple roles of the mathematics educator, including the role of policy-informer. Further to this point, our analysis of the research highlighted the contrast between the testing and accountability agendas of governments (and perhaps the OECD) and the reform-based approaches to education favoured by teacher educators and researchers. Our final observation takes the form of a question to fellow mathematics educators: In our constant quest to *improve* ITE—through curriculum, pedagogy, and practices to enhance the learning of pre-service teachers—is our research unintentionally reinforcing the deficit view of teacher quality emanating from the policy and politics space?

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Chapter 6
Teachers' Professional Learning and Development in Mathematics Education

Janette Bobis, Berinderjeet Kaur, Katherin Cartwright, and Lisa Darragh

Abstract In this chapter we review Australasian research surrounding practising teachers' professional learning and development in mathematics education. We adopt a broad view of teacher professional learning by considering all aspects of teacher learning and development as well as publications that sought to conceptualise and theorise teacher learning. To help structure the chapter, we developed a framework that allowed us to interrogate each report of teacher learning in a systematic way according to six components: purpose; context and learning focus; models; theories of learning; scope, scale and duration; and impact. Discussion of the literature according to this framework assisted in the identification of particular patterns and trends in teacher professional learning and development research in Australasia in the period 2016–2019. Drawing upon our discussion of research literature, the concluding section draws together the most prevalent aspects of professional learning research in the current review period, makes comparisons with what was reported in previous reviews and provides thoughts about where future research in teacher professional learning in mathematics education might develop.

Keywords Teacher learning · Professional development · Learning theories · Mathematics teachers · Practising teachers · Quality professional learning

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1 Introduction

In this chapter we focus on research concerning the professional learning and development of practising teachers. The preferred terminology to describe the activities and experiences teachers undertake to improve their knowledge, beliefs, and practices has changed over the years with the use of terms such as inservice training and professional development (PD) being increasingly superseded by the preferred generic term of professional learning (PL). In their chapter on practicing teachers' education and development in the 2012–2015 review, Beswick, Anderson, and Hurst (2016) referred to learning as the "process by which teachers acquire new knowledge, skills, affects or behaviours" and therefore "a means by which teachers develop" (p. 330). PL is a more accurate term to reflect the key characteristics of reflective practice, critical thinking and continuing learning that is typical of what are considered to be effective or quality teacher learning experiences than the term PD. In reviewing particular studies in this chapter, we use the term adopted by the original authors. However, we acknowledge that there are times when teachers participate in professional activities that do not lead to any changes of beliefs, knowledge or practices and therefore cannot be considered PL.

Similar to Beswick et al. (2016), we adopt a broad view of teacher learning by considering not only all aspects of teacher development but also publications that sought to conceptualise and theorise teacher learning. Consequently, the research reviewed varied enormously, making the decision about chapter structure and sorting studies for discussion challenging. Previous reviews have focused on PL programme design features, such as scale (Anderson, Bobis, & Way, 2008) and others on subject matter focus (Beswick et al., 2016). Following a wide review of the literature that included reviews of PL conducted by others incorporating research beyond the remit for this chapter (e.g., Kennedy, 2016), we developed and refined a framework through our collaborative analysis and discussion. The emerging six categories of our analysis framework allowed us to interrogate each report of PL in a systematic way, thus assisting in the identification of particular patterns and trends in PL research in Australasia in the period 2016–2019. Most reports of research usually have more than one focus and can therefore be discussed according to a number of the framework's categories. Hence, we chose to refer to aspects of outputs when relevant and to report in detail one or two publications under each category that exemplified the characteristics of that particular component of our framework. This structure necessitates that some reports feature more than once across the chapter but with a different aspect as the focus of discussion.

The chapter comprises four main sections. Following this introduction, the second section considers important issues identified in the literature surrounding effective and quality PL. Section 3 introduces the components of our analysis framework before using each component to structure our review of research. The final section highlights the issues and complexity surrounding teacher PL research and provides thoughts about where future research in this area might develop.

2 Quality and Effective Professional Learning

The predominant discourse in reviews of education, policy documents and education research literature positions quality teaching as key to good student learning (Organisation for Economic and Cooperation and Development [OECD], 2016; Australian Institute for Teaching and School Leadership [AITSL], 2018). A globally accepted means by which teaching quality can be enhanced is through effective teacher PL. This chapter was written from the premise that a deeper and better understanding of teacher learning and development is critical to improving teaching quality with the ultimate goal of enhanced student outcomes. Much of the research reviewed in this chapter stems from this premise and the idea of contributing to an evidence base that will inform the effective learning of teachers.

The characteristics of effective PL are well acknowledged in the literature with the list of key design elements remaining fairly consistent over the past few decades. Summaries of these characteristics typically include opportunities for active learning by teachers, extended timeframes, a shared purpose, informed by research, collaborative learning and research with other teachers (OECD, 2017). It was noted by Beswick et al. (2016) that PL reported by Australasian researchers in the period 2012–2015 consistently incorporated a variety of design features from the list. This practice was also noticed in the current review period. In their study of a large-scale and sustained PD project involving clusters of primary and secondary schools, Goos, Bennison, and Proffitt-White (2018) refer to previous research on effective PL as informing their framework for analysing the factors that contributed to the sustainability and scaling up of the initiative. Similarly, Anderson (2017) generally acknowledged the agreed upon features of effective PL used to design programmes of development for teachers as part of the STEM Teacher Enrichment Academy (see Chap. 3 for further details of STEM PL research). While explicitly acknowledging components of effective PL such as active participation and collaborative learning that were incorporated into an intervention designed to shift teachers' beliefs about student engagement in mathematics, Bobis, Way, Anderson, and Martin (2016) remind us that adherence to design elements on this list does not guarantee PL effectiveness.

Australasian and other researchers (e.g., Kennedy, 2016) are starting to question the generalisability of a 'one-size fits all' list of design elements or criteria to determine PL effectiveness. Beswick, Fraser, and Crowley (2017) clarify that for PL to be considered effective, it must achieve its intended goals. Given the difficulty often associated with measuring achievement of goals such as enhanced student learning, Beswick et al. (2017) contend "that quality PL according to these criteria is likely to be effective but effectiveness cannot be claimed in the absence of rigorous evaluative evidence" (p. 169). They argue that it is more useful to conceptualise PL in terms of quality and that PL incorporating these established characteristics is associated with high quality teacher learning. Endeavouring to identify teachers' perceptions of the characteristics of quality PL, Beswick et al. (2017) surveyed 109 Australian teachers—85% of whom were secondary mathematics teachers. Using an online questionnaire, teachers were asked to rate on a 5-point Likert scale the contribution of each of 16 features to the PL being the ‘best ever’ they had experienced. The 16 features were derived from the literature on effective (quality) PL. Contrary to the widely accepted list of design elements in the effectiveness literature, almost half of those surveyed nominated one-off PL as the best they had ever experienced. The researchers suggested that such infrequent PL may meet the needs of teachers particularly when they structure their own learning such that the infrequent but formal PL experiences concur with their informal learning experiences and together form a coherent programme of PL. Beswick et al. (2016) argued PL that ‘break the rules’ by not complying with established lists of effective PL characteristics justify the need for a more “nuanced approach to conceptualising the PL quality including in terms of its effectiveness that takes account of the aims and context of particular initiatives” (p. 348).

In Australia the current arrangements for teacher registration and the various associated accountability measures are framed by government agencies as mechanisms for ensuring the ongoing quality of the teaching workforce (Bahr & Mellor, 2016). At least 100 hours of PL over a five-year period are required for teachers to maintain their registration, the majority of which must be accredited by registered authorities. With registration compliance conceptualised as the number of PL hours, rather than the quality of the PL, it is conceivable that Australian teachers might be discouraged from engaging in more intensive long-term experiences and make choices based on the ease with which they can satisfy regulatory requirements rather than on their actual learning needs or practical value they might derive from the PL.

Like Australia, Singapore teachers must also undertake a set number of hours of PL that is funded by the Ministry of Education—100 hours per annum. Unlike Australia, Kaur and Wong (2017) note that the *Teach Less, Learn More* (TLLM) initiative of the Singapore Ministry of Education shifted the focus from quantity to quality learning for both teachers and their students. As part of the emphasis on sustained and quality teacher PL, teachers are provided with time to meet and undertake structured approaches to their PL (such as lesson study) during school hours. Kaur and Wong argue that the systematic approach to the professional development of all teachers in Singapore has created a culture of life-long learning in teachers who view the opportunity of PL as their “entitlement” (p. 99). Juxtaposing the two cases of Australia and Singapore with different mandated hours of PL, highlights the impact that context and culture can have on how mandatory requirements such as PL hours are perceived and enacted upon.

New Zealand, by contrast, does not set a certain number of PL hours for teachers. Rather, funding is allocated to schools or groups of schools (grouped as *Kahui Ako* or communities of learners). This enables the New Zealand government to set areas of national priority for PL. In 2017, one of three priority areas was mathematics, but at the time of writing this review, the priorities were being changed to much broader areas of learning that are not subject specific (see Ministry of Education New Zealand, n.d.). These changes may impact on the type of mathematics PL available to teachers in the next review period.

3 Professional Learning Initiatives

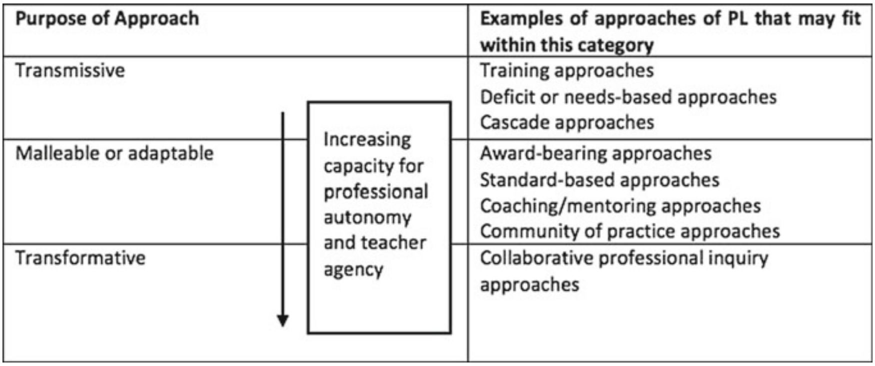
In this section we commence our analysis of PL initiatives reported in the Australasian research literature for the review period 2016–2019. Our analysis is structured according to the framework developed to assist achieving a systematic review process. The framework consists of six categories:

(i) *Purpose.* In this category we examine the broad approach to teacher learning adopted by PL initiatives that emanate from their key purpose. Approaches range from transmissive to more autonomous approaches of teacher learning.
(ii) *Context and learning focus*. Issues of context at global, national, local, and individual teacher levels are explored. Meanwhile, the focus of PL attends to the finer-grained aspects such as specific content areas of the curriculum (e.g., algebra) or particular issues impacting student learning (e.g., engagement).
(iii) *Models.* Prevalent models that researchers have used to represent the relationships between PL components and to guide their research are examined in this category.
(iv) *Theories of learning.* This category appraises the learning theories researchers used to guide their research and interpret their data.
(v) *Scope, scale and duration.* In this category we consider patterns and trends evident in PL research based on the scope, scale and duration of PL initiatives.
(vi) *Impact*. The final category examines the impact and outcomes reported by studies of PL using Guskey's (2016) five levels of data to organise the discussion.

3.1 Purpose of PL Approaches

Teacher PL occurs in many forms and is delivered for a range of purposes. However, the overarching purpose of most PL initiatives is to have an impact on teacher professional growth (Loong, Vale, Herbert, Bragg, & Widjaja, 2017). The approach taken to achieve such a purpose is often greatly influenced by how the process of teacher change is conceived by those who design and deliver the PL. We adapted Kennedy's (2014) spectrum of Continuing Professional Development (CPD) (represented in Fig. 6.1) to guide our initial grouping of PL approaches for discussion. Kennedy's spectrum was chosen because it reflects a continuum of change that is marked by an increase in teacher capacity for professional autonomy and agency.

Changes in teacher thinking about both their identity as teachers of mathematics, and their classroom practices, "depends on the individual and context" (Lomas, 2018, p. 495). This section addresses a shift in focus suggested by Lomas, where instead of stating whether a PL is effective or not, we move to discuss "*how* programs work in particular settings" (p. 495) (emphasis in original) with the ultimate goal of enhancing student outcomes. Reasons why an individual teacher, or group of



Purpose of Approach		Examples of approaches of PL that may fit within this category
Transmissive	Increasing capacity for professional autonomy and teacher agency	Training approaches Deficit or needs-based approaches Cascade approaches
Malleable or adaptable		Award-bearing approaches Standard-based approaches Coaching/mentoring approaches Community of practice approaches
Transformative		Collaborative professional inquiry approaches

Fig. 6.1 Spectrum of PL approaches adapted from Kennedy (2014)

teachers, participate in a particular PL programme or event are varied and are often influenced by who is driving the change in practice (for example, the teacher or the school). The purpose and ownership of participation in the PL can often affect the uptake and/or implementation of PL by teachers where an approach that is "easy to implement and results in positive student responses is crucial" (Carter, Cooper, & Anderson, 2016, p. 168) in convincing teachers of the value of the approach being utilised.

While Kennedy's (2014) original spectrum referred to 'models' of CPD, we use the term 'approaches' to convey the active nature of PL regarding both a teacher's choice of the type of PL they elect to attend, and the delivery style of the PL itself. The term also serves to distinguish between the overarching approach towards teacher learning from our definition and discussion of PL models in Sect. 3.3. The spectrum highlights that there are many approaches to PL, each of which has the potential to meet individual teachers' specific needs. For this reason, the spectrum does not preface one approach over another. Kennedy states:

> It is absolutely essential to acknowledge that no one individual model [or approach] of CPD on its own can be seen to support a particular purpose of CPD; rather, the categories are designed to help us analyse patterns and trends in our own CPD experiences as individuals and to analyse institution-wide and system-wide approaches. I am not suggesting that all CPD experiences must be transformative in nature, but rather that a transformative purpose, or orientation, will privilege particular models of CPD, while still acknowledging that some skills may well be best learned or refreshed through more transmissive approaches to learning. (p. 694)

The following subsections present examples from the literature and explores how they fit within each purpose of approach suggested in Fig. 6.1.

3.1.1 Transmissive Purpose

Transmissive approaches of PL are particularly efficient at introducing teachers to educational reforms such as new curricula content or policy. Content-specific PL that meets a teacher's immediate need is a vital element within a wider PL programme. In reviewing the literature there is an ever-increasing body of research being undertaken with the purpose of extending teachers' Mathematics Content Knowledge (MCK) (Hilton, Hilton, Dole, & Goos, 2016; Loong et al., 2017; Sullivan, 2018). Much of the research involved targeted professional learning on specific content areas, such as computational thinking (Bower, Wood, Lai, Howe, Lister, Masson, et al., 2017) and mathematical or proportional reasoning (Bragg, Herbert, Loong, Vale, & Widjaja, 2016; Hilton et al., 2016; Muir, Beswick, Callingham, & Jade, 2016; Sullivan, Holmes, Ingram, Linsell, Livy, & McCormack, 2016). In some circumstances these focus areas were selected based on teacher need, in others on student data. Either way, a common purpose across the research was to improve student outcomes, understanding that "the difficulties experienced by students in developing … foundational concepts have been attributed to the ways in which these areas are taught" (Hilton et al., 2016, p. 195).

These research projects may in the past have been attributed to a deficit approach within a transmissive purpose to develop deep knowledge in teachers around a specific area. However, they have generally not been undertaken based solely on an individual teacher's lack of MCK. The term *deficit* could possibly be rephrased as a *needs-based* approach that is more encompassing of teachers' collective needs. There is also evidence of cascade approaches (for example, train the trainer) for the purpose of addressing numeracy needs of students through the implementation of interventions that include elements of teacher PL. For example, the *YuMi Deadly Maths* programme reported on by Carter, Cooper, and Anderson (2016) "seek[s] to improve the capacity of teachers to equip every student with the mathematical knowledge needed" (p. 166). This programme utilised a 'train the trainer' model that involved a combination of centrally organised PL along with on-site shoulder-to-shoulder support for teachers. Although aspects of this programme, and other similar research projects, could fit within a transmissive purpose, there are elements that also align with a malleable or adaptable purpose when teachers themselves are involved with action-research or ongoing mentoring.

3.1.2 Malleable or Adaptable Purpose

Adaptable approaches allow increased teacher agency over the PL by blending elements of transmissive PL on building teacher knowledge, with more transformative practices that promote teachers as leaders within the school setting, resulting in higher self-efficacy (Proffitt-White, 2017). Although school or systemic student performance and assessment data may have been the catalyst for the PL, the flexibility and allowance for collegiality within the PL design led to professional growth of teachers where they experienced increased capacity for professional autonomy.

Classroom mentoring in the form of lesson planning (Davidson, 2017; Olteanu, 2017), using challenging tasks (Sullivan, 2018; Sullivan et al., 2016), conducting lesson observations (Russo & Hopkins, 2017) and viewing demonstration lessons (Loong et al., 2017) are illustrative of teachers taking an active role in PL decisions. Mentoring approaches draw strength from both transmissive and transformative purposes of PL to develop teachers' knowledge and pedagogies so future PL can be structured and individually tailored. In situ mentoring and community of practice approaches are becoming more prevalent with the purpose of "building a culture of teacher collaboration" (Kaur & Wong, 2017, p. 99). Kilpatrick and Fraser (2018) identify professional networks and learning communities as successful strategies for shared PL. Professional learning communities differ from communities of practice in that they focus on *learning* more so than *practice* and often build and change over time organically where learning 'branches-off' as further action-research or classroom inquiry is undertaken. Learning communities have a clearly transformative purpose and this approach may be better situated in the transformative category as evidenced in Singapore's Professional Learning Communities (PLC) where teachers engage in PL to suit their individual needs, and teachers work collaboratively to develop themselves through a number of ways (Kaur & Wong, 2017).

3.1.3 Transformative Purpose

Collaborative inquiry approaches aimed at building whole-school change were also evident in the review period. Anthony, Hunter, and Hunter's (2016) inquiry communities project was also inclusive of mentoring, making it similar to the programmes previously highlighted within an adaptable purpose. However, the focus on developing "relational agency" (p. 119) as part of collaborative consultation is more transformative in nature. All participants, inclusive of teachers, mentors, leaders and researchers, became stakeholders in each other's practice acknowledging that "teacher learning involves a process of social participation within communities of practice" (Anthony et al., 2016, p. 119). The transformative purpose of supporting teachers in becoming self-directed learners, where teachers feel confident to continue their own development of understanding alongside student agency, was evident in PL research during the review period. For instance, Geiger, Muir, and Lamb (2017) proposed a new PL approach shifting from a PL cycle where the "researcher provides input about alternative pedagogies that teachers then trial in their classrooms and receive feedback from the researcher" (p. 458) to the use of video for teacher self-analysis where "in-the-moment" decisions are studied as an aspect of self-reflective practice. Self-reflection and self-scaffolding were also emphasised by Hollingsworth and Clarke (2017) where teachers sought feedback on specific elements of their classroom practice while being observed by the researchers. Hollingsworth and Clarke utilised technology in the form of video-recorded lessons as PL to achieve their purpose for teachers to "sharply focus their attention on those targeted areas" (p. 469) during self-reflection.

3.1.4 The Purpose of PL Approaches in the Future

As the relationship between researcher and classroom practitioner is strengthened through a diverse range of PL, the *purpose of approach* categories presented in Fig. 6.1 are constantly shifting and blending. In reviewing the current body of research, characteristics were noted that situate many studies across the categories of purpose where classroom practitioners and researchers became mutual benefactors of the PL, each influencing the other's practices. 'Observation' as an approach to PL could be included in the transformative category with an increase in research on teacher noticing. Teacher observation of their own or peer's classroom practices and pedagogy, and noticing of children's thinking and reasoning, have a positive impact on the development of teacher agency (Bragg et al., 2016; Chan, Clarke, Clarke, Roche, Cao, & Peter-Koop, 2018; Choy, 2016, Loong et al., 2017). Teacher noticing is becoming more prevalent in mathematics research and is supportive of the practical, in-the-moment benefits of researchers and teachers collaborating in classrooms.

Essentially, the purpose of any PL approach needs to match teachers' needs. Beswick and Fraser (2018) suggest that allowing teachers to be curators of their own learning has the potential for sustained teacher learning; as teacher confidence grows it can lead to increased professional autonomy. An increased need for individualisation of PL for teachers and a movement towards self-identified PL may lead to a future of increased teacher agency regarding PL purpose and choice. Self-identified PL via face-to-face and online PL courses is worthy of research in the future. However, as discussed earlier, we need to be mindful of other purposes that may take precedence and change the purpose-culture of PL, particularly as the international focus on improving teacher standards through accreditation processes increases. Although recommended for further exploration in the previous RiMEA chapter concerning teachers' professional development (Beswick et al., 2016), impacting factors on PL such as the experience of the teacher and school context, still warrant further research when discussing *how* and *why* teachers choose PL activities.

3.2 Context and Learning Focus

The purpose and learning focus of PL is influenced by its context. Some of the articles we reviewed directly addressed the issue of context, whilst in others it was implicit. Context is complex and may be considered at each of global, national, and local levels.

The global context reflects worldwide trends in PL research. It is clear that 'reform mathematics' remains a strong international discourse that motivates PL programmes. Here, the research centres on modifying teacher practices, often to focus on the use of challenging tasks or promoting collaborative problem solving (Anthony, Hunter, & Hunter, 2017; Bobis & Tregoning, 2019; Makar & Fielding-Wells 2018;

Russo, 2019; Russo & Hopkins, 2017). Additionally, the continuing increase of digital technologies available in the teaching and learning of mathematics has led to a variety of teacher-learning around the implementation of these technologies (see also Chap. 4) and lately a move toward computational thinking is related to teachers' pedagogical capabilities (Bower et al., 2017). International trends, such as Japanese Lesson Study and teacher noticing are also evident in the Australasian research field (see Sect. 3.3 for details).

At the national level, we saw some variation in the different foci of PL research. Recently introduced curricula in Australia and Singapore, for example, have led to programmes of PL that develop content knowledge or pedagogical content knowledge in areas such as algebra (Wilkie, 2016) or focus on enactment of the curriculum more generally (Collis, 2016; Kaur, Tay, Toh, Leong, & Lee, 2018). Related to curriculum, the Australian PL programme, *Let's Count*, has led to a range of associated research (Fenton, MacDonald, & McFarland, 2016; Perry & MacDonald, 2015) exploring the shifts in educators' and parents' beliefs, attitudes and expectations of very young children's learning of mathematics (see Chap. 9 for further research involving early childhood education contexts).

A number of studies looked at Australian contexts in comparison to other places, for example with Canada (Bragg et al., 2016; Loong et al., 2017), or with China and Germany (Chan et al., 2018). These studies tended to explore the impact of PL programmes on effective teacher practice. Bragg and colleagues focused on teacher noticing of mathematical reasoning and Chan et al. utilised teachers' collaborative reflections on their own practice as a way to learn from the act of teaching.

From Singapore we saw Ministry-driven projects of PL such as *Teaching for Metacognition* (Wong & Kaur, 2018) in addition to studies exploring the implementation of a new mathematics curriculum (Kaur, Tay, Toh, Leong, & Lee, 2018). The three main PL providers in Singapore (National Institute of Education, the Academy of Singapore Teachers and six Centres of Excellence) work together with schools to make teacher-learning cohere with teachers' needs and interests, guided by the 'Teacher Growth Model' (Bautista, Wong, & Gopinathan, 2015).

In New Zealand, by contrast, PL provision has become privatised and largely removed from the public sector. Government funding is given directly to schools that engage in teacher inquiry cycles. This process involves identifying needs specific to their school staff and then making a plan of learning based on these needs. At this point, a private (accredited) PL provider is contracted. One result of this process is that universities are divorced from the PL and therefore much PL in New Zealand is not systematically researched, unless by the teachers themselves via masters or doctoral study (e.g., Eden, 2018) or the 'Teacher-led Innovation Fund' (see www.education.govt.nz). The *Developing Mathematical Inquiry Communities* (DMIC) programme, based at Massey University, is an exception to this, as a large body of research accompanies the PL (e.g., Anthony et al., 2017; Hunter, Hunter, Anthony, & McChesney, 2018). This programme is an example of addressing local needs, as a key aim of the PL centres on teaching mathematics in such a way that adequately engages learners from marginalised groups, specifically incorporating Pasifika culture in the mathematics programme.

Research on PL with a similar learning focus can be seen in Australia with *The Accelerated Inclusive Mathematics—Early Understandings* (AIM EU) project, which aims to better engage Indigenous and low SES students via culturally relevant mathematics pedagogy (Anderson et al., 2017). This project stemmed from *YuMi Deadly Maths*, which caters to similar groups of marginalised students, with an aim to improve their future employment and life chances (Carter et al., 2016). Another area of research interest in Australia that impacts on marginalised students is teaching in remote areas. Kilpatrick and Fraser (2018), for example, address the issues of rural practice such as isolation and out of field teaching with a PL initiative called *STEMCrAft*.

The above are examples of PL that considers the local context, something Beswick et al. (2017) discuss. Research studies catering to the local context are evident in collaborations between mathematics education researchers and teachers (e.g., Peña, Cortina, & Visnovska, 2018; Visnovska & Cortina, 2018). Wong and Kaur (2018) discuss the way in which PL worldwide is moving away from university-based forms of knowledge production in favour of school-based. These are 'in-situ' and led by teachers with support from university scholars. Research on professional learning communities (PLCs), or networked learning communities (Seto, 2019), similarly cater to the local context and attend to the way teachers learn within their locality and respond to their individual or institutional needs (Kaur & Wong, 2017; Kilpatrick & Fraser, 2018; Prodromou, Robutti, & Panero, 2018).

As noted in Sect. 3.1.3 where we considered the purposes of PL, we found a range of research that emphasised teacher reflection as a learning focus (Downton, Giumelli, McHugh, Roosen, Meredith, Caleta, et al., 2018; Geiger et al., 2017; Hollingsworth & Clarke, 2017; Kaur, Bhardwaj, & Wong, 2017; Russo, 2019; Wong & Kaur, 2018). Correspondingly, one may wish to consider the individual teacher's 'internal' context. These studies tend to have the learning of students as the focus, via the enactment of reflection on the part of teachers.

Zooming in on the classroom context, there was a range of Australasian research with a learning focus on a specific content area, such as algebra (Wilkie, 2016), proportional reasoning (Hilton et al., 2016), and multiplicative thinking (Downton, Giumelli, McHugh, Roosen, Meredith, Caleta, et al., 2019). Other foci for PL included financial literacy (Sawatzki & Sullivan, 2017), gifted education (Peters & Jolly, 2018) statistical literacy (Pierce, Chick & Wander, 2015) and research with a focus on teachers' own mathematics content knowledge (Sullivan, 2018). At the classroom level, there continues to be research interest in project-based learning (Muir et al., 2016), for example *Big Picture* (Callingham, Beswick, & Ferme, 2015), and the similar inquiry-based learning which has been made a national priority in Australia during the time of our review (Makar & Fielding-Wells, 2018). Learning mathematics via inquiry involves solving complex problems that contain ambiguities. It was found that teachers need time and support to adopt these inquiry practices in their classrooms (Makar & Fielding-Wells, 2018).

In contrast to national agendas and local initiatives, there was a notable absence of research on informal teacher learning, i.e., learning not associated with any large-scale programme of PL and that is instead driven by individual or groups of teachers.

We suggest that blogs, Twitter feeds, and Facebook groups, to name a few examples, may be fruitful contexts in which researchers could gain an understanding of learning that mathematics teachers engage with on their own.

Finally, an emergent context, which may see more research in coming years, is the virtual context. For example, in the last review period 2012–2015, Ball, Steinle, and Chang (2015) created a virtual-learning environment in order to deepen teachers' pedagogical content knowledge (PCK). However, no similar studies were identified by Australasian researchers in the current review period. Importantly, it appears Australasian authors have not yet fully explored the intersection of PL with advances in technology (also see Chap. 4).

3.3 Models Guiding PL Design, Research and Analysis

In this section we examine prevalent models (also referred to as frameworks and conceptual frameworks) of professional learning that researchers used in the review period to design, guide and evaluate teacher PL initiatives. We use the term 'model', rather than 'framework', to distinguish our discussion of PL models from the overarching framework used to structure our analysis of the research literature in this chapter.

We adopt Boylan, Coldwell, Maxwell, & Jordan's (2018) definition of 'model' as a "non-unique, partial representation of a system, object and event process or idea" (p. 122). PL models usually aim to identify and represent the interrelationships between different PL components such as teacher beliefs and prior knowledge and the influences these components might have on teacher learning. Proponents of the various PL models emphasise different components and may use different terms to describe similar components or processes of PL. The focus on particular components, processes and influences is generally a result of the theory(ies) informing a model's development (see Sect. 3.4 for a discussion of relevant learning theories). This selective focus means that no one model can comprehensively represent or be used to examine all components of PL. The inability to comprehensively represent all PL components in one model is a possible reason why many researchers in the review period adapted existing models. Such adaptations often enabled researchers to more closely scrutinise particular component of PL. For instance, Clark and Peterson's (1986) model of teacher thought and action of instructional planning was frequently cited by researchers—all of whom adapted (e.g., Bobis, Downton, Hughes, Livy, McCormick, Russo, et al., 2019) or augmented (e.g., Sullivan, 2018) the model. Sullivan, for instance, considered the model appropriate for his research focus on challenging tasks because it represented a process for anticipating reasons why teachers undertaking PL might have been reluctant to incorporate challenging tasks and associated pedagogies into their repertoires. However, in spotlighting the importance of teacher content knowledge in this process, Sullivan found it necessary to augment the knowledge component of Clark and Peterson's model with Hill, Ball, and Schilling's (2008) conceptualisation of mathematical knowledge for teaching.

The interconnected model of teacher professional growth proposed by Clarke and Hollingsworth (2002) (see Fig. 6.2) was a popular choice of PL model for researchers in the review period (e.g., Downton et al., 2018; Hughes, 2019; Lomas, 2018; Roche & Gervasoni, 2017). Its popularity is attributed to its alignment with contemporary perspectives of teacher change as professional growth, acknowledging teachers as active and reflective learners who are influenced by society and their learning environment. Downton et al. drew upon this model to inform the PL design and associated research study that explored the impact of school-based PL on primary teachers' mathematical knowledge of multiplicative thinking. The model was also an appropriate tool to guide data analysis as the researchers focused on changes in teacher knowledge and its impact on their enactment of new mathematical practices in the classroom.

The interconnected model of teacher professional growth also provided the lens for Lomas (2018) to describe and interpret the understandings and perspectives constructed by two Year 5/6 case study teachers as they implemented an innovative mathematics curriculum. Reflecting upon her findings, Lomas proposed structural changes to the Clarke and Hollingsworth (2002) model by expanding the personal domain component to represent interactions occurring among teachers' knowledge,

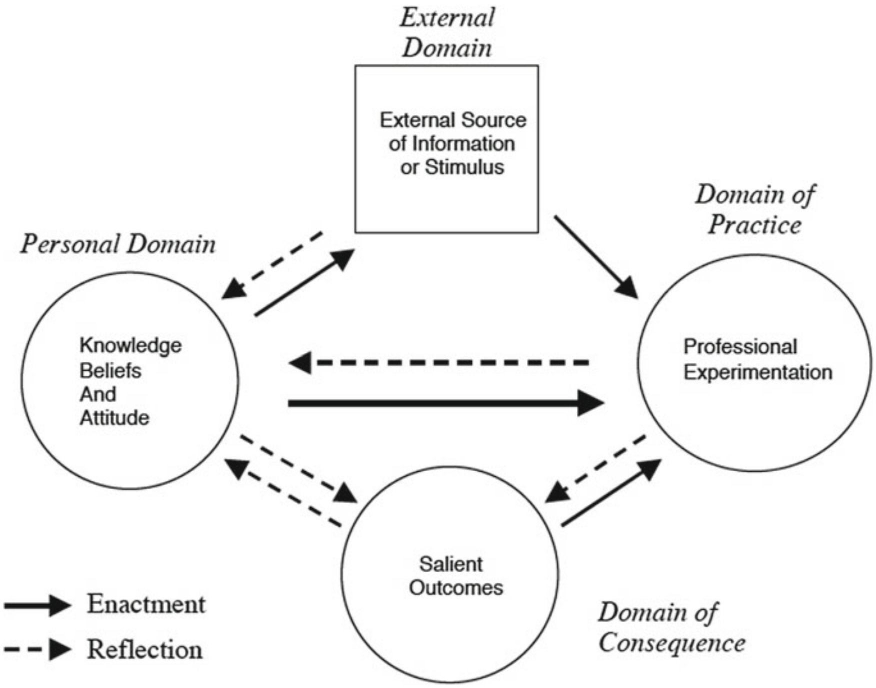


Fig. 6.2 Clarke and Hollingsworth's (2002, p. 951) interconnected model of teacher professional growth, with permission from Elsevier

beliefs and attitudes. She claimed that such refinements to the model "expanded the extent to which data generated in this study could be analysed" (p. 501).

Taking account of the growing body of research surrounding teacher noticing (e.g., Herbert & Bragg, 2017), Chan et al. (2018) built on the Clarke and Hollingsworth (2002) model to propose a new model that conceptualises teacher learning as situated in their daily practice. Chan et al. proposed that when "teaching a lesson, teachers learn from the things to which they attend" (p. 92) and that a teacher's attention, and the significance attributed to the objects of that attention, are influenced by a teacher's prior knowledge and beliefs, the intentions of the lesson, and the environment in which the lesson takes place. The model of teacher learning as situated in practice was used by Chan and colleagues to guide the analysis and interpretation of data from case studies and an online survey of mathematics teachers from Australia, China and Germany. The researchers' aim was to develop a better understanding of teacher in situ learning and thus inform how best to enhance the capacity of teachers to learn from their daily practice.

Lesson study was another popular in situ model used by researchers to design PL and guide the analysis of teacher learning through their reflections of their everyday practice (e.g., Choy, 2016; Groves & Doig, 2016; Groves, Doig, Vale, & Widjaja, 2016). Starting with the lesson study cycle components of planning, teaching and reviewing a lesson, Choy drew upon the teacher noticing literature to elaborate each component to explicitly focus on the improvement of teachers' mathematical task design knowledge and skills. The new model to emerge was used by Choy to provide a broad view of teachers' noticing through an entire lesson study cycle, as well as "a close-up view of noticing at each stage of the lesson cycle" (p. 429).

Influenced by social learning theories that emphasise the development of teachers in the context of complex relationships and communities of practice, Prodromou et al. (2018) drew upon the meta-didactical transposition (MDT) model (Arzarello et al., 2014) to describe and analyse the professional development process when secondary mathematics teachers from Italy and Australia worked with a community of researchers. Like other users of PL models, Prodromou et al. found that drawing upon just one model was insufficient for studying the complexity of the PL process. The researchers integrated the notion of *emergence* with the MDT to explicitly shed light on the interactions and practices of individual teachers.

The PL models mentioned to this point were all referenced as part of studies of specific PL initiatives. Conversely, Beswick and Fraser (2018) proposed a model for career-long PL. Their model was proposed to help explain the prevalence and relative satisfaction with one-off PL by Australian teachers of mathematics. Adapting Huberman's (1995) open collective cycle of teacher PL, the cyclic model begins with a teacher perceiving a problem. Beswick and Fraser explain that each cycle might take a few weeks to several years and represent a teacher's involvement in a specific PL initiative or his/her individual work to resolve the problem. The resolution of the problem involves a process incorporating input from external sources (a colleague or expert) followed by some level of diagnosis and development of some new methods that are trialled and reflected upon to determine the method's effectiveness for resolving the initial problem. The significance of this model is that it can apply to a wide range of PL approaches that teachers experience throughout their career, from self-initiated reflections on practice (e.g., Russo, 2019) to action research projects (e.g., Hurst, 2018) and to system-wide PL initiatives (e.g., Bobis, 2019; Bobis & Tregoning, 2019; Goos, Bennison, & Proffitt-White, 2018).

While our focus in this chapter is on PL for teachers of mathematics, it is both interesting and important to note that all the models used by researchers (that we could detect from the literature reviewed) were general models of PL, meaning that the models were intended to have broader applicability and were not specific to a mathematics context. In some instances (e.g., Choy, 2016) the general model was modified for a very specific mathematical purpose. It is likely that adding specificity to a model in this way might limit its usefulness to a wider audience of researchers and designers of PL and is a possible explanation as to why most developers of such models choose generic designs. Nevertheless, PL models were, and will continue to be, important and useful tools to inform the design and examination of PL.

3.4 Theories of Learning Underpinning PL

The choice of learning theories central to the studies reviewed was generally determined by the PL model guiding the design of the research. As noted previously, Clarke and Hollingsworth's (2002) model was utilised or adapted in many of the studies; therefore, theories of learning underpinning this model specifically influenced much of the research reviewed (see Sect. 3.3 for a detailed analysis of this and other models). Theories adopted within the research studies utilising this model support cognitive and/or situative perspectives on learning where PL activities regarding teacher knowledge and practice are viewed as "mediators of change" (Clarke & Hollingsworth, 2002, p. 956). These theoretical perspectives underpin the research design, data collection, and methods of analysis evident in the research.

This section illustrates the diversity of foci learning theories afforded across the body of research. Theories pertaining to student learning are also included in this section since a number of studies viewed improvement in student learning outcomes as a mediator of change to teacher beliefs and attitudes, which aligns to Guskey's (1986) process of teacher change.

3.4.1 Theories of Teacher Learning

Hollingsworth and Clarke (2017) suggest that teacher change occurs through teacher reflection and enactment involving features of the teacher's world, such as their knowledge, belief and attitudes, and their classroom experiences. Teachers in Hollingsworth and Clarke's study selected a specific focus for their own professional learning. Teachers then observed and analysed videos of their teaching practice, engaging in teacher feedback discussions with a researcher as part of a learning community or community of practice. According to Hollingsworth and Clarke, this supportive collaboration has the potential to build theory and express this theory in the "language of teacher learning and everyday classroom practice" (p. 462). Beswick et al.'s (2017) research was also informed by theory that emphasises teacher reflective practices. This focus supports a constructivist view of learning incorporating a notion of "reflection-in-action and reflection-on-action" (p. 170) situating PL in the classroom context. Although not mentioned specifically as a learning theory, reflective practices were also highlighted in Kilpatrick and Fraser's (2018) research implementing a STEM framework for peer mentoring. Reflective practices were seen as beneficial in learning communities where expert thinking could be captured, critical reflection practices could be modelled, and "trusting relationships with experienced teachers" (p. 3) and mentors could be formed. The making of connections with experienced teachers or "more accomplished colleagues" (Goos et al., 2018, p. 143) as part of coaching and teacher networks is also presented as a positive professional learning activity supporting teacher change. Sustained networks between schools as support structures for PL acted as mediators of change as Goos et al. reported, "such networks helped teachers feel safe and supported in order to take risks in changing their practice" (p. 143). Tully, Poladian, and Anderson (2017) promoted the benefits of communities of practice within the *Inspiring Mathematics and Science in Teacher Education* (IMSITE) programme for both pre-service and in-service mathematics teachers "as they engage in critical dialog and reflection for ongoing professional learning" (p. 523). These communities fostered co-learning and co-mentoring processes within three networks across specific spaces for learning. From the in-service teachers' perspectives, this learning partnership afforded the development of meaningful mentoring relationships and allowed the teachers to feel valued as members of the profession with knowledge and practices to share.

Examples of social learning theories were likewise present in research that recognised the complexity of teacher professional learning and the multitude of factors that influenced and impacted teacher learning. Prodromou et al. (2018) considered teacher learning to be influenced by the teacher, the school, and the learning activity itself and how these elements interact. Prodromou et al. explored the "actions and interactions among teachers, researcher and technological tools" (p. 447) noting that it is the type of task chosen and the technique employed, and the justification of these components, that needs investigating and analysing. A 'task' and 'technique' focus incorporates teachers' mathematical content knowledge and their pedagogical content knowledge. This focus is visible in the research design of Sullivan's (2018) study on improving teacher knowledge of mathematics through the selection and use of challenging tasks. Sullivan's approach used an iterative cycle of interventions addressing the issue of practice, "the connection between teacher mathematical knowledge and teaching" (p. 3).

As well as social theories of learning, there was also evidence of personal theories of learning. In particular, research conducted by Russo (2019) was underpinned by a "self-reflective component" (p. 17) of inquiry. Although his study involved co-teachers, their role was passive. Whereas Russo employed self–journaling, self–observation, and self–reflection as a "valuable lens through which to consider" (Russo, 2019, p. 17) his own teaching. Russo concluded that teacher reflections "could be a powerful process for both improving one's own teaching practice" (p. 23) and supporting the development of other classroom teachers.

Instructional leadership is another factor that has "been shown to have an impact on teacher change" (Roche & Gervasoni, 2017, p. 442) resulting in improved student outcomes. Roche and Gervasoni targeted school leaders for their PL project with the belief that school leaders could "support and initiate whole-school reform in mathematics teaching and learning" (p. 442). While this PL approach led to changes in teachers' practices, Roche and Gervasoni acknowledged that the greatest impact occurred when leaders participated in the PL and when PL was situated in their own school context. The researchers noted that the capacity of the leaders to "translate and reproduce" (p. 442) key messages from the initial PL was a factor for its success. Instructional leadership was also explored by Sexton and Lamb (2017) in their investigation of a school-based mathematics instructional leader. The researchers drew upon activity theory to analyse the motivations and tools used by the instructional leader to influence teachers' affect and develop shared understandings.

3.4.2 Theories of Student Learning

PL outcomes include improved mathematics learning by students (Beswick et al., 2017) together with teacher professional growth. Therefore, knowledge and application of student learning theory is a fundamental aspect of PL research. Carter et al.'s (2016) research was informed by a social constructivist epistemology of knowledge creation where "the importance of culture and context" (p. 166) were influencing factors on student learning alongside "personal experiences and collaboration with more knowledgeable others (teachers)" (p. 166). In their research the "social capital that students bring to the classroom" (Carter et al., 2016, p. 167) is valued, as they report on PL involving a teaching cycle that supports making connections between mathematics and students' interests based on cultural and environmental factors. Research of this nature has a dual purpose; to equally equip teachers and students to be successful in their capacity to understand mathematical concepts and to build confidence in their mathematical abilities leading to growth in self-efficacy. This duality is illustrated in current research on challenging tasks (Sullivan et al., 2016; Sullivan, 2018; Bobis et al., 2019). The use of challenging tasks provides students with opportunities for prolonged thinking. Bobis et al. argue that challenging tasks require struggle on behalf of students, and that this struggle can assist in the development of conceptual understanding. Hence, as teachers learned the benefits of allowing students time to struggle, they also had to learn new practices to ensure they could effectively manage student struggle in the classroom. Similarly, in Sullivan et al.'s (2016) research, ideas around student learning are theorised where student persistence with tasks provides opportunities for students to engage with the content for themselves. Teacher learning through PL about posing challenging tasks combined with appropriate prompts can lead to students learning from the "thinking activated by working on the first task … then applying that learning" (p. 672) to new situations.

Theories of learning that underpin research in this review period make strong connections between changes in teacher knowledge and practice and the resulting effect on students' knowledge and understanding. An emerging theme is one of student mindsets and the processes by which learning in mathematics occurs, such as struggle, persistence and time for thinking and sharing knowledge. Knowledge of these emerging factors influencing student learning theory seems reflected in theories of teacher learning where teachers are also benefiting from experiencing the struggle and perseverance with the mathematics themselves.

3.5 Scope, Scale and Duration of PL

In this section, we examine the scope, scale and duration of PL programmes with the goal of identifying particular patterns or trends in PL research in Australasia during the period 2016–2019.

From the PL programmes reviewed, it is apparent that their scope was mainly micro, meaning that they were specific to selection, creation and enactment of instructional related aspects. In the PL programmes, there was a focus on micro-level interactions between teachers and curriculum materials that impact classroom practice of mathematics teachers. In some of these programmes teachers not only engaged in PL but also contributed towards the PL of other teachers through the artefacts they created and the knowledge they acquired. In the *STEM: Critical Appraisal for Teachers* (STEMCrAfT) project (Beswick et al., 2016) a group of 15 very experienced and highly qualified teachers of mathematics and science worked collaboratively over a period of two days. They codified their tacit knowledge about the use of resources in their teaching. The codification contributed towards a draft STEMCrAfT framework, which made expert teachers' knowledge and thinking explicit thereby helping less experienced teachers, who also participated in the project, think systematically through issues that underpin their decisions about the use of a particular resource in their specific context with their students. The final STEMCrAfT framework has contributed to the PL of mathematics teachers, ranging from out-of-field teachers to teams of teachers in schools. Likewise, during the third phase of the *Teaching for Metacognition* project (Kaur et al., 2017), participants were empowered to contribute towards the PL of fellow teachers in their schools and elsewhere. They did this by sharing their learning through workshops held in their respective schools and presenting at conferences. The PL was specific to the facilitation of lessons that imbue metacognitive activities so as to nurture metacognition amongst learners in mathematics lessons.

PL programmes reviewed varied in scale, from small to large. Often the scale was dependent on the intent of the programme and initiatives that drove the PL. For example, in the *STEMCrAfT* project (Beswick et al., 2016), only a small number of teachers were involved to work alongside PL leaders to create a tool for PL of other teachers. In each of the following professional learning projects: *Implementing Structured Problem-Solving Mathematics Lessons Through Lesson Study* project

(Groves et al., 2016), *Developing Mathematical Inquiry Communities* (Anthony, et al., 2017) and *Teaching for Metacognition* project (Kaur et al., 2017) groups of teachers from several schools participated in the PL projects together. These projects appear to be researched for proof-of-concept that is pilot tests to evaluate the efficacy of a PL programme. In contrast, the *Teachers First Model* of PL (Proffitt-White, 2017) was a large-scale PL project driven by a need to improve effective delivery of mathematics lessons across 219 schools in a region. It involved clusters of teachers working collaboratively with a common aim of improving classroom instruction. The PL was scaled-up systematically. As noted by the researchers, the model of PL continuously evolved as it resolved issues as they emerged. One such issue was that as the PL was free, credibility of the "free service" had to be established. This was done by inviting Australia's leading academics to present at the regional conferences organised by the PL project. These presentations supported both the design and quality of the activities and assessments created by the project. The researchers also noted that several factors facilitated the scalability of the project. These factors were supporting teachers in their learning with knowledge that would help them in their day-to-day mathematics instruction, providing them with time to work with peers to talk and work towards common goals and empowering them to make decisions about pedagogies best suited for their students (Profitt-White, 2017). Similarly, the *YuMi Deadly Maths* (YDM) PL programme (Carter et al., 2016), has scaled up into over 200 schools since its genesis in 2010. The YDM's pedagogical approach enhances mathematics learning outcomes and closes the gap between Indigenous and low socioeconomic status with other schools. Carter et al. note that the pedagogical approach that is easy to implement and results in positive student responses has been instrumental in persuading teachers to adopt it. The process of scaling up involves training a small (core) group of teachers in a school and empowering them to be change agents and researchers in their respective schools with sustained support from the YuMi Deadly Centre of the Queensland University of Technology.

The duration of PL programmes reviewed varied significantly. Some were short-term whilst others were long-term. Some were incidental where PL took place during classroom instruction time whilst others were planned activities ranging from a few hours (e.g., Beswick & Fraser, 2018) or days (e.g., Beswick et al., 2016), to school semesters or years (e.g., Roche & Gervasoni, 2017; Kaur et al., 2017; Groves et al., 2016; Restani, Hunter, & Hunter, 2019). Beswick and Fraser's (2018) model of career-long PL shows that PL of teachers may take place in cycles and that a cycle may take anything from a few weeks to several years. Depending on the needs of teachers and practicalities such as finance, finding relief teachers and fitting PL into their already busy schedules, teachers will engage in an appropriate one-off PL session or spaced PL (Beswick & Fraser, 2018). In an audit of Australian PL available for mathematics teachers, Reaburn, Kilpatrick, Fraser, Beswick, and Muir (2016) noted that 61% of the PL programmes on offer were either one-off or annual events. Beswick et al. (2017) speculate that limited experience of PL that is sustained over time and/or collaboratively may be the reason for teachers rating one-off PL as their best ever experience of quality PL.

3.6 Impact and Outcomes of PL

In this section we use Guskey's (2016) five levels of data for PL programmes to discuss the evaluation of Australasia PL and possible impacts of such programmes reported during the period 2016–2019. Guskey's levels form a hierarchy, from simple to more complex with each succeeding level, requiring more time and resources to gather the data. Level 1 looks at participants' reactions to the PL experience. In some ways it is a measure of "happiness quotients that reveal only the entertainment value of an experience or activity, not its quality or worth" (p. 33) and data for this level are easily collected at the end of the PL using surveys. Such data are helpful to improve the design and facilitation of PL. None of the reviewed studies reported evaluation of PL at this level.

Level 2 focuses on measuring the new knowledge, skills, and attitudes or dispositions that participants gain from the PL (Guskey, 2002). Oral or written personal reflections or examination of curriculum materials designed for use by participants in PL help researchers document their learning. Most of the reviewed studies reported PL of teachers being evaluated at this level (Davidson, 2017; Groves & Doig, 2016; Roche & Gervasoni, 2017; Wong & Kaur, 2018). For most of the studies, self-reports in the form of reflective journals, surveys and semi-structured interviews were used to collect data for evaluation of PL. Generally, in all the reviewed studies the PL evaluations affirmed positive outcomes with regards to teacher practice and learning environments for student learning.

At Level 3, the attention shifts from participants to organisational dimensions that may be vital to the success of the professional learning experience. At this level, the focus is on organisational characteristics necessary for success, such as "Did the professional learning promote changes that were aligned with the mission of the school? Were sufficient resources made available, including time for sharing and reflection (Langer & Colton, 2005)? Were successes recognised and shared?" (Guskey, 2016, p. 37). Although none of the studies reported on organisational characteristics related to PL programmes, there were two studies we reviewed that possibly evaluated PL at this level. In Muir, Livy, Herbert, and Callingham (2018) it is apparent that school-based PL had a positive impact on the good NAPLAN numeracy results of the three schools in the study. It was the school-wide explicit improvement agenda which allowed teachers to successfully put into practice their PL experiences. However, in the second study although Groves and Doig (2016) noted that the rigour of the post-lesson analysis fed directly into the practice of all teachers, it was envisaged that this PL experience was not feasible in the context of Australian schools. The major constraints in adapting and implementing the Japanese Lesson Study were finding suitable problem-solving tasks to match the Australian curriculum, and the Australian teaching culture that emphasises small-group rather than whole-class teaching (Groves & Doig, 2016). In Muir et al. data were collected using a case study methodology involving interviews with schools' principals and schools' leaders through interviews. Whilst in Groves et al., data were collected through semi-structured interviews with teachers who participated in the study.

Level 4 data help to answer the question "Did the new knowledge and skills that participants learned make a difference in their professional practice?" (Guskey, 2016, p. 35). Although in some of the PL studies reviewed (e.g., Proffitt-White, 2017; Wong & Kaur, 2018) teacher or school reports about teachers integrating their new knowledge into classroom practice at the end of the PL programmes or soon after were available, the data are not commensurate with Level 4. At this level, data must be collected after a lapse of time to check for sustainability of practice using robust methods such as observations that must be as unobtrusive as possible. None of the studies reviewed reported evaluation of PL at this level. Though the main aim of PL is to improve student learning outcomes, none of the studies evaluated PL robustly at Guskey's most demanding level—Level 5. None-the-less in a few studies teachers did collect evidence about their students' learning in ways they deemed fit for their own use. In the study by Proffitt-White, teachers noticed a change in their students "have a go" (p.19) attitude. They administered open-ended tasks and monitored student attempts over a year, as well as daily feedback between teachers and students. Wong and Kaur also reported that "students appeared to be metacognating—'thinking about their work'. They were questioning, critiquing their solutions and identifying gaps in their knowledge" (p. 432).

4 Conclusion and Future Directions

Reviewing Australasian research on the PL and PD of practising teachers between 2016 and 2019 two striking features that stood out were the incredible variety of ways in which teacher learning occurs and the extent to which the field has moved beyond the traditional 'one-size fits all' inservice teacher training approaches of the past. Comparing the content of past RiMEA chapters that have focused on teacher learning (e.g., Anderson, et al., 2008; Beswick et al., 2016; Bobis, Higgins, Cavanagh, & Roche, 2012) with that of this chapter, it is possible to identify some enduring aspects of PL as well as those that are just emerging or appear to have lacked the attention of Australasian researchers. In this final section, we draw together the most prevalent aspects of PL research in the current review period, make comparisons with what was reported in previous reviews and provide thoughts about where future research in this area might develop.

4.1 Enduring Aspects

There is no doubt that teacher PL continues to be a major focus in Australasian mathematics education research as evident by the sheer volume of literature reviewed for this and previous review periods. While commonalities of 'effective' PL were noted in the 2004–2007 review of research, Anderson et al. (2008) supported the notion of 'rich ingredients'. The authors found that across a range of studies involving programmes of PL, developers selected specific ‘ingredients’ that seemed to suit their context. Further down the track, Beswick and colleagues in their 2012–2015 review provided a critical perspective of the so-called agreed upon characteristics of effective PL, highlighting issues associated with the evidence needed to declare PL ‘effective’. While it was noted that the trend to incorporate a variety of characteristics from this list into PL designs continued from the previous review period to the current one (e.g., Anderson, 2017; Goos et al., 2018), we also found a growing level of scrutiny surrounding an established list of characteristics (e.g., Bobis et al., 2016). Such scrutiny was fuelled by evidence that PL programs that ‘break the rules’, such as one-off PL, were considered to be effective by teachers (Beswick et al., 2017).

Related to the issue of effectiveness, is the enduring international discourse present in reviews and mathematics education research literature linking teacher PL to quality teaching and improved student learning. The focus on quality teaching in policy documents around the world and teacher accreditation requirements such as those in Australia and Singapore, means that the need to substantiate the quality and effectiveness of teacher PL initiatives is likely to continue to drive research on PL well into the future.

Beswick et al. (2016) recognised that PL models were, and speculated that they will continue to be, important and useful tools to inform the design and examination of PL. The current review proved the accuracy of their speculation. The interconnected model of teacher professional growth by Clarke & Hollingsworth (2002) continued to be a popular choice by researchers (e.g., Downton et al., 2018; Hughes, 2019; Lomas, 2018; Roche & Gervasoni, 2017), as was Clark and Peterson’s (1986) model of teacher thought and action (e.g., Bobis & Tregoning, 2019; Sullivan, 2018). As noted earlier in this chapter, most researchers adapted these models to make them more suited to their own context or amenable to their data, such as in the case of Lomas. Other researchers focused solely on a small component of a model to drill-down to the fine-grained aspects of the component in question (e.g., Sullivan, 2018). It may be that the adaptability and generalisability of these and other popular models is a quality that will ensure their endurance well into the future of PL research. However, as noted in Sect. 3.3 and discussed below, new models of PL have also emerged in the current review period.

Sustainability and the scale-up of PL has been an enduring issue; featuring in the 2008, 2016 and the current review of teacher PL research. The 2016 review focused on questions of whether particular PL projects could be sustained or scaled-up to increase their impact on student learning. Given the range of PL initiatives in the previous review periods that were not featured in the current review, we could infer that few (if any) PL initiatives have been sustained, let alone scaled-up. Reflections upon this current review leads us to suggest that it is important for researchers to specify which aspect of a PL they are examining for its sustainability and scalability—the programme in its entirety, its impact on teacher learning, its impact on students, or something else. For instance, the 2008 review was dominated by references to large-scale, systemic PL initiatives such as *Count Me In Too [CMIT]* in New South Wales [NSW], the *Early Numeracy Research Project [ENRP]* in Victoria, and the *Numeracy Development Projects* in New Zealand. While none of these projects were mentioned in the 2016 review, there is little doubt that their legacies live on in the form of curricula content or the practices of classroom teachers. We suggest that although concerns surrounding the sustainability and scalability of PL might be enduring ones, there needs to be discussion and further research surrounding the optimum time for specific programmes to exist before they either become embedded in official curricula or a subsequent PL initiative, accepted practices or obsolete due to new knowledge and technologies. New programmes have evolved from the good practices of past programmes. For example, the large-scale NSW Department of Education PL initiative, *Building Numeracy Leaders* (Bobis, 2019; Bobis & Tregoning, 2019), builds upon many of its predecessors', namely *CMIT* and *ENRP*. In this sense, past PL initiatives can have a sustained impact in one form or another.

4.2 Emerging Interests and Gaps in the Research

As previously mentioned, new models of PL emerged in the current review period. Such models were generally developed to reflect new foci in PL research. For example, Chan et al.'s (2018) new model addresses both the growing interest in teacher noticing as PL (e.g., Choy, 2016; Herbert & Bragg, 2017) and the importance of teacher learning situated in everyday practice (Chan et al., 2018). Similarly, a PL model recognising career-long learning (Beswick & Fraser, 2018) emerged from an increased need for the individualisation of PL for teachers and a movement towards self-identified PL. It is likely that changing teacher accreditation requirements and the desire to increase teacher agency regarding PL purpose and choice may impact what models of PL look like in the future.

In contrast to national agendas surrounding teacher accreditation and despite the emergence of PL models recognising the importance of in situ teacher learning, the ways in which teachers learn from their daily practice was under-represented in the literature. There was also a notable absence of research on informal teacher learning; that is, learning not associated with any large-scale programme of formal PL but instead driven by individual or groups of teachers and situated in their daily practices. In their 2008–2011 review of teacher professional knowledge, Bobis et al. (2012) acknowledged an evidence void surrounding teacher learning from their own practice. This void is still evident in the current review period. With the rapid increase in social media as a new and flexible vehicle by which teachers can learn, we suggest that blogs, Twitter feeds, and Facebook groups, to name a few examples, may be fruitful contexts in which researchers could gain an understanding of individualised teacher learning that is informal and self-directed.

Similarly, Bobis et al. (2012) noted the lack of Australasian research linking the impact of growth in teacher knowledge with student learning. In the current review, the absence of PL studies documenting the sustained effects of PL on either teaching quality or student outcomes in a systematic or rigorous manner (i.e., Guskey's most demanding level of PL—level 5) was also noted. PL initiatives provided by private institutions or large public systems of education were clearly operating in the review period in different regions of Australasia as evident from a desktop search of institutional websites (e.g., https://education.nsw.gov.au/teaching-and-learning/curriculum/literacy-and-numeracy/professional-learning). However, a research component may not have been attached to ascertain the impact of such PL, as is the case of most centrally funded PL initiatives currently occurring in New Zealand (see Ministry of Education, n.d.).

Finally, another notable evidence void concerns teacher PL policy—its development and its implementation at national, state, system or institutional levels. For example, many of the studies in this review referenced national mandated policy (e.g., AITSL) as a justification for their research (e.g., Beswick & Fraser, 2018). However, we could not find any studies that actually focused on the development of PL policy, how institutions interpret and implement such policies, or their impact. Surely, if mathematics educators are to better understand policy and its implications for teacher PL, there is a need for research that does not just respond to policies but explores problems of policy and their impact on teachers and schools.

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Chapter 7
Researching the Affective Domain in Mathematics Education

Naomi Ingram, Vesife Hatisaru, Peter Grootenboer, and Kim Beswick

Abstract This chapter backgrounds and critically reviews Australasian research in the affective domain in mathematics education from 2016–2019. It first locates the body of research within broader considerations of an affective domain and then explores the individual affective aspects of beliefs, self-efficacy, identity, attitudes, motivation and interest, feelings, anxiety and emotions, and engagement. Consideration is given to the methodologies used in research in the affective domain during this time period.

Keywords Mathematics education · Affect · Beliefs · Identity · Attitude · Motivation · Interest · Engagement · Feelings · Emotions · Anxiety

1 Introduction

This is the fifth consecutive chapter dedicated to affect within the four-yearly review, *Research in Mathematics Education in Australasia.* Over the last 20 years, this set of chapters has mapped the evolution of Australasian affective research related to mathematics education. Research reviewed within these chapters has variously included one or more of a broad list of affective constructs, including beliefs, values, attitudes, emotions, feelings, anxiety, self-concept, self-efficacy, identity, motivation, engagement, optimism and pessimism, and attributions of success and failure.

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Affect is accepted to be an integral part of students' learning of mathematics (Hannula, 2014), and there are compelling connections between affect and learning outcomes such as achievement, engagement and participation (Grootenboer & Marshman, 2016). In the last four years, there have been several indicators of the continued prevalence of research in the affective domain within the Australasian mathematics education community. A book published by Grootenboer and Marshman (2016) explored the development of students' affective views and responses towards mathematics and mathematics learning in the middle years. A special issue of the *Mathematics Education Research Journal* (*MERJ*) explored foundations of engagement in mathematics. Associated with the *Mathematics Education Research Group of Australasia* (MERGA) conferences, a keynote by Ingram (2017) explored the theories, relevance and prevalence of research in the affective domain and critiqued the integration of affect within mathematics education research. Furthermore, within this time period, eight percent of the conference papers have had an explicitly affective focus; slightly under the long-term trend of 10.2% (Ingram, 2017).

Researchers doing work in affect in mathematics education continue to negotiate the socially constructed meanings of affect. This negotiation is often dependent on whether they are informed by ideas from psychology, sociology, neurophysiology, education, or physiology (Hannula, 2012). There is resulting variety, and at times, muddiness in researchers' theoretical perspectives and the definitions used. Indeed, the authors of the previous review chapters (Attard, Ingram, Forgasz, Leder, & Grootenboer, 2016; Grootenboer, Lomas, & Ingram, 2008; Lomas, Grootenboer, & Attard, 2012; Schuck & Grootenboer, 2004) all acknowledge, in some form, the difficult task of defining the affective domain, its associated theories, and the "porous and interconnected" (Attard et al., 2016, p. 74) affective elements.

There is regular (e.g., Orellana & Barkatsas, 2018; Skilling, Bobis, Anderson, Martin, & Way, 2016) reference to McLeod's (1992) seminal conception of the affective domain, which was informed by research on problem solving. He conceptualised *affect* as a range of beliefs, feelings and moods that go beyond the cognitive domain, and the *affective domain* as including the interacting elements of beliefs, attitudes, and emotions represented along a continuum of levels of cognition and stability. This conceptualisation captured the complexity of the affective domain and allowed for relationships between elements to be explored; important in understanding students' mathematical learning (Grootenboer & Marshman, 2016). Leder and Grootenboer's (2005) useful diagram (previously captured in the review by Grootenboer et al., 2008) represented this continuum and also included and positioned values. Known tensions with McLeod's (1992) definition have been acknowledged within this research period. Grootenboer and Marshman (2016) outlined these well; these tensions include the overlapping nature of affective constructs and difficulty in identifying where constructs such as motivation and identity fit within McLeod's conceptualisation.

Ingram (2017), also acknowledging these tensions, suggested the consideration of Hannula's (2012) representation of a meta-theory of mathematics-related affect to further capture the complexity of the affective domain. This meta-theory enables connections to be made across different theories of mathematical-related affect and across different eras of research in the affective domain. This conceptualisation maps the affective domain along three dimensions: aspects of cognition, motivation and emotion (similar to McLeod's three-pronged affective domain but using motivation as a more agentic and dynamic affective construct than attitude); movement between state and trait; and, strands of physiological, psychological and social theories of affect (Goldin et al., 2016).

A number of Australasian researchers within the review period have made reference to this useful conceptualisation. Woodward, Beswick, and Oates (2018), for example, used it to position their work on productive dispositions. Ingram's (2017), Ingram et al. (2019) research aligned with these dimensions by describing individuals' relationships with mathematics as providing the context for and being shaped by their engagement in the subject. In contrast, Roth and Walshaw (2019) suggested the need for a quite different approach to affect, principally emotions, based on Vygotskian ideas. They argued that emotions and intellect are inseparable, as are emotions and activity or experience. They proposed a "psychology of drama" in which the whole person and their relations with others in context are considered together, and foreshadow further theoretical developments that may feature in subsequent reviews.

The previous chapters have identified a number of themes and gaps in the research. Schuck and Grootenboer (2004) noted that the extensive literature on attitudes had been supplemented by research on beliefs. Four years later, beliefs dominated, although identity, motivation and engagement were noted as emerging constructs within research in the affective domain (Grootenboer et al., 2008). During 2008–2011, there was a growing focus on identity and international theorising of affective concepts was filtering into Australasian research (Lomas et al., 2012). Four years ago, there was a notable shift in attention towards student engagement. Over the 20 years the attention given across sectors and ages of participants have become more balanced, although it seems the call for research into mathematical affect among early childhood students and teachers had not been answered prior to the current review period.

For this review, research was sought that had a significant affective dimension, from publications in books, to journals such as MERJ and *Mathematics Teacher Education and Development* (MTED) to conference proceedings such as those of MERGA, the *International Group for the Psychology of Mathematics Education* (PME) and *Mathematical Views* (MAVI), and databases used to locate Australasian research within other national and international journals and books. MERGA members were also invited to submit relevant articles for review. How the researcher defined the affective construct, how research was grounded within the affective literature, and methodology, participants, and research findings were noted.

In this chapter, we present the Australasian literature on the affective domain related to the categories: (1) beliefs and identity; (2) attitudes; (3) motivation and interest; (4) engagement, and feelings; and, (5) emotions and anxiety. Because of the overlapping nature of the affective constructs it is somewhat difficult to explore them in a linear manner, and we have noted when these overlaps, and opportunities for breadth, occur within the research. We also note when research has been done in a more holistic manner. We then explore the methodologies and methods employed during this research period.

2 Beliefs

Beliefs have continued to be an area of considerable interest to Australasian mathematics education researchers over the period of this review. Issues arising from the use of multiple terms that can be interpreted as, or encompassing, beliefs (e.g., dispositions, perceptions, views), and the failure of some authors to define clearly the construct on which they focus, persisted. Several researchers have dealt with multiple aspects of affect and have attempted to distinguish among them, but this was challenging. For example, Itter and Meyer (2017) appeared to rely on the claim that attitudes and beliefs are “overlapping constructs” (Grootenboer & Marshman, 2016), perhaps to avoid explicitly defining attitudes. Instead, they equated positive and negative attitudes with particular sets of beliefs, used “attitudes and beliefs” as a single construct, and referred to perceptions in the title of their paper with no explanation of how perceptions connect with either attitudes or beliefs. In this review, the inclusion of research focussed on identity in a section on beliefs is consistent with a psychological view of identity as beliefs about oneself, but we are aware that this is contentious and not a stance with which some of the researchers whose work we reviewed would concur.

In Hannula’s (2012) representation of the affective domain, beliefs are situated within the cognitive, psychological, trait area with Chick and Beswick (2018) reiterating earlier arguments (e.g., Beswick, Callingham, & Watson, 2012) that they are distinguishable from knowledge only by the degree of consensus that they attract. Theoretical attention to beliefs within the scope of this review appears confined to Beswick’s (2018) use of Davis’s (2004) work on the application of complexity theory to mathematics education, to consider an individual’s beliefs as comprising a complex system with emergent properties that can include other beliefs as well as attitudes. Beswick argued that this approach both explains the unpredictability of interventions on participants’ beliefs and points to ways in which the shared beliefs of groups might be considered and influenced.

Whose beliefs were studied (e.g., teachers, students), and the range of objects of the beliefs studied was varied, including engagement (e.g., Bobis, Way, Anderson, & Martin, 2016), numeracy (e.g., Forgasz & Hall, 2016), student capability (e.g., Beswick, 2018), and self-efficacy (e.g., Lomas & Clark, 2016). These foci of beliefs research reflect broader shifts of interest in mathematics education research. Consistent with this is the considerable interest in identity, a topic that Darragh (2016, p. 19) described as “a topic *de jour*”. We briefly outline the major alternate perspectives on identity at the start of that subsection. Before that, other beliefs research is reviewed according to the informants in the research: practising teachers, preservice teachers, students, and mathematics teacher educators. Studies that included parents or other non-teaching adults are included in the section on students' beliefs because students were also participants in these studies.

2.1 Practising Teachers' Beliefs

Research on practising teachers' beliefs in the review period continued to include investigations of beliefs about mathematics, mathematics teaching, and mathematics learning (e.g., Beswick, 2018; Bobis, et al., 2016; Lomas & Clarke, 2016)—the categories proposed by Ernest (1989) as relevant to mathematics teaching. These studies typically included attempts to influence these beliefs with a view to influencing teachers' practice. For example, case studies were presented by Lomas and Clarke (2016), and Lomas (2017) on the impact that mathematics curricula can have on the beliefs of Year 5 and 6 teachers. They reported success when teachers were open to change and able to assimilate the proposed changes into existing belief systems (Lomas & Clarke, 2016) but less success if the teacher was resistant (Lomas, 2017). This group of studies also illustrates the close connection between beliefs and knowledge in belief research that investigates the impacts of interventions on both.

Teachers' beliefs about student capability have been identified by Beswick (2017, 2018) and Scherer, Beswick, DeBlois, Healy, and Moser-Opitz (2016) as crucial to the opportunities to learn that teachers provide. In particular, Beswick (2017) found that the secondary mathematics teachers in her study were unlikely to offer tasks that allowed students to demonstrate or to develop mathematical proficiency as defined in the *Australian Curriculum: Mathematics* (Australian Curriculum, Assessment and Reporting Authority, 2014) to students who were perceived as poor students. Such tasks were believed by teachers to be appropriate only for students who already demonstrated proficiency (Beswick, 2017).

Bobis et al. (2016) reported on changes in three upper primary teachers' beliefs about student engagement during a 10-week action-learning based intervention. Like Lomas (2017) they found differences in the responsiveness of teachers according to a range of personal characteristics that included the extent to which they believed student engagement was important and believed it to be their responsibility, their own self-efficacy in relation to influencing student engagement and their confidence in their ability to learn new mathematical content. Drawing from a larger sample of teachers involved in the same study, Skilling et al. (2016) also linked teachers' beliefs about engagement, including their self-efficacy to influence student outcomes, to the extent to which they were willing to adopt practices likely to enhance student engagement. Teachers' self-efficacy—their beliefs about their capacity to have an impact—also featured in other papers dealing with teacher beliefs. Beswick (2017), for example, attributed teachers' beliefs that skill-based repetitive tasks were appropriate for low attainers rather than tasks that might enhance students' mathematical proficiency to lack of knowledge, and hence lack of self-efficacy, in relation to teaching mathematical proficiency. Lomas and Clarke (2016) referred to the difficulty of changing an individual's beliefs about him or herself. Beswick (2018) explained this in terms of Green's (1971) characterisation of some beliefs as more central, that is more interconnected with other beliefs. Beliefs about oneself are, she claimed, in that category. Nevertheless, teacher's self-efficacy can improve when they see evidence that new approaches positively impact their students' mathematics learning (Lomas & Clarke, 2016).

Most beliefs research focussed on individuals, but there has been some interest in the beliefs of groups of teachers. Beswick (2018), informed by previous work that considered a group of teachers as a complex system (Beswick, 2016), described the beliefs of one secondary mathematics teacher as an individual's beliefs system nested within the collection of beliefs held by a group.

2.2 Preservice Teachers' Beliefs

Relatively few papers published in the review period reported on the beliefs of preservice teachers. Self-efficacy, or closely related concepts, was a consistent feature of these studies with two of them (Brown & O'Keeffe, 2016; Forgasz & Hall, 2019) set in the context of a new or impending nationally mandated (in Australia) Numeracy Test for Initial Teacher Education.

The participants in Forgasz and Hall's (2019) study were enrolled in a compulsory numeracy unit and included both primary and secondary preservice teachers. Forgasz and Hall (2019) reported changes in participants' views (beliefs) about numeracy and its relationship with mathematics, and increased confidence (self-efficacy) in their ability to incorporate numeracy in their teaching.

Itter and Meyers (2017) focussed on the interrelationships among beliefs, attitudes, and emotions, and followed Grootenboer and Marshman (2016) in considering beliefs and attitudes to be overlapping constructs because beliefs influence the ways in which experiences are perceived and hence contribute to the development of attitudes. They reported on the 111 of 152 (73%) of their preservice primary teachers who reported negative or neutral attitudes towards, and beliefs about, mathematics, and identified among their respondents self-limiting beliefs about their capacity to learn mathematics. Participants attributed these to experiences of learning mathematics at school that were characterised by boredom, difficulty, and irrelevance, and feelings that included fear, loathing, frustration, dislike and alienation.

Ingram, Linsell, and Offen (2018), took a holistic approach over three years and explored the relationships 83 primary preservice teachers had with mathematics and mathematics teaching, separating beliefs from knowledge, and included aspects of beliefs, knowledge, feelings, identities and engagement within their relationship with the construct of mathematics. They found the preservice teachers' relationships with mathematics and mathematics teaching became more positive as their teaching identity grew during their experience, which included interventions that addressed their mathematical content knowledge, the explicit teaching of affective aspects, and positive role modelling.

2.3 Students' Beliefs

Studies of student beliefs focussed on their beliefs about mathematics and mathematics learning and frequently reported gender differences. Self-efficacy was another important theme in relation to student beliefs.

The beliefs of middle school students were the subject of a book by Grootenboer and Marshman (2016) in which they drew together findings from a number of studies. Findings included that students believe mathematics to be mainly about Number and that learning the multiplication tables is the most important aspect of the subject. The belief that boys are better at mathematics than girls tended to be held by some student groups (Grootenboer & Marshman, 2016). Forgasz and Leder (2017) also found gender differences among 15-year-olds in their analysis of PISA data from Australia, Canada, and the United Kingdom (UK). Boys were more likely than girls to believe that mathematics was important and would be useful for their careers, and that their parents similarly believed that mathematics was important to their futures. In their review of large-scale test data, aimed at uncovering the extent to which such assessments can be considered gender neutral, Leder and Forgasz (2018) found persistent "subtle" gender differences across many countries in mathematics performance that were most often in favour of males. They also noted that in parts of communities in many countries, gender-stereotyping of mathematics as a male domain persisted and they wondered about the impacts of these gendered perceptions on girls as they study mathematics. This is particularly pertinent given Grootenboer and Marshman (2016) reported middle school students believed boys to be better than girls at mathematics.

Bartley and Ingram (2018) collected data on the self-efficacy and emotional arousal (considered on a scale from very calm to very anxious) in relation to mathematics from 84 parent-child pairs and investigated how these variables were manifested in the context of parents helping their Year 8 child with mathematics homework. They found that, based on the almost three-quarters of parents who did help their child with homework, there was a positive correlation between parents being calm and children reporting greater self-efficacy. Children could accurately detect their parents' emotional arousal, and this contributed to their beliefs about their parent's ability and willingness to assist with homework.

Very few studies focussed on the beliefs of post-secondary students not enrolled in initial teacher education programs. Two exceptions were those of Murphy and Wood (2017) and Murphy (2018) concerning tertiary mathematics students. Murphy (2018) confirmed well known positive correlations between self-efficacy and mathematics achievement but added nuance to this by identifying self-beliefs related specifically to selection processes—choices made about the social and physical environment, and activities, as particularly important. That is, students with high selection process self-beliefs make choices about time management and study patterns that make success more likely.

2.4 Mathematics Teacher Educators

Research on mathematics teacher educators was an emerging field that gained in attention over the review period. Much of the focus of research in the area has been on mathematics teacher educators' knowledge and ways in which this can be conceptualised in relation to mathematics teachers' knowledge (Beswick & Goos, 2018). Building on work on the interrelationship of mathematics teachers' beliefs and knowledge, some conceptualisations of mathematics teacher educator knowledge have incorporated beliefs. For example, Chick and Beswick (2018) presented an adaptation for mathematics teacher educators of Chick's (2007) framework for mathematics teachers' Pedagogical Content Knowledge (PCK). Unlike the earlier teacher framework, their conception of mathematics teacher educator knowledge included beliefs about the nature of the content that they are teaching to preservice teachers (i.e., PCK for teaching mathematics to school students). Beswick and Goos (2018) urged mathematics teacher educator researchers to learn from research on affect related to mathematics teachers and hence not underestimate the importance of the affective domain, including beliefs, to mathematics teacher educators work.

Marshman and Goos (2018) reported on the beliefs about the nature of mathematics, mathematics teaching, and mathematics learning of Australian mathematicians, statisticians and mathematics educators based on 82 complete responses to a survey. It is worth noting that all these participants could be considered mathematics teacher educators because they all were teaching mathematics to preservice mathematics teachers (Beswick & Goos, 2018). Marshman and Goos (2018) were motivated by a concern for how preservice secondary teachers who experience teaching from both mathematicians and mathematics educators might be impacted by possibly differing beliefs across these groups. Although most respondents appeared to hold problem-solving beliefs (Ernest, 1989) about the discipline, there were differences between those with educational qualifications compared with those without, and between those responsible for teaching mathematics pedagogy to preservice teachers and those who taught mathematics (or statistics) content. The authors concluded that postgraduate study in education might increase the tendency to have problem-solving views of mathematics, whereas postgraduate study of mathematics might support an emphasis on the correctness of content and its sequencing.

2.5 Identity

As already mentioned, there is variation in the way in which identity is conceptualised by mathematics education researchers, some drawing upon psychological perspectives to consider identity as beliefs about self (e.g., Beswick, 2018), and others taking sociological perspectives (e.g., Darragh, 2016; Walshaw, 2016). Darragh (2016) described the distinction in terms of an acquisition-action distinction with those approaching the topic from a psychological perspective seeing identity as something that is acquired or possessed, whereas others, including Darragh (2016) advocated for identity as an action or process. Darragh (2016) further pointed out there is a danger of theoretical and methodological incoherence when researchers use terms like identity without carefully considering the implications of their stance, and explicitly stating what that is.

Researchers did not always define the way in which they were using identity, but it was implicit in the way they used it. Bennison (2017), for example, presented a framework comprised of Five Domains of Influence—life history, knowledge, affective, social and context—and noted the situated, complex and changing nature of identity as described by Wenger (1998). Although she did not explicitly define identity, sociological influences were apparent, and this continued through her use of Valsiner's (1997) Zone Theory to understand shifts (a process) in one teacher's identity as an embedder-of-numeracy. Similarly, Beswick (2018) did not define identity but referred to it in the context of teachers' belief systems. Although she emphasised the dynamic nature of belief systems, she described them as something an individual possesses.

3 Attitudes

Over the time period, there appears to have been renewed interest in the construct of attitudes, although it has been variously defined, or left undefined. For example, Woodward et al. (2018) accepted having productive dispositions as encompassing positive attitudes. Both Tran and Javed (2017) and Ferme (2018) did not present their conceptualisation of attitude but only presented the scales of their data collection instruments (see below). Many studies used 'attitude' as an outcome and did not ground the concept in affective theory. Further clarity continues to be needed in distinguishing between attitude and other affective constructs, especially between attitudes, beliefs and confidence.

A range of research methods were used to study attitudes, although questionnaires remained the most common. Larkin and Jorgensen (2016) used video-diaries to explore primary school students' attitudes and emotions to mathematics. One finding was that the Year 3 students expressed quite negative views of mathematics. To the authors, this suggested that students develop poor attitudes and emotions about mathematics in the early years of schooling. Later, to better understand the data, Jorgensen and Larkin (2017) used two paradigms (psychology and sociology) to explain students' attitudes towards mathematics from different social backgrounds. Upon this further analysis, the authors found explicit patterns in students' responses that were associated with the social contexts of students' mathematical experiences. Use of these two paradigms was valuable for seeing the nuances within classrooms and their impacts on students.

Bakar, Way, and Bobis (2016) explored six-year-olds' dispositions towards drawing in mathematics—where this was defined as a willingness or reluctance to draw.

The children did not spontaneously produce drawings as part of mathematical activity, which the researchers attributed to the children's lack of experience in drawing for the purpose of mathematical representation. Further research was suggested to determine effective teaching strategies and learning experiences that would build students' confidence and skill in using drawing as a problem-solving tool.

Another study focused on making sustainable changes in students' dispositions, defined as the student actions and attitudes that improve their learning (Long, 2016). This study aimed to provide an intervention (The Prepare 2 Learn Program) that supported students approximately six months behind their year level to reach the expected level or beyond, as well as positively changing students' dispositions towards mathematics. This was done by preparing the students for their mainstream classes and making them aware their actions and attitudes could positively impact their own learning. The author found positive changes in students' dispositions toward mathematics and mathematics learning. Koch (2018) studied students' confidence in their mathematical ability and attitude to mathematics before and after an intervention across 120 schools in Australia. The intervention concerned the creation of geometric shapes, describing them and discovering and generalising patterns. Also included in the intervention was the use of growth mindset strategies and specified teaching approaches. Similarly, the researcher found a positive change in students' confidence and attitude following the intervention.

Studying preservice teachers' attitudes was of interest during the review period. Itter and Meyers (2017), for instance, investigated third year preservice teachers' attitudes towards mathematics through a written reflection task requiring the participants to describe their views about mathematics and the factors that they thought had shaped their views. The researchers found the participants' attitudes varied on a continuum from highly positive to neutral or to highly negative. Indeed, many of the participants held beliefs and attitudes so negative that they were frequently described 'hatred' towards mathematics. One of the sources of participants' negative attitudes and beliefs related to their perceptions of mathematics as a difficult, boring, and irrelevant subject was found to be exposure in secondary school to teacher-centred approaches that emphasised speed and process over understanding and relevance. Tran and Javed (2017) examined attitudes of preservice teachers who were perceived to have low academic ability in mathematics to inform their decision-making on the design of mathematics units, and to support the candidates in transitioning into higher education programs. To measure the attitudes towards mathematics, they used a survey developed by Palacios, Arias, and Arias (Palacios et al. 2013) that comprised four scales: perception about mathematical incompetence, enjoyment of mathematics, perception about utility, and mathematical self-concept. According to the authors, and contrary to the previous literature that indicated preservice teachers had negative attitudes to mathematics, most participants reported they could succeed in doing mathematics, if they tried hard and showed positive attitudes to the subject. This required challenging the preservice teachers about their attitudes towards mathematics and introducing them to growth mindset activities, and providing supporting structures such as giving regular and frequent feedback. Matthews, Boden, and Visnovska (2018) examined preservice teachers' attitudes to and understanding of uses of technology in teaching and learning mathematics. The researchers assumed that technology itself might be a source of anxiety in addition to mathematics in preservice teachers. The survey instrument, therefore, included questions assessing the participants' level of anxiety towards mathematics aiming to study its hinder impact on using technology in mathematics teaching. All participants showed positive attitudes towards mathematics, however, it was noted that of the 87 students who enrolled in a mathematics elective course during their one-year Graduate Diploma program, only eight completed and returned the survey.

Ferme (2018) researched the effect of having a Science, Technology Engineering or Mathematics (STEM) specialisation in their initial teacher education, on teachers' attitudes (and confidence) towards teaching numeracy. Forty-seven teachers from eight regional and metropolitan government secondary schools (Grades 7 to 12) in New South Wales were surveyed. Although the author did not define 'confidence' or 'attitudes', the survey included six confidence and eleven attitude items adapted from instruments used in past studies exploring teacher confidence and beliefs (e.g., Beswick, Watson, & Brown, 2006; Watson, 2001). The results indicated that, compared to other secondary teaching areas such as the humanities and physical education, secondary STEM teachers had greater confidence and tended to have more positive attitudes towards numeracy. As an aspect of a larger research project, Thiele, Dole, Carmichael, Simpson, and O'Toole (2019) examined how the attitudes of the teachers about teaching and learning number facts in Year 3 and 4 were impacted by the project. Following Dole and Beswick (2002), the research instrument involved Likert items such as, "I enjoy teaching number facts to my students" and "I have the pedagogical content knowledge to teach number fact fluency to my students" (p. 718). A positive change in the teachers' attitudes about teaching and learning number facts measured by those items was evident.

4 Motivation and Interest

Research on motivation related to mathematical learning has continued in the present review period. In some studies, motivation was used together or interchangeably with 'interest' or 'aspiration' (Anderson, Holmes, Tully, & Williams, 2017; Li, 2019) and 'beliefs' (Poh & Dindyal, 2016). The construct of aspiration was sometimes operationalised as 'switching off', (i.e., not being interested in pursuing mathematics in future academic life) and was found to be influenced by motivational factors (namely self-efficacy and valuing of mathematics) (Collie, Martin, Bobis, Way, & Anderson, 2019). Schukajlow, Rakoczy, and Pekrun (2017) accepted motivation and emotions as distinct constructs but observed that motivation conceptually overlaps with emotions. They found emotions and motivation both to be important prerequisites, mediators and outcomes of learning and achievement. An iterative relationship between the teacher and student motivation was suggested in research by Russo and Russo (2019). When teachers implemented inquiry-based tasks related to their personal interest (e.g., basketball, robotics, dance) and when students enjoyed them, the teachers enjoyed teaching mathematics, which could further drive student enjoyment. At the same time, students were found to be more positive and motivated about learning mathematics when inquiry-based learning approaches were used.

Orellana and Barkatsas (2018), carefully positioned the construct of motivation within the affective domain and explored the factor structure of students' motivation and perception. The authors used items from a student survey used in their Reframing Mathematical Futures II (RMFII) Project. Data were collected from 442 students in Years 7 to 10 across Australia, and resulted in the identification of four factors: Intrinsic and Cognitive Value of Mathematics, Instrumental Value of Mathematics, Mathematics Effort, and Social Impact of School Mathematics. In addition, statistically significant differences were found between state and year levels for some of these factors.

Wilkie and Sullivan (2018) investigated 3500 middle school students' motivation in mathematics by examining their writing, including about one wish they had for their mathematics learning. The authors aimed to study how the students' self-reports were related to intrinsic and extrinsic motivational aspects in learning mathematics. They found that some students wanted changes within themselves such as to be smarter and to learn more efficiently, whereas others wished for changes in their learning environment such as the opportunity to do more cooperative, challenging or interesting work. The students' willingness to share was an indication of the usefulness of the free-format questions for eliciting student voice in researching mathematical affect.

Li (2019) developed an instrument (Mathematics Enrolment Choice Motivation) to explore motivational factors that influence Year 11 students' decisions about whether or not to pursue studying mathematics. Li (2019) acknowledged the inherent value of using multiple theoretical lenses to explain the differences in mathematics enrolment choice and found five clear factor structures of this instrument: self-concept, self-efficacy, subjective value, anxiety and learning mathematics.

Muir (2018) examined the role of the teacher in implementing a flipped classroom through a case study of one secondary mathematics teacher's classroom. Muir used self-determination theory, which posits that individuals need to experience: competence, autonomy and relatedness, and found these needs to be catered for in flipped classrooms. The online survey included a motivational aspect, and its analysis showed the teacher's use of the flipped classroom approach helped students academically, mainly because the teacher had established a healthy relationship with the students, and this meant students were motivated to watch the videos prepared by their teacher and to engage in class.

Carmichael, Callingham, and Watt (2017) used 471 matched student-teacher data pairs from a 2015 Australian national survey to explore the extent to which teacher enthusiasm impacts the classroom motivational environments. The authors used achievement goal theory to provide a framework for analysing the motivational environment of the mathematics classroom. One of the results was that student perceptions of teachers' enthusiasm for teaching mathematics positively predicted students' perceptions of a classroom mastery environment, which in turn predicted student interest. They identified that interest has both state and trait-like properties, and defined *situational interest* as the interest felt within a classroom environment, and the deep and enduring interest built up over time as *individual interest*. Furthermore, the authors viewed interest as directed by affective (including emotional and value dimensions) and cognitive control systems, and noted that studies generally focus on exploring affective control systems:

> The emotional dimension includes the experience of enjoyment and excitement, the value dimension the extent to which the knowledge and practice of mathematics is associated with conceptions of self, and the cognitive dimension a desire to acquire new knowledge about the subject. (p. 449)

There was a small number of other studies related to interest. For example, as part of a MERGA Symposium (Anderson, Holmes, Tully, & Williams, 2017), Williams presented the results of student and teacher interviews, which provided evidence that using interdisciplinary projects that emphasise design thinking can increase students' interest in STEM careers. Poh and Dindyal (2016) used the rich history of Calculus as a pedagogical tool in a quasi-experimental design in a secondary school context. Many students found the lesson package on the history of Calculus enjoyable and interesting which increased their interest in and motivation to read more about Calculus and improved their Calculus scores.

5 Engagement

Engagement in mathematics, defined as an individual's involvement in mathematical activity (Durksen et al., 2017) has been a prominent area of study in mathematics education research since the last review period. Students' engagement has been deemed particularly important because of its perceived role in improving student outcomes in mathematics (Coupland et al., 2017), and the part it plays in participation in science, technology, engineering and mathematics-related disciplines (Watt & Goos, 2017); a role acknowledged in Australian federal budgets (Coupland et al., 2017). Engagement-related research in the review period was anchored by the publication of a Special Issue of the *MERJ* (introduced by Carmichael & Callingham, 2017). Empirical studies explored in this issue were grounded in theoretical frameworks related to engagement in mathematics and the discussion was relevant to the wider field of mathematics education research.

The prominence of studies that include engagement as a central component is due partly to the concept's definition, which can support multidimensional approaches. Several researchers based their conceptualisation of engagement (e.g., Attard, 2018a; Durksen et al., 2017; Watt & Goos, 2017) on the work of Fredricks, Blumenfeld, and Paris (2004). This broad approach considers individuals' engagement to have affective, cognitive and behavioural dimensions and associated contextual influences, arguably encompassing what might be considered as other aspects of the affective domain:

> In mathematics, behavioural engagement refers to the extent to which students participate, including actual or intended enrolments, and degree of effort applied. Affective engagement includes the emotional dimension of interest, student enjoyment, and can extend to identification with the school culture [. . .] Cognitive engagement taps students' personal investment, including self-regulatory strategies. (Watt & Goos, 2017, p. 134)

Using this approach, Attard (2018a), defined engagement as the actions and behaviours that are the result of a student's motivation. She conceived of it operating at cognitive, affective and operative levels (see Attard's Framework for Engagement with Mathematics, 2012), and conducted an action research project that explored whether the combination of financial literacy and mathematics could improve student engagement with mathematics. Attard (2018a) found that student engagement improved when they were engaged in problem solving in real-life contexts. Carmichael, Muir, and Callingham (2017) used data from the 2015 national survey to explore the impact of within-school autonomy on students' goal orientations and engagement. Their large data set included students' survey responses to items including what the researchers described as engagement items reflecting emotional interest ("I like mathematics more than my other subjects at school") and cognitive interest (e.g., "After maths class I am curious about we are going to do in the next lesson"). They found that students in high-autonomy schools were more likely to describe a classroom motivational climate with a greater focus on mastery, higher levels of teacher enthusiasm, and a greater emphasis on caring in their school than did their peers in lower-autonomy schools. In terms of the emotional and cognitive components of engagement, the study did not indicate a link between within-school autonomy and engagement (or interest).

A body of research within the review period explored teachers' pedagogical practice using engagement as a learning outcome. In the case of rural schools, in addition to teacher-related factors (teacher capacity, differentiation in teaching strategies), valuing mathematics, careers education and vocational education and training were also found to positively contribute to student engagement and mathematics performance (Murphy, 2019).

Durksen et al. (2017) used interrelated but distinguishable indicators of motivation and engagement to better understand students' experiences in the classroom. They presented a strong theoretical base for their work, informed by the Motivation and Engagement Wheel (Martin, 2007), which combines theories of achievement motivation, attribution theory, self-determination theory, self-efficacy theory, and self-worth motivation theory. These researchers also considered additional emotions beyond those identified in the Wheel (e.g., anxiety, value). Furthermore, they used the Classroom Assessment and Scoring system (Pianta & Hamre, 2009) to assess observable teacher-level supports in the classroom, measuring them across the three domains defined by Pianta, Hamre, and Mintz (2012): emotional support, classroom organisation, and instructional support. Their mixed-methods study of 4383 middle years (aged 10–13) students in 257 classrooms, was designed to discover broad pedagogical characteristics of mathematics teachers to inform an intervention programme aimed at arresting the decline in mathematics achievement and engagement during the middle years of schooling. As part of this study, they explored the perceptions of six upper primary and secondary teachers to explore how they motivated and engaged students such that students had higher-than-average levels of motivation and engagement. They found that confidence, classroom climate, contact and connection, and within classroom organisation, were key indicators important for student motivation and engagement in mathematics. Other researchers who sought to improve students' engagement in learning mathematics through pedagogical practices were Fielding-Wells, O'Brien, and Makar (2017) who explored the use of inquiry-based learning, and Brown (2017) who investigated the protentional of collective argumentation.

A further strand evident in the engagement literature during this research period was the use of engagement to assess an aspect of using technology within the classroom. Attard (2018b), for example, used Fredricks et al.'s (2004) multidimensional view of engagement to inform her conceptualisation of a Framework for Engagement with Mathematics (Attard, 2012). The framework indicates the conditions needed for engaging learning experiences in mathematics to occur. She used it as a lens to assist in determining how the use of mobile technologies impacts students' engagement with mathematics. Ingram, Williamson-Leadley, and Pratt (2016), and Ingram, Pratt, and Williamson-Leadley (2018) used Ingram's conceptualisation of students' engagement skills to explicate the quality of engagement skill that a student needs to thrive in mathematics. They used teachers' and students' perspectives to explore whether the use of Show and Tell apps enables teachers to enact effective pedagogy and students to demonstrate engagement in problem solving. Laird and Grootenboer (2018) reported on the design of a data collection instrument to capture students' engagement in mathematics. Other uses of engagement within the technology in mathematics field included the work of Calder and Murphy (2017, 2018), Muir (2017, 2018), Muir and Geiger (2016), Symons and Pierce (2018), and Sedaghatjou and Rodney (2018).

Other researchers examined students' engagement in relation to pedagogical practice, including using challenging tasks (Russo & Hopkins, 2017), a reform-oriented approach (McCormick, 2018), and the five-question approach (Ley, Attard, & Holmes, 2018). Ingram, Holmes, Linsell, Livy, and Sullivan (2016) found that, when teachers arranged for and encouraged students to work independently and cooperatively, asked questions, provided materials, and asked students to reflect on their engagement, students were encouraged to persist with a challenging task.

Engagement was often viewed as participation in mathematics courses, especially when these becomes non-compulsory (e.g., Anderson, Holmes, Tully, & Williams, 2017; Bennison, Goos, & Bielinski, 2018). Bennison et al. (2018) used a sociocultural approach to identify practices that might promote sustained engagement with mathematics among students from disadvantaged backgrounds. They identified engagement and motivation as one set of affective factors influencing students' aspirations in relation to mathematics and noted the value of Rogoff's (1995) person-in-context perspective because of the important linkages between students' individual cognitions and embedded contextual influences, and students' socially constructed aspirations. They collected data on students' perceptions about strategies and factors, which lead to increased participation in mathematics. Six themes emerged as potential institutional practices contributing to increased enrolments: curriculum organisation across year levels, staffing of mathematics classes, culture of the mathematics department, STEM program, and provision of appropriate tasks and resources. A school culture, at the institutional level, of high expectations and belief in students was deemed to be important.

Not evident in research conducted in this review period is the consideration of the connection between engagement and emotions during engagement with a mathematical task (state). Also there is little evidence of research into students' or teachers' perceptions of what engagement is.

6 Feelings, Emotions and Anxiety

There have been a relatively small number of studies that have focused on feelings about mathematics either at the level of an individual's overall relationship with the subject (trait) or the feelings that an individual experiences when engaging in mathematics (state). There were, however, connections with research related to teachers' enthusiasm as previously discussed (e.g., Carmichael, Callingham, et al., 2017a, Carmichael, Muir, et al., 2017b).

Mathematics anxiety, conceptualised predominantly as an emotion (Ashcraft & Krause, 2007), was of interest in relation to preservice teachers. For instance, Brown and O'Keeffe (2016) explored how increasing preservice teachers' conceptual understanding of mathematics can impact on their perceived levels of mathematics anxiety. Brown and O'Keefe held fortnightly sessions with five preservice teachers, mostly centred on fractions, percentages and statistics. The sessions did not include formal teaching practices, but rather, were based on Bandura's (1997) two principles of social learning theory: "working together as a supportive group of peers and building on each other's experiences of success and positive reinforcement and affirmation to boost their confidence" (p. 160). Using the analysis of pre- and post-survey data related to procedural and conceptual mathematical knowledge and beliefs regarding performing various mathematical activities, the authors concluded that building effective support around Bandura's (1997) two principals of self-efficacy could be helpful in terms of ensuring that preservice teachers build strategies to help themselves succeed.

Perkins (2016) researched the potential impact of a mentoring program on eight preservice primary teachers' self-identified mathematics anxiety and their levels of confidence to teach mathematics. The author found some shifts in participants' confidence to teach mathematics and concluded that matching preservice teachers with confident, expert teachers could support them in overcoming their lack of confidence and their mathematics anxiety.

Wilson (2016, 2017) made important contributions to the frameworks available to study the potential causes of mathematics anxiety, namely the Quality of Life framework and the Stages of Humiliation model. By asking the participants to identify a critical incident that impacted on the way they felt, Wilson (2017) identified humiliation as an important element in incidents that the preservice teachers described.

Wilson (2016) proposed that "challenging the perceptions of past experiences identified via critical incidents and the feelings that they invoke, will assist preservice teachers in addressing mathematics anxiety and becoming more effective teachers of mathematics" (p. 621).

In the context of an education program addressing teaching skills and confidence issues in mathematics and science areas, Axelsen, Galligan, and Woolcott (2018) used reflective practices to uncover types of emotions and themes from student chosen critical moments. It was found that the strongest common emotions that the preservice teachers associated with teaching mathematics was anxiety. It was importantly noted that:

> … while the preservice teachers felt anxiety or felt they displayed anxiety while teaching a mathematics lesson, other people (i.e. other preservice teachers and their mentors) observing were less likely to observe displays of anxiety. Here the use of critical moments and engaging with video recordings in reflective practice is important for helping to demonstrate to preservice teachers that while they may feel certain negative emotions and indeed feel they are displaying those emotions, often those feelings are being professionally and well contained. This intervention therefore helps preservice teachers to build a certain level of confidence for the realisation that their teaching 'performances' are often better than they perceived. (p. 136)

Several studies focused on school students. O'Keeffe, White, Panizzon, Elliott, and Semmens (2018) examined a large group (n = 1240) of Years 7 and 8 students' mathematics self-efficacy, self-concept and anxiety through a survey based on the PISA 2012 survey. Based on the results, the researchers underlined that mathematics anxiety remained an issue for almost one third of students in schools in South Australia and that this is a deeper issue for female students. Prodromou and Frederiksen (2018) made pedagogically valuable suggestions for reducing mathematics anxiety. Employing a design research methodology, they worked with an experienced practising teacher and studied his 26 Year 6 students' level of anxiety (if any) using a mathematics test and an anxiety questionnaire. The authors suggested that using pictures, graphs or visuals in mathematics teaching might help students who are anxious. Prodromou and Frederiksen concluded that the study "might help the teachers to be more aware of mathematics anxiety and it will hopefully lead to an open discussion so it can address the needs of the students" (p. 645). Another study compared the efficacy of personal videoconferencing tuition with a face-to-face model, in terms of impacts on mathematics anxiety for upper primary and middle school students who had mathematical learning difficulties (Kestel, 2019). The findings suggested that, after the personal videoconferencing tuition program, the participants showed significantly lower levels of anxiety, implying that these types of programmes might decrease mathematics anxiety in students with mathematical learning difficulties.

7 Methodology and Methods

The research methodologies and methods employed in this review period were generally from a small range similar to those used previously. Overall, the nature and size of the participant groups in the studies were similar to those identified in previous four-yearly reviews. In this section, a summary of the methodologies, methods and participant groups is provided with brief comment, before some emerging issues are critically discussed. The summaries have been developed drawing on the information available in the approximately 80 relevant published research reports reviewed. These were not always clear, so the aggregated information should be seen as indicative of overall trends.

The reported methodologies employed included a fairly small range with an emphasis on qualitative approaches (see Table 1).

Although here the term 'qualitative' is rather broad, it seems to refer to studies that are descriptive and/or interpretive in nature, and when combined with the 'case studies' (which were almost all qualitative) accounted for nearly 60% of the studies reported. The studies reported as 'design research' tended to be where an intervention or innovation was trialled, and the outcomes monitored, and the quantitative studies were predominately questionnaires administered to a large sample.

Within the methodologies noted above, the following research methods were noted (see Table 2).

A small number of studies used methods like "drawings" (e.g., Bakar, Way, & Bobis, 2016), tests, and "video diaries". Larkin and Jorgensen (2016), for instance, used video-diaries to explore primary school students' attitudes and emotions to mathematics by having students make video-recordings in a "mathematical thinking space" (a tent). The methodological implication of the study was that iPads were a useful tool for collecting student data.

Table 1 Methodologies

Qualitative (not further defined)	37%
Case study	21%
Mixed methods	14%
Quantitative	10%
Design research	9%
Theoretical or literature review	9%

Table 2 Methods

Survey/Questionnaire	32%
Interviews	28%
Observations	18%
Written reflections	12%
Documents/Artefacts	6%

Of note here is the relatively high percentage of surveys/questionnaires given the small number of quantitative studies noted above, and the predominance of small samples (to be outlined in the next section). Also, it appears that, at least at this crude level of analysis, standard research methods still dominate research into the affective dimension of mathematics education.

It is perhaps striking that ‘action research’ does not feature more prominently here (although design-based research can be seen as a form of action research), particularly given the perennial and consistent nature of the affective issues noted previously. To address these long-standing and pernicious concerns in mathematics education, site-based action research, where educators and researchers (and perhaps managers and students) work collaboratively to engage in research and development in response to local affective mathematical concerns, would seem to be a viable and worthwhile methodology to employ.

7.1 Participants

The sample size for the studies of affect and mathematics education over the review period ranged from n = 1 to n = 3660, although smaller samples were the most common (see Table 3).

Of note, 16% of the studies had a solitary participant, and while it is not problematic to have one participant, the proportion seems quite high. Given the predominance of qualitative studies noted previously, it is not surprising that the largest share of the studies had a relatively small sample. Furthermore, the nature of these samples were mostly teachers (see Table 4).

As can be seen in Table 3, nearly 70% of the studies reported were undertaken with teacher (practising and preservice) participants, and although teachers are crucial in promoting positive affective development in mathematics, this does seem to be a disproportionally high share of the research undertaken. Finally, the studies were conducted across all educational stages (see Table 5), with some studies focusing

Table 3 Study sample sizes

1–10	45%
11–50	18%
51–100	13%
101–200	8%
200+	16%

Table 4 Participants

Teachers	49%
Preservice teachers	20%
Students	26%

Table 5 Educational stages

Early childhood	1%
Primary school	48%
Middle school	13%
Secondary school	31%
Tertiary	6%

across levels.

In the previous summary tables smaller numbers have been omitted, but in Table 5 early childhood is included as it seemed striking that there was only one reported study with this focus. It is important to note that the 'tertiary' category focused on studies in tertiary mathematics courses, but not preservice teacher education (even though it is tertiary).

8 Methodological Issues

In evaluating the research designs of the relevant studies over the review period, there were some issues that emerged, and these are now briefly outlined. First, as was noted above, there is an apparent research gap related to studies in early childhood contexts. The long standing and continuing work of Perry, Gervasoni, and Hampshire, with O'Neill (2016), into students' mathematics education prior to primary school reported some results regarding "dispositions", but this was the lone study.

Second, as was evident in Table 1, there were quite a few mixed methods studies (14%), although many of these had small participant numbers. While this is not the place to discuss issues regarding mixed method research in general, a concern relates to the veracity and viability of the quantitative part of mixed method studies when the sample is small. Perhaps in this area of mathematics education research, the term mixed methods has come to be used quite broadly and generally, when many of the studies are actually qualitative with some minor descriptive statistics used to augment or contextualise the findings.

Third, historically many quantitative studies of affect in mathematics education used well-established scales such as the Fennema-Sherman scales (e.g., Forgasz, Leder, & Kaur, 2001). However, these are generally quite old and often developed for another context, and while some contemporary instruments have been developed (e.g., Grootenboer & Marshman, 2016), none of them seem to have been used widely. To provide some robustness and capacity for comparison, some modern instruments that relate to the Australasian context might be useful to advance research here. Perhaps some of the ones reported over the review period might be picked up by others to this end.

Fourth, as was highlighted, there were a disproportionate number of studies with a single participant, and 45% with ten or fewer. There is no doubt that to understand the complex issues related to affect in mathematics education, in-depth studies with a small number of participants are useful and necessary. However, alone they lack the gravitas to advance knowledge and practice, and so perhaps these have limited value, and it may be a useful exercise to try and undertake a meta-analysis of these studies to see if there are any insights from across them. At this stage in the field's development (we have noted its inclusion in these reviews for two decades) it might be expected that larger studies that explore the extent of relevance of variables uncovered in small scales studies would be in order.

Fifth, not only were the studies mostly small scale, there was a continuing trend to centre on teacher participants. Of course, teachers are the crucial and most significant factor in students' mathematics learning, but the focus of education is students, and so it would seem important that a higher percentage of studies focus on the development of their affective qualities vis-à-vis mathematics. Currently it is only around 25%.

Finally, given the fairly well-understood problems with poor affective responses to mathematics, it seems that perhaps it is timely to have studies focussed on ways to ameliorate these and to facilitate positive mathematical affect. Indeed, given the long-standing nature of these issues, it would appear that there is still important research and development work to be done so well-known concerns can be addressed, and informed change initiated. To this end, maybe it is time for some robust design-based research and action research to be undertaken through collaboration between researchers, teachers, and educational systems.

9 Concluding Comments

Reviewing studies of affect in mathematics education continues to be challenging because of the unclear and overlapping definitions employed. We note that studies that do not focus on affect, yet use its constructs, are particularly sparse in terms of definitions, theoretical framing and positioning in relation to learning. However, when a broad approach to affect is taken, these overlaps can be opportunities for getting a useful picture of how improvements can be made to student learning. That said, in any approach, careful definition of the affective constructs and considered theoretical underpinnings are vital in order to compare and grow this research field, and to make robust recommendations for development.

Many of the studies assumed a relationship between positive affect and positive student outcomes, defined mainly to be increased learning and increased participation in mathematics courses. Achievement used as an indicator that student learning has taken place is useful, but there is a paucity of studies that link affect with achievement. It is pleasing to see studies that explore the behavioural aspects of engagement—the state of doing of mathematics within a specific situation. This serves as a useful proxy for learning with the assumption that the more one does something, the more they might improve. Further separating individuals' overall relationships with mathematics (trait) from the context of the moment—the moment when they engage in the mathematics (state) is therefore useful.

Of course, mention must again be made of the dearth of studies that explored the affective domain in the context of student and teachers in the years prior to formal schooling. The recent publication of a special issue that explores affect and mathematics in young children in *Educational Studies in Mathematics* (Batchelor, Torbeyns, & Vershaffel, 2019), will hopefully provide a catalyst for this work. Understanding how individuals are first introduced to mathematics and how they build their understanding in those very early years is the cornerstone of understanding students' mathematical journeys and the pathways they choose in later years.

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Chapter 8
Equity, Social Justice, and Ethics

Colleen Vale, Robin Averill, Jennifer Hall, Helen Forgasz, and Gilah Leder

Abstract Recent Australasian research on equitable, socially just, and ethical mathematics teaching and learning is reviewed and critiqued in this chapter. The literature surveyed includes studies in which researchers reported on the degree of equity for Australasian communities previously identified as disadvantaged in mathematics: girls and women; low socio-economic students; Indigenous, Māori, and Pasifika students; and rural and remote students. Studies of teaching practices and whole-school approaches to improve the outcomes of students in schools in disadvantaged communities are discussed, as are pre-service teacher education programs for teaching and working within these communities. In the reviewed work, researchers drew on various theoretical frameworks for equity, social justice, and ethical practice. In several studies, cultural responsiveness for mathematics learning was explored and the researchers drew attention to the importance of ensuring participation of disadvantaged and marginalised communities. While many of the studies reviewed were small scale, there was also evidence of longitudinal and multiple case study research. Sustained further research is needed to address diversity for social justice in mathematics education within disadvantaged communities, including non-binary gender, cultural diverse, and rural communities.

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1 Introduction

In the chapter on equity, social justice, and ethics in mathematics education published in the previous MERGA 4-yearly review, Vale, Atweh, Averill, and Skourdoumbis (2016) concluded that "equity, social justice and ethics concerns remain high in Australasian mathematics education research" (p. 113). Then, and now, the scope and extent of disadvantage experienced by select students are not readily quantified, though data from large-scale national and international mathematics and numeracy tests provide some provocative snapshots. For example, researchers analysing data from tests such as the Programme for International Student Assessment (PISA) report not only overall performance data, but also provide results for constituent subgroups, including by ethnicity, language background, socio-economic level, gender, and geographic location. Through the inclusion of affective measures, possible links between these factors and students' mathematics achievement can be explored. Inspection of these reports readily reveals that certain groups, on average, consistently perform below or above their peers, both nationally and internationally. These results show the importance of continued emphasis on equity, social justice, and ethics in mathematics education in Australasia. Such work is not straightforward, as "creating, supporting, and sustaining a culture of access and equity require [*sic*] being responsive to students' backgrounds, experiences, cultural perspectives, traditions, and knowledge when designing and implementing a mathematics program and assessing its effectiveness" (National Council of Teachers of Mathematics [NCTM], 2014, p. 1).

The aim of this chapter is to review and critique research focussed on the creation, enactment, and maintenance of equitable, ethical, and socially just educational provisions, in and beyond the mathematics classroom. Issues of particular relevance to Indigenous, Māori, and Pacific Nations learners comprise an important component of this discussion. Research with a focus on these groups is included here, rather than in a separate chapter as in the previous 4-yearly review (Meaney, Averill, McMurchy-Pilkington, & Trinick, 2016). While acknowledging that both Vale et al. (2016) and Meaney et al. (2016) drew heavily on Fraser's (2005, 2013) model of social justice in their analyses of relevant research, this theoretical framework did not unduly shape or constrain the evaluation of the research monitored for this chapter. In some studies reviewed in this chapter, the researchers drew on Fraser's theory of social justice; however, we were also keen to identify other frameworks informing research in this field, notably non-binary gender frameworks and decolonial theory.

In our literature search, we identified a large number of scholarly publications related to the field. We have accepted a core instruction issued by the editors of the previous 4-yearly review: "Since it is impossible to report on all publications of Australasian research in this period," they admonished, "chapter authors are asked to be selective in the research they reported" (Makar et al., 2016, p. 3). Accordingly, we have attempted to balance the need to focus on robust research, reports of international tests, and theoretically-driven initiatives, while giving voice to smaller but promising research studies. Hence, we generally excluded from our review partial reports or incomplete descriptions of small studies on a single strategy, unless they seemed likely to move the field forward. Although many of the publications reviewed relate to multiple equity issues, we have pragmatically used the main topics to cluster our comments.

We begin by identifying and defining the equity factors explored and providing policy context relevant to studies of these equity groups. Then, we discuss studies of equity factors concerning students' participation, achievement, and attitude. In the third section, we focus on studies of socially just and ethical teaching of mathematics. In some of these studies, the researchers focussed on particular disadvantaged communities, while other studies were situated in disadvantaged communities and thus relate to multiple equity factors, such as low socio-economic status (SES) and rural location. In the final section, about teacher knowledge and school change, we discuss studies where researchers investigated initial teacher education to prepare teachers for teaching in disadvantaged communities, as well as professional learning and whole-school change models to implement socially just practices. The chapter culminates with a discussion organised according to equity groups and a conclusion.

2 Equity Factors

In the previous RiMEA chapter on equity, social justice, and ethics, four equity factors were the focus of the review: SES, gender, ethnicity, and geographic location. In this review, we note the various definitions and measures used to categorise students in large- and small-scale quantitative studies and to describe participants in small scale qualitative or intervention studies. In small-scale studies, researchers typically used descriptors for SES, such as low SES, while, in large-scale international studies, proxies, such as the number of books in the house, were used as an indicator of SES. Several categories are used for ethnicity and/or language background: Indigenous/non-Indigenous, language background other than English (LBOTE)/non-LBOTE, English spoken at home/other language spoken at home, and foreign-born/first-generation/Australian-born parents. In small-scale studies, researchers identified specific ethnic communities (e.g., Māori, speakers of Kimberley Kriol). Geographic location classifications vary according to the study; researchers typically used descriptors such as metropolitan, provincial, and remote. Distances from a metropolitan location are used to define these terms, but these distances vary according to state and study. In most studies, gender is defined by binary categories: male/female or girl/boy. The problems with these categories for gender and other equity factors are discussed later in the chapter.

In the various qualitative studies reviewed in this chapter, researchers noted the interrelatedness of these factors and typically described their participants and schools as being "disadvantaged schools" due to the existence of one or more equity factors. Warren and Miller (2016) provided a particular definition for "marginalised" communities as "disadvantaged in all aspects of life" (p. 2), including racial isolation, social exclusion, unemployment, financial dependence, drug and alcohol abuse, and low education levels. Schools servicing such communities also report a high turnover of staff and employment of inexperienced and under-qualified staff with no connections to or understandings of the community (Jorgensen, 2017).

As noted, this chapter also includes a review of the studies involving Indigenous and Māori students. Understanding the political context in Australia and New Zealand is necessary in order to review research involving Indigenous and Māori students. Aotearoa New Zealand was founded on a treaty, Te Tiriti o Waitangi, between Indigenous Māori and the British crown. Māori are a diverse group, with differences in culture and language within and between iwi (tribes). With the ideals of the treaty yet to be fully realised, much of New Zealand society largely reflects Eurocentric structures, institutions, and processes. Education policy and research in New Zealand is increasingly focussed on seeking to ensure that the treaty is honoured. The term "Pasifika" is used in New Zealand to refer to people from the Pacific region who live in New Zealand and have family and cultural connections to Pacific Island Nations (Ministry of Education New Zealand, 2018). Māori and Pasifika students constitute 24% and 10%, respectively, of the student population in New Zealand (Education Counts, 2019).

In Australia, Aboriginal and Torres Strait Islanders are 5.6% of the student population (Australian Bureau of Statistics, 2018). Unlike New Zealand, Australia's Indigenous people were not recognised until 1967 when they were "counted" as part of the population. After 200 years of colonisation, there is no treaty or recognition of Aboriginal and Torres Strait Islander tribes as the first sovereign Nations of Australia. In 2017, the First Nations people came together and produced the *Uluru Statement from the Heart*, calling for recognition in the Australian Constitution and their voice to be heard and matter:

> We seek constitutional reforms to empower our people and take a rightful place in our own country. When we have power over our destiny our children will flourish. They will walk in two worlds and their culture will be a gift to their country. We call for the establishment of a First Nations Voice enshrined in the Constitution. ("Uluru Statement from the Heart," 2017, p. 1)

In this statement, teachers and educators are called on to provide a voice for First Nations people in their school communities and practices so that First Nations children and students can retain their cultural identity and excel, including in Western mathematics. Recognising and providing voice to Indigenous, Māori, Pasifika, and Papuan communities was critical in the studies focussed on enhancing opportunities for students in these disadvantaged or marginalised communities (e.g., Edmonds-Wathen, Owens, & Bino, 2019; Jorgensen, 2018; Trinick, Fairhall, & Meaney, 2016; Warren & Miller, 2016).

3 Mathematics Participation, Achievement, and Attitude

Reports of national and international mathematics and numeracy testing include analyses by a range of equity variables. In the review period, there were also a few research studies in which various aspects of mathematics education were explored with multiple equity variables involving primary, secondary, and university students.

Large-scale testing regimes remain contentious. While the intended purpose of tests may be educationally sound, there can be a range of unintended consequences. The foci of scholarly critiques of large-scale testing include how the data are interpreted and reported, the pressures on students to perform well, time and money wasted on preparing students for the test, and the deleterious impacts on low-performing schools, as well as on teachers and principals (e.g., Forgasz & Leder, 2011; Lingard, Thompson, & Sellar, 2016). It should be remembered, however, that it was the early analyses of large-scale mathematics test data disaggregated by gender ("Sex" was the variable used at the time) that led to research efforts to find explanations for the gender differences identified. Responsible and appropriate interpretation of large-scale data can provide effective overviews of the status of equitable educational outcomes.

3.1 Equity in School Mathematics Achievement

To provide an Australasian overview of mathematics achievement by a range of equity variables, results were drawn from the 2015 Programme for International Student Assessment (PISA; May, Flockton, & Kirkham, 2016; National Center for Education Statistics, n.d.; Organisation for Economic Co-operation and Development [OECD], 2016; Thomson, De Bortoli, & Underwood, 2017) and the 2015 Trends in Mathematics and Science Study (TIMSS; Mullis, Martin, Foy, & Hooper, 2016; Thomson, Wernert, O'Grady, & Rodrigues, 2017). The second PISA financial literacy assessment was conducted in 2015. Of the 15 participating countries (Thomson & De Bortoli, 2017), Australia, but not Singapore or New Zealand, took part. Also, it is only in Australia that annual national testing of mathematics (called numeracy) is conducted. The latest (2018) National Assessment Program Literacy and Numeracy (NAPLAN) numeracy results (Australian Curriculum, Assessment and Reporting Authority, 2018) were examined.

Gender differences in achievement in large-scale international and national tests varied according to test and country. Boys outperformed girls for PISA and TIMSS (Year 4) in Australia and New Zealand, for TIMSS (Year 8) and NAPLAN (all year levels) in Australia, but these differences were not statistically significant. Exceptions included TIMSS in Singapore for Year 4 and Year 8 and New Zealand for Year 8, where girls outperformed boys. In Australia, girls scored significantly higher than boys on the PISA financial literacy test.

Findings were consistent across all tests regarding Indigeneity. For each of the tests, Australian and New Zealand non-Indigenous students scored significantly higher than Indigenous (First Nations or Māori) students. However, findings regarding ethnicity or language background varied according to test and country. In Australia, students who spoke English at home scored lower than those who spoke another language on Year 8 TIMSS and NAPLAN. However, findings were the opposite for PISA financial literacy, as LBOTE students scored lower than non-LBOTE students. In New Zealand, Asian students scored highest on PISA, followed by Pākehā (New Zealand European) students and Māori students; Pasifika students scored lowest.

As noted, different measures of SES were used to compare achievement by SES. However, the results were consistent across all studies, with students from higher SES backgrounds achieving significantly higher than students from lower SES backgrounds. Each of the tests conducted in Australia showed that achievement decreased as distance from metropolitan capital city increased.

In summary, the following patterns were generally found: non-Indigenous students outperformed Indigenous students, students from higher SES backgrounds outperformed students from lower SES backgrounds, and students from metropolitan schools outperformed students attending non-metropolitan schools. Findings with respect to language background (or language spoken at home) and gender were inconsistent and varied according to the test and/or country.

3.2 Equity in Upper Secondary School Mathematics Participation and Achievement

Equity factors impacting student participation and achievement in upper secondary mathematics were investigated. Murphy (2018) focussed on the mathematics subjects that provide pathways to tertiary mathematics. He reported that access to, participation in, and achievement in upper secondary mathematics were lower in low SES schools compared to high SES schools, and in non-metropolitan schools compared to metropolitan schools. Importantly, "a non-metropolitan location can mitigate the apparent influence of school SES" (p. 588), as the students from higher SES schools in non-metropolitan locations did not participate and perform to similar levels as students in high SES metropolitan schools. Watson et al. (2016) reported on Tasmanian rural students' perceptions of the factors influencing their intentions to complete secondary school and study mathematics. The researchers reported findings related to gender, SES, and rural and remote location. From their regression analysis, they identified "Friends, English and Mathematics Ability, Other Activities and Teacher Support as the best predictors of students' aspiration to continue schooling" (p. 4).

3.3 Equity in University-Level Participation in Mathematics and Related Fields

In the following sections, we highlight gender equity-related issues regarding university enrolment and graduation in Australia, New Zealand, and Singapore.

In Australia, only 33% of undergraduate students in the mathematical sciences are women (Australian Mathematical Sciences Institute [AMSI], 2017), compared to 58% of undergraduate domestic students who are women (Larkins, 2018). Thus, women are underrepresented in mathematical sciences programs. Overall, from 2001 to 2014/2015, the number of students graduating with bachelor's degrees in the mathematical sciences has declined, whereas the reverse trend is true for honours degrees (AMSI, 2017). This increase is due to increased numbers of graduations by men, as there have been decreased numbers of graduations by women (AMSI, 2017). At the doctoral level, from 2001 to 2015, there was an overall increase (as well as an increase for men and women, when considered separately) in the number of degree completions (AMSI, 2017). Notably, the increased number of doctoral completions by women was due to international students; there was a slight decline in the number of completions by domestic students (AMSI, 2017). These statistics raise questions regarding the pathways of women who are domestic students in mathematics.

In New Zealand, in 2017, only 4.1% of students enrolled in bachelor's degree programs, 1.5% of students in master's degree programs, and 1.9% of students in doctoral programs were in the mathematical sciences (Education Counts, 2018). Statistics related to gender were only provided for domestic students, of whom, in 2017, 35.2% of bachelor's students, 36.8% of master's students, and 41.2% of doctoral students in the mathematical sciences were women (Education Counts, 2018). In contrast, 61.7% of bachelor's students, 61.4% of master's students, and 58.3% of doctoral students, across all fields of study, were women (Education Counts, 2018). Thus, as in Australia, women are underrepresented in the mathematical sciences at the university level in New Zealand.

In Singapore, statistics about university programs in the natural, physical, and mathematical sciences are combined, so it is difficult to make comparisons to the statistics from Australia and New Zealand, where data regarding the mathematical sciences are provided separately. In 2017, 58.1% of students enrolling in and 64.2% of students graduating from these programs were women, compared to 50.0% and 51.6% women overall in university programs (Ministry of Education Singapore, 2018). Hence, women are slightly overrepresented—both relative to the population in general and to the population of university students—in these fields of study.

3.4 Gender Equity in Participation, Achievement, and Attitude

Approximately 20 publications about gender and mathematics education were analysed. The research reported in all but two of these publications was conducted in Australia; the other research was conducted in New Zealand. In the following sections, the findings and methodologies of "gender issues" publications are discussed.

Concerningly, the patterns shown in the analyses of achievement, participation, and attitudinal data show little improvement from earlier studies, in terms of gender equity. As discussed, boys generally outperformed girls on large-scale mathematics/numeracy assessments, such as PISA and NAPLAN, over the past several years (Forgasz & Leder, 2017; Li, 2018). However, at the school level, the patterns were very different, with girls performing better than or as well as boys on classroom assessments (Hopkins & Bayliss, 2017; Sikora & Pitt, 2019). Such differences between classroom and large-scale assessment outcomes by gender have been frequently reported, both in Australasia and around the world (Leder, 2019).

With regard to studying mathematics when it is no longer compulsory (typically Years 11 and 12), concerning patterns were also found: Boys were more likely than girls to study mathematics at these levels and to select higher-level mathematics classes (Forgasz & Leder, 2019; Li, 2018; Sikora & Pitt, 2019). Forgasz and Leder (2019) examined enrolments in Victorian single-sex schools in the Victoria Certificate of Education STEM subjects (2001–2017). For the mathematics subjects, they found no difference in the gendered pattern of enrolment between single-sex and co-educational schools for the lowest and highest level mathematics subjects. For the middle level mathematics subject in Victoria (Mathematical Methods), a higher proportion of girls in single-sex schools than in co-educational schools was enrolled. Forgasz and Leder (2019) concluded that "increasing females' participation in STEM-related subjects and career paths does not lie in perpetuating the naive belief that single-sex educational settings are the answer" (p. 15).

Other focus areas in the literature were attitudinal factors and views of mathematics generally (by students) and of gender and mathematics (by adults). Boys were more likely than girls to report positive relationships with mathematics and to view themselves positively as mathematics learners (Forgasz & Leder, 2017; O'Keeffe, White, Panizzon, & Elliott, 2018). Interventions, such as informing students about growth mindset, were shown to affect girls (positively) more than boys, an encouraging finding (Koch, 2018). When examining adults' views—both those of parents (Glynn, 2019) and of the general public (Forgasz & Leder, 2017)—results were mixed, with participants typically holding gender-neutral views; however, when gendered views were found, they typically favoured boys.

Further studies pertained to parental involvement in children's mathematics learning (Bartley & Ingram, 2018) and the gendered experiences of undergraduate mathematics majors (Hall, Robinson, Flegg, & Wilkinson, 2019). Regarding the former, children viewed their fathers more positively than they viewed their mothers with respect to mathematics, and fathers were more likely than mothers to be involved in children's mathematics learning. Regarding the latter, gender-stereotyped experiences were reported by third-year students, but not by first-year students; all of the negative gender-related experiences were reported by or about women, which raises concerns about the university mathematics environment and, more broadly, pipeline issues.

The "gender issues" studies typically involved traditional types of data collection, such as interviews (e.g., Glynn, 2019), questionnaires using scale factors (e.g., Forgasz & Leder, 2017), and analyses of large-scale datasets (e.g., Li, 2018). Only a few studies featured other types of data collection. For instance, Hall et al. (2019) used photovoice, a methodology in which participants take photographs to represent key themes/ideas; then, the photographs are used as discussion prompts in focus group interviews. In King's (2018) study with elementary students regarding a gendered interaction in a group work situation, video-stimulated reflective dialogue was used. Given the lack of diversity found in the methodologies and methods of the studies examined, we recommend the use of more varied methodologies and methods to support the exploration of "gender issues" in mathematics in novel and nuanced ways. Additionally, we recommend involving participants besides primary and secondary school students, who featured in most studies, to learn about the gendered experiences of other groups (e.g., pre-service teachers, preschool children).

4 Socially Just and Ethical Mathematics Teaching

Studies of socially just and ethical teaching practice were conducted with students and teachers in disadvantaged communities, primarily classrooms and school communities with Indigenous, Māori, or Pasifika learners. Most studies were based in Australia and New Zealand, with a few based in Papua New Guinea and Tonga. From the publications reviewed, there are positive signs of increased focus on Indigenous and other cultures and languages in mathematics education policy and practice, in relation to learners, learning, and parental involvement (e.g., Anderson, Stütz, Cooper, & Nason, 2017; Averill, 2018a; Edmonds-Wathen et al., 2019; Hunter et al., 2016; Trinick, Meaney, & Fairhall, 2016).

Some researchers explored socially just and/or ethical teaching in classrooms and communities where a combination of equity factors was evident (e.g., low SES and rural, rural and Indigenous, low SES and mixed ethnic backgrounds). SES featured in several studies that were examined, but it was not a variable of analysis in the majority of these studies. Rather, the contextual settings of the research conducted and reported were low SES schools. Enhancing students' financial literacy was the major focus of some studies in these contexts (e.g., Attard, 2018; Sawatzki, 2017). In other studies, researchers focussed on culturally inclusive practices and/or compassionate or caring teaching practices (e.g., Blue, O'Brien, & Makar, 2018), or were concerned with acknowledging rurality as the learning context (e.g., Roberts, 2017).

In most of the studies about teaching practices in disadvantaged communities, researchers used qualitative methodologies, including case studies, ethnography, action research, and design-based research. In four studies (Allen & Taplin, 2017; Finau, Treagust, Won, & Chandrasegaran, 2018; Kennedy, 2018; Salgado, 2016), researchers investigated intervention programs and used quantitative or mixed methods to analyse the outcomes of these interventions.

Across the reviewed literature, diverse theoretical framings and methodologies were found. It is positive that culturally-appropriate methodologies such as Kaupapa Māori methodology were used in some studies (e.g., Hāwera & Taylor, 2017). The theoretical framing of a small number of publications was unclear; thus, making theoretical frameworks more explicit should be a priority for further research. Several publications were based on research- and culturally-informed reflections as opposed to empirical studies. The ideas from this work should prove useful for informing empirical studies.

4.1 Ethnomathematics and Decolonisation Approaches

In the reviewed literature, authors identified and sought to find ways to address the diverse challenges that exist in reflecting, responding to, and incorporating Indigenous and other cultures and languages in mathematics education. Focus areas included incorporation of contexts and pedagogies from, linked to, or consistent with those of the target students' heritage cultures. Factors challenging progress towards enhancing equitable access to mathematics learning included tensions regarding who holds power in policy making, mathematics teaching, and schools; the different status held by Western, Indigenous, and Pacific knowledge systems within English-medium formal education; and the empowerment (or lack thereof) of student involvement in critical reflection on the knowledge to which they are exposed and the processes by which this knowledge is presented (e.g., Edmonds-Wathen, 2017; Meaney, Trinick, & Fairhall, 2017; Trinick et al., 2016). There were calls by researchers in the reviewed literature for Indigenous community control of decision-making; for questioning of curriculum, pedagogical, and assessment regimes and the pervasive focus on learning Western mathematics; and for revisiting where "blame" may lie for achievement limitations (Meaney & Trinick, 2018). However, as acknowledged by these researchers, there are substantial complexities in relation to changing policy to enable substantive utilisation of the languages and cultures of marginalised communities in mathematics teaching and learning (Meaney et al., 2017). Despite such challenges, mathematics education can play an exciting role in revitalising language and culture. For example, Trinick (2019) used Fraser's (2005) theoretical framework to describe how first-hand experiences of developing lexicon, curricula, and teacher capacity to teach mathematics in the medium of the Māori language helped to revitalise the Māori language.

In New Guinea, Papuan language (Tok Ples) is the language of instruction for the first few years of primary schooling. To support teachers to use the home language for mathematics learning, Edmonds-Wathen et al. (2019) used an ethnomathematics approach to identify Tok Ples words and representations to make connections among language, cultural knowledge, cultural ontology, and Western mathematics, and to develop resources for teachers. The researchers drew on the mathematical knowledge systems of Papuan cultures documented by other researchers, such as Owens (2017), who described various cyclic counting systems (5, 10, 20 systems and base-6 systems) used in Pasifika cultures. Owens (2017) argued that the study of Indigenous and Pasifika number systems would provide enrichment for Indigenous and Pasifika students.

Teaching using contexts linked to the cultural capital of target student groups was investigated in several studies (e.g., Hunter & Miller, 2018; Saunders, Averill, & McRae, 2018; Siemon, 2017; Trinick et al., 2016). A three-step approach to introducing Indigenous ethnomathematical practices, to ensure that the integrity of the practices are maintained while mathematics learning is enabled and student critical awareness is enhanced, was proposed by Trinick et al. (2016). An example of this process was explained in relation to providing focus for a contemporary pāngarau/mathematics classroom through aspects of cultural and mathematical knowledge embedded in highly valued artifacts such as a wharenui (meeting house). In other work (e.g., Hunter & Miller, 2018), researchers reported less focus on the cultural meanings of culturally-embedded contexts, privileging instead the Western mathematics that could be drawn from the context. For example, in their study involving 27 Year 2 students, Hunter and Miller (2018) focussed on using Pacific and Māori patterns to explore sequencing, enabling students to draw from their cultural capital, rather than linking more strongly to the cultural opportunities afforded by the patterning contexts.

Enhancing the responsiveness of teaching to the Indigenous and Pasifika cultures of their students through developing deeper understandings and using pedagogies linked to students' heritage cultures was also explored in the studies examined. Rather than mathematics learning being seen as culture-free, in a range of studies, researchers illustrated the belief that teaching and learning are cultural experiences best tied to the cultures of learners, such as by incorporating collaborative learning activities (e.g., Hill, Hunter, & Hunter, 2019; Hunter & Hunter, 2017; Hunter, Hunter, Anthony, & McChesney, 2018). Indicators of the Māori term and cultural competency "ako" were identified through a study involving the lead researcher and her Year 9 mathematics class using cogenerative dialogue, student questionnaires, reflective teacher notes, and discussion of key themes with cultural advisors (Saunders et al., 2018). Resulting indicators of ako included kaiako (teacher/s) encouraging ākonga (students) to teach and learn from each other, holding high expectations of ākonga learning, and seeking and responding to ākonga and whānau (family) voice about learning. A key strength of this work is that it provides one way of interrogating a powerful culturally-embedded concept within the Eurocentric setting of a school mathematics classroom. Learner engagement and classroom management improved as relationships with students and their families, grounded by students' perspectives, developed over the study.

Research conducted in Australia also led to reassessing the meaning of culturally responsive pedagogies (Jorgensen & Lowrie, 2018; Siemon, 2017). Drawing on her experiences of being in the community and observing Yolgnu teacher assistants, Siemon (2017) shared stories that show the value of observation and imitation for the learning of mathematics in an Indigenous community. She made a distinction between pedagogical practices advocated for Indigenous students and pedagogical practices of Indigenous knowledge systems. Siemon (2017) reported that the use of first language and genuine engagement with community are important to avoid being disrespectful when trying to use culturally responsive pedagogies. Similarly, Jorgensen and Lowrie (2018) found that enabling the use of pedagogical practices of an Indigenous community assisted learning of symmetry. They found that the Western terminology of "symmetry," "mirror," and "reflect," and the use of mirrors did not support students' understanding. Rather, physically copying and reflecting the shapes of images of dancers, including Indigenous dancers, resulted in successful engagement with the concept.

In each of these studies, the researchers reported on the importance of valuing and using the language and pedagogical/knowledge systems of Māori, Pasifika, Indigenous, or Papuan communities for the learning of mathematics. The researchers argued for the need to establish respectful relationships with, and involvement of, the community in order to inform the language and pedagogical approaches to be used with authenticity in the classroom. These studies illustrate a shift toward the use of decolonial theories and methodologies in education (Tuhiwai Smith, 2012). Other studies in which researchers explored culturally responsive practices for Indigenous and Pasifika, low SES, or rural communities were more focussed on the tasks used and on describing student learning. These studies are discussed in the next section.

4.2 Culturally Responsive Teaching Practices

In the reviewed literature, researchers have explored culturally responsive practices in communities disadvantaged by multiple equity factors (e.g., SES and rural location). In some of these studies, the researchers focussed on particular mathematical topics or proficiencies.

In their book *Mathematics at the Margins*, which was a report on a 4-year longitudinal study of students' knowledge and teachers' knowledge and practice, Warren and Miller (2016) recognised that teachers working in marginalised schools need knowledge of the community context and culture, and need to hold high expectations of these students. The researchers argued that equitable teaching practices "support mathematical reasoning, conceptual understanding and discourse" (p. 22). Warren and Miller used design-based research involving culturally appropriate activities, developed with the engagement of Indigenous Education Officers (teaching aides) and based on their RoleM (representations, oral language, and engagement mathematics) model. Warren and Miller described a five-step learning trajectory for teachers, from “gaining teachers’ interest” to “holding higher expectations for students,” that coincided with a five-step learning trajectory for students, from “students engaging in learning” to “students engaging in higher cognitive mathematics” (p. 96). Culturally appropriate resources were “rich in representations that were familiar to students” (p. 98).

As part of this longitudinal study, Miller (2016) studied young Australian Indigenous students’ engagement in the mathematical discourse of pattern generalisation. She reported findings of a learning trajectory for Year 3 Indigenous students en route to generalising growing patterns and functional thinking. Indigenous education officers at the school participated in the analysis of teacher and student interactions to ensure that cultural nuances of these interactions were included and identified. Miller, Warren, and Armour (2018) explored the cultural discourse that occurred at the boundary between Indigenous and Western knowledge systems for the generalising project. They found two teaching and learning actions that created space for mathematical discourse involved in pattern generalisation: acknowledging cultural ontology and acknowledging semiotic systems. Allowing students to use storytelling together with hands-on materials enabled them to use the oral traditions of their culture to engage with Western mathematics. Gesture (i.e., semiotic mediation) was also critical to students’ talk and storytelling. Following cultural ontological practices, students interacted with each other to copy or imitate others’ work and demonstrate group ownership. The authors concluded that when teachers have an awareness of students’ cultural ontology, they can better interpret the classroom discourse in which students display their knowledge and understanding.

Further insights into pedagogies with the potential for culturally responsive practice include using rich, open investigative tasks, which provide opportunities for student autonomy and decision-making through offering multiple interpretations and multiple solution strategies (Averill, 2018a). Such tasks were explored using kapuapa Māori methodology to explore the learning of four Indigenous Year 3 and 4 students in a Māori-medium context in which show and tell technology was used in a successful intervention to develop both mathematics understanding and mathematical language development (Allen & Taplin, 2017). Using narrative literature review, arguments for including singing, story-telling, metaphor, and dance—pedagogies consistent with those of many heritage cultures (Averill, 2018b; Taeao & Averill, 2019)—as pedagogies for mathematics, included their potential for developing students’ mathematical understanding and enjoyment, alongside their holistic wellbeing. The exploration of mathematical ideas using the Samoan dance, the sāsā, was discussed by Taeao and Averill (2019), but was not explored with students in this study. In accordance with Siemon (2017) and Jorgensen and Lowrie (2018), investigation of mathematics learning and student affect in relation to such pedagogical approaches is warranted.

In a few studies, researchers explored the nature and use of culturally or socially responsive curriculum in low SES or rural locations. Salgado (2016) used quantitative methods to explore the impact of tasks based on familiar contexts on Year 10 students’ performance. He found that a more familiar problem context did not improve performance for these low SES students. However, Sawatzki (2017), investigating the role of familiarity in financial literacy tasks with primary students in low SES rural locations, found that students valued tasks that were authentic, familiar, and relevant to their lives.

Roberts (2017) argued that much of the research into social justice was “spatially blind,” that is, not concerned with the needs of students from rural locations. He researched rural teachers’ and leaders’ perceptions of the curriculum and found that those who had been in rural schools longer or were in schools furthest from a metropolitan area were more likely to believe that the Australian curriculum did not value or account for local knowledge or learning needs. Roberts (2017) argued there is a need to value “knowledge produced in, for and with rural [communities]” (p. 34). Further research is clearly needed to better understand this knowledge and these communities’ needs.

4.3 Ethical and Caring Approaches

Some of the researchers who conducted culturally responsive studies paid particular attention to the development of respectful relationships and to adopting a caring approach. Rather than focussing on the suitability of tasks, problems, or materials, these researchers focussed on fostering productive learning relationships in the mathematics classroom. A few studies were identified in which the authors focussed on, or reflected, ethical and/or caring pedagogical approaches.

Specifically, Blue et al. (2018) observed and video-recorded a Year 4 inquiry-based mathematics lesson focussing on money and financial mathematics. The researchers adopted the theory of practice architectures to examine and analyse practices in the classroom that might promote a critically compassionate approach to financial decision-making. Several classroom practices were identified that might promote a compassionate approach to learning about financial literacy: positive and collaborative engagement with peers, focussing on connecting the task with ethical and social considerations, and recognising that financial decisions can impact others.

Gibbs and Hunter (2018) used a socio-constructivist view of mathematics learning and a qualitative case study approach to focus on classroom factors that might inhibit or enhance students’ participation in mathematical inquiry, as well as the actions that teachers take to promote equitable participation. In a small primary school with predominantly Māori and Pasifika students from low SES backgrounds, two students with achievement, status, and power issues with mathematics were observed and interviewed. The teacher’s actions and classroom practices were not seen to address the inequities or to promote the participation of the students. The researchers concluded that unless teachers intervened to address such issues, the mathematics underachievement of diverse students would persist.

Based on a number of linked studies, Hunter et al. (2016) revealed that there are continuing tensions related to cross-cultural misinterpretations, resulting in inequitable practices for Pasifika students. Hunter et al. provided examples that illustrate that when educators relate to Pasifika students “as culturally located people with rich funds of knowledge to contribute” (p. 208), there can be equitable outcomes for the students and their families. Together, these studies illustrate that in order to enact socially just practices in teaching, teachers must use culturally responsive pedagogies in order to construct productive and compassionate relationships in the classroom as well as respectful academically-focussed relationships and practice.

4.4 Intervention Approaches

Two studies of intervention programs were reviewed: one that involved high-achieving students in Tonga and one that involved low-achieving students in a disadvantaged community in Australia. Using an acceleration programme with Year 8 Tongan students, Finau et al. (2018) indicated that targeted, informed interventions can create positive effects on mathematics achievement, self-regulation, motivation, and ways of learning. Kennedy’s (2018) intervention study was conducted in six low-performing primary schools in South Australia. Prior to the intervention, teachers engaged in professional learning to familiarise themselves with the researcher’s model for learning for conceptual change within challenging mathematical tasks. A standardised test was employed to gauge students’ learning gains over two years. Low-performing students’ gains were beyond expected growth levels, which Kennedy attributed to student engagement with challenging tasks.

Overall, there is an increased understanding that mathematics education researchers’ responses towards enhancing equity of opportunity for mathematics achievement include relational components and components related to cultural knowledge and language, and that there are complexities around how cultural knowledge can be appropriately incorporated into learning programmes. Consistent with findings in the last RiMEA chapter on Indigenous learners, and international literature more widely (Meaney et al., 2016), with exceptions such as Allen and Taplin (2017), there are again some gaps regarding quantitative studies that show increased achievement of learners in target groups and the positive effects of the teacher development discussed. The absence of quantitative studies is perhaps not surprising, as there are complexities around the meaning of and priorities for assessment across cultural groups. Hence, finding suitable assessment tasks is challenging. Additional research into the focus areas of the material reviewed and ways to spread the effects of the affordances found would be useful.

5 Teacher Knowledge and School Change for Socially Just Policy and Practice

There has been continued interest in researching teacher knowledge of marginalised learners and the ways in which initial and professional teacher development can enhance marginalised learners' opportunities. Not so prevalent in this group of work as for the previous review period are studies involving initial teacher education. In previous related chapters, authors called for more focus on developing capabilities with mathematics content (e.g., Meaney et al., 2016), and some studies in this time period reflect this focus (e.g., Hāwera & Taylor, 2017; Trinick et al., 2016; Trinick & Meaney, 2017). An emerging focus of research was the use of case studies to investigate the culture, knowledge, and practices of schools that demonstrated success or improvement in participation or achievement outcomes for disadvantaged or marginalised students (e.g., Bennison, Goos, & Bielinski, 2018; Jorgensen, 2018; Muir, Livy, Herbert, & Callingham, 2018).

5.1 School Improvement and Whole-School Change

Regarding participation in senior school mathematics, Bennison et al. (2018) investigated effective practices in low SES schools with increased enrolments in the highest level senior mathematics subject. They interviewed mathematics teachers, guidance counsellors, and students to identify effective practices to promote "sustained interest and engagement in mathematics involving the study of calculus" (p. 154). The researchers identified the following factors that contributed to student participation: "curriculum organisation across year levels, staffing of mathematics classes, culture of the Mathematics Department, STEM program, and provision of appropriate tasks and resources" (p. 157). They also noted that the culture in the school involved teachers holding high expectations of the students. Murphy (2019) also found that several factors contributed to relatively high participation and achievement rates in senior mathematics for a small rural P-12 school, including differentiated learning, student self-directed learning, and co-planning and teaching across year levels. This school employed a higher than average class time devoted to learning mathematics from P-12. The teachers at the school endeavoured to show the relevance of mathematics to the real world, but no examples were provided, and it is not known whether these related to their local rural context.

Joseph (2019) investigated factors associated with the high performance on the NAPLAN by students at nine disadvantaged Victorian primary schools. Interviews were conducted with school principals and staff, and literacy and numeracy (i.e., English and mathematics) lessons were observed. Six common themes emerged from the research: clear and consistent discipline practices founded in high expectations, direct and explicit instruction, experienced and autonomous school leadership, data-informed practice, teacher collaboration and professional learning, and comprehensive reading instruction. Similar findings were reported by Muir et al. (2018) in their multiple case study of three P-12 schools where students demonstrated above-expected growth in NAPLAN numeracy scores from Year 3 to Year 5 and from Year 7 to Year 9. Two schools were private schools with above-average SES; the third was a small government school with below-average SES. A range of data on school curriculum and practices was collected and school leaders were interviewed. Notably, the presence of qualified secondary mathematics teachers was one of the factors that leaders at all schools attributed to growth in achievement in the secondary years. High expectations and a range of assessment tools were used in the two higher SES schools. Consistent whole-school approaches to teaching, including collaboration between teachers across grades, mentoring, and in-school professional learning, were features of the low SES school. Fluid groupings of students and/or differentiation were not common but did occur in the low SES school and one of the private schools.

In case studies of 35 schools in remote and very remote Aboriginal communities from five states and territories, Jorgenson (2018) found that developing language resources and strategies to scaffold Aboriginal students' transition from their home language (or Kriol) to Standard Australian English enabled these schools to provide successful numeracy experiences and outcomes for Aboriginal students. In addition to the production and use of language-rich resources and the use of learning intentions to document mathematical language, Aboriginal education workers (teacher assistants) were central to the success in these schools. They worked with the teachers to plan lessons and materials, co-teach, or act as translators in individual or whole-class discussions.

In other studies, researchers addressed whole-school change through engagement with teacher professional learning at one school or a cluster of schools. In studies about community and parent engagement, researchers addressed power relations and cultural recognition and participation whilst developing cultural competence in working in Indigenous, Māori, or Pasifika communities (Cooper & Carter, 2016) or rural communities (Proffitt-White, 2017).

Armour, Warren, and Miller (2016; see also Warren & Miller, 2016) found that including Indigenous education officers (or teacher aides) in teacher professional learning not only improved their confidence and contribution in the classrooms, but also allowed them and their students to begin to "walk" between the two knowledge worlds—Indigenous knowledge and Western knowledge. The education officers assisted with the design of culturally appropriate materials for lessons, and the interpretation and analysis of interactions between students, and between students and teachers.

Cooper and Carter (2016) foregrounded their study of whole-school change by emphasising that the only acceptable research involving Indigenous and low SES students is that in which researchers use decolonising methodologies (Tuhiwai Smith, 2012) to engage, empower, and benefit research participants. Cooper and Carter (2016) reported on a school improvement program using YuMi Deadly Maths (YDM). They explain that "YuMi" is a Torres Strait Islander Creole term meaning "you and me" and "deadly" is a term for "smart" used by Australian First Nations people. YDM focuses on three big ideas of Western mathematics: (1) a structure of mathematical ideas, (2) a language for concisely describing real-life situations, and (3) a tool for problem solving. In the YDM program, sequences of lessons built on each other to deepen students' understanding of structure for particular concepts, "chunk knowledge" using common concepts, and connect the big ideas across mathematics topics. Cooper and Carter (2016) worked with an Aboriginal mathematician, Matthews, using his ontological framework of mathematics (2009) to frame their materials and RAMR (reality, abstraction, mathematics, reflection) program. In Matthews' (2009) cycle of reality–mathematics–reality:

> Both abstraction and reflection are *creative and problem-solving* acts; mathematics as a language and structure is built around *symbols* that carry concepts, strategies and relationships from reality to abstraction and back to reality; and the mathematics and how it is used in reality is framed by the *cultural bias* of the person creating the abstraction and reflection. (Cooper & Carter, 2016, p. 176, emphasis in original)

The researchers found that the effectiveness of the YDM program depended on the support of the principal, continuity of staff, and at least two or three teachers with enthusiasm for the project. Using YDM methods improved mathematics teaching and learning for all students at the schools (Carter, Cooper, & Anderson, 2016).

A cluster of researchers described the rationale for and results from the Developing Mathematical Inquiry Communities professional development project being conducted in New Zealand (e.g., Gibbs & Hunter, 2018; Hunter et al., 2016, 2018). Key findings include that when educators reflect the languages and cultures of Pasifika students in their work and explicitly establish respectful and reciprocal relationships with these students and their families, learning can be enhanced and cultural identity affirmed (Hunter et al., 2016). Through a substantial literature review on parental involvement in mathematics learning, Averill, Metson, and Bailey (2016) advocated for expectations on teachers of policy relating to involving family in the teaching of Māori and Pasifika learners, school-wide commitment, learning-focussed parent-teacher partnership and decision-making, and purposeful home-based learning activities are necessary. Challenges to strong parental input include varied expectations, language and cultural differences, and the time needed to develop strong parent-teacher relationships and curriculum-related understanding. Empirical studies are needed to explore these findings in relation to enhancing equity of access to learning and achievement.

As reported by Vale and Drake (2019), schools in rural and remote Indigenous communities are known to have difficulty in attracting and retaining teachers. Using case studies of out-of-field and beginning teachers in remote, rural secondary schools, Vale and Drake (2019) reported that schools need to provide a culture of support if out-of-field mathematics teachers are to successfully transition into (and remain in) their new community and new field of teaching. The kinds of support requested by the beginning and out-of-field teachers in the study included leadership and mentoring from experienced teachers, accessible curriculum and teaching resources that provide sufficient detail for the out-of-field teacher to interpret and enact, and collaborative planning practices.

The Teachers First Initiative (Proffitt-White, 2017) was designed to draw on active principal networks in rural and regional Queensland to enact a cluster-model approach to school improvement. Five clusters of primary and secondary school leaders, organised geographically, focussed on trusting teachers to develop and provide effective mathematics teaching. Proffitt-White (2017) reported enhanced teacher enthusiasm and confidence, and a shift away from exclusive use of explicit teaching to include a stronger focus on proficiencies in the Australian Curriculum, especially problem-solving and reasoning. The creation of cross-school protocols for formative assessment and reflection on practice involving classroom visits and observations by peers developed "a culture of trust, a willingness to not only listen to ideas but to have the collegial support to try things out" (Proffitt-White, 2017, p. 20).

5.2 Initial Mathematics Teacher Education

In studies of initial mathematics teacher education concerning social justice, all of which occurred in New Zealand, researchers were concerned with developing teachers' cultural competence (see also Chap. 5). Most teachers in New Zealand are New Zealand European, with relatively small proportions of Māori and Pasifika teachers compared to the student population. Hence, while variation in educator knowledge bases about Indigenous and Pacific languages, contexts, and cultures exists, many marginalised students are taught by teachers without deep understanding of these students' out-of-school lives. New Zealand education policy and support resources are aimed to ensure greater educational opportunities for Māori learners through enhancing teaching and school leadership practices (e.g., Education Council, 2017; Ministry of Education New Zealand, 2008, 2018). For example, teachers of Māori learners are encouraged to reflect socially-embedded praxis with an emphasis on care, respect, reciprocity, communication, student autonomy, involvement of whānau (family), and new learning being built on the experiences and knowledge of learners (Ministry of Education New Zealand, 2011). Teachers are also encouraged to reflect the values and competencies important to Pasifika people in their work with Pasifika learners, identified from a wide range of research and consultation (Ministry of Education New Zealand, 2018).

Collectively, Pasifika are the fastest growing ethnic group in New Zealand (Statistics New Zealand, 2013); however, Pasifika achievement lags that of non-Pasifika students on many measures. The New Zealand government policy and policy support materials are designed to prioritise improvement of learning opportunities for Pasifika learners through expecting teaching to be underpinned by a Pasifika values base and to be culturally responsive (Ministry of Education New Zealand, 2013, 2018). The material focussed on mathematics learning of New Zealand Indigenous and Pasifika learners reviewed for this chapter indicates positive signs of increased focus on Indigenous and other cultures and languages in mathematics learning. Examples include exploration of the enactment of education policies and resources in relation to learners, learning, initial teacher education, professional development, and parental involvement.

In some of the reviewed literature, researchers focussed on the importance of reflecting cultural competencies in mathematics education in both initial teacher education and professional development, and the potential effects of doing so (e.g., Averill, Drake, Anderson, & Anthony, 2016; Averill, Metson, et al., 2016; Edmonds-Wathen, Owens, Bino, & Muke, 2018; Hāwera & Taylor, 2017; Saunders et al., 2018; Wilson, McChesney, & Brown, 2017). For example, using Kaupapa Māori methodology (Bishop, 1996; Smith, 1995) and focus group discussion, Hāwera and Taylor (2017) explored six Indigenous student teachers' views about the usefulness of their compulsory mathematics methods courses for supporting their practicum teaching. Using the cultural competencies for teachers of Māori learners (Ministry of Education New Zealand, 2011) as an analysis tool, Hāwera and Taylor (2017) found that the student teachers valued emphases on developing both cultural competencies and content/pedagogical knowledge in their coursework for their development as teachers. Specifically mentioned were course foci on nurturing an ethic of care and respectful classroom relationships, establishing prior knowledge, developing knowledge of content and assessment practices, language learning, planning and pedagogy, and exposing student teachers to different ways to teach mathematics. Participants felt that more assistance in their courses in drawing key ideas from course texts and in preparation for navigating their relationships with their practicum teachers could further strengthen links between course and practicum work.

In their study involving 13 first-year English-medium pre-service teachers, Wilson, McChesney, and Brown (2017) found that student teachers were able to use *Tātaiako: Cultural Competencies for Teachers of Māori Learners* (Ministry of Education New Zealand, 2011) to inform their practice in a range of ways. During their first-year mathematics methods course, the Māori strategy manager discussed the competencies with student teachers. The mathematics lecturer then led a discussion about mathematics resources that could be used to help demonstrate the cultural competencies in mathematics teaching, and encouraged critical consideration of these in relation to cultural integrity and mathematical authenticity. Analysis of students' journal and planning documents showed that student teachers used *Tātaiako* to identify suitable learning and teaching practices, and for their language development. Research about the extent to which these understandings are transferred into teaching practice is needed.

Tātaiako (Ministry of Education New Zealand, 2011) was also used in a study of in-the-moment coaching of student teachers' mathematics teaching (Averill, Drake, et al., 2016). Using design study and incorporating videos of rehearsals of practice, reflective debriefs, and student teacher surveys across a range of courses over 4 years, questioning, rather than directive instructions, was used as much as possible to encourage and acknowledge student teachers' pedagogical decision-making. Using questions to interrogate practice showed promise for modelling and developing two cultural competencies in particular: ako (roughly translated as the reciprocity of teaching and learning) and wānanga (roughly translated as co-construction). Other values vital for culturally responsive practice, such as respect and empowerment, were also a focus. Limitations to transferring such findings into practice included available course time and student teachers' prior understanding of the competencies.

In each study, research of successful mathematics teaching of Māori learners and government policy in which language and cultural pedagogies and knowledge systems are acknowledged and valued were drawn together to investigate the development of knowledge and practices of pre-service teachers. The findings show the relevance of *Tātaiako* (Ministry of Education, 2011) for initial teacher education practices and the impact of research in the social justice field on those beginning their teaching careers.

6 Discussion

In this section, we comment on the overriding themes and significant findings from the publications discussed, and provide a critique of the focus of research, theoretical frameworks, and methodologies used, organised by equity group.

6.1 Gender

Concerningly, in nearly all of the publications, the authors did not define what they meant by gender and did not provide a theoretical perspective on their gender-related views. Additionally, many authors were inconsistent with their use of gender-related terminology, such as using the phrase "sex differences research" and then referring to "boys" and "girls." Only Hall et al. (2019) discussed non-binary genders when reporting their findings; hence, in other studies, this gendered group was marginalised and the gender binary was reified. While there are admittedly challenges regarding this issue when using large-scale datasets, in smaller-scale studies, such ideas should be taken into consideration so that the views and experiences of students of all genders are explored. In addition to these issues, nearly all authors treated gender groups as homogeneous, rather than examining differences within the groups (See Sikora & Pitt, 2019, for a counter-example). Hence, it appears that the suggestions made by Damarin and Erchick (2010), Esmonde (2011), and Glasser and Smith (2008) approximately a decade ago have not been adopted by Australasian researchers investigating gender issues in mathematics teaching and learning. In future research, we encourage gender issues researchers to provide operational definitions of gender and related terms, be consistent with their use of gendered language, include participants with non-binary genders, and examine patterns within gendered groups, rather than strictly between groups.

6.2 Indigenous and Culturally Marginalised Students

In research published in this time period regarding the mathematics education of Indigenous learners, authors continued to focus on ways to redress imbalances in educational opportunities experienced by Indigenous and other culturally marginalised students. In the reviewed research, authors continued to explore the theme of reducing cultural conflict and culturally-linked differences between students' school and outside of school experiences and learning. The study by Jorgensen (2018) illustrates a shift towards researching practices that have been shown to enhance Indigenous students' mathematics learning outcomes.

As called for in past RiMEA chapters on Indigenous learners, authors (Meaney et al., 2016; Meaney, McMurchy-Pilkington, & Trinick, 2012) recommended the active participation of Indigenous education officers, teacher aides, or mathematicians in recognising and enabling Indigenous ontology to contribute to teaching and teacher learning brings further meaning to Fraser's (2005, 2013) notion of redistribution, recognition, and participation for social justice in education and teaching practices. Much of the literature reviewed in the current period resulted from teams in which at least one team member was a member of the Indigenous or cultural community. Such representation at all stages of the research must become the norm, expected in ethics and research funding applications. For research teams that do not include members of the Indigenous of cultural community being investigated, it is important that consultation strategies used to maximise the suitability of research decisions are clearly outlined.

Generic terms such as "Indigenous," "Pasifika," and "Pacific" run the risk of conveying homogeneity. Overall, authors of the reviewed material promoted understanding of the diversity within each target group through their descriptions of the target groups (e.g., Hunter et al., 2016; Siemon, 2017; Taeao & Averill, 2019). Such descriptions help to develop understanding of the complexities of cultural diversities and the nuances of research in this area.

The reviewed material has useful messages relevant to mathematics education in general. Specific examples include findings in relation to content knowledge development, choice of contexts for and styles of learning activities, using student and family voice, and using questioning in school classrooms and in initial teacher education (e.g., Averill, Drake, et al., 2016; Hāwera & Taylor, 2017; Hunter et al., 2016; Trinick et al., 2016; Wilson, McChesney, & Brown, 2017).

6.3 Low SES Students

There were very few studies in which researchers focussed specifically on teaching and learning of low SES students. The exceptions are those in which researchers explored ethical and caring approaches in disadvantaged communities (e.g., Blue et al., 2018; Sawatzki, 2017). There seems to be an assumption that the issues pertaining to the teaching and learning of low SES students are addressed by studies of under- or low-achieving students or students with learning difficulties. In a few studies (e.g., Bennison et al., 2018; Joseph, 2019), researchers have begun to explore practices that have been shown to improve student participation or achievement in schools in low SES communities. We need more studies that illustrate how to develop engagement, positive dispositions, and success for students from low SES communities, and identify and celebrate high achievement of students from low SES communities.

6.4 Rural and Remote Students

International, national, and small studies of students from schools in rural locations continue to show lower achievement across all year levels studied and low participation in senior secondary schooling and mathematics. In studies on schools and students in rural locations, researchers used quantitative methods to report on participation, achievement, and attitude gaps. For instance, Watson et al. (2016) reported findings of rural students' perceptions of mathematics and schooling that informed the design of programs to improve participation of rural students in senior secondary schooling and mathematics.

With continued high levels of incidence of out-of-field mathematics teaching in secondary schools, examining ways to support in-field and out-of-field mathematics teachers in rural and remote communities is important. Proffitt-White (2017) described a cluster model for whole-school improvement in rural and regional schools; however, the support that this model provided to out-of-field mathematics teachers was not identified. More studies are needed into ways to support and retain secondary teachers of mathematics, including out-of-field teachers, particularly those in rural locations. Such studies could include engaging with parents and the community, and providing meaningful contexts for student exploration and modelling of mathematical concepts. Investigating the meaning of rurality and the implications for student participation and learning would guide the use of contexts for mathematics learning that are meaningful for students in rural schools.

7 Concluding Comments

Mathematics continues to be widely recognised as a critical filter to further educational and career opportunities. Recording and monitoring the design and implementation of programs aimed at managing and redressing disadvantages encountered by students in their learning of mathematics thus remains of critical importance.

Documentation of factors that might impede optimal access by students to mathematics taught in classrooms continues to be fuelled and reinforced by the reporting of data from large-scale national and international testing of mathematics. The choice of equity variables used in analyses of the data collected is noteworthy and influential. However, subtle differences in the definitions used in these large-scale tests need to be recognised, both in comparisons of data considered and in the interpretation of findings reported.

The broad and generalised findings from large-scale assessments such as those reported at the outset of the chapter have undoubtedly served as significant prompts and offered a relevant context for the design and conduct of the smaller studies covered in the remainder of the chapter. These more targeted studies not only enabled a more nuanced sketching of outcomes but also collectively contributed to further confirmation, or challenge, to the broader and more global findings presented. Many of the New Zealand studies reported were motivated by a commitment to a treaty partnership and to addressing historical imbalance in how heritage cultures, especially Māori, are acknowledged, reflected, supported, and sustained in educational settings. Australian researchers working with Indigenous school communities have also tried to give voice to Indigenous communities, acknowledge Indigenous ontology, and use culturally responsive approaches. Taken together, these studies provide useful messages relevant to mathematics education more generally, such as content knowledge development of teachers, choice of contexts for and styles of learning activities, and teacher questioning strategies. The scarcity of work focussed on early childhood students and teachers should not go unnoticed.

The challenges faced by educational researchers have been captured eloquently by Melhuish and Thanheiser (2018): “In education research,” they argued,

> it is impossible to have the clear, delineated, randomized studies that may exist in the hard sciences. Each study is situated in any number of contextual variables, from the particular group of students and teachers to the nature of any particular school setting (p. 104).

These complexities and constraints are also evident in the 4-year snapshot of research referred to in this chapter—work in which researchers drew on a variety of theoretical and methodological perspectives. While the impact of multiple equity variables on students’ lives was often acknowledged, explorations of single-variable effects on performance still dominated in the material surveyed. Replications or small extensions of earlier research were particularly prevalent.

Cai et al. (2018) noted that replication studies fall into two categories: an exact replication of an original study or research involving conceptual replication including exploring, from a different perspective, the contexts in which the original results were obtained. For this chapter, with its emphasis on research set in different countries, the latter interpretation is particularly relevant. While replication studies will no doubt continue to be mounted in the 4-year period ahead, thoughtful planning and justification should precede such work.

The difficulty of gaining funds for longitudinal studies has been raised before, within and beyond the MERGA 4-yearly reviews. Yet, our field will surely be enriched by the findings of an ambitious longitudinal study in which researchers follow students, retrospectively or pro-actively (e.g., Warren & Miller, 2016), from groups still considered to be disadvantaged, as they move through the educational system and grapple with mathematics and its role in their subsequent development. Productive and pragmatic approaches for such an investigation could be sourced from earlier, ground-breaking studies. Csikszentmihalyi, Rathunde, and Whalen's (1993) exploration of a diverse range of factors that supported or impeded high school students' longer-term success in mathematics, as well as in other subjects, is worthy of renewed inspection. Could this ambitious 5-year longitudinal project be fruitfully adapted and modified to document, showcase, and better understand how and when students faced with equity hurdles can be successful as they progress through school? Bloom's (1985) retrospective study of adults successful in a number of fields, including mathematics, is also worth revisiting. In brief, we echo the comments of others engaged in educational research that longitudinal studies should be encouraged and, critically, be appropriately funded.

To conclude, the survey of recent research reported in this chapter not only adds to the existing body of work on mathematics education and equity, social justice, and ethical issues, but, we hope and anticipate, will also serve as a catalyst for future work.

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Chapter 9
Mathematics Learning and Education from Birth to Eight Years

Ann Downton, Amy MacDonald, Jill Cheeseman, James Russo, and Jane McChesney

Abstract This chapter presents a critical review and celebration of the most significant Australasian early childhood mathematics education research that has been published over the period 2016–2019. We utilise the internationally-accepted definition of 'early childhood' as the age range birth to eight years, encompassing prior to school settings, school settings, as well as home and community contexts. Eminent scholars in the field have undertaken the research presented in this chapter in conjunction with a range of stakeholders in early childhood mathematics education, including teachers, families and children. This chapter is structured according to six key themes which emerged from the preliminary analysis and categorisation of the current research across the Australasian region; namely: Mathematics content; Curriculum, policy and assessment; Aspects of teaching and learning; Home and prior to school contexts; Australian and New Zealand Indigenous education; and Emerging areas of research. Indeed, this review highlights several very promising new areas of research, for example: mathematics education for children aged birth to two years; and innovative research methodologies such as 'camera glasses' and 'trolley cams' utilised in everyday context. The new areas discussed in this chapter highlight the growing interest in, and opportunities for, research in the early years space. However, with the emergence of new areas of research there has been a decline in other areas such as pattern and algebra, geometry, and length measurement. From the synthesis of the research literature the following findings are evident. First, young

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children are often capable of mathematical thinking from a very early age, which suggests a mismatch between the intended curriculum and children's capabilities when they start school. Second, the current education policies within Australia and New Zealand have yet to bridge the mathematical transition from early childhood settings to school. Third, the contrast between a holistic appreciation of the mathematics surrounding children in prior to school settings is in stark contrast to school settings where mathematics is formalised and segmented and less richly experiential. The chapter concludes with a discussion of recommendations for future research in this field.

Keywords Assessment of early mathematics learning · Early childhood curricula · Mathematics content · Mathematics curricula · Pedagogies in early childhood mathematics · Transition to school · Young Indigenous mathematics learners

1 Introduction

Early childhood mathematics education research is burgeoning in Australasia and internationally, and this is reflective of the recent injection of National government funding in this area. While the national curriculum for prior to school in Australia and the continued influence of *Te Whariki* in New Zealand have stimulated research in the field, the political initiatives in early childhood education make research findings in the field particularly relevant for our future planning in early childhood mathematics. A distinct feature of this chapter is the proposal of an alternative title to "Mathematics Education in the Early Years" which implies the focus is solely on 3–8 year-olds, and emphasises 'formal' contexts (e.g., school) over 'informal' context (e.g., home). This alternative title is broader and provides more consideration to future mathematics research in the birth to three-year old space. While building on the work of equivalent chapters in previous editions of RiMEA, the structure of the chapter highlights the research and policy developments, as well as emerging research initiatives related to, for example, birth to two years, cognitive development, and innovative research methodologies.

As was the case in the previous RiMEA, we again celebrate the contributions of Australasian early childhood mathematics education researchers to three significant books focused on mathematics in the early years: *Contemporary research and perspectives on early childhood mathematics* (Elia, Mulligan, Anderson, Baccaglini-Franks, & Benz, 2018), *Engaging families as children's first mathematics educators* (Phillipson, Gervasoni, & Sullivan, 2017), and *Forging connections in early mathematics teaching and learning* (Kinnear, Lai, & Muir, 2018). These texts are collections of international research edited by members of Mathematics Education Research Group of Australasia (MERGA), demonstrating the leadership of Australasian researchers in the field of early childhood mathematics education. In addition, the location of papers accepted and published in refereed journal in this review period varied widely in terms of likely audiences and methodological approaches.

Some were aimed at the mathematics education community (e.g., *Educational Studies in Mathematics*; *Mathematics Education Research Journal*; *Mathematics Thinking and Learning; ZDM Mathematics Education*); the early childhood community (e.g., *Asia-Pacific Journal of Research in Early Childhood Education*; *Australasian Journal of Early Childhood*; *European Early Childhood Research Journal*; *Journal of Early Childhood Research*); the educational research community more widely (e.g., *Journal of Educational Psychology; Learning and Instruction; New Zealand Journal of Educational Studies; Statistics Education Research Journal*) and those beyond the education research community (e.g., *Frontiers in Psychology; Journal of Experimental Psychology; Social Science and Medicine*). Such an extensive distribution reflects the quantum of early childhood mathematics education research emanating from Australasia in recent years.

The chapter is structured according to six key themes, which emerged from the preliminary analysis and categorisation of the current research across the Australasian region. These themes are: Mathematics content; Curriculum, policy and assessment; Aspects of teaching and learning; Home and prior to school contexts; Australian and New Zealand Indigenous education; and Emerging areas of research. Within these themes subheadings have been used to indicate specific foci in each area. For example, within the theme of mathematics content there are subthemes related to number, statistics and probability, and measurement. In some instances further categorisation was warranted to indicate specific foci. For example, studies within number there are three categories: classroom based research; cognitive science research; and system-based research.

Much of the material reviewed in this chapter is generally considered only once; that is, only under one heading or subheading—even though a close read of the work might reveal that an alternative categorisation would not be unreasonable. Due to the nature of some studies some overlap is unavoidable. For example, both assessment and pedagogies are referred to across themes and subthemes. While the articles reviewed in this chapter are relatively evenly distributed, the largest groups appear within the Mathematics Content section, and the Home and Prior to School Contexts section. Similarly, across the chapter some studies are reviewed in greater depth than others due to the nature of the studies. Interestingly, since the last review, we could not identify any empirical studies in the early years focused on geometry. We suggest that this might be a fruitful area of focus for Australasian early childhood mathematics researchers in the near future, particularly given the recent work of Lowrie and colleagues, in particular, exploring the links between spatial reasoning and mathematics (see Chap. 10, this volume).

2 Mathematics Content

In order to highlight the coverage of research across different content areas in the early years, this section of the review is devoted to mathematics content. The section has been divided into three subsections: number, statistics and probability, and measurement. The titles of these subsections are indicative of the content areas that have been the focus of research. The majority of studies in the early years with a specific content focus have related to number, with a smaller number of studies relating to statistics and probability, and measurement.

2.1 Number

There have been a number of studies undertaken within the MERGA community that have considered the learning of number in the early years since the previous review. Such studies can loosely be categorised into three methodological orientations: classroom-based research, cognitive science based research and large-scale research.

2.1.1 Classroom-Based Research

The first methodological orientation considered in the number sub-section is classroom-based research. Several classroom-based research studies have been published since the previous review. Generally, these studies have focused on observing and/or developing teacher professional knowledge and behaviour, and how this connects to the student learning experience.

Adams (2018) explored the collaborative learning, teaching and assessment of two five year-old children (Mary and Bill) in an Australian international school setting as they engaged with mathematical ideas. The specific focus of her study was on the children's conceptual understanding of addition as gleaned through observing their interactions across a range of classroom experiences. Using observational techniques (e.g., field notes, video), Adams found that although both students were already fluent in adding numbers to 10, Mary acted as the more knowledgeable other to support Bill's developing understanding of addition within his zone of proximal development. However, according to Adams, the teacher remained unaware that Mary was supporting Bill in this manner, and tended to group both students together as highly able. This issue was further compounded by the limited nature of the formal assessment that concluded the unit of work, which involved closed tasks that were insufficiently challenging; that is, rather than focusing on Mary and Bill's potential as learners, the assessment was focused on "yesterday's development" (Vygotsky, 1987, p. 211, cited in Adams, 2018). Adams concludes that it is critical that teachers understand the power of the zone of proximal development to foster learning during peer interactions.

The apparent teacher-centred practices observed by Adams (2018) contrasts notably with the pedagogical approaches adopted by the teachers in Cheeseman's (2018) study. Cheeseman presented two illustrative vignettes developed from the classrooms of foundation teachers identified as being highly effective. Her particular focus was on how such teachers use particular pedagogies to build a community of learners, including: the use of questioning to elicit student thinking, refraining from ‘telling’ children what to do, supporting children to reason mathematically, and encouraging children to engage critically with the mathematical ideas presented by their classmates. Cheeseman found that both teachers used these aforementioned pedagogies to become genuine participants in the mathematical inquiry taking place in the classroom. She concluded by offering three recommendations for classroom practitioners based on her work with highly effective mathematics teachers. Teachers should endeavour to “plan time for close mathematical conservations with children, expect thinking of children, including conjecturing, reasoning, justifying, and consider tasks and their potential to engage and extend children’s thinking” (Cheeseman, 2018, p. 22).

Bicknell, Young-Loveridge, and Nguyen (2016) provided further evidence that including challenging mathematical tasks and adopting ambitious pedagogical approaches when working with young children can lead to substantial learning benefits. Adopting a design-based research methodology where the research team worked in collaboration with the classroom teacher to develop and deliver an instructional sequence, Bicknell et al. (2016) investigated the impact of introducing five year-old New Zealand children in their first year of schooling to multiplication and division problems. The experienced teacher participating in the study had not previously explicitly introduced problems with this structure to children of this age group. The instructional sequence used in the project with 15 participating children emphasised the design and ordering of tasks, the importance of (multiple) representations for supporting children’s thinking, and the role of the teacher in supporting student discourse, and the development of the classroom mathematics registrar. Pre-post comparisons indicated that participating in the instructional sequence resulted in substantial improvements in student mathematical performance. The authors concluded that these findings challenge the assumption that addition and subtraction instruction should precede multiplication and division, and suggested there are benefits to exposing students to mathematical problems with these ostensibly more complex structures at younger ages, than is normally indicated in curriculum documents. These findings and conclusions have been reiterated by these authors in more recent studies (Young-Loveridge & Bicknell, 2018). When reflecting on the success of the project, the authors concluded that design-based research can support rich, deep teacher-researcher collaborations that deliver a range of benefits to all stakeholders (Bicknell & Young-Loveridge, 2017).

Like other papers discussed in this subsection, Bobis and Way (2018) emphasised the important role of the educator or teacher in supporting the student to reason mathematically when presented with a challenging mathematical task. The authors presented a classroom episode that sought to illustrate the role student-generated representations play in their mathematical development. They illustrated how exposure to a partial array to solve a problem (how many textbooks can we fit on this table, with no overlap and leaving no gaps) prompted a five year-old child, Emma, to draw a picture to represent the problem situation. Emma’s decision to draw a picture was driven by the very practical constraint that there were insufficient textbooks to cover the entire table. The picture created by Emma transformed the partial array into a complete array, and noted how many textbooks there were in each row (12) and column (5). Using a counting by 5s strategy, Emma worked out that 60 textbooks were needed to cover the table. It was noteworthy that the fact that Emma was unable to completely model the problem situation appeared to prompt her to adopt a more abstract, explicitly mathematical representation. Beyond the choice of task and materials, the teacher also supported Emma to articulate the connection between her pictorial representation and the underlying mathematical structure of problem situation she was representing.

2.1.2 Cognitive-Science Based Research

The second methodological orientation considered in the number sub-section is cognitive science based research. Australian-based researcher Robert Reeve has published a series of studies with colleagues (Jacob Paul in particular) examining the relationship between several specific mathematical abilities and mathematical performance. In contrast to the classroom-based research described previously, these studies are instead very much within the domain of cognitive science.

Chew, Forte, and Reeve (2016) undertook a study examining the relationship between non-symbolic (e.g., •• vs. ••••••) and symbolic (e.g., 2 vs. 6) magnitude judgement abilities in young children. Although symbolic magnitude judgement and mathematical ability more generally are consistently found to be positively correlated, previous research has presented conflicting evidence regarding the extent to which these two constructs are related to non-symbolic magnitude judgement. The authors contend that different learner profiles may account for these ambiguous relationships. Through utilising latent profile analysis with a sample of 124 children (5–7-year-olds), it was revealed that children with an imbalance between their symbolic magnitude abilities and their non-symbolic abilities tended to demonstrate lower levels of mathematical performance. That is, the learner profile characterised by children who were substantially more proficient in identifying the larger of two digits (symbolic magnitude ability) than in identifying the larger of two quantities (non-symbolic magnitude ability) were less accurate with single-digit addition problems and when required to read multi-digit numbers. Likewise, the learner profile characterised by children who were substantially more proficient in identifying the larger of two quantities than in identifying the larger of two digits also demonstrated lower levels of accuracy on addition and place-value assessments. The implication is that an asymmetrical development between the development of symbolic and non-symbolic magnitude abilities is problematic; and that both these abilities are therefore important to the undertaking of more complex mathematical tasks.

There is further evidence in support of the idea that non-symbolic magnitude processing is related to early mathematical performance; in this instance, number fact retrieval. Adopting a longitudinal design, Paul, Gray, Butterworth, & Reeve (2019), assessed 267 six year-olds (at Time 1), across a two- year timespan to explore the interrelationships between general math and reading ability, core number abilities (dot enumeration tasks, number comparisons tasks) and early mathematical competence (single-digit addition facts, reading multi-digit numbers). They found that general mathematical ability and performance on dot enumeration tasks predicted speed of addition fact recall, whilst general mathematical ability and general reading ability predicted success with accurately reading multi-digit numbers. Similarly, Reeve, Gray, Butterworth, and Paul (2018) provide further evidence that subitising is linked to efficient single-digit addition strategies and extend this finding to incorporate multi-digit addition strategies.

Paul and Reeve (2016) investigated the strategies children adopted when confronted with single-digit addition problems. They were interested in ascertaining whether the types of strategies children employed reflected differences in general reasoning and/or cognitive abilities, or whether they were uniquely tied to their skill with solving single-digit addition problems. They found that the tendency to utilise more sophisticated single-digit addition strategies (e.g., retrieval and decomposition) was related to visuospatial reasoning, suggesting that reasoning should be considered alongside visuospatial working memory as an important predictor of early mathematical performance. Related research has found that the primacy of visuospatial reasoning for supporting efficient addition strategies is robust across cultural groups, including Indigenous children living in remote communities (Reeve, Reynolds, Paul, & Butterworth, 2018).

2.1.3 Large-Scale Research

The third methodological orientation considered in the number sub-section is large-scale research related to sector programs or initiatives. Two notable studies have been included in the review. Although both these studies can be broadly described as large-scale research, the focus of the research and methodologies differ significantly.

Gould (2017) explained the difficulties of learning the counting sequence in English, particularly in contrast to many Asian languages. For example, it has been claimed that English-language speakers effectively need to learn 20 discrete terms to count to 20, whereas Chinese-based languages generally require knowledge of only 10 discrete terms in order to count to 99 (Fuson, 1988, cited in Gould, 2017). However, Gould noted that further research is needed into understanding how children's counting develops over time, particularly in relation to refining the teaching-learning paths that best describe how English-language speaking children acquire the oral count. To address this issue, he undertook a longitudinal analysis of over 800 Australian students to examine how their highest oral count evolved over their first three terms of schooling. He found that, contrary to previous research with smaller samples of children (e.g., Siegler & Robinson, 1982, cited in Gould, 2017), there was evidence that children did not learn the teen numbers discretely, instead relying on semi-regular patterns when possible (e.g., the numbers 16–19).

An important finding from Gould's (2017) study is the need to distinguish between what he terms "rest points" and "hurdles" when considering the decade numbers and the numbers immediately preceding decade numbers (p. 102). Specifically, the first two decade numbers, 10–20, are best understood as rest points, as far more students extinguished counting on these numbers than on the number immediately preceding them (9–19). Gould notes that in the case of 10–20, it is (for some students) “as if the counting has arrived at a destination point” (p. 103). By contrast, entering the next two decade numbers, 30–40, are best understood as hurdles; considerably more students extinguished counting on the preceding numbers (29, 39) than on the decade. Similarly, 100 presents as a rest point, whereas the transition from 109 to 110 is another notable hurdle, as is the transition from 199 to 200. Gould suggests that such hurdles can be understood as “missing terms in a pattern that is yet to be observed” (p. 105). Gould also noted that, at times, apparent hurdle numbers (e.g., the transition from 29 to 30) are better understood as “potholes” (p. 108); if the student is corrected or supported to state the next number in the sequence, they can continue counting far beyond this number (e.g., to 100). The presence of potholes suggests that the student has knowledge of the relevant patterns, and can benefit substantially from explicit targeted teaching to address this gap in their knowledge.

The other study was undertaken by Gervasoni, Roche, Giumelli, and McHugh (2019), and involved assessing the impact of the Extending Mathematical Understanding (EMU) intervention, designed to support students vulnerable in the early years of schooling. The authors found that the 342 Year 1 children who participated in the program across the course of a year were less likely to be vulnerable in at least one of the four number learning domains (counting, place value, addition and subtraction, multiplication and division). Moreover, although the mean number domains in which they were vulnerable remained higher than for non-EMU participants, the gap had closed substantially compared with the difference prior to the intervention. It can be suggested that the EMU intervention, and similar approaches focused on addressing learning gaps through targeting conceptual understanding, have substantial potential to reduce the tail of under-achievement in mathematics.

2.2 *Statistics and Probability*

Following the trend from the previous RiMEA, statistics continue to be a focused area of research in early childhood mathematics education, as researchers recognise that young children can engage with sophisticated ideas of statistical inquiry (Fielding-Wells, 2018); informal inferential reasoning (Makar, 2016, 2018; Oslington, Mulligan, & Van Bergen, 2018); and variation (Watson, 2016). A common thread throughout these studies was a focus on real life contexts or the use of picture storybooks as a stimulus for statistical inquiries (Kinnear, 2018). While Makar (2016) maintains that inference is at the heart of statistics, work in statistical inference is least understood in the early years, where children have had little if any experience with data handling. Fielding-Wells (2018) purported a similar view, that statistics is most commonly taught superficially and limited to the collection of data, construction of simple data representations and a literal reading of the information represented.

2.2.1 Inferential Reasoning and Statistical Inquiry

Makar and colleagues interpret informal statistical inference as distinct from (formal or informal) descriptive statistics as a claim that includes three characteristics: make a prediction that extends beyond the data set, use data as evidence of that prediction, and articulate their predictions with uncertainty (Makar & Rubin, 2009). Recent studies in Australasia indicate that some children in the early years of schooling are capable of inferential reasoning demonstrated by their ability to analyse complex data sets and engage in data modelling (e.g., English, 2013; Kinnear, 2013; Makar, 2016; Mulligan, 2015; Oslington et al., 2018).

Inferential Reasoning

Informed by previous work on informal inferential reasoning (English, 2013; Makar, 2014) Makar (2016) undertook an exploratory longitudinal study to investigate Foundation (5 year-olds) and Year 1 (6–7 year-old) children's emergent informal inferential reasoning in statistics in a supported inquiry-based environment. The study involved two phases: the first phase (Foundation children) was on sense making rather than formal knowledge, with the aim to build the skills that would support informal statistical inference and scaffold children's use of probabilistic language; the second phase (these children in Year 1), with the aim to engage them in an extended statistical investigation in which they were encouraged to make data-based inferences in an inquiry-based task. In both phases, Makar (2016) identified evidence of the aforementioned three characteristics required for making informal statistical inferences, identified by Makar and Rubin (2009). However, her findings indicated that informal statistical inference requires several skills and understandings beyond those characteristics, such as, "articulating observations, recording, organizing data using invented methods, working with data aggregates, and engaging with the variability of data" (p. 18). Furthermore, at 5–6 years of age these skills are informal and apply to both descriptive and inferential statistics, and to determine any relationship between the two, Makar (2016) indicated that additional research is required.

Oslington et al. (2018) also explored young children's reasoning through data exploration, specifically related to mathematical model building and data interpretation, using nine high-ability Year 1 students over the course of 16 lessons. The findings indicate that these young children were capable of developing and applying a model using a complex undefined set. However, as was the case in Makar's (2016) study it was necessary to devote time to foundation skills for predicting, checking and developing inferential reasoning. While Oslington et al. acknowledge a limitation of the study was the small sample size that focused on highly gifted children and as such the findings are not generalizable, their findings are consistent with those of earlier studies, and indicate that the capacity of Year 1 students in regular classrooms is underestimated. They concur with others (e.g., English, 2013; Makar, 2014, 2016; Makar & Rubin, 2009; Mulligan, 2015) that "incorporating modelling activities into the early years of schooling to develop rule-based models and reasoning skills …is critical to advancing students' critical numeracy capabilities" (Oslington et al., 2018, p. 210).

Statistical Inquiry

In relation to statistical inquiries, Fielding-Wells (2018) acknowledged that recent research indicates that young children (5–6 year-olds) can engage in statistical inquiries (Fielding-Wells, 2010; Fielding-Wells & Makar, 2013; McPhee & Makar, 2014). However, little research had been undertaken related to how teachers scaffold children in their initial inquiries.

Fielding-Wells (2018) argued that to implement statistical inquiries requires a pedagogical shift in teacher practice, from a teacher-led approach focused on data collection, display and literal interpretation, to a rich inquiry approach which involves the use of an "investigative cycle" (p. 111). She examined the ways in which a teacher, experienced in using an inquiry approach, facilitated a statistical inquiry with very young children (5–6 year-olds). The aim of her study was to gain insights into the scaffolding practices the teacher used to engage the children with *ill-structured* statistical problems using the statistical inquiry process. The findings indicated the teacher's use of questioning and feedback were the main means of scaffolding the children's learning through the statistical inquiry process. While it was possible to measure the amount of teacher scaffolding in relation to contingency (type, timing and strength of support) in a single unit of work, Fielding-Wells (2018) acknowledged it did not provide a realistic measure of fading (support is gradually withdrawn) or transfer (accountability is shifter to the learner); to do so requires a longitudinal study. The findings also revealed that a shift to statistical inquiry requires teachers to have a good foundation in the process of statistical inquiry, knowledge of statistics, and facility with scaffolding children through the process. Several future research opportunities were identified from the study including: a focus on insights into teacher scaffolding of statistical inquiry, and "the identification of mechanisms for teachers to identify class ZPD accurately so as to provide the least amount of support necessary to progress students" (Fielding-Wells, 2018, p. 126).

In contrast, Makar's (2018) statistical inquiry of younger children (4–5 year-olds), focused on the use of a problem context as a scaffold for working with powerful statistical ideas and structures. More specifically, she explored the theoretical notion of "statistical content-structures", which typify the specific features of problem contexts that can create opportunities for children to "engage with key statistical ideas and structures (concepts with their related characteristics, representations and processes)" (p. 4). The teacher in the study chose a person problem context (eye colour) to scaffold and informally introduce the five statistical ideas and structures: variability, aggregate, population, data and representation, which are essential foundations for understanding statistical concepts and practices. Makar's analysis included a mapping of the context elements illustrating the children's emerging reasoning about the context and the related statistical structures they engaged with, albeit informally at this stage. Makar (2018) maintained that the purpose of the study was not to provide evidence of individual success, but to expose and engage young children in experiences in which they can reason with powerful statistical structures and to show that the problem context can be a powerful scaffold for young children's learning.

2.2.2 Variation

Watson's (2018) study explored the concepts of variation and expectations with seven 6 year-old children (five boys, two girls) in a Foundation class (first year of school) using four interview protocols (lollies, book reviews, transport, and weather) that she had devised and used with older students. Her study was informed by a claim she made in an earlier study (Watson, 2005), in which she indicated that children begin to develop the concept of variation much earlier than is traditionally taught. Traditionally, measures associated with expectation (e.g., mean) are introduced before measures of variation (e.g., standard variation).

Student interview data were analysed in relation to their understanding of data, "variation (capacity to describe variation in the data presented or created) and expectation (use of variation implicitly or explicitly in the context to make predictions that reflect meaningful expectations)" (p. 63). The results indicate that children's familiarity with the context influenced their ability to understand the questions asked, and while the children had an understanding of the variation in outcomes, they did not have the language to explain the random behaviour. However, some children's reasoning was restricted to the context of the protocol presented, rather than on the data, which Kinnear (2013) referred to as *abductive reasoning*. For example, with the book and transport protocols children's expectations expressed as predictions were based on imaginary situations, within or outside the context, rather than based on reasoning associated with the data. Reasons for this include lack of familiarity with the library and characters in a book, and awareness of the route to school or time taken to travel to school. Examples of this were evident in earlier studies (e.g., Ben-Zvi, Aridor, Makar, & Bakker, 2012), and suggest that young children's thinking progresses from "imaginary reasoning" outside of the context presented, to "abductive reasoning" using only the context presented, to early "statistical (or inferential) reasoning using the data within the context in decision-making" (Watson, 2018, p. 71). As variation was prevalent throughout the interviews, more so than expectation, Watson (2018) posited that variation is the foundation of all statistical inquiry, which supports her earlier claim that an appreciation of variation is the starting point for children's engagement with the practice of statistics (Watson, 2005). Further research with young children is needed to ascertain this suggested pathway and ways to scaffold children into the "practice of statistics" (Watson, 2018, p. 71).

2.2.3 Context of Picture Storybooks

Kinnear (2018) reported a small study with 5 year-old children, in which she used picture storybooks to fulfil two contextual roles: as a “data context” and as “task context” for statistical problems and statistical problem solving. The findings indicated that some of the picture storybooks stimulated both affective and cognitive interest in the data context and task context of the modelling activities related to a statistical problem. However, Kinnear acknowledged that finding suitable children’s literature to support young children’s statistical learning was problematic for teachers and researchers and recommends that further research on the characteristics of picture storybooks that stimulate interest in both data context and task context, is required.

2.3 Measurement

Since the previous review period, research within Australasia about young children’s measurement understandings has continued to predominately relate to mass. However, there is an emergence of research on time (MacDonald & Murphy) and on using measurement as a context to explore young children’s number learning (Cheeseman, Benz, & Pullen, 2018). Cheeseman and colleagues have been the main researchers in this area along with MacDonald and Murphy. In relation to mass measurement, Cheeseman and colleagues continued to explore young children’s understanding of mass measurement across two different, but related studies (Cheeseman & McDonough, 2016; Cheeseman, McDonough, & Golemac, 2017) that build on their earlier work (Cheeseman, McDonough, & Clarke, 2011; Cheeseman, McDonough, & Ferguson, 2014). Three foci were evident in these studies: use of measuring instruments; investigative play; and role of the teacher to elicit children’s mathematical reasoning. MacDonald and Murphy’s (2018) study on time focused on children’s understanding of clocks and the structural features of an analogue clocks. Common to the mass and time studies was a focus on children’s perceptions, representations and understanding of the tools used to measure.

2.3.1 Measurement as a Vehicle for Number Learning

Drawing on their collective research in measurement and recognition that mathematical concepts develop from an early age, Cheeseman, Benz, and Pullen, (2018) designed an intervention program focused on measurement, which replaced the traditional number program. Their justification for such a study was threefold: first, young children’s early mathematical experiences are often in real-life measurement contexts; second, young children have intuitive and informal capabilities in spatial and geometry concepts as well as numeric and quantitative concepts; and third, mathematics learning in pre-school is in stark contrast to that of school, where it is heavily number based.

The year-long design research project was conducted in a Foundation classroom by two teachers with 40 children (5–6 year olds) entering their first year of formal schooling. The majority of children were from low socio-economic backgrounds, and a high proportion came from non-English speaking backgrounds and newly arrived migrant families. Results of the one-to-one interviews on number concepts indicated an improvement in children's counting and place value number knowledge, within a measurement-based program. In fact, students performed better than the control group on place value, which Cheeseman et al. (2018) attributed to the need to use larger numbers in the measurement context. The findings also indicated that a core of children could not verbalise the number sequence to 20 at the end of the year, which Cheeseman et al. contend is a "fluency skill that can be developed with practice" (p. 113). Despite the results related to children's counting skills, Cheeseman et al. maintained that providing a measurement-focused program allowed the children to explore number in meaningful ways, and stimulated some children to go beyond the intended curriculum in relation to measurement outcomes (use direct and indirect comparison), to understanding iteration of units; and quantifying and comparing measures. This study raises several issues related to the type of mathematics experiences young children have in their first year of schooling, and how best to cater for children from disadvantaged backgrounds who may also be linguistically disadvantaged.

2.3.2 Mass Measurement

In relation to mass measurement, Cheeseman and colleagues continued to explore young children's understanding of mass measurement across two different, but related studies (Cheeseman & McDonough, 2016; Cheeseman, McDonough, & Golemac, 2017) that build on their earlier work (Cheeseman, McDonough & Clarke, 2011; Cheeseman, McDonough, & Ferguson, 2014). The measuring instruments were a focus of both studies, as was the role of the teacher to elicit children's mathematical reasoning.

Cheeseman and McDonough (2016) highlight the need to foster young children's mathematical curiosity and suggested the reconceptualising of the primary mathematics classroom in relation to Askew's (2012) teaching tripod of Tasks, Tools and Talk. Cheeseman and McDonough (2016) researched classrooms of children (5–6 year olds) as they engaged in mass measurement through investigative play, using the tools of balance scales and a variety of materials. The researchers were particularly interested in how the children engaged with challenging tasks, explored the balance scales, and the role of the teacher during the investigation. Teachers in classrooms they observed: "encouraged exploration and investigation; used probing questions; listened to children's talk; expected exploration of mathematical reasoning; and expressed genuine appreciation for logical thinking and experimentation" (p. 143). While the researchers found the use of the tools provided a stimulus for investigating mass, in each classroom the teacher played a critical role in relation to the choice of tasks and fostering the children's curiosity and mathematics thinking.

The implications of this research for classroom teachers were threefold: "to offer children challenging tasks and interesting mathematical tools: listen carefully and be aware of the learning potential children bring to the task; and encourage curiosity by showing a real interest in children's investigations" (p. 151).

Extending on their previous research (Cheeseman, McDonough, & Ferguson, 2014), in which they observed 6–8 year olds using measuring instruments such as pan balances for measuring mass, Cheeseman, McDonough, and Golemac (2017) observed 12 children (5–7 year olds) exploring mass measurement using home-made suspended balances. Their study focused on the importance of eliciting and valuing children's thinking and the use of 'thinking conversations' (Lee, 2012), as a data collection tool. The role of the adults was to "listen, interact appropriately, probe children's thinking and seek explanation for their actions" (p. 147) as they captured the children's conversations. This small study emphasised the importance of the "thinking conversations" teachers and researchers conduct with young children to "elicit thinking and to stimulate mathematising" (p. 155). The findings indicate that young children can engage in big mathematical ideas such as equivalence, make mathematical generalisation, reason about mathematical investigations and justify their reasoning. Moreover, the findings highlight the power of investigative play; the importance of the teacher attending to what children notice, and eliciting children's mathematical reasoning.

2.3.3 Time Measurement

There is very little recent research around measurement related to time, in particular young children's understanding of clocks since the seminal work of Pengelly (1985) which focused on young children's understanding of a clock face through the use of young children's (aged 3–7) creation of clock faces. From the analysis, she developed a sequence of understanding of the clock face. Smith and MacDonald's (2009) study of drawings of 4–6 year olds challenged Pengally's developmental sequence, which did not include a focus on hands. More recently MacDonald and Murphy (2018) collected drawings from 132 children within their first six weeks of Foundation (first year of Primary School) and used a coding system based on three structural features of an analogue clock: numbers, hands and partitioning. Drawings were coded according to the degree that these features were evident within the drawings. The results revealed that the majority of children start school with some ability to represent one of more structural features of a clock. However, the Australian Curriculum: Mathematics (ACARA, 2016) makes no explicit mention of clocks in the first year of schooling, which highlights a mismatch between the intended curriculum and children's mathematical ability when they start school. MacDonald and Murphy's found that children's ability to represent the structural features of an analogue clock does not progress linearly as purported by Pengelly and that young children attend to different features of clocks; and therefore have alternative developmental journeys.

3 Curriculum, Policy and Assessment

The period of the present review has been one of great change and some uncertainty for early childhood education policy in Australasia, and also assessment in the early years. Since the last RiMEA period there has been less research on specific assessment practices, and a shift to identifying cognitive markers and children's learning potential, as well as having a balanced approach to assessment, pedagogical practices, policies and programs.

3.1 Australian Curriculum Policy

As highlighted in the 2012–2015 review chapter on mathematics education in the early years (MacDonald, Goff, Dockett, & Perry, 2016), the last four years have continued to be a "tumultuous time" for the Australasian early education sector. In Australia, two national curricula in Australia (Early Years Learning Framework and Australian Curriculum) continue to provide a platform for further implementation and associated research. Changes to early childhood policy have been made with a particular focus on raising the quality standards of the early childhood education sector (Australian Children's Education and Care Quality Authority [ACECQA], 2018). Curriculum and policy developments have been represented through research focusing on mathematics education and transitions from prior-to-school settings to primary school. Hard, Lee, and Dockett (2018) noted that despite efforts to promote the complementarity of the early childhood and school curricula, there is little explicit alignment between the two. This lack of alignment has implications for transitions in mathematics education, as many children experience a disconnection between the mathematics they know in early childhood, and that which they experience upon school entry (Perry, MacDonald, & Gervasoni, 2015).

3.2 New Zealand Curriculum Policy

In Aotearoa New Zealand, 2016–2019 has been a period of intensive change for teachers in the early years, particularly since the release of a report on mathematics in early childhood services in 2016 by the Education Review Office (ERO). The report claimed that despite "evidence of good practice in many settings, the ERO has found that mathematical teaching practice in the early years remains variable" (p. 29) and advocated that teacher leaders focus on: greater emphasis on opportunities for mathematical learning within their settings; more inclusion of meaningful contexts for mathematics learning; and ways to enhance teachers' mathematical knowledge.

During the same period time, the early childhood curriculum *Te Whāriki* (Ministry of Education, 1996) was reviewed, and a draft released for consultation. The updated and final version *Te Whāriki* was published in early 2017 and distributed to the sector (Ministry of Education, 2017). From an analysis of the mathematics focus of *Te Whāriki*, McChesney (2017) identified that much of the original orientation to mathematical learning remained. For example, mathematics continues to be explicit in two of the five strands (Mana Reo—Communication and Mana Aotūroa—Exploration). As an illustration of the orientation towards the learning of mathematics, one of the learning outcomes is "Recognising mathematical symbols and concepts and using them with enjoyment, meaning and purpose" (p. 42). Associated with this learning outcome, in *Evidence of learning and developmen*t is "Familiarity with numbers and their uses by exploring and observing the use of numbers in activities that have meaning and purpose (and) Recognition that numbers can amuse, delight, comfort, illuminate, inform and excite" (p. 42). The updated *Te Whāriki* is therefore an opportunity for New Zealand early childhood educators to revisit the existing mathematics framework of Te kākano, and to expand and develop further resources (McChesney, 2017).

The change in government in late 2017 has seen intensive activity for those in the early years, particularly the early childhood sector, with the publication of a draft Strategic Plan for early learning 2019–2029, requiring many educators to provide consultation and feedback (Ministry of Education, 2018b). Similarly, the abolition of The National Standards for national assessment and reporting (Ministry of Education, 2018a) of students in the first three years of primary school (age range 5–7 years) sent a clear message to teachers, and parents of more effective assessment processes and presented the sector with another opportunity "to reconsider our mathematics programmes" (Bailey, 2018, p. 86). Much of this policy work, including ongoing consultation with teachers, parents and communities, continues through 2019 and beyond.

3.3 Assessment Practices

Assessment of children's mathematical learning was the subject of research in the review period. Profiling young children in order to predict their future mathematical success seems to have become of some interest to researchers. Gray and Reeve (2016), writing in the field of experimental psychology, investigated cognitive markers for profiling the mathematical ability of preschool children. They found there were two number markers that were predictive of children's early mathematical competence: dot enumeration and the spontaneous focus on number. The authors also suggested that there was no unitary mathematical ability of preschool children and that "different combinations of cognitive abilities may underlie different competency profiles" (p. 16). The study has implications for mathematics education as it adds to the extant literature that suggests there is no singular mathematics "ability".

Moss, Bruce, and Bobis (2016) noted the critical importance of not only young children's mathematical thinking on entry to school but of the rate of learning in the first two years of school as a predictor of long-term mathematical success. These authors strongly recommend adoption of enriched and expanded pedagogy, curriculum and mathematics programs, in the early years. These recommendations align with those of English (2016) who argued the need to consider maximizing young children's mathematical talents might be maximised once they are revealed.

In her overview of the special issue of ZDM, English (2016) alluded to concerns about environments that "push teachers to increasingly structured assessments" (p. 1079) which cannot be exclusively relied upon to determine children's mathematical competencies or determine their learning potential. Instead, teachers and researchers need to draw on a rich bank of diverse assessment practices such as clinical interviews, observations, representations, use of technology, and testing that not only reveals children's capabilities but also to consider how we might use these insights to further their mathematical learning. English contended that broadening assessment practices in early childhood mathematics education will also broaden "our knowledge of young learners' capable minds" (2016, p. 1086) and acknowledged that teachers' must be equipped with the "knowledge and skills to understand and interpret the development of their students' mathematical knowledge and reasoning" (p. 1800).

4 Aspects of Teaching and Learning

The previous sections have focused on mathematical content, policy and assessment. Each of these sections highlighted aspects of teacher practice or factors that impact on their practice. A common theme throughout the content section was the reference to play-based learning, the use of challenging tasks, the role of the teacher to observe, asking questions to elicit children's thinking and notice the connections children are making in the course of a lesson or learning experience. Unlike the 2012–2015 review in which context was a focus of much of the research, in this current review there is a growing emphasis on lesson design and use of challenging tasks, and play-based learning, that maximise young children's mathematical capabilities. In this section we provide two contrasting aspects of teaching and learning. The first focuses on lesson design and challenging tasks as part of a sequence of learning in the early years of formal schooling, whereas the other focuses on play based learning in the pre-school settings.

4.1 Lesson Design and Sequencing Learning

Challenging tasks and the research findings surrounding their characteristics and use have appeared in the literature hitherto. In this review period, Russo and Hopkins (2017a, 2017b, 2017c, 2019a, 2019b) studied 75 Year 1 and Year 2 students' reflections on challenging tasks, teachers' perceptions of teaching challenging tasks, and the effect that lesson structure has on student outcomes. The authors described how Russo delivered two units of work (28 lessons) to three classes of students, systematically adopting either a “discussion first” or a “task first” approach to teaching mathematics. The distinguishing feature between the two lesson structures was whether or not students had a chance to explore the mathematical concepts in a whole group setting before being given a challenging mathematical task. The findings of the study (2019b) suggested that both approaches resulted in large improvements in mathematical performance, although there was some evidence that the discussion first approach was more effective in improving mathematical fluency.

Students’ reflections on learning with challenging tasks (Russo & Hopkins, 2017c) provided evidence that students embraced struggle and persisted with the tasks. Many students reported that they enjoyed being challenged by the tasks. Interestingly, the authors noted that students were evenly divided over which lesson structure they preferred, however tended to have a strong preference for one lesson structure over the other.

The three classroom teachers were also asked their perceptions of their students when observing challenging tasks taught by Russo. Teachers thought that each of the two lesson structures had distinct strengths. The task first approach was perceived as better able to foster creativity, promote mathematical discussion, build persistence, and effectively engage students through challenge. Conversely, there was some evidence that the discussion first approach was seen to be a more focused, efficient approach to instruction (Russo & Hopkins, 2017b). More generally, regardless of how the lesson was structured, the teacher-participants described their students as autonomous, persistent and highly engaged. Teachers explained the positive student reactions as reflecting: a classroom culture that embraced struggle, high teacher expectations, and consistent classroom routines (Russo & Hopkins, 2019a).

The authors concluded that there is more than one way to teach with challenging tasks to generate sizeable learning gains (Russo & Hopkins, 2017a). However, they emphasised that this does not imply that the two lessons structures are equivalent. They argued that the presence of apparent trade-offs between efficiency and creativity (for example), combined with the fact that students have highly divergent preferences for how lessons involving challenging tasks should be structured, implies that teachers should strive to provide students with opportunities to experience both types of lesson structures.

4.2 Play-Based Learning

The role of play in early childhood mathematics education is the focus of several studies over the review period. Some studies have been reported in the measurement subsection (e.g., Cheeseman and colleagues), which focused on the use of investigative play in the context of mass measurement. While each of the studies presented in this subsection relate to play-based learning in mathematics, they have different research foci. Cohrssen and colleagues considered the impact on the educator’s practice and their use of formative assessment practices; Colliver and colleague considered the impact of providing young children with everyday mathematics experiences of adults; Marcus and colleagues investigated young children's noticing of the mathematics as they engage in play; and Moss and colleagues considered the importance of play in young children's development of disposition and habits of mind relating to mathematics learning.

Cohrssen, Church, and Taylor (2016) studied the implementation of a play-based suite of activities by six educators of 3–5 year-old children in pre-schools. The aim of the study was to observe the effect of providing play-based mathematical activities together with clear learning objectives, step-by-step implementation instructions, and descriptions of the underpinning mathematical thinking to early childhood educators. The findings showed that when educators implemented the activities: their professional development was supported, their practice changed, and their confidence and formative assessment of children's knowledge grew. In addition, children became more enthusiastic and motivated participants in mathematical activities. The findings also highlighted an on-going need to challenge educators' beliefs and to consider new pedagogical practices if they are to meet the demands of early childhood mathematics education and cater for the future mathematics learning needs of young children, in pre-school settings.

Colliver (2017) and Colliver and Arguel (2018) undertook a small study that sought to inspire child-initiated mathematical and literacy-based play by four year-olds in early childhood centres. Educators and family members were given scripts to learn in order to demonstrate in pairs and verbalise everyday problems and solutions for children to hear. The study tested the effects of children's observations of the practices demonstrated. Results indicated that children were influenced to: play with the same mathematical materials, and extend their play-based exploration of pattern and mathematical concepts. While the scope of the research was very limited, it did suggest that by involving young children in the everyday mathematics of adults, children played for longer and were curious about mathematics. Further work is needed to find effective ways to translate this study into the broader community in its present form.

Another small pilot study reported by Marcus, Perry, Dockett, and MacDonald (2016), conducted with six pre-school children and six children in their first year of school over a six month period, investigated what young children notice about mathematics as they played and interacted with other children. The adults' (teacher and parents) role in such a situation was to facilitate the noticing, which is different from the teacher noticing in the research literature. Marcus et al. found that the children in both contexts noticed, explored and talked about mathematics in their own play and that of their peers. While the authors acknowledged that this was a small study it provides an impetus for a larger scaled study across a diverse range of contexts.

In contrast to the aforementioned studies, Moss et al. (2016) in their review of challenges and developments in Early Years mathematics (5–8 year-olds) questioned the widespread acceptance of a play approach to mathematics together with the belief that all mathematics activity should emerge from child-directed play. These authors described the centrality of play for the development of dispositions and habits of mind valued in mathematics education, such as curiosity, creativity, and acceptance of multiple possible answers to the same problem. However, they also noted that, while children learn from play, it appears that children can learn much more with “artful guidance and challenging activities provided by their teachers” (p. 162). They also raised concerns about the lack of explicit mathematics content in early childhood educators programs, and research on the process of teaching mathematics in early childhood settings and in the first years of schooling. Moss et al. (2016) recommend research into professional development programs for educators and teachers of young children and the “study and implementation of pedagogies that foster deep mathematics understanding” (p. 182).

These studies highlight the important role the adult plays in enhancing young children’s mathematics learning and the value of play-based mathematics learning. A key finding of the study by Cohrssen et al. (2016) was the need for increased professional learning related to their own mathematics content knowledge and play based mathematics pedagogies and planning; a consideration for future research.

5 Home and Prior to School Context

The previous section focused on aspects of children’s learning within formal settings. The range of studies presented in this section acknowledges the important contribution families and the home context plays in the mathematics learning of young children, and the various ways of supporting parents’ roles in enhancing mathematical experiences for children.

The publication of a substantial book, *Engaging families as children’s first mathematics educators* (Phillipson, Sullivan, & Gervasoni, 2017), reflects the prominence of this area of research in mathematics education in the early years. Written for an audience of professionals and educators with a clear focus on children from birth to 5 years, the book clearly positions the importance of families in the mathematics education of children.

The extensive research base of the *Let’s Count* project continues to both provide opportunities for further analysis and a springboard for related studies. For example, in Fenton, MacDonald, and McFarland’s (2016) study educators were supported to “act as mentors to parents and family members of the children in their setting” (p. 47). The educator had two meetings with parents and provided options of sample ideas for family gatherings, or a workshop where activities can be carried out at home, and information pertaining to mathematics underpinning the home activities. The qualitative data from the educator and parents were analysed using a strength-based analysis framework and the results compiled into individualised planning documents, that included relevant mathematical concepts, which were then shared with the parents. Such an approach was found to be useful as a means of fostering early mathematics learning. This case study also illustrates a reciprocity of expert knowledge; the early years educator in terms of mathematical learning and curriculum knowledge; and parents’ insights about their children’s interests and responses.

Another example of using data from the *Let's Count* study was the further analysis of the interviews with parents (Perry, Gervasoni, Hampshire, & O'Neill, 2016). From the analysis the authors identified two elements; the alignment of educators and family members with a focus on mathematics language; and "the ease with which the children assume language when it is introduced in a relevant and meaningful way" (p. 81). Similarly, further theory building from the *Let's Count* study (Gervasoni & Perry, 2016; Perry et al., 2016) was found in Perry and Dockett's (2018) work, in which they drew upon the four elements of Bronfenbrenner's bioecological model (person, process, context and time) and integrated them into the Let's Count programme through the mantra: "notice, explore, and talk about mathematics" (p. 613). Perry and Dockett emphasised that it was the "confluence of these elements that provides the greatest explanatory power" of *Let's Count* (p. 613), and they acknowledge that "working collaboratively with those who are in a position to facilitate meaningful, ongoing, regular, reciprocal and increasingly complex mathematical interactions with mathematics at their core" (p. 615), is required when support young children's mathematical development.

Smaller studies have investigated how contexts from children's lives provide opportunities for mathematical activity. Shopping was one context that was the focus of a pilot study with six families (MacDonald, Fenton, & Davidson, 2018). While undertaking a normal family shopping trip, the shopping trolley held a "trolley cam" to record actions and speech of the family. Two older children (approximately 8 years old) also volunteered to wear glasses with an inbuilt camera. Multiple vignettes of mathematical actions and conversations were analysed using Bishop's (1988) six mathematical activities of counting, measuring, locating, designing, explaining, and playing. The study found evidence of all of Bishop's activities within the context of shopping with locating being the most prevalent activity for both younger and older children, while both children and adults engaged in mathematical noticing in various ways (i.e., verbal and non-verbal forms) and times while shopping together. This study also contributes to theory building through the use of Mason's framework of *intentional noticing* (2002) and Perry and Gervasoni's (2012) *notice, explore, talk about mathematics* adaptation for young children's learning. This study provides further evidence of mathematical activities and conversations in authentic family contexts, which contribute to children's ongoing mathematics learning beyond the home. A related family context is swimming where Jorgensen (2017) expanded on her earlier work that identified swimming as a source of mathematical capital that adds to children's mathematical knowledge. Similarly, Mousley (2017) revisited the role of stories and narrative in home experiences, and her study reminds us of the powerful influence of stories and narratives that connect over time, and among different people and places.

A common thread for a number of studies was the design, implementation and research of ways to support parents in enhancing mathematical experiences for children. Muir (2018) illustrated this ongoing focus with a synthesis of earlier projects in the book *Forging connections in early mathematics teaching and learning* (Kinnear, Lai, & Muir, 2018). Initiatives such as *take home* numeracy bags, and a maths club, were designed to bridge home and school. Muir found that parents noticed lesson features, which related to the main task of the lesson and to the various social interactions of the children. In an unrelated study, the research team of another study (Niklas, Cohrssen, & Tayler, 2016a, 2016b, 2016c) adapted the design of their literacy study to focus on numeracy. Based on an approach described as a "light touch-low cost" intervention (p. 136), thirty-seven parents from a larger study self-selected to attend a parent meeting where the researchers provided supporting materials about supporting children's numeracy skills in the home environment. This study found that the children in the intervention study showed greater gains in their mathematical competencies than the non-intervention children and that parents enhanced the home numeracy environment for their children (Niklas et al., 2016a).

In stark contrast to the aforementioned studies, Glynn (2019) investigated whether parents have gendered constructs in relation to their children's interests and abilities in mathematics. Sixty-one parents of children in the first year of primary across three schools completed an online *Who and Mathematics* survey and seven of these parents participated in an interview to obtain insights into the reasons behind their beliefs. While more parents perceived girls to enjoy challenging mathematics problem more than boys, the majority believed that boys enjoy mathematics more than girls and are expected to do well. The findings revealed there is still stereotypical gendered thinking from parents about mathematical aptitude and highlights the need for parents and educators to recognise messages being conveyed to young children about mathematical capabilities and the impact these messages have on self-concept.

6 Cultural/Indigenous Contexts and Pedagogies

Australasian researchers continue to make new contributions in this area particularly related to the importance of cultural connectedness; and need for culturally relevant mathematics pedagogies in pre-school and school settings for young indigenous learners and those from diverse cultural backgrounds. Key researchers in this area continue to explore the pivotal role played by culture when young Indigenous, Pāsifika, and Māori students explore growing patterns.

As indicated in the equivalent section in previous RiMEA review "young indigenous Australian students are capable and proficient users of mathematics" (MacDonald et al., 2016, p. 180). Building on prior research related to Indigenous children's learning of growing patterns and Miller's (2014) "cultural learning semiotic model", Miller, Warren, and Armour (2018) explored teaching and learning processes that facilitated 16 Indigenous students (7–9 year-olds), to engage in the mathematical discourse of pattern generalisation. The findings of this study provided a major contribution to understanding student learning; the importance of creating spaces for both cultural and mathematical discourse; and the sharing of sign systems, in this case drawings or models of the growing pattern. In addition, Miller et al. found that Indigenous students bring their own cultural ontology (ways of being, knowing and doing) to the classroom and that the cultural backgrounds of the students and teacher influence the teaching and learning process.

Two related New Zealand studies (Hunter & Miller, 2018; Miller & Hunter, 2017) also focused on growing patterns with children of Pāsifika, Māori and South East Asia backgrounds. Hunter and Miller (2018) conducted a series of ten lessons focused on how Pāsifika patterning tasks might support young children's understanding of growing patterns. The findings indicate that student thinking moved from counting, to additive, to multiplicative thinking, and many students began to make generalisations. The findings also illustrate how an authentic cultural pattern can be used to develop early algebraic reasoning, and that young children can identify pattern structure and articulate algebraic relationships. Miller and Hunter (2017) conducted one-to-one task based interviews with students that focused on four pattern tasks, two from Pāsifika and Māori culture and two patterns used in typical New Zealand or Western mathematics lessons. Findings from this study concur with others (Hunter & Miller, 2018; Miller et al., 2018) that these students can engage in early algebraic concepts, such as growing patterns prior to formal instruction and have greater success when patterns come from a familiar context; an important consideration for teachers and researchers when introducing culturally diverse students to growing patterns.

Further research about the importance of cultural connectedness has been contributed from different locations in the Pacific to a significant publication *History of number: Evidence from Papua New Guinea and Oceania* (Owens, Lean, Paraide, & Muke, 2017). In relation to the early years, Paraide and Owens (2017) noted that "the integration of Indigenous and Western number is limited in formal learning at the lower primary level" (p. 244) and argued that such integration is possible if the teachers are adequately supported in implementing this integration. In another study related to number, Paraide (2017) found that the traditional number system appeared sophisticatedly adapted to the Tolai's cultural context. In particular, that Tolai number and measurement knowledge are frequently still used across a range of day-to-day activities and therefore should remain central to mathematics education in this community.

Anderson, Stütz, Cooper, and Nason (2017) similarly argued that teachers can be supported to implement culturally relevant mathematics pedagogy for young Indigenous learners. This is of particular importance in light of a study by Edmonds-Wathen (2017) in which a critical analysis of curriculum support resource for Indigenous students revealed that:

> …a discourse of developmental imperatives and completely inarticulated expectations leads to a focus on teaching English language and concepts, rather than on how students' prior knowledge might be used as a resource in a culturally responsive approach. (p. 36)

Anderson et al. (2017) proposed a theoretical framework to scaffold the design of programs that focus on the needs of Indigenous and low-socioeconomic status children in the first three years of school. Reflecting on the initial stages of the development of the framework, they stressed that a focus on both culture and mathematics in teacher learning, and addressing teachers' beliefs and conceptions about working with community, are central issues to be addressed with teachers of young Indigenous and low-SES students.

7 Emerging Areas of Research

The equivalent 2012–2015 review chapter on mathematics education in the early years (MacDonald et al., 2016) concluded with a list of emerging areas worthy of focus in the coming years. It is pleasing to see that several of these recommended areas of research (namely; data, mathematical reasoning, and roles of families and communities in young children's mathematical learning) are now represented by publications included in the current review. This current review has thus identified several new emerging areas that are beginning to gain attention in early childhood mathematics education research; specifically:

- Early mathematics learning and later achievement;
- Influence of early childhood education and care settings on children's mathematics;
- Relationship between self-regulation and mathematics;
- Birth to 2 years; and
- Innovative research methodologies.

The last four years has seen several publications in these growing areas of interest, and these are canvassed in the following sections. The authors of this chapter also acknowledge a further two emerging areas of research, namely, spatial reasoning and early STEM learning, and learning technologies in early childhood mathematics; however, these two areas are reviewed in detail in Chaps. 10 and 13, respectively, of this volume.

7.1 Early Mathematics Learning and Later Achievement

The 2016–2019 review period has seen new attention focused on early mathematics learning and development. Studies in this area have a range of different foci, but all have an interest in understanding the relationship between children's development, generally, and children's *mathematical* development, specifically. Much of this attention has been generated as a result of international studies (in particular, that of Duncan et al., 2007), which have demonstrated that early mathematical competence is a predictor of later achievement, both in mathematics and more generally. This relationship was examined in an Australian context by MacDonald and Carmichael (2018) who interrogated data from a nationally representative sample of 2,343 children participating in the Longitudinal Study of Australian Children (LSAC). Their study examined the relationship between mathematical competencies at age 4–5, and later results of Years 3, 5, 7, and 9 *National Assessment Program—Literacy and Numeracy* (NAPLAN) numeracy tests. Consistent with international studies, they found moderate correlations between preschool mathematics competencies and later mathematics achievement. However, their analysis of individual growth trajectories revealed that while early competencies predicted *initial* achievement, they did not predict subsequent growth—rather, performance at each year level predicted the next. Thus, the findings of MacDonald and Carmichael highlight the need for quality early childhood mathematics education programs for getting children's mathematical development "off to a strong start"; but the quality of mathematics education in the schooling years is also critical.

7.2 Influence of Early Childhood Education and Care Settings on Children's Mathematics

Within the review period, two studies have looked at the influence of the early childhood setting on children's mathematical competence. Hildenbrand, Niklas, Cohrssen, and Tayler (2017) investigated the relationship between children's attendance at different types of early childhood education and care programs (i.e., long day care, family day care, kindergarten) and their mathematical skills. They analysed data from 1,314 children participating in the E4Kids study at two measurement points and found that at first measurement, children attending predominantly informal care outperformed children attending predominantly formal care or a mix of care types. However, at the second measurement point one-year later, mathematical development did not differ between the care types. Another study utilising E4Kids study data was that of Niklas and Tayler (2018) who examined the impact of the quality of the early childhood room and group composition on children's mathematical abilities. While child and family characteristics explained much of the variance in children's mathematical abilities, room characteristics added significantly to the explained variance. Further, higher program quality was associated with greater mathematical abilities. These findings reinforce the need for quality early childhood mathematics programs, and teacher professional learning.

7.3 Relationship Between Self-regulation and Mathematical Development

The relationship between self-regulation and mathematical development has also been of interest during the review period, and offers a different perspective on children's pathways to mathematical achievement. Williams, White, and MacDonald (2016) utilised LSAC data for 5,107 children to examine whether mathematics achievement at age 8–9 differs by gender, how mathematics achievement is associated with self-regulatory pathways from age 2 to 3, and whether these pathways differ by gender. They found that attentional regulation was directly associated with mathematics achievement, and emotional regulation indirectly so. Moreover, these pathways were not moderated by gender. Williams et al. have suggested that embedding self-regulatory support in early childhood education programs may assist children's mathematical development. Pearce et al. (2016) also examined relationships between self-regulation and mathematics achievement, but with a focus on socio-economic inequalities. However, they found that cognitive ability, rather than self-regulatory ability, had the greatest effect on pathways between socio-economic disadvantage and mathematics achievement. They argue that interventions to improve cognitive ability, rather than self-regulation, serve to have the greater potential for improving the mathematics achievement of socio-economically disadvantaged children.

7.4 Birth to 2 Years

Cognition research has demonstrated that from birth, infants are capable of detecting numerical correspondences and abstract properties of objects and events (Starkey, Spelke, & Gelman, 1990). However, research focused on mathematics *education* for very young children—in particular, children younger than two years of age—is an emerging field of interest. Indeed, in MacDonald and Murphy's (2019) systematic review of the research concerning mathematics education for children under four years of age, only three papers out of the 103 reviewed (4%) focused on children under two years of age. This was consistent with Linder and Simpson's (2017) systematic review concerning early childhood mathematics educators which found that only 5% of papers focused on educators of children aged birth to two years. The three Birth to Two papers included in MacDonald and Murphy's review were from outside of Australasia; indeed, there were only six Australian studies and one New Zealand study included in the review overall which canvassed 2013–2018.

A new focus on Birth to Two is afforded through MacDonald's (2018–2020) Australian Research Council-funded study examining mathematics education for babies and toddlers. Findings from a national survey of early childhood educators indicate that educators of children under three years of age report high levels of comfort in planning and implementing mathematical learning experiences with these very young children (MacDonald, 2019). Educators report a good degree of utilisation of everyday resources and activities, which has been advocated for in existing studies (for example, Gervasoni & Perry, 2015). Moreover, findings suggest that educators are comfortable in gaining ideas from babies and toddlers, which is an important step in providing meaningful mathematics education for these very young children (MacDonald, 2019).

It would appear that there is great potential for Australasian researchers to contribute to the emerging field of Birth to Two mathematics education, and we would expect to see Australasian researchers contributing to this space in the coming years.

7.5 Innovative Research Methodologies

The 2016–2019 review period saw the emergence of several innovative methodologies in early childhood mathematics research. One such example is MacDonald, Fenton, and Davidson's (2018) use of "trolley-cam" to capture young children's mathematical interactions while shopping with their family. Families were invited to complete their shopping using a shopping trolley mounted with a custom-built Go-Pro© camera rig. The trolley-cam was specially designed to capture the interactions of the family with the store environment, within close range (approximately arms' length). This methodological approach revealed that young children and their families notice, explore, and talk about a great deal of implicit and explicit mathematics whilst shopping together. All of the children displayed instances of mathematical noticing, with the children indicating what they had noticed in both verbal and non-verbal forms (MacDonald et al., 2018).

The same study by MacDonald et al. also utilised "camera glasses" (i.e., glasses with an in-built video camera) in order to capture the child's perspective of the shopping experience (Davidson, MacDonald, & Fenton, 2018; MacDonald et al., 2018). This innovative research method enabled the researchers to view mathematical interactions from the "eyes of the child", thus offering a new perspective on mathematical conversations and actions in everyday experiences such as shopping.

Although not a "new" methodological approach (see, for example, MacDonald, 2013; MacDonald & Lowrie, 2011), the review period saw renewed focus upon the use of children's drawings to ascertain young children's experiences with, and understandings about, mathematical ideas. Indeed, both the 2018 and 2019 MERGA conferences contained symposia explicitly focused on the use of drawings in early childhood mathematics education research (see Way et al., 2018; MacDonald et al., 2019). A particular innovation related to children's drawings has been Way and Thom's (2019) use of a digital pen to capture children's mathematical drawing processes. Digital pens have the ability to capture both mark-making and sound, and thus create "pencasts" that can be replayed. As Way and Thom explain, the use of a digital pen allows for a "layered" approach to data analysis, whereby the pencast can be replayed without sound in order to interpret the drawing itself, before adding the synchronised soundtrack, which enables analysis of the child's verbal explanation (Way & Thom, 2019).

These examples demonstrate the scope for creative thinking with regards to "making visible" the ways in which young children experience mathematics. It is anticipated that these studies will spark future research adopting similar methodologies, and it will be interesting to see what other innovative research approaches emerge over the coming years.

8 Concluding Remarks

The review of literature presented in this chapter indicates there is a broad research agenda and much to celebrate about Australasian early years mathematics education research. Two particular emerging areas–Birth to Two, and use of technological tools to capture and "make visible" the ways in which young children experience mathematics with their families–offer Australasian researchers new opportunities to contribute to the early years space in the coming years.

As was evident in the previous RiMEA review, Australasian researchers demonstrate strength in research that identifies young children's mathematics capabilities, particularly in the content areas of number, mass measurement, time, and statistics in the early years of primary school; in both home and school contexts. System-based research related to number learning continues to be a focus, whereas content areas prominent in the previous review, such as pattern and algebra, and geometry were not apparent in this current review period, possibly reflecting the changing nature of the research agenda. The emergence of cognitive science based research associated with young children's number learning, and the relationship between mathematics abilities and mathematical performance, reflect the broadening nature of research in early years mathematics learning and development. Within the measurement domain an emergent area of research was the use of measurement as a context to explore young children's number learning, within the first year of school. Critical issues raised and considerations for future research include: the type of mathematics experiences young children have in their first year of school; and how best to cater for children from disadvantaged backgrounds who are also linguistically disadvantaged.

Several studies highlighted that young children are capable of more than the intended curriculum stipulates and can engage in mathematical inquiry and challenging tasks, and explain and justify their reasoning. Throughout these studies was a strong thread of research related to pedagogical practices such as the use of investigative play, mathematical inquiry and investigations, challenging tasks, and problem solving. There was also emphasis on teacher observing, listening, noticing, questioning to probe student thinking, and student discourse in supporting children to reason mathematically and to engage critically with the mathematical ideas presented. A clear recommendation from these studies is that young children need to engage in rich learning experiences that maximise their mathematical capabilities.

Unlike previous reviews, in this current review period there was little research pertaining to professional learning. Several studies indicated the need for pre-school educators and early years teachers to consider new pedagogical practices and their own dispositions and beliefs towards mathematics if they are to meet the needs of early childhood mathematics education, and cater for the future mathematics learning needs of young children. Aligned with this is the need for quality early childhood mathematics programs. These findings highlight a need for increased professional learning and research of early years teachers and pre-school educators related to their own mathematics content knowledge, pedagogical practices, and mathematics planning and assessment practices.

There has been considerable research related to supporting young Indigenous students' mathematics learning, the importance of cultural connectedness, and the need for culturally relevant mathematics pedagogies in pre-school and school settings for young Indigenous learners and those from diverse cultural backgrounds. However, ongoing research is needed into ways to support teachers to develop programs and practices that cater for Indigenous learners, those from different cultural backgrounds, and low-SES students.

Two other areas of ongoing research relate to (1) the impact of curriculum and policy on mathematics education in the early years both within the Australian and New Zealand settings; and (2) assessment, in particular the emphasis on structured assessment, which seeks to measure predictors of children's future mathematical success. However, several researchers indicate that testing alone is insufficient and that broadening assessment practices in early childhood mathematics education will also broaden teachers and researchers' knowledge of the capabilities of young children and their capacity to reason mathematically. Both areas provide potential for future research.

The review of research presented in this chapter has contributed new knowledge to the field of early childhood mathematics education; and provided insights and ideas for further research. Some further research might incorporate innovative methodologies to examine:

- Features of quality early childhood mathematics experiences that stimulate children's mathematical reasoning, build children's resilience, and encourage problem solving;
- Early childhood educators' practices, knowledge, beliefs and dispositions towards mathematics and their impact on young children's love of mathematics;
- Pedagogical practices required to scaffold children's understanding of early mathematics; for example, aspects of measurement, time, geometry and statistical inquiry; and
- Curricula that build children's experiences and knowledge to transition from the intuitive and informal to the formal mathematics and include programs that cater for Indigenous learners, those from different cultural backgrounds, and low socio-economic backgrounds.

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Chapter 10
The Re-emergence of Spatial Reasoning Within Primary Years Mathematics Education

Geoff Woolcott, Tracy Logan, Margaret Marshman, Ajay Ramful, Robert Whannell, and Tom Lowrie

Abstract This chapter presents a review of the re-emergence of spatial reasoning in Australasia as a potentially powerful but under-utilised bridging mechanism between real-world experiences and mathematics teaching and learning. This is the first time a chapter has been dedicated solely to spatial reasoning in the Mathematics Education Research Group of Australasia's (MERGA's) four yearly review and hence the chapter outlines preliminary studies that have formed the basis for the research profiled in the 2016–2019 period. The focus on primary years (Foundation to Year six) mathematics reflects a resurgence of insights from the 1980s amplified as a research focus on the interaction of spatial reasoning and mathematics development during childhood. Because mathematical concept formation is connected to interaction with the three-dimensional world in both a mathematical and non-mathematical way it will be important to spatialise the primary curriculum. The review includes coverage of the work of established Australasian research projects, along with smaller studies and literature emanating from intervention programs that are not nominally spatial, but have spatial underpinnings or spatial reasoning components. While further research

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is needed to explore teacher knowledge and practice, this chapter acknowledges the valuable contributions and global influence of re-emerging Australasian research.

Keywords Spatial reasoning · Primary years mathematics · Mathematics development · Intervention programs · Re-emergence

1 Introduction

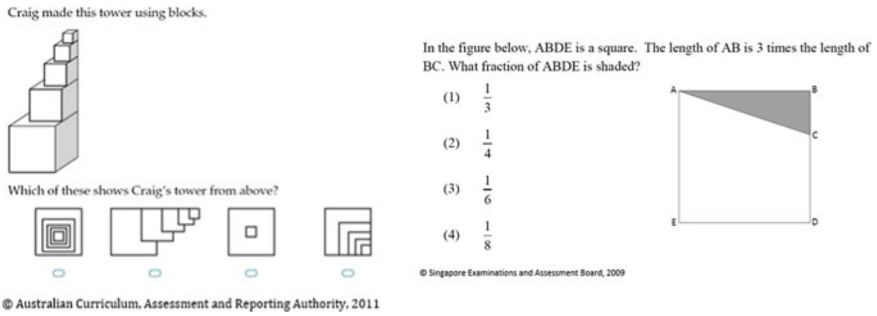
Spatial reasoning is re-emerging as an area of mathematics education research, largely because of a resurgence of research showing that spatial ability and mathematics performance are highly correlated (e.g., see Lowrie, Logan, & Ramful, 2017; Mulligan & Woolcott, 2015). Indeed, data from *Programme for International Student Assessment* [PISA] studies show that high performing countries in mathematics such as China (Shanghai and Hong Kong), Singapore and Korea do much better in the Space and Shape component compared to the three other components of the international assessment, that is, Change and Relationship, Quantity, and Uncertainty and Data (Thomson, De Bortoli, & Buckely, 2013). Further, a significant number of longitudinal studies have shown that spatial reasoning, like mathematics, can predict creative and scholarly achievements over lifetime periods and in other academic disciplines (Bruce et al., 2017; Davis, & The Spatial Reasoning Study Group, 2015; Uttal, Miller, & Newcombe, 2013). Recent studies continue to emphasise the crucial underpinning of spatial reasoning as a marker of success in secondary and tertiary education environments (Newcombe, 2013; Sinton, 2014; Wai, Lubinski, & Benbow, 2009; Woolcott, 2018).

Although spatial reasoning (or spatial thinking) as a concept has been defined in varying ways, (e.g., see Buckley, Seery, & Canty, 2018; Davis et al., 2015; Höffler, 2010; Tosto et al., 2014) at the core they refer to the spatial manipulation of objects and their relations. It may be quite challenging to set strict delimiters as to what are the characteristic attributes of spatial reasoning. Spatial reasoning involves skills such as locating, orienting, rotating, decomposing, recomposing, navigating, patterning, scaling and recognising symmetry. In this review, we subscribe to the following conceptualisation from the Spatial Reasoning Study Group (SRSG), an international consortium dedicated to spatial reasoning research: “the ability to recognize and (mentally) manipulate the spatial properties of objects and the spatial relations among objects” (Bruce et al., 2017, p. 147). This definition has been used in database searches, examination of seminal work in the field and discussion among the team members.

Recently reviewed publications suggest a shift in curriculum foci to strands concerned with spatial reasoning, such as Measurement, Space and Geometry in mathematics (Atweh, Goos, Jorgensen, & Siemon, 2012). There has been an emerging focus on spatiality (spatial reasoning, spatial thinking or spatial sense) as a fundamental component of pre-school and primary years education (Davis et al., 2015). This focus has shown that, as well as having strong correlations to mathematics achievement, spatial reasoning is malleable (Uttal, Meadows, et al., 2013) and, since it can be taught, it needs to be connected with a diversity of mathematics learning environments. The connection of spatial reasoning to curriculum, however, lags behind other areas that have focused on real-world alignment of learning, such as numeracy across the curriculum (e.g., Geiger, Goos, & Dole, 2014) (See Chap. 4, this Volume).

In the Australasian context, spatial reasoning has been in the limelight both on the academic front as well as in terms of curriculum inclusion. The work of Alan Bishop, Ken Clements and Glen Lean in the 1980s (e.g., Bishop, 1980; Lean, 1984; Lean & Clements, 1981) set forth the foundational milestones that paved the way to opening the research agenda on spatial reasoning in Australasia. That spatial reasoning has been a continuous element of interest since that time, can be inferred from the various presentations in the Mathematics Education Research Group of Australasia [MERGA] conferences for the past 30 years.

The conception of spatial reasoning by curriculum developers or examiners, however, tends to vary across cultures and is reflected in the type of spatial tasks posed in national examinations or in school textbooks. Compared to several Asian mathematics curricula which have put more emphasis on Geometry, Space appears to have received higher curricular esteem in the Australian context. For instance, cross-cultural research conducted between Australia and Singapore has shown that the type of spatial tasks in the Australian and Singapore national examinations are remarkably different (e.g., see Lowrie, Logan, & Ramful, 2016a, 2016b). The Australian spatial tasks are purely spatial in the sense that the numerical calculations are secondary to the problem (if, indeed, there are any calculations at all, see Fig. 10.1). In the Singapore tasks, the spatial components tend to be integrated in the problem context with an accompanying load of numerical calculation. Given the varying positions of the respective countries on international assessments such as PISA, research is again moving toward better understanding the relationship between spatial reasoning and mathematics education in the primary years. This research could lead to spatial reasoning being a consideration in a future curriculum review.



Craig made this tower using blocks.

Which of these shows Craig's tower from above?

○ ○ ○ ○

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In the figure below, ABDE is a square. The length of AB is 3 times the length of BC. What fraction of ABDE is shaded?

(1) $\frac{1}{3}$

(2) $\frac{1}{4}$

(3) $\frac{1}{6}$

(4) $\frac{1}{8}$

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Fig. 10.1 Differences in spatial tasks across cultures, with permission from ACARA

2 Overview of the Chapter

In this chapter, research endeavours that investigated a broad range of spatial reasoning concepts are examined. The focus is predominantly on the primary years of schooling, however, the connection to the 'bookend years' of pre-school and early high school and the global spatial-mathematics education space are discussed. The chapter provides a sharing of expert knowledge in terms of discipline content, ongoing projects, and professional expertise that outlines the re-emerging importance of spatial reasoning as a fundamental component of mathematics education in the primary years. The review process engaged a hybrid, multidisciplinary academic team following a collaborative protocol described by Lake et al. (2017, 2018). This protocol allowed the review team to share a range of practical strategies as well as "significant skills and/or expert knowledge to the project, in terms of discipline content, project management, or professional expertise" (Lake et al., 2018, p. 17). The review articles, therefore, were drawn not only from conventional literature searches, but also from bibliographies in well-known centres dedicated (in part or whole) to studies of spatial reasoning in various contexts, including school-based intervention frameworks with a spatial reasoning component. As a result, the studies included add context to the review, capturing data related to the database searches.

The chapter is organised in five sections, beginning with the foregoing introduction and ending with some concluding remarks. Section two provides a background on spatial reasoning research and section three provides Australasian perspectives on spatial reasoning research prior to 2016, together outlining how a global re-emergence has manifested within mathematics research in Australasia. These two sections focus on the interactions between, and across research collaborations that have had an impact on spatial reasoning as being fundamental to mathematics education; in particular how spatial reasoning is seen to underpin mathematical development in the primary classroom. Section four outlines spatial reasoning research and mathematics development in Australasian classrooms since 2016, focussing on primary years. This section outlines how spatial reasoning is being considered and enabled in today's primary classrooms, along with education in the bookend years, highlighting major research thrusts in Australasia and their influence on research and practice in neighbouring regions.

3 Background on Spatial Reasoning

Historically, the interest in spatial reasoning started in the field of psychology and was mainly investigated with an adult population. Spatial reasoning as a form of intelligence gained importance with the development of spatial tests as an entry requirement in particular professions (e.g., see McGee, 1979; Piaget & Inhelder, 1948). Although progress was slow, spatial reasoning gradually became part of the schooling curriculum and attracted interest from mathematics educators. The investigation of spatial reasoning within the mathematics classroom developed largely from examination of studies of geometry by teaching mathematicians (e.g., Hilbert & Cohn-Vossen, 1952) as well as through investigative studies on spatial (sometimes termed visuo-spatial) abilities and related tasks by mathematics educators and psychologists (see e.g., Bishop, 1980; Clements & Battista, 1986, 1992).

Spatial reasoning implies making sense of objects and space; that is, how we mentally insert ourselves into a three-dimensional context to solve a problem. It is also recognised that there are numerous and diverse spatial elements potentially contributing to a child's development both prior to schooling and after a child begins school, even though the link to mathematics or mathematical thinking remains elusive (e.g., see discussion in Davis et al., 2015). Several recent publications have stressed the significance of incorporating spatial reasoning into primary mathematics classroom teaching, describing a process of spatialising the mathematics curriculum, where the spatial reasoning that underpins mathematics success has a stronger presence in classroom mathematics lessons and where this presence can influence student learning (Bruce et al., 2017; Davis et al., 2015; Hawes, Moss, Caswell, Naqvi, & MacKinnon, 2017). Recent studies have argued that a spatial reasoning focus that embraces this diversity as an untapped support for mathematics learning has the potential to go beyond geometry and conventional mathematics teaching. Hence, enabling students to access complex ideas in non-traditional ways, rather than focussing on computation, memorisation, and repetition, opens doors to other ways of engaging in mathematics (Mulligan, 2015; Mulligan & Woolcott, 2015).

The movement towards curriculum 'spatialisation' developed from a resurgence of interest in early studies, particularly from those of the 1980s (e.g., Bishop, 1980; Clements & Battista, 1986; Presmeg, 1989; van Hiele, 1986). Since 1995, enabled by the internet revolution (Tronco, 2010), an increasing number of studies globally have engaged a spatial reasoning focus in establishing connections across curricula and to the real world, including collaborations based in: North America (Bruce et al., 2017; Davis et al., 2015; Hegarty, Goodchild, Janelle, & Doehner, 2013; Janelle, Grossner, & Lenaburg, 2012); Asia (Chao & Lui, 2017; Hsieh, Lin, Chang, Huang, & Hung, 2017); Europe (Höffler, 2010; Tosto et al., 2014); and, Australasia (Lowrie & Jorgensen, 2018; Mulligan, Woolcott, Mitchelmore, & Davis, 2018; Owens, 2015a, 2015b).

A significant theme has also developed from the idea of embodiment—humans as dynamic bodies that interact with other bodies on various scales, from micro to macro—where conceptual blending allows conceptual development based on combinations of original embodied concepts derived from interactions with the three-dimensional world (e.g., see Lakoff & Núñez, 2000; Mowat & Davis, 2010). Those from an embodied perspective would argue that activities such as sliding and rolling are fundamental for conceptual development. Based on this understanding of embodiment as a cornerstone of learning and memory, de Freitas and Sinclair (2013) put forward the view that an embodied cognitive perspective embraces both a body *in* mathematics as well as a body *of* mathematics. Davis et al. (2015) carry forward this perspective in arguing that principles of embodiment, in particular enactivist principles relating perception, movement and object manipulation (Maturana & Varela, 1991), are central to conceptualisation and reasoning. From this perspective, Davis et al. (2015) argue that, not only must an investigation of spatial reasoning in young children be required, but that such investigation must be reflected in mathematics curriculum (see also Francis, Khan, & Davis, 2016; Khan, Francis & Davis, 2015).

Emerging from studies based largely in cognitive psychology is the notion that spatial reasoning is important for conceptual development, especially in early years education (Verdine, Golinkoff, Hirsh-Pasek, & Newcombe, 2017), and that spatial reasoning is also malleable, that is, it is anchored to experience and can be improved with training (Uttal, Meadows, et al., 2013). A recent focus has been on studies that relate spatial training to primary years mathematics, but with some studies moving from the formal experimental approaches of psychology (e.g., Cheng & Mix, 2014) to the more informal context of classroom experiments (Hawes et al., 2017; Lowrie, Logan & Hegarty, 2019; Lowrie, Logan, & Ramful, 2016b, Lowrie, Logan, & Ramful, 2017).

4 Australasian Perspectives on Spatial Reasoning Research Prior to 2016

It can be argued that the application of spatial reasoning research in modern Australasian classrooms developed largely from studies in the teaching and learning of geometry and psychological studies of perception and spatial relationships, rather than studies on spatial reasoning per se (e.g., Boger, 1952; Elkind, Barocas, & Rosenthal, 1968; Inskeep, 1968; Oldham, 1937; Trump, 1946). The Bishop review (1980) of spatial abilities and mathematics, however, marks a point in history where a broader view of spatial reasoning was employed in contexts other than geometry and psychology. Subsequently, spatial abilities were expressed in classroom activities across number, arithmetic, and geometry, and with less reliance on the experimental approaches common in psychology.

Although later studies by Clements and colleagues (e.g., Clements & Battista, 1986, 1992) remained focused on geometry and its connection to spatial reasoning in the classroom, Clements began an important research link between Australasian studies of spatial reasoning and those of the rest of the world. The research collaboration of Clements with Lean (e.g., Lean & Clements, 1981) was dedicated to the study of links between spatial reasoning and mathematical reasoning in the emerging school system of Papua New Guinea. Lean followed this with further publications about spatial reasoning in school contexts (e.g., Lean, 1984) and he was a major influence on Owens, who also worked in Papua New Guinea and published on spatial reasoning topics (e.g., Owens, 1992).

Research focused on the link between space and geometry was taken up by Pegg and colleagues (e.g., Pegg, 1992; Pegg & Davey, 1991; Pegg & Woolley, 1994) as well as Mitchelmore and colleagues (e.g., Mitchelmore 1976, 1980a, 1980b, 1984; Mulligan, 1985, 1992), with evidence of reciprocal influence through citation or cross citation, for example, of Mitchelmore (1976) in Bishop (1980). The 1990s also witnessed the work on space and representations by Lowrie and others in Australasia, in particular on graphicacy and mathematical structures (e.g., Lowrie, 1994) and representation of space (Mitchelmore, 1980a; Owens & Clements, 1998; Owens & Outhred, 1997). The early Australasian work in the field was encapsulated in previous Research in Mathematics Education in Australasia [RiMEA] chapters by Owens, Mitchelmore, Outhred, and Pegg (1996) and Lowrie and Owens (2000), although these were not dedicated to spatial reasoning, instead embracing its conceptualisation within space, geometry, measurement and visualisation.

Spatial reasoning research was carried forward in Australasia through extensive collaborative research involving Mulligan, Mitchelmore, and colleagues, Lowrie, Diezman, Jorgensen, Logan, and colleagues, and Owens and colleagues, with many of these researchers each having more than 200 publications and presentations related to spatial reasoning grounded in the period prior to 2016 and extending into the present. Many of these Australasian collaborations have been linked, either by publication or funding, to international collaborations (see e.g., Mulligan, 2015 and the ZDM special edition on spatial reasoning in 2015), aligning with the beginnings of an emphasis on the redefinition of mathematics as a broad assemblage of concepts that includes a basis in spatial reasoning, rather than mathematics being narrowly defined in terms of number and arithmetic (Mulligan, 2015; Mulligan & Woolcott, 2015).

In Australasia, research on spatial reasoning in the classroom became increasingly focused on primary school contexts, with insights providing potential evidence that appropriate and validated spatial reasoning programs for young children underpin their mathematics performance (e.g., Lowrie, Harris, Logan, & Hegarty, 2019; Mulligan, 2016; Mulligan, English, Mitchelmore, & Crevensten, 2013; Mulligan, Mitchelmore, & Stephanou, 2015). These insights are supported by studies of the international SRSG, which has Australian members, based in the education of young children (e.g., Davis et al., 2015), and by increased collaboration between the Science, Technology, Education and Mathematics [STEM] Education Research Centre [SERC, University of Canberra] and founding members of both the Spatial Thinking Lab in the Center for Spatial Studies [CSS] at the University of California, Santa Barbara (e.g., Hegarty et al., 2013) and the Spatial Intelligence and Learning Center [SILC], now at Northwestern University (e.g., Uttal, Miller, et al., 2013). Additional support for the spatial underpinnings of classroom mathematics emanates from intervention programs, not nominally spatial, but based in, or having spatial reasoning components (see e.g., Hung, Hwang, Lee, & Su 2012; Owens, 2015b).

From the late 2000s, spatial reasoning studies also developed a connection across classrooms with the renewed focus on what is now called STEM, a connectivity theme not obviously relevant prior to 2000 (Lowrie, Logan, & Larkin, 2017). For example, the longitudinal STEM studies of Wai and colleagues (e.g., Wai, Lubinski, & Benbow, 2009; Wai, Lubinski, Benbow, & Steiger, 2010) had a distinctly spatial focus and flavour and, in 2012, the CSS published a portal to instructional resources on spatial concepts in STEM (Janelle et al., 2012). STEM was also a focus of studies from Uttal and colleagues linking spatial reasoning and STEM (Uttal & Cohen, 2012; Uttal, Miller, et al., 2013) as well as studies of spatial thinking as fundamental to STEM education (Taylor & Hutton, 2013). Several later publications began to elaborate how research on spatial reasoning could contribute to improvements in STEM education (e.g., Khine, 2016).

More recently, the increased interest in the impact of spatial reasoning on mathematics is reflected in the extended research of these now influential global collaborations, such as the SRSG (e.g., Bruce et al., 2017), SERC (e.g., Lowrie & Jorgensen, 2018; Lowrie, Logan, & Ramful, 2017; Lowrie, Harris, et al., 2019), the Spatial Thinking Lab (within the CSS, e.g., see Kuhn et al., 2016) and SILC (e.g., Hamdan & Gunderson, 2017).

5 Spatial Reasoning Research and Mathematics Development in Australasian Classrooms Since 2016

The recent literature on spatial reasoning and mathematics education identified large-scale projects and smaller studies being undertaken on the topic, with some studies continuing beyond 2019. The findings from these projects and studies have been collated under themes to identify the impact of spatial reasoning on mathematics curriculum and teaching and learning. The main themes are discussed in turn.

5.1 Spatial Reasoning Intervention Programs

With the heightened understanding of the nature and relevance of spatial reasoning, current research in primary classrooms tends to focus on implementation designs, especially in an era where STEM is gaining more recognition. (See also Chap. 3, this Volume). This research and implementation interest is reflected in Australian government recognition of the importance of spatial reasoning in a primary classroom context, as seen in two 2017–2020 Australian Research Council Discovery Projects [ARCDP] investigating the influence of spatial reasoning on school mathematics, awarded to Mulligan, Woolcott, Mitchelmore, and Davis; and to Lowrie and Jorgensen. Both projects have an intervention focus as well as a planned program of curriculum linkages that attempt to show where spatial reasoning programs may be most effective.

The ARCDP project led by Mulligan, involving two other members of the SRSG, Woolcott and Davis, builds on their research in spatial reasoning contexts as well as the collaboration of Mulligan with Mitchelmore, established in the 1980s. The project is designed to generate an “innovative knowledge framework based on spatial reasoning that identifies new pathways for mathematics learning, pedagogy and curriculum" (Mulligan et al., 2018, p. 77). The project involves the design, implementation and evaluation of a Spatial Reasoning Mathematics Program [SRMP] in Years 3 to 5, extending the reach of the earlier government supported Pattern and Structure Mathematics Project, a suite of related studies with 4 to 8-year olds. The earlier project developed an interview-based assessment, the Pattern and Structure Assessment [PASA], and a pedagogical program, the Pattern and Structure Mathematics Awareness Program [PASMAP] (Mulligan et al., 2013, 2015). The overall aim of the recent project is to provide resources for the development of critical spatial skills for students, increased teacher capability and informed policy and curriculum across STEM education. Initial findings relating to students' spatial skills and teachers' pedagogical practices are yet to be published. Mulligan's ARCDP project aims to develop a Spatial Reasoning Mathematics Knowledge Framework aligned with the *Australian Curriculum: Mathematics* (Mulligan et al., 2018) and may possibly motivate a curriculum rethink.

The ARCDP project led by Lowrie is focused on the contribution of spatial reasoning to "specific differences in achievement based on variables, such as gender and socio-economic background, in terms of mathematics performance" (Lowrie & Jorgensen, 2018, p. 65). The project has a focus on improving visuo-spatial skills and competencies in students who may be excluded from school mathematics because of social disadvantage (e.g., low socio-economic status, indigenous or female) given that acquisition of such skills and competencies is a strong predictor of mathematics competence (Lubinski, 2010). The project is investigating the role and nature of spatial reasoning in mathematical development and the long-term effect of a spatial reasoning intervention program, and builds on spatial reasoning interventions and equity projects carried out by Lowrie, Jorgensen and colleagues (e.g., Jorgensen & Lowrie, 2013; Logan, Lowrie, & Diezmann, 2014; Lowrie 2012; Lowrie, Logan, & Ramful, 2017; Lowrie, Harris, et al., 2019). The project has a specific focus on Australian Indigenous children (e.g., Lowrie & Jorgensen, 2018; Jorgensen & Lowrie, 2018; Jorgensen, Grootenboer, Niesche, & Lerman, 2010), who may have stronger spatial reasoning than non-Indigenous children (Kearins, 1986; Watson & Chambers 1989). As well, the project has a focus on spatial reasoning and gender, which has long been a focus of research and more recently in the Australasian context (Logan & Lowrie, 2017; Lowrie, Logan, & Ramful, 2017; Winarti & Patahuddin, 2019).

5.2 *Embodiment and Early STEM Australia [ELSA]*

The principles of embodiment are also being explored in projects with a spatial reasoning focus, most notably the Early Learning STEM Australia [ELSA] project, a design research project commissioned by the Australian Government and headed by Lowrie [SERC]. ELSA is designed to give preschool children an awareness of STEM Practices (Lowrie, Leonard, & Fitzgerald, 2018; Lowrie & Logan, 2019), and has a decidedly spatial focus, with embodied learning based in structured play and lived experiences: "ELSA is building on children's inquisitive nature, enhancing play with learning opportunities and giving educators the tools to capitalise on these learning opportunities" (https://www.canberra.edu.au/about-uc/media/newsroom/2018/march/uc-led-stem-pilot-is-childs-play). Like other spatially-based projects in Australasia, ELSA has a strong basis in evidence from research implementation that outlines how spatial reasoning can influence primary school mathematics performance and achievement (e.g., Lowrie, Logan, & Ramful, 2017; Mulligan et al., 2018; Owens, 2015b).

In this way, spatial reasoning is developing in a context of classroom instruction while engaging multiple methodologies in both research and practice, including evidence of the positive effect of structured play and embodied learning (e.g., Cohrssen, de Quadros-Wander, Page, & Klarin, 2017; Highfield & Mulligan, 2008; Highfield, Mulligan & Hedberg, 2008; Lowrie, Diezman, & Logan, 2012), as well as the instructional design paradigms and models of practices developed in related Australasian projects (Lowrie et al., 2018b; Mulligan et al., 2013, 2015; Woolcott, Chamberlain, & Mulligan, 2015).

5.3 Spatial Reasoning and Geometry

The link between spatial reasoning and geometry has been heavily researched. Here, however, the focus has shifted from how geometry is spatial, to how does one's spatial reasoning influence his/her geometric understanding. In their overview of the current research into geometry, Sinclair et al. (2016) identified that spatial reasoning is having an increasing impact on the field. They advocate for future research to consider how aspects of geometric reasoning, such as spatial reasoning, might help build mathematics understanding more generally:

> A valuable focus of future research might be to investigate how geometric ways of thinking, including visuospatial reasoning and diagramming, may serve not only to improve geometric understanding, but also mathematical understanding more generally, and may even broaden the range of learners who might become interested in, and excel at, mathematics. (p. 285)

This link has been reinforced in research undertaken by Livy, Downton, Reinhold, and Wöller (2018), Ma'rifatin, Amin, and Siswono (2018), Patahuddin, Logan, and Ramful (2018), Wright (2016), and Wright and Smith (2017), using case studies and small scale interviews to consider the spatial reasoning required to work with, and mentally manipulate 2D shapes and 3D objects. The qualitative nature of these studies provided insights into the kinds of mental manipulations and problem-solving strategies students utilised when working with geometric shapes and objects.

Working with a year 4 student, Livy et al. (2018) identified the problematic nature of the prototypical shape and object when working spatially. The idea of a rectangular prism that did not have a square as its base shape was foreign to this student. His spatial reasoning wasn't supported with a range of visual examples, illustrating a disconnect between the 2D and 3D representations of prisms. The *Australian Curriculum: Mathematics* however does not specifically connect 2D and 3D until Year 5 and the New Zealand Mathematics Curriculum does not specifically connect 2D and 3D. Despite this lack within curriculum, the connection between 2D and 3D representations were reported in Ma'rifatin et al. (2018), Wright (2016) and Wright and Smith (2017), identifying that higher performing students use a combination of both mental and physical spatial manoeuvres and analytic problem solving to complete net folding tasks, where there is a transition from 2D to 3D. In these studies, students who were able to visually fold nets into the respective 3D object would assign a fixed face on the net as the base of the object, an analytic approach to problem solving. This approach allowed for mental spatial reasoning in manoeuvring the other faces into position, seeing where they would attach to the base; the more central that fixed face was to the centre of the net, the easier the spatial manoeuvres were. There was also a relationship between the use of gesture and the mental manipulations as students attempted to demonstrate, using their hands, how the nets would fold. The most successful and fluent students were able to rely less on spatial manoeuvres and more on analytic thinking as they were able to see patterns across and between the nets as they visualised the whole 3D object and connected the 2D parts with that whole. Those students who struggled with this task were unable to see the relationship between the 2D net and the 3D object.

Patahuddin et al. (2018) worked with slightly older children to investigate how they solved tasks related to finding area of composite 2D shapes when no numerical features were provided. Tasks required students to describe how they would decompose the shape in order to find the area and interviews were developed to elicit the students' spatial thinking as opposed to their understanding of the area formula. Three key ideas in spatial reasoning were reported: figure-ground perception; local and global processing; and, gesturing. The ability to foreground and background various shapes with a larger image was critical to identifying the area of a shape within that image. Both local and global processing was evident across the student cohort, where they either worked with smaller parts of image separately and collectively or they extended the task beyond the given image to enclose the graphic in order to find the area. Two main types of gestures were evident in the student thinking, the first being rotating the images and shapes to identify and dis-embed known shapes. The second approach was to gather perceptual information from the drawn objects using their fingers. Tracing the shapes helped to make them explicit and allowed the students to focus their attention on the relevant information. This study is important for the primary years since many of these spatial skills could be developed throughout the primary curriculum in conjunction with the formulae for various geometric rules.

5.4 Spatial Reasoning in Mathematics Assessment

Another theme to emerge was associated with spatial reasoning in mathematics assessment, specifically large-scale assessment. A number of studies have considered the association of primary students' spatial reasoning and their mathematics assessment performance, predominantly utilising assessment items from the Australian national numeracy assessment, National Assessment Plan—Literacy and Numeracy [NAPLAN], or similar. Ramful, Lowrie, and Logan (2017) designed and validated a spatial reasoning instrument to measure middle school students' spatial reasoning ability. The instrument was inspired by and measured against the psychological instruments commonly use to measure spatial visualisation, mental rotation and spatial orientation. However, it was also designed to reflect current national assessment items for the middle years students so as to provide a familiar context for children of this age. This instrument has been utilised throughout a number of other studies both in Australia and internationally.

Working with NAPLAN-like items, Lowrie and Logan (2017) identified that students' spatial reasoning is influential in performance across mathematics assessment tasks when presented in both digital and pencil-and-paper format. The importance of spatial reasoning in assessment strategy has also been investigated (Forndran, Lowrie, & Harris, 2019) Findings indicated that students with lower levels of spatial reasoning performed better in the digital format, while the high spatial ability cohort performed best in the pencil-and-paper format. Also utilising the national assessment items, Logan and Lowrie (2017) investigated gender differences on spatial NAPLAN items across grade levels. Males outperformed females on some spatial tasks across the grade levels but there were also differences in the processing of those spatial items, indicating that females could benefit from some specific teaching related to spatial reasoning.

Linking with the gender findings is a thematic review by Reilly, Neumann, and Andrews (2017), who report that males tend to have higher levels of spatial ability and higher levels of confidence with spatial tasks. The review indicates that this gap surfaces in early childhood and precedes gender differences in mathematics and science. They advocate that recent research has identified that concentrated spatial teaching and various intervention programs have been influential in bridging this gender gap.

These findings could have implications for national assessment in Australasia as the move into digital assessment becomes commonplace (e.g., see Lowrie & Logan, 2019). It is possible that the digital assessment platform may be overly influential for those with higher or lower levels of spatial reasoning or for females or males. However, the somewhat contradictory findings from the above studies indicate that further research is still needed in this area.

5.5 *Spatial Reasoning in Other Areas of Mathematics and Beyond*

Several other projects within Australasia are engaging spatial reasoning in the context of primary teacher education, but in a number of differing ways. At the University of the Sunshine Coast, Marshman and colleagues (e.g., Marshman, Woolcott, & Dole,

2017) are giving primary initial teacher educators and pre-service teachers experiences in spatially-based problem solving by way of the multiuser virtual environment, CAVE2^{TM}, a 320-degree, cylindrical 3D virtual environment. Such environments offer collaborative, immersive experiences that would otherwise be unavailable as real-world experiences. The CAVE experience is being used to examine the capacity of teachers to design and solve rich spatial reasoning tasks and activities. Similarly, Leonard and colleagues at the University of South Australia are examining primary school student-teacher interactions with a number of spatially based virtual environments, for example, examining relationships between planets and their orbital paths, as well as digital interactions (Fowler, O'Keefe, Cutting, & Leonard, 2019).

Research is also exploring spatial reasoning within everyday contexts. A number of studies have looked at spatial reasoning within other projects, for example, through the use of mathematical modelling in the major STEM-related projects *It's part of my life: Engaging university and community to enhance science and mathematics education* (Axelsen, Galligan, & Woolcott, 2017; Galligan et al., 2019; Woolcott et al., 2017), the examination of spatial relationships in *Regional Universities Network (RUN) Maths and Science Digital Classroom: A connected model for all of Australia* (Woolcott, 2018) and *Inspiring Science & Mathematics Education* (Blom, Pentland, & Woolcott, 2018). While such studies were not nominally spatial, it is clear from the preliminary studies that having spatiality as an intrinsic part of classroom activities can motivate both classroom students and teachers within mathematics lessons. Having a spatial component in classroom learning activities can also help teachers establish a repertoire of problem-solving approaches for investigating the surrounding environment, approaches that lend themselves to such mental activities as mathematical modelling (Galligan et al., 2019). Several conference papers at MERGA 42 indicate a renewed focus on spatial components of mathematics learning activities (Cutting, 2019; Gronow, Cavanagh, & Mulligan, 2019; Ho & McMaster, 2019; Murphy, 2019; Seah & Horne, 2019).

Another emerging theme is the, albeit slow, coming together of spatial reasoning studies conducted within mathematics education and those studies that sit in other domains (Bruce et al., 2017). Mathematics education researchers are starting to examine a large body of educational research in geography, design and technology, engineering and geosciences, and other domains that utilise spatial reasoning. Examples of this inter-disciplinary research can be found in geospatial modelling (e.g., Sethuramasamyraja, Sachidhanantham, & Wample, 2010), digital design and 3D printing (e.g., Verner & Merksamer, 2015), and the teaching of geography (e.g., Lee, Jo, Xuan, & Zhou, 2018; National Research Council, 2006; Shin, Milson, & Smith, 2016).

Within economies with a large agricultural component, such as Australia, spatially-based technology in the form of the 'SMARTFarm' is merging technological innovation with farming practices (Krintz et al., 2016) and some educational institutions are working with schools in bringing this innovation into mathematics classrooms. An example of this is the Growing Regional and Agricultural Students in Science (GRASS) project at the University of New England (UNE). The project engages with teachers and students both in regional and rural schools and on the

UNE campus with a variety of technology related spatial inquiry-based activities set in agriculture science contexts. One focus of the program is teacher professional development, with many hundreds of teachers attending on-campus professional development programs. A number of learning modules for use in schools have been developed and are available online (https://smartfarmhub.education/). Students, for example, can use GPS technology to track spatial location or to map agricultural land.

5.6 Australasian Influences: Spatial Reasoning Research in the Asian Region

There have also been reciprocal research relationships developing between spatial reasoning researchers in Australasia and researchers in neighbouring countries. For example, Lowrie and colleagues have developed strong ties with researchers in Indonesia (Patahuddin et al., 2018; Patahuddin, Suwarsono, & Johar, 2018), Singapore (e.g., Ho & Lowrie, 2014; Logan, Lowrie, & Ramful, 2017) and the Philippines (Ogena, Ubiña-Balagtas, & Diaz, 2018) through international collaborations. There are also recent studies in spatial reasoning and mathematics education that have cited Australasian or related studies, for example from Indonesia (Khairunnisak, Elizar, Johar, & Utami, 2018; Kurniadi, Putri, Ilma, Hartono, & Abels, 2013; Ma'rifatin et al., 2018; Revina, Zulkardi, Darmawijoyo, & Galen, 2014; Septia, Prahmana, Pebrianto, & Wahyu, 2018; Shanty & Wijaya, 2012) and Malaysia (Hamid & Idris, 2017; Saha, Ayub, & Tarmizi, 2010; Wahab, Abdullah, Mokhtar, Atan, & Abu, 2017). Although not a prime area of research, spatial reasoning appears as a topic of interest in several Asian countries. Research interests appear to be predominantly guided by the relationship between spatial ability and performance in geometry (Ma'rifatin et al., 2018; Patahuddin et al., 2018; Winarti, 2018).

6 Areas for Further Research

There was little evidence of research relating to spatial reasoning among New Zealand researchers. Whilst this was somewhat surprising, it may be an area for further research for the New Zealand mathematics education community. Given their strong cultural focus and integration (e.g., see Cunningham, 2019; Sharma, 2019), there may be opportunities to understand how spatial reasoning impacts on mathematics education within this environment.

A significant advance is in recognising that both classroom teachers and initial teacher educators may need to become involved in the dynamic teaching environments in which spatial reasoning is being presented so that they engage with interventions and training programs within classrooms. In both the Lowrie and Mulligan

ARCDPs, classroom teachers have an active role where they have both feed forward and feedback interactions with researchers. The effectiveness of this type of interaction in improving learning is supported by Australasian studies (e.g., Galligan et al., 2019; Jorgensen et al., 2010; Jorgensen & Lowrie, 2017; Lowrie, 2014; Mulligan et al., 2018; Owens, 2015a, 2015b). Given the importance of teacher disposition on student learning (e.g., Yeigh et al., 2016), further research needs to be conducted on classroom teachers' levels of spatial reasoning and confidence to teach spatial concepts. Similarly, by building on the work being undertaken by Marshman et al. (2017), further research could be conducted with pre-service teacher education to better equip our new teachers with both the mathematical and spatial concepts required to teach primary students.

7 Concluding Remarks

Spatial reasoning has re-emerged as a focus of mathematics teaching in primary schools in Australia and the neighbouring region, reflecting a worldwide trend influenced by reciprocal global research interactions and enhanced by technological improvements, especially with the advent of the Internet. There is now a re-emerging view that the teaching of spatial reasoning can not only enhance learning in mathematics and across the curriculum, but also in STEM and seemingly non-related learning domains such as reading. This spatial basis for learning is directly related to how children experience the world in their pre-school years, and the future focus should be on optimising spatial reasoning skills through accommodation of what students know before they come to school. A future challenge is upskilling teachers and pre-service teachers, based on what is becoming a solid research base, so that they are prepared to lay the spatial foundations necessary for their students' success in mathematics and related STEM fields.

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Chapter 11
Research into Teaching and Learning of Tertiary Mathematics and Statistics

Linda Galligan, Mary Coupland, Peter K. Dunn, Paul Hernandez Martinez, and Greg Oates

Abstract Reviewed in this chapter is the growing depth and variety of research being undertaken in the tertiary mathematics field. In particular, the scope of the research and issues being examined have been highlighted in sections:

- Tertiary mathematics education. The focus is strategies for teaching and learning in mathematics education and the professional learning of the lecturers and tutors.
- Mathematical content. Papers reviewed are those that focus on certain content areas in tertiary mathematics, both practical and theoretical.
- Tertiary statistics education. The authors reviewed the small but growing area of research that examines the teaching and learning of statistics, content areas of statistics education and some of the innovations being used in this area of teaching and learning.
- Transitions and support. The focus is school to university transition and the support structures being implemented to provide academic support for undergraduate students of mathematics.
- Service teaching. The authors consider papers that link mathematics into service areas such as engineering and the health sciences.

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1 Introduction

This chapter reviews the growing field of research into learning and teaching of mathematics and statistics at the tertiary level. A 2012–2015 review of the literature (Coupland, Dunn, Galligan, Oates, & Trenholm, 2016) suggested that research in this field would grow as collaborations between mathematicians, statisticians and mathematics educators increased. Indeed, since that review, the field has grown and evolved. For example, King and Cattlin (2017) described the development of a national network for mathematicians teaching undergraduate mathematics in Australian universities, and the subsequent emergence of a community of practice. Their case study of the First Year in Mathematics network suggested that supportive connections enabled the development of a strong sense of identity and recognition of common challenges across institutional boundaries. The breadth of studies represented in this review show that other undergraduate mathematics forums and publishing platforms, such as those as considered in the 2012–2015 RiMEA, may be facilitating the growth of a similar community of practice on a wider international scale.

In this literature review the authors present an overview of developments in this area of research from 2016 to 2019, since the last review of the field. To capture the depth and variety of research being undertaken, this review examines research related to five main areas of tertiary mathematics: mathematics education; mathematical content; statistics education; transitions and support; and service teaching. The final section then provides suggestions for future research. Space limitations necessitate favouring journal articles over refereed conference presentations and sometimes only one of a set of related papers by the same author or group was included.

2 Tertiary Mathematics Education

In this section, research that explores developments in the teaching and learning of mathematics is reviewed. Areas of interest to researchers have included pedagogical strategies and technologies being used to improve educational outcomes; innovations in assessment; and the use of theoretical concepts and frameworks to research students' or lecturers' beliefs and practices in an educational context.

2.1 Pedagogical Strategies

Many researchers investigated how the delivery of a program affects student engagement and outcomes. In this space, a growing number of researchers are interested in flipped classroom delivery. Kensington-Miller, Novak, and Evans (2016) reported on a case study of lecturers designing and introducing flipped lectures where student argument and debate were prominent. A subsequent paper (Novak, Kensington-Miller, & Evans, 2017) reported the results of introducing this flipped lecture model in a large service course: greater student engagement and an increased understanding of the material; however, some students remained unconvinced by this pedagogical approach.

Another research area has been the use of online and blended approaches. Trenholm, Peschke, and Chinnappan (2019) reviewed the literature on fully online (FO) mathematics instruction. They found that students in FO mathematics courses tend to be more dissatisfied compared to students in face-to-face (F2F) instruction and retention rates were lower. They also found that some studies suggested that students performed more poorly in FO mathematics courses compared to those in F2F instruction, but others found that student outcomes in FO instruction were neither worse nor better. Loch, Borland, and Sukhorukova (2019) discussed a mix of online and F2F instruction (‘blended learning’) in teaching a second-year mathematics subject. They reported that the new delivery method resulted in fewer students failing and more students gaining top grades.

Related to blended learning, researchers also studied the use of recorded video lecture (RVL) in mathematics instruction. Trenholm et al. (2019) found that a reduction in live lecture attendance coupled with a dependence on RVL was associated with an increase in surface approaches to student learning. They concluded that regular use of RVL may be depressing the quality of student learning. Tisdell and Loch (2017) studied the perceived level of usefulness of closed captions in online mathematics videos. They found that students broadly agreed that captions were useful in allowing flexibility on ‘when’ and ‘where’ videos were watched, understanding speakers’ accents and clarifying explanations that were difficult to hear in the recording. Another study (Yang, Fu, Hwang, & Yang, 2017) trialled an online, two-tier diagnostic test with feedback on a first-year calculus course. The authors found that this approach improved students’ learning performance and also their confidence in learning calculus within a F2F traditional lecture. These studies suggest that blended learning approaches might have great potential but there is scope to research such approaches in more depth.

Another topic of interest to researchers are the types of resources and strategies used in tertiary mathematics education. Woodcock (2017) argued for avoiding predictability in question structures by asking questions ‘backwards’ to how they might traditionally be asked. Traditional questions typically pose a well-defined problem for which a single correct answer exists; a ‘backwards’ question may have multiple correct answers. When a question is asked ‘backwards’ a student thus has to demonstrate a much broader understanding of the topic, often having to display comprehension of multiple mathematical ideas to produce a correct solution. Klymchuk

(2017) researched the regular use of mathematics puzzles in the teaching and learning of second-year engineering mathematics, finding that solving puzzles enhanced most students' problem-solving and generic thinking skills. Tisdell (2019a) used an online music video to teach some of characteristics of the number *e*, which many students found 'fun'. Maciejewski and Merchant (2016) analysed the nature of the tasks given to students in the four years of an undergraduate mathematics degree, the students' approach to learning, and how these variables related to their grades. The authors concluded that a superficial learning approach resulted in poorer grades in upper years. Oates, Paterson, Reilly, and Woods (2016) explored different student collaborative approaches used in undergraduate mathematics courses, arguing that working collaboratively enhances students' understanding of the material and the enjoyment they experienced in the process. Shepstone (2017) found a high correlation between time spent working on tutorial problems and student improvement in mathematics. He argued that students need to undertake directed practice with relevant feedback in the mathematics that they need to know. These studies explore a variety of resources and strategies that carry the potential for engagement, deep learning and subsequent student attainment; however further research is needed to better understand the potential outcomes of these resources and strategies, as well as the contexts in which they might or might not work.

Researchers have also been interested in using digital technology resources to investigate mathematical learning (Chap. 13, this Volume gives a comprehensive review on the use of technology in university mathematics). Geogebra is software that has been the focus of many research studies. Researchers have used Geogebra to: teach the motion of falling bodies by simulating the experiments of Galileo (Ponce Campuzano, Matthews, & Adams, 2018; enhance students' insights into line integrals of vector fields (Ponce Campuzano et al., 2019); and teach basic algebra and calculus (Getenet, 2018). These studies found that using Geogebra enabled students to experiment and to visualise complex mathematical concepts. However, Getenet (2018) also found that for the software to be effective, learners should have prior knowledge of how it works, and teachers should understand their students' prior knowledge of the software. This highlights the need for further research into the use of similar software in developing deeper mathematical understanding.

2.2 Assessment

In contrast to the last review, limited research had assessment as its central focus; however, some research analysed assessment as part of a wider pedagogical strategy or an element of a blended system. Such research focussed on the effectiveness of systems in discriminating between deep and superficial learning (Easdown, Roberts, & Corran, 2018); and computer-aided systems that generate individualised assessments that are automatically marked (Herbert, Demskoi, & Cullis, 2019). The dearth of research into tertiary mathematics-related assessment suggests this in an area where future research efforts could be focussed.

2.3 Professional Learning, Beliefs and Practices: Theoretical Perspectives

Many researchers used theoretical concepts or frameworks to explore students' or lecturers' beliefs and practices. Researchers that focussed on students used a variety of concepts to explain their learning and engagement with different aspects of their mathematical education. Maciejewski, Roberts, and Addis (2016) used the concept of episodic future thinking (simulations of imagined future events) to explore if students imagine a solution before engaging in solving a problem. They found that all of their participants engaged in episodic future thinking by: anticipating what actions would be needed to complete a mathematical task; imagining solving a mathematical problem in a social situation; adapting past experiences of working with similar mathematics; experiencing emotions associated with solving the mathematical task; and/or anticipating failure to make progress. Kontorovich (2019b) used the concept of epistemological status (the degree to which a solution fulfills the solver's intellectual and psychological needs in a particular problem situation) to hypothesise that students' decisions to check their solution to a problem is shaped by its epistemological status. Tisdell (2019b) explored some mathematical examples using Schoenfeld's models of problem-solving, arguing for the importance of the individuality of the learner, their unique perspectives, and their personal knowledge in the process of solving mathematical problems.

Oates and Evans (2017) used Schoenfeld's ROG framework (resources, orientations and goals) to discuss a model for professional development in which collaboration between lecturers and mathematics education researchers was a key characteristic. They concluded good evidence exists that the use of this model is effective, positively received by lecturers and viable as an ongoing basis for professional development in university mathematics teaching. Surith (2017) explored lecturers' knowledge that is withheld during their lecturing practices. Reasons given by lecturers to withhold knowledge included: curricular reasons such as 'content not in the syllabus'; pedagogical reasons such as 'creating a self-learning opportunity for students'; and lecturers' reasons such as gaps in their knowledge, or having the flexibility in using mathematical knowledge according to the context of the lecture. The research described in this section demonstrates that theory-driven analyses continue to advance our understanding of teaching and learning in mathematics education.

3 Mathematical Content

The focus of this section is on research that has explored specific content areas in tertiary mathematics. Strong interest in mathematical content at the tertiary level has continued, mostly from a practical teaching perspective, but with a growing development of theoretical and conceptual studies.

3.1 Practical Approaches to Content Delivery

Many papers described innovative and alternative pedagogical approaches across a variety of content areas, for example complex functions, differential equations, integration, linear algebra, limits and differentiation, proof of trigonometric identities, and roots of real numbers. Tisdell (various) published a series of eleven papers over this period (not all discussed here). Nine of these have examined pedagogical approaches to the teaching of linear, first- and second-order differential equations, with reference to specific examples of particular sets of equations (e.g., Bernoulli equations, Tisdell, 2017a; harmonic motion, Tisdell, 2018) and methods of solving (e.g., mnemonic instruction and the SHIELDS acronym, Tisdell, 2019c; Rivera-Figueroa and Rivera-Rebolledo, Tisdell, 2019d). These studies establish links between different existing methods of solving differential equations, argue for the practical importance of history and theory when teaching and learning iterative methods, and showcase how students and teachers can use technology (for example Maple code) to experiment, explore and learn. Tisdell (2017b) has also contributed a paper challenging the approaches traditionally adopted to the teaching of double and triple integrals. He critically reflects on predominant methods of rearranging the order of integration and the privileging of graphical methods, proposing an alternate pedagogical approach to solve some of the classical problems involving double and triple integrals.

Hannah, Stewart, and Thomas (2016) built on their previous studies in linear algebra with a teaching intervention based on Tall's three worlds (embodied, symbolic and formal) of mathematical thinking. In this case study, they adopted multiple approaches using a combination of formal definitions, traditional by-hand calculations, and technology to help students build connections between Tall's three worlds. Kontorovich (2019a) examined the procedural knowledge students can develop in a typical first year linear algebra course. He found an inverse relationship between the efficiency of the methods that students used and the number of students who used them. The implication is that university teachers need to draw explicit attention to the common difficulties that students experience in linear algebra.

Three studies described ways of scaffolding and developing student understanding of specific content. Ponce Campuzano (2019) investigated the use of phase portraits to visualise and investigate isolated singular points of complex functions. He suggested that by analysing the representations of singularities and the behaviour of the functions near their singularities, students can make conjectures and test them mathematically, which in turn can help to create significant connections between visual representations, algebraic calculations, and abstract mathematical concepts. Ahmad (2018) observed that many students, even at the graduate level, know little about the existence and multiplicity of real roots of real numbers (for example the fifth root of –2) and even those who may know the answers are unable to give a logical explanation for the validity of their answers. Ollerton (2018) noted that proofs of trigonometric identities in early undergraduate courses are typically approached algebraically using complex exponential forms of the trigonometric functions and the Euler formula. He proposes wider use of geometric approaches with several examples from lesser known sources.

The studies collectively described in this section contribute to the growth of our undergraduate community of practice, adding to our shared understanding of mathematical content and pedagogy. They extend the theoretical underpinnings of our practice, are grounded in historical and contemporary practice and demonstrate the authors' belief that teaching multiple ways to solve the same problem has both academic and social value. They provide a strong platform to provoke discussions of alternative approaches and effective evidence with examples for colleagues wishing to pursue similar practical agendas.

3.2 Theoretical and Conceptual Perspectives

Threshold concepts have been identified as an emerging field of study in previous reviews, principally regarding financial mathematics. Oates, Reaburn, Brideson, and Dharmasada (2018) and Reaburn, Oates, Dharmasada, and Brideson (2018) extended such investigations to focus on asymptotes, limits and differentiation. Their studies revealed considerable student confusion with these concepts in their first-year course, despite having met them at school. Reaburn et al. (2018) recommended that lecturers adopt different approaches to teaching these concepts, giving students difficult examples that may cause conflicts in their thinking. Both studies suggested greater communication is needed between teachers at both levels about how such potential threshold concepts are developed.

Several studies focussed on student misconceptions and confusion in the areas of complex numbers, reciprocal and inverse functions, sub-spaces, and graph theory. Moala, Yoon, and Kontorovich, (2018) posited an emerging prototype with a contextualised graph theory problem. They found that students frequently maintained a feature of a concept's example and engaged in a 'patching process' to preserve inappropriate features in subsequent generated examples. Two studies (Chin & Jiew, 2018; Kontorovich, 2018b) focussed on complex numbers, both of which described carefully chosen examples where student answers revealed interesting misconceptions or inconsistencies. Kontorovich (2017) further revealed student confusion with the superscript (-1), which can be interpreted as a reciprocal or an inverse function. He found students struggled with choosing the contextually correct interpretation, and some students used both interpretations in their solutions. For sub-spaces, Kontorovich (2018a) identified five unconventional tacit models that governed students' reasoning and found that, similar to Chin and Jiew's (2018) findings with complex numbers, students' prior experiences in identifying concept examples affected their explanations for the emergence of the sub-space conception. While sub-spaces and graph theory are unique to undergraduate mathematics, like threshold concepts considered earlier, the examples and findings from the complex number and inverse and reciprocal functions studies should help inform the future teaching of these concepts and should provide a useful focus for conversations between upper secondary and tertiary teachers with respect to consistent pedagogical approaches.

The final group of studies in this section consider theoretical perspectives. Two studies by Radmehr and Drake (2017a, 2017b) examined students' metacognitive thinking in integral calculus. The first study (2017a) explored students' understanding of the relationships between definite integrals and areas under curve(s), with specific attention to students' understanding of the Fundamental Theorem of Calculus (FTC). The findings of this first study suggested students' metacognitive experiences and skills need further development. The second study (2017b) provided a path for such metacognitive development. It unpacked the knowledge dimension for the Revised Bloom's Taxonomy (RBT), identifying, defining and exemplifying eleven sub-types of the knowledge dimension to support the teaching and learning of integral calculus and the development of students' metacognition and metacognitive knowledge.

Pinto and Scheiner (2016) and Scheiner (2018) gave a theoretical consideration of mathematics cognition, using the evolving framework of structural abstraction as a theoretical lens to investigate how mathematics major university students understand the limit concept of a sequence. Their analysis revealed unsettled issues about structural abstraction and provided new directions for advancing our understanding of this kind of abstraction. Scheiner (2018) further contrasted the assumption that mathematics cognition involves the attempt to recognise a previously unnoticed meaning of a concept with the suggestion that mathematics cognition may be reconsidered as a process of ascribing meaning to the objects of one's thinking. He concluded that mathematics cognition is as much concerned with creating a meaning of a concept as it is with comprehending it.

The studies described in this section further add to Sect. 2.3 on theoretical perspectives of lecturers' and students' beliefs and practices. As tertiary mathematics education continues to strive for relevance in the 21st century, such research is important in helping us explore how theory-driven analyses may take tertiary mathematics research in new directions.

4 Tertiary Statistics Education

Research in statistical education has traditionally focussed on introductory classes, however the topics have broadened recently. In this section the small but growing area of research into tertiary statistics education is reviewed. Areas of interest to researchers have included the teaching and learning of statistics, content areas of statistics education and innovations being used in tertiary statistics education.

4.1 Towards Probability

In recent years, statistical education has emphasised connecting statistics and probability (Pfannkuch, Ben-Zvi, & Budgett (2018); Pfannkuch, Budgett, Fewster, Fitch, Pattenwise, Wild et al., 2016). Pfannkuch et al. (2016) examined this shift and its implications for tertiary statistics education in the 21st century. The authors found that many introductory probability courses reside mainly in the mathematical world and focus on developing probability knowledge from a theoretical modelling perspective. The authors thus suggested that better development of probabilistic thinking may occur if courses could balance theoretical mathematical models, constructing empirical models, and exploring the behaviour of models. Pfannkuch et al. (2018) explored this paradigm shift further, discussing why the shift has occurred and the frameworks being developed to understand student reasoning with statistical models and modelling. Budgett and Pfannkuch (2018) then examined the case of linking the Poisson and exponential distributions using prototypical software, using real and simulated data. According to these researchers, the activities appeared to help students appreciate the difficult concepts of randomness and distribution. The continuing shift towards connecting statistics and probability in tertiary mathematics education, combined with increased emphasis on the need for deep statistical literacy in the 21st century, will undoubtedly provide researchers a space for ongoing research into the consequences of this shift and related educational developments.

4.2 The Use of Technology

Evaluating the use of software and technology in statistical education is a popular research topic. Wild (2018) explored the advantages and disadvantages of point-and-click and programming-based statistical software and presented the author's iNZight software that draws on the advantages of both (www.stat.auckland.ac.nz/~wild/iNZight/). Bulmer and Doyle (2018) discussed using *The Island* (a synthetic learning environment comprising a virtual human population, used for simulating data collection; http://islands.smp.uq.edu.au/login.php), showing how *The Island*, can now produce qualitative data.

Some researchers evaluated curriculum and course redesign after incorporating technology. Dunn, Donnison, Cole, and Bulmer (2017) revised an epidemiology and biostatistics course to include *The Island* to increase authenticity in assessment and teaching; they evaluated how student behaviour and understanding of course content changed as a consequence. The authors reported favourable responses from the students to the curriculum changes and *The Island*, with objective data showing high levels of engagement. Tirlea, Baglin, Huynh, and Elphinstone (2016) evaluated the use of *The Island* using an experimental study, comparing an interactive classroom exercise for taking a sample from a population to an online simulation method using *The Island* for sampling. The authors found that students in both groups performed similarly in terms of knowledge acquisition. In another evaluation of technology Tirlea, Muir, Huynh, and Elphinstone (2018) studied a Student Response System (Socrative: https://socrative.com) as an in-class interaction tool and concluded it has the potential to increase engagement in statistics classrooms. Khan and Watson (2018) compared two semesters of traditional teaching with two semesters using the flipped classroom approach, concluding that students were more engaged and had better understanding of concepts and improved results when the flipped model was used. Research in this area highlights the potential for technology to support statistics education; however, as sample sizes are often small in statistics education research, ongoing studies are needed to test and support the findings made in these current studies.

4.3 Understanding Statistics

Statistics education research continues to focus on the deep understanding of concepts. These studies can be grouped into a number of areas including language, statistical literacy, specific core concepts and directed support to improve attitude and belief. Statistical literacy and language were explored by Dunn, Carey, Richardson, and McDonald (2016), who discussed numerous conflicts, difficulties and contradictions between the language of general English, mathematical English, discipline-specific English and statistical English. The authors discussed ways to help students navigate this language confusion in statistics without offering any evidenced advice. Carey and Dunn (2018) found that using teaching techniques for teaching foreign languages (such as Jigsaw and Think-Pair-Share methods) increased student's engagement in learning statistical language, but the statistics tutors needed training to use the techniques effectively in the classroom.

Utilisation the *Guidelines for Assessment and Instruction in Statistics Education* (Aliaga et al., 2005; GAISE College Report ASA Revision Committee, 2016) recommendations were evaluated in two studies. Paul and Cunnington (2017) revised a statistics course explicitly around the GAISE recommendations. Using the *Students' Attitudes Towards Statistics Models* tool and student feedback, they found slightly improved attitudes towards statistics in the GAISE-focussed course. Dunn, Carey, Farrar, Richardson, and McDonald (2017) evaluated 25 introductory statistics textbooks in light of the GAISE recommendations and found that the textbook authors were adept at promoting statistical literacy and using real data and using it well, but were less adept at emphasising concepts over procedures. They noted that glossaries were rarely included in introductory statistics textbook, despite the language challenges of statistics and the emphasis of GAISE on statistical literacy.

Reaburn (2017) studied the understanding of P-values by statistics *instructors* (lecturers and tutors) and found that many held misconceptions about P-values. Similarly, Reaburn, Holland, Oates, and Stojanovski (2019) studied students' understanding of confidence intervals and found many misunderstandings. The authors suggest that confidence intervals may be considered a threshold concept in statistics. Using the experiences of students in an introductory statistics course, Pfannkuch et al. (2018) examined how a graphical display of two-way tables of counts (an *eikosogram*) helped students to understand proportional reasoning, conditional and joint probabilities, and independence. A different view of understanding concepts was taken by Dunn, Marshman, and McDougall (2018) who evaluated five Wikipedia entries about introductory statistics topics. They found that Wikipedia performs poorly, with errors and poor diagrams particularly problematic. They therefore argued that Wikipedia should be actively discouraged as a learning resource.

In the undergraduate setting, Intepe, Shearman, and Rylands (2019) studied support for first-year mathematics and statistics students. The authors found (in general) that students of all mathematical backgrounds who attended support workshops achieved better results, though attendance overall was very poor. Carey, Grainger, and Christie (2018) discussed a data-literacy course for pre-service teachers (PSTs), finding that PSTs' beliefs in their skills generally improved after taking the course, though these skills were not objectively evaluated. In the postgraduate setting, Baglin, Hart, and Stow (2017) explored statistical knowledge of Higher Degree by Research (HDR) students and supervisors and found that HDR students and supervisors need more statistics training and access to statistical consultancy. Bhowmik, Meyer, and Phillips (2019) discussed the use of blended learning in teaching post-graduate students in statistics. Their success may suggest a model which might have a role in the statistical training of HDR students.

Morphett, Giagos, Gunn, and Reid (2018) evaluated two marking approaches (rubric; traditional marking) for written statistics assignments consisting of a series of questions and found that the methods had a similar inter-rater and intra-rater reliability. More broadly, Chiesi, Primi, Bilgin, de Carmen Fabrizio, Gozlu et al. (2016) found that the Approaches and Study Skills Inventory for Students instrument, which can be used to describe and study learning approaches to statistics, is a valid tool across different languages and educational contexts. In undertaking their assessment of the instrument, the researchers used large samples from five countries (Argentina, Italy, Australia, Turkey, Vietnam) confirming that the instrument remains a valid tool for statistical education researchers.

A common (but not universal) thread in the research into tertiary statistics education is that small sample sizes of students mean that making strong conclusions is difficult. While this observation about the small sample sizes is not a criticism of the studies reviewed in this section, it does acknowledge the ongoing difficulty in obtaining enough consenting students for research into tertiary statistics education.

5 Transitions and Support

Academic support for undergraduate students of mathematics is provided in different ways and research into this support often takes the form of describing innovations. The research reviewed in this section describes support and enabling innovations, both designed to assist students, either concurrent to studies or prior to studies. It also reviews the small number of studies that have examined programs designed to support students in their transitions from school to university and then from university to the workplace.

5.1 Support

In the area of academic support, researchers have explored the effectiveness of different programs aimed at improving the mathematics readiness of first year students. Building on from research reported in RIMEA in 2016, Rylands and Shearman (2018) have continued to examine ways to support beginning students. Reporting on an intervention in which all new engineering students were required to enrol in a preliminary mathematics subject, Rylands and Shearman found that the variables 'placement test mark' and 'number of support workshops attended' had a significant influence on the final mark students achieved for the compulsory subject. Fox (2019) examined a similar initiative, the Mathematics Benchmarking Quiz, finding it a useful a diagnostic test for identifying 'at risk' students who then benefited from enrolment in a support program. Hillock and Khan (2019) provided thirteen semesters of data to support their claim that a support learning tutorial (SLT) designed to assist students in the first tertiary level mathematics subject was effective in increasing the likelihood that students, especially those judged to be underprepared, would pass.

Study centres are a form of support that have been of interest to researchers. In a case study of mathematics study centres, Nicholas and Rylands (2017) made the case for continuing the type of F2F study support that these centres provide to students, describing the important role such support centres play in encouraging students in the *doing* of mathematics. This paper is a valuable starting point for those needing to make a case in their own institutions for the role of study centres in supporting student engagement in mathematics. Also describing the importance of drop-in centres, Edwards and Carroll (2018) analysed the visit records from first year students who used a drop-in centre. The authors found that compared to students who did not visit the centre, students who visited performed, on average, 8.5% points higher in summative assessments. Female students and older students were also more likely to seek support and more likely to seek support further in advance of assessments.

Offering intensive delivery of first year mathematics subjects over summer to improve retention is another type of support. Using eight years of student records, Easdown, Papadopoulos, and Zheng (2019) found that students who repeat certain fundamental subjects over summer were more likely to pass than students repeating in a regular term. The authors proposed that students' motivation, smaller class sizes, qualities of teaching staff and the shorter, more intensive mode were possible contributors to this result.

In a larger study that reviewed support interventions globally, Lake et al. (2017) undertook a systematic meta-analysis of interventions that enhance mathematics learning at university. The conclusions and recommendations do not appear to offer anything new to researchers in this field; however, from a ‘students at-risk’ perspective, the results reinforce the benefits of diagnostic testing to identify students who need support, rather than simply assuming that all disadvantaged demographic groups will require mathematics support.

5.2 Enabling Mathematics and Academic Numeracy

In contrast to support programs, enabling mathematics subjects are often located in pre-university courses as pathways or included as subjects in a degree program. Brady (2016, p. 176) defined academic numeracy as “the capacity to confidently and competently use mathematics at university level, and to be able to apply, interpret, critique and communicate mathematical concepts in particular disciplinary contexts” and argues for the need to explicitly teach academic numeracy in the university context. The author described how an Australian university addressed this need by introducing a credit-bearing academic numeracy module. Since the introduction of this module, Brady noted uptake has increased across the university, with more course coordinators from different faculties mandating the topic as core for their students.

Irwin, Baker, and Carter (2018) explored numeracy preparation in enabling education programs. From the 27 enabling programs across Australia, they explored practitioner perceptions and practices. Not surprisingly, a key finding from the audit was that much diversity, complexity and context-dependency exists in the programs. The audit showed that relationships with discipline-based academics in the same institution were mostly informal, and topics commonly included in such programs were arithmetic, number, algebra and statistics. Taylor and Brickhill (2018) described a 13-week mathematics-enabling program for students intending to enrol in undergraduate courses. Notably, mathematics problem solving was not identified as a key category by Irwin et al. (2018), underlining the innovative nature of this program described by Taylor and Brickhill. Jain and Rogers (2019) described the development of an academic numeracy unit that is part of an enabling program. While entry-level mathematics was previously taught in a way that imitated high school mathematics, the new approach has critical thinking at its core. The paper describes the intention, content and delivery of the new unit; however, evaluation of its impact is minimal.

Through a phenomenographic study based on the experiences of students attending a satellite campus in a low SES area, Elsom, Greenaway, and Marshman (2017) described the experiences of non-traditional students in a tertiary entrance program that includes mathematics. Grounded entirely in the students’ words, the research uses three thematic categories to describe the ways that students experienced the program: stairway (to be climbed), doorway (to be passed through), and hallway (offering opportunities for exploration along the journey). The authors suggested the model is a useful way to categorise students, to identify their experiences and develop strategies to support them. Also using qualitative research, Mann (2019) explored how students developed quantitative literacy for learning mathematics in an enabling course at university. She found seven interrelated student-centred categories for developing this literacy: relating; holding interest; exploring ways; taking time; practising; working through confusion; and tailoring ways of learning mathematics provided through teachers and others such as peers, family, and friends.

Due to the ongoing concern about the preparedness students for the quantitative skills needed for university (McMillan & Edwards, 2019), both enabling and support research will continue into the future. What is perhaps needed is a national collaboration to increase understanding of the issues involved.

5.3 Transitions

Research in this area explores both the school to university transition and the transition from university to work. From the school to university perspective, Kouvela, Hernandez-Martinez, and Croft (2017), Kouvela, Hernandez-Martinez, and Croft (2018) used Bernstein's concepts of classification and framing in educational transmissions to analyse the messages given to first year mathematics students by their lecturers about what and how they should study. The researchers found that students' identities as mathematics learners, formed by their school experiences, influenced how the messages were received and acted upon (or not). Anderton, Hine, and Joyce (2017) used a statistical approach to supplement the increasing body of knowledge predicting the success of first-year university students (measured by Grade Point Average, GPA) based on high school subject selection. The researchers found that studying higher levels of mathematics, physical science, and the completion of both advanced level mathematics and one science course were all associated with higher GPAs.

Successful transitioning from university to work requires attention to workplace skills in addition to disciplinary knowledge. King, Varsavsky, Belward, and Matthews (2017) surveyed 144 final-year mathematics students about seven graduate skills. They found that while students valued all of the skills, fewer than 30% of the students agreed that ethical thinking and communication were included in their studies, and even fewer felt these topics were assessed. The implications of these findings for curriculum design are clear if graduate employability is to be raised.

6 Service Teaching

Past reviews have noted research related to service teaching into engineering and nursing. Topics of preparedness and contextualisation that have been highlighted in these disciplines are now emerging in new contexts of sport and exercise science (SES), paramedicine, agriculture and biomedical science. This section reviews the research conducted in these 'service' disciplines.

In this area of research, understanding of student preparedness may be enhanced by using data analytics tools such as network and relative risk analysis (Woolcott, Chamberlain, Whannell, & Galligan, 2018). Using these tools, Woolcott et al. (2018) found particular mathematics service courses with a high number of network connections may be useful targets for a coordinated, strategic intervention. Taking a leadership approach to preparedness, Wilkes and Reid (2019) offered the Distributed Leadership Framework as a means to address issues in quantitative skills for agricultural degrees that potentially has implications for a university-wide approach to dealing with the general underpreparedness of students.

6.1 Engineering

Only one research project was found relating to engineering (with two papers discussing elements of the project: Dunn, Loch, & Scott, 2018; Loch & Lambert, 2016). The project utilised 'student-as-partners' in the development of a series of student-produced videos on the relevance of mathematics to engineering. In analysing the project, Loch and Lambert (2016) reported on interviews with the students who produced the videos ($n = 5$), as well as focus group interviews with the students who viewed the videos in class (two focus groups with two participants each). One of the main findings was that the participants felt students should be producing the videos and not staff. Dunn et al. (2018) reported on a pre- (start-of-semester) and post- (end-of-semester) survey that invited students enrolled in a first-year mathematics subject to reflect on the relevance of the individual mathematical topics taught, as well as the use of student-produced videos in the teaching of mathematical content. In relation to the student-produced videos, the results from the surveys were mixed, indicating that it was unclear whether the student-produced videos provided any advantage over staff-produced videos. Unsurprisingly, student perceptions on the relevance of the individual mathematical topics taught showed that all the topics perceived to be low in relevance at the start of semester increased in scores in terms of relevance at the end of the semester.

6.2 Nursing

A paper by Galligan et al. (2017) discussed the outcomes, from a nursing student context, of a project that investigated the perceptions of lecturers and students in relation to students' mathematical readiness in engineering, business, education and nursing after having completed a semester of studies. Interestingly, the nursing students appeared overconfident in their graphing skills; and while they often found algebra an obstacle in their mathematics learning, they were often unaware of the skills they already possessed. The impact of mathematics anxiety on dosage calculations was the focus of two papers (Choudry & Malthus, 2017; Williams & Davis, 2016). The latter is a scoping review paper that would be useful for those investigating this topic further. The former paper highlighted that teaching methods (unsurprisingly) are important when addressing mathematics anxiety.

6.3 Health Sciences

Paramedicine is a relatively new area in the higher education field and a paper by Bell and Latham (2018) offered game theory, gamification, and mastery learning as approaches to improving numeracy skills of paramedicine students. In a study of the numeracy levels of exercise science students, Green et al. (2018) tested the numeracy level of first, second and third level SES students. The results showed that students' highest levels of mathematics study at high school were strongly associated with their mathematical and numeracy performance relevant to learning in their SES degree (evident across all three years). The study also found a strong positive association between numeracy performance and students' self-ratings of mathematical proficiencies, which suggests students make accurate judgements of their mathematics readiness.

Moving from readiness to development of quantitative skills in the degree, a review of thirteen life science degree programs in Australia by Matthews, Belward, Coady, Rylands, and Simbag (2016), which was an extension of previous work, revealed a limited presence of quantitative skills in students undertaking these programs. Their study suggested that insufficient learning opportunities exist for students to adequately build their quantitative skills across the degree program. In another study of quantitative preparedness in the health sciences area of biomedical science, Carnie and Morphett (2017) investigated the effects of biological or mathematical background (having studied high school biology and/or high school mathematics), prior mathematical achievement (in high school mathematics), and gender on a student achievement in mathematics courses specifically developed for biology students. While no effect of prior biology study on achievement in the subject was found, a student's level of achievement in the subject was correlated positively with their achievement in school mathematics. The authors also found evidence that completing additional non-calculus mathematics at school had no effect on achievement in the undergraduate biomedical mathematics subject. Another study in health sciences by Joyce, Hine, and Anderton (2017) similarly found a negative impact on students, particularly in SES, physical education, and nursing, if they had a lower level of mathematics completed at high school.

This section on service teaching links closely with the previous section on support and enabling research and all addresses a critical issue of underpreparedness. As we continue to observe a percentage of students embarking on tertiary study without an adequate level of mathematics, research investigating or describing innovations or interventions, such as support teaching, that address the issue of underpreparedness will continue to an important and relevant area of research.

7 Suggestions for Further Research

To further research in the area of tertiary mathematics and statistics education we suggest:

1. The co-evolution of pen-enabled technology, the flipped classroom, and blended learning to create learning cultures to assist students to gain a deep understanding of mathematics.
2. Better understandings of some of the higher-level contexts of mathematics and the application of mathematics and statistics in work-integrated learning contexts and HDR research.
3. Research needs to explore curriculum, models and benefits related to the integration of learning in STEM. Given the push for more work integrated learning in the STEM in Australian universities (Edwards, Perkins, Pearce, & Hong, 2015) it is surprising there is not more research in this area.
4. The continued exploration of frameworks that can assist in deep understanding of mathematics. While threshold concepts were mentioned in the last review, this approach has extended to other areas of mathematics.
5. Investigations need to move beyond highlighting quantitative underpreparedness. Both national and case study investigations may indeed answer recent national concerns of declining numbers of students taking high level mathematics at school (McMillan & Edwards, 2019).
6. Further research could utilise data analytics tools and leadership frameworks which would assist universities in positive curriculum change.

In the introduction, we suggested collaborations between mathematicians, statisticians and mathematics educators are increasing, producing broader and deeper research. In the next four years we hope that this collaboration expands to include a greater number of secondary/tertiary research relationships. This is particularly important considering a recent report (Hinz, Walker, & Witter, 2019) highlighting the need for teachers to have both deeper content and pedagogical knowledge. We also hope that more productive relationships with industry will help tertiary mathematics and statistics educators, and their students, to better understand the role of mathematics in twenty-first century society.

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Chapter 12
Innovative and Powerful Pedagogical Practices in Mathematics Education

Jodie Hunter, Jodie Miller, Ban Heng Choy, and Roberta Hunter

Abstract Powerful and innovative pedagogical practices are necessary for all students to learn mathematics successfully and equip them for the future. In this chapter, we review Australasian studies that provide evidence of pedagogical practices that support creative and flexible mathematical thinkers for the 21st century. The review is structured around three key themes that were evident in the research literature. The first theme is the need to develop innovative learning environments that benefit all learners. The second theme is centred on how both tasks and tools can be used to support powerful pedagogical practices. Finally, the third theme reviews the challenges of developing innovative mathematical learning environments. We argue for the need for effective pedagogy for *all* learners and a need for ambitious, future-focused teaching in mathematics education.

Keywords Pedagogy · Inquiry · Problem-solving · Tasks · Learning environments · Equity

1 Introduction

All students should have access to high-quality mathematics curricular, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential. This chapter focuses on a review of the powerful and innovative pedagogical practices that teachers use to ensure that all students learn

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mathematics successfully. We take the position that both powerful and innovative pedagogical practices are necessary to sufficiently equip students for the future. Key components of innovative and powerful pedagogical practices include a focus on inquiry, problem-solving, critical thinking, and creativity. Within the chapter, we draw on Australasian studies that provide evidence over the previous 4 years (2016–2019) of the need to continue exploring research around those pedagogical practices that support creative and flexible mathematical thinkers for the 21st century.

Within the review, we have included a range of studies that include both small scale and larger projects undertaken by Australasian researchers. The review of literature focuses on research studies that have been undertaken across years of schooling including early years through to secondary schooling and initial teacher education, however the key focus is on the compulsory years of schooling. Also included are some studies which have focused on teacher professional development to shift pedagogical practices. The first section of the chapter examines the development of innovative learning environments for all learners. The following section focuses on mathematical tasks and tools in relation to powerful pedagogical practices. The final section reviews some of the challenges of developing innovative learning environments and makes suggestions for next steps.

This focus of the chapter is timely given ongoing concerns about effective pedagogy for *all* learners and a need for ambitious teaching in mathematics education. As highlighted by Chan, Clarke, and Cao (2018), the highest priority in the field of education is the study of the nature and promotion of learning. Fitting within this area and priority, this chapter focuses on Australasian studies that provide evidence of powerful pedagogical practices to influence positive outcomes for all learners in mathematics classrooms.

2 Developing Innovative Learning Environments for *All* Learners

Both Australasian researchers (English & Kirshner, 2015; Hunter & Hunter, 2017, 2018) and international researchers (Celedón-Pattichis et al., 2018) recognise that systemic change is required to ensure that every student has access to innovative learning environments, where high quality mathematics teaching is provided for all. As part of this review, we have highlighted three main ways in which classrooms can be innovatively structured to support all learners to engage in productive mathematical activity. First, we examined the development of productive discourse, mathematical reasoning, the use of mathematical language, as well as ways in which diverse learners can be supported and facilitated to equitably participate in productive discourse. Next, we looked into ways in which teachers can equip students to develop 21st century skills such as critical thinking, creativity, and intellectual risk-taking. Finally, we focus on how the use of innovative and powerful pedagogies can engage learners in mathematics and support them to develop a mathematical disposition.

2.1 Developing Productive Reasoned Discourse in the Mathematics Classroom

An ongoing theme in the MERGA four yearly reviews is that within classrooms that represent innovative and powerful mathematical learning environments, students are facilitated to engage in effective communication. This includes both the provision of opportunities for students to engage in mathematical reasoning and the use of pedagogical actions by educators that develop student use of productive and reasoned mathematical discourse. An important element and outcome of these actions is an associated shift in student roles and dispositions.

Mathematical reasoning is an aspect included across curriculum documents both in Australasia and internationally. McCluskey, Mulligan, and Mitchelmore (2016) highlight reasoning as one of the four key mathematical proficiencies included in the Australian Curriculum Mathematics (ACM) (Australian Curriculum, Assessment and Reporting Authority, 2015). These researchers argue that although within the ACM the proficiencies are described individually, to build and deepen conceptual understanding in mathematics, teachers should take a holistic approach by focusing on the interrelationships between the proficiencies. A key finding of their analysis of ACM was the under-representation of key terms associated with reasoning and a lack of emphasis on this proficiency in relation to the emphasis on other proficiencies such as problem-solving. The authors propose a pedagogical model drawing on an integrated view of mathematical proficiencies and note the need for appropriate professional learning promoting the use of reasoning in mathematics classrooms.

To develop children's use of mathematical reasoning within the classroom requires both professional development for educators and significant shifts in teacher pedagogy to align with innovative and ambitious practices. Mathematics educators and researchers need to consider ways in which to support teachers to observe, reflect upon and change practice. In some cases, similar to McCluskey and colleagues, researchers describe the use of scaffolds to support teachers to induct students into the use of a range of reasoning practices. For example, Hunter, Hunter, Anthony, and McChesney (2018) describe the use of a Communication and Participation framework which supports teachers to engage students in a range of mathematical practices. Likewise, Brown (2017) promotes the use of an approach called collective argumentation which draws on a set of interactive principles. Using data drawn from one primary classroom Brown illustrated that when expectations were placed on students to explain, justify and present their reasoning to the whole class, student engagement was increased.

In other research studies, the focus has been on effective professional development for educators to develop their understanding and attention to mathematical reasoning. For example, Bragg and her colleagues (2016) explored the use of demonstration lessons to focus teacher attention on student reasoning. They contend that this proves an effective way for teachers to develop a professional eye in relation to assessing students' mathematical reasoning. With the aim of gaining insights into how teachers assess mathematical reasoning, as well as the challenges they faced, Herbert (2019) designed a professional learning programme to trial some of the tasks and resources as part of the reSolve project. Besides attending a workshop on mathematical reasoning, teachers had opportunities to teach and observe other teachers at their school using tasks developed by the research team before assessing students' reasoning using the rubrics developed by the researchers during the post-lesson discussion. This was then followed up with a second round of lesson observation and post-lesson discussions using tasks sourced or crafted by the teachers. Such an approach provides a platform for teachers to learn from the researchers and their colleagues as they try to make sense of their students' reasoning. Moreover, Herbert's (2019) study revealed several challenges faced by teachers as they learn to assess students' reasoning. Developing strategies to overcome or mitigate these challenges will likely be critical as mathematics educators learn to support teachers to enact and assess ambitious teaching practices that promote reasoning.

Attention to language is also an important element within research focused on student reasoning. A number of researchers (e.g., Bicknell, Young-Loveridge, & Nguyen, 2016; Bragg et al., 2016) note the importance of inducting students into the use of mathematical language both to progress their learning and for them to competently share their mathematical reasoning. Some research studies (e.g., Bragg et al., 2016) in this area highlight a continuing discrepancy between effective practice advocated by mathematics education researchers and the perceptions of practitioners. For example, Bragg and colleagues (2016) found that teachers frequently commented on the need for students to develop mathematical language prior to engaging in reasoning activities. This is contrary to recommendations within mathematics education research that students learn to use mathematical language associated with reasoning while participating in reasoning activities.

The centrality of dialogic practices to the teaching and learning of mathematics is highlighted in research studies (e.g., Attard, Edwards-Groves, & Grootenboer, 2018; Hunter, 2017). These practices have the potential to transform pedagogies, support classroom environments with deep mathematical discussions, and facilitate student engagement in developing and explaining their mathematical reasoning. However, powerful and innovative pedagogies such as rich and robust dialogical interaction are challenging for teachers to develop in their classrooms. A study by Attard et al. (2018) examining the dialogical interactions in a Year 6 classroom found that most of the exchanges occurring followed an initiation-response-feedback pattern with high teacher control. The researchers argued that the pattern of interaction lacked a connection to mathematics and consequently distorted students' experience of mathematics in practice and additionally limited "students' capacities for developing deep mathematical knowledge and producing extended turns aimed at deepening reason (for example) about specific mathematics concepts" (p. 128).

Evident in research studies is that developing interactive reasoned dialogue in the classroom requires a shift in the role of students and teachers and a deliberate change in pedagogical actions. Hunter (2017) examined a classroom in which the teacher undertook actions to deliberately address classroom and mathematical practices. Within the changing classroom context, she mapped shifts in student perceptions and their recall of mathematical reasoning. Initially, although the teacher used paired or group-work and whole class discussions, there were limited opportunities for student engagement in mathematical reasoning. By focusing student attention on productive ways to talk, facilitating students to give mathematical explanations, and developing mathematical argumentation, the teacher successfully shifted the student role to one of responsibility for providing mathematical reasoning and justification. Interestingly, as the year progressed, students more readily recalled both their own and their peers' solution strategies and mathematical reasoning. They also shifted to an understanding of how explaining ideas and listening to others explain their ideas could be used as a reflective tool.

For many children, the opportunities they are offered to participate in communicating their mathematical reasoning has strong links to both their mathematical achievement and the development of a mathematical disposition. However, Hunter and Hunter (2018) signal that the increased focus on developing mathematics classrooms, in which student voice dominates, can contribute to classrooms as ongoing sites of inequity. This is due to the privileging of some students and their "voice" while others are marginalised. Research studies (e.g., Hunter & Hunter, 2018; Jorgensen, 2018; Miller, Warren, & Armour, 2018) provide us with examples of how students can be facilitated to participate and engage in discourse in equitable ways. For example, Hunter and Hunter (2018) describe how a recent significant shift in policy from a focus on individuality within the mathematics classrooms towards considering mathematics classrooms as learning communities can be a way forward. They describe the need for teachers to proactively create space and reposition all students to access the discourse. The use of a strength-based approach underpins this work. These researchers promote the need for teachers to draw on and use the core values of Pāsifika students and other diverse learners to shape the social and socio-mathematical norms used in the mathematical discourse.

Other studies (e.g., Jorgensen, 2018) focus on the importance of the use of language as a resource to scaffold mathematical learning for Indigenous learners. In her large-scale study in remote and very remote contexts in Australia, Jorgensen (2018) examined successful numeracy practices. She noted the importance of teachers drawing on resources to scaffold the transition from home language to Standard Australian English (SAE) as students participated in mathematics lessons. Jorgensen (2018) identifies two key resources used by teachers—human resources, that is local Aboriginal people who work alongside teachers in classrooms and—pedagogical (material) resources. Teachers who worked alongside Aboriginal people who have access to the home language of students, were able to enhance their classroom practices as both educators (teacher and Aboriginal teacher assistants) translated between home language and mathematical language to support the learning of mathematics for all students. This in turn provided access to the students in these mathematics classrooms to engage in rich discourse practices that both empowered and supported their learning of mathematics.

Similarly, Miller et al. (2018) examined mathematical discourse with young Aboriginal and Torres Strait Islander students as they participated in mathematical patterning activities that supported the development of mathematical generalisations.

They drew on the theoretical constructs of semiotics and Indigenous research perspectives throughout the study both in relation to task design, ensuring that the tasks were accessible, and analysis of the data. The researchers argued for a broader definition of discourse, advocating for a nexus of Indigenous ontology, mathematics discourse, and semiotics to create an equitable space where young Indigenous students share their knowledge of mathematics. Apparent in the findings were the ways in which young students were able to generalise mathematical patterns drawing on both language and gesture. As such, this entails a shift from a focus purely on the spoken (Western norm) to that which encompasses gesture as a means to articulate mathematical knowledge. The implication for teachers is the need to use gesture as well as a need to focus more broadly on the full sign systems utilised by students to articulate their knowledge. This will assist teachers to determine what young students know. Finally, the authors present a framework that demonstrates the interconnectedness of Indigenous knowledge with mathematics and semiotics that create an equitable space for young Indigenous students to participate in mathematics discourse.

2.2 Equipping Students with 21st Century Skills in the Mathematics Classroom: Critical Thinking, Creativity, and Intellectual Risk-Taking

In recent years, we have had a volatile global landscape characterised by rapid technological advances and societal shifts. To cope with these challenges, educators are developing ways to equip and empower students with 21st century skills—competencies required to thrive in an ever-changing environment—in the classrooms. In mathematics, an important component of innovative learning environments lies in its affordance for fostering mathematical communication, reasoning, and problem solving. A connective thread across recent research studies is the importance of developing specific skills such as creativity, critical thinking, and intellectual risk-taking in order to empower students to succeed within problem-based or inquiry learning environments. Hurst and Hurrell (2016) go so far as to argue that these thinking and reasoning skills are essential factors in all mathematics classrooms.

By drawing on innovative practices within the classroom, mathematics can be framed as a creative enterprise and students can be positioned to view mathematical problem-solving as a process of creativity and exploration. This requires teachers to structure mathematics lessons in ways which allow students to solve problems, explain reasoning, and importantly explore the related mathematics further. Parish (2016) provides an example of an intervention where a teacher implemented a structure similar to that outlined above with the whole class. The focus of the study was on a highly capable student. The study demonstrates how the introduction of creativity facilitated shifts in the student's disposition from a focus on speed, correct answers, and high grades to exploring beyond the set mathematical task and drawing on creativity and challenge.

A key element of successful learning within problem-based classrooms is transforming students from passive learners to active learners. In a study set in Thailand by Promsawan and Katwibun (2017), the researchers described how within many Thailand schools, students remain passive learners. However, in contrast these researchers illustrated how a group of Year 11 students in a problem-based learning environment; one which drew on real world problems, were able to positively self-regulate. The shift towards self-regulation matched the press by teachers on the students to take an active rather than passive role in their learning. More recently, Naimsamphao and Katwibun (2019) investigated a group of 40 Grade 10 students who took part in a series of 12 Problem-based Learning (PBL) lessons, and found high levels of student agency and authority in all the steps of the PBL process. Such shifts towards dimensions of classroom environments that promote mathematical reasoning are promising and more work could be done to investigate the success factors in task design and implementation.

Another expected outcome for students in problem-based classroom environments as highlighted by Siriwat and Katwibun (2017) is the development of critical thinking. These researchers explored the critical thinking of forty-seven 11th Grade students in a problem-based mathematics classroom. Both a critical thinking test and observations were used at the end of one month to assess the students' critical thinking across five steps (or what they describe as a ladder) of problem-based learning. Their results showed that the students were able to draw on critical thinking across all dimensions. They showed particular strengths in explaining the situation or context but were weaker in drawing conclusions.

The use of mathematical evidence for addressing a mathematical question and coming to a reasoned conclusion is problematic—in that students do not see the need for evidence to support an assertion. In trying to understand how experienced inquiry teachers focus their students on an evidence-based approach to mathematics, Fielding-Wells and Fry (2019) developed and trialled a *Framework for Evidence focus during Guided Inquiry* with an experienced teacher to develop evidence usage as a classroom norm. The framework provides a series of actions focusing on the use of evidence in the classrooms and the trial revealed insights into students' use of evidence as they engage in mathematical inquiry. However, students may not always focus on the correct features of mathematical evidence. In an exploratory study which involved students in critiquing three fictitious students' claims, Wilkie and Ayalon (2019) suggested that task design—the number and choice of fictitious task responses or solutions—plays a critical role in drawing students' attention to or away from the important features of mathematical evidence.

2.3 Using Innovative and Powerful Pedagogical Practices to Engage All Students and Support Their Development of a Mathematical Disposition

Engaging students to learn and develop strong mathematical dispositions across all year levels is a critical element of innovative and powerful pedagogical practices. This includes addressing the social value of mathematics, fostering the engagement of students in mathematics, and supporting their development of a positive mathematical disposition.

Studies with children in early years settings support the importance of fostering problem-solving activities through child-initiated play (Colliver, 2018), in particular ascribing social value to numeracy application as an approach to teaching in the early years. This differs to common approaches to early mathematics teaching which are often adult-initiated and have the potential for young children to lose interest. Colliver's study (2018) sought to determine if young children were more likely to display numeracy-related concepts in free play if they were exposed to adult demonstrations of numeracy activities; fostering an interest in mathematics in everyday life. The demonstrations provided social value to numeracy activities and appeared to influence the increased numeracy related activities in children's free play at preschool.

In the middle and later years of schooling, fostering the engagement of students in mathematics is particularly important, given the documented decline in student engagement with mathematics in the middle years. While this decline relates to multiple factors, a key component is the use of teaching practices that disregard student motivation and engagement (Bobis, Way, Anderson, & Martin, 2016). In addition, it has been identified that institutional practices (structural school practices) impact on students from low socio-economic backgrounds, in relation to participation and engagement with mathematics in secondary schooling (Bennison, Goos, & Bielinski, 2018). Researchers identified five themes that impact on students' sustained participation in mathematics involving the study of calculus in Year 11 Mathematics B classes in Australia. These institutional practices included: (a) curriculum organisation across year levels with clear pathways from Years 7–10 in preparation for Year 11; (b), staffing of mathematics classes, including strategic choices of teachers and mentoring for new teachers; (c) culture of the mathematics department, that fosters a collaborative environment which extends to the classroom; (d) a well-structured STEM program that encourages participation in senior science and mathematics; and, (e), provisions of appropriate tasks, such as problem-solving tasks in each lesson, and the use of, and access to, a wide range of resources for students (Bennison et al., 2018, p. 157).

One promising way to engage students in mathematics is to use an instructional approach that positions problem solving at the centre of mathematics instruction.

For example, Sangkaew and Katwibun (2019) implemented a series of ten problem-based learning lesson plans and found that their students' self-beliefs, including self-construal and self-efficacy, were positive in all steps of the problem-based learning process. Besides the process, it may be useful to engage students in learning experiences built around contexts in which the students are personally engaged. For example, Russo and Russo (2019) investigated students' intrinsic motivation to learn mathematics by developing integrated mathematics units around topic areas of students' personal interests or passion, and found that students attributed their positive engagement with mathematics to the contexts in which the mathematics was presented.

3 Tools That Support the Development of Innovative and Powerful Pedagogical Practices in the Mathematics Classroom

In this second section, we look at the tools that support the development of innovative and powerful pedagogical practices in the mathematics classroom. A common thread across many studies is the importance of task design including both the challenge of tasks and the contextual basis of tasks. Linked with the use of challenging, meaningful tasks is the importance of teacher noticing of key mathematical ideas both in the planning of tasks and in the moment noticing of student reasoning during teaching. Finally, we examine the use of representations, and the connections between different representations as important aspects of classrooms that draw on innovative and powerful pedagogical practices.

3.1 The Use of Challenging Tasks

The use of challenging tasks has had increased research interest with significant building on the earlier comprehensive and seminal work of Peter Sullivan and colleagues. Russo and Hopkins (2019) describe the way in which incorporating enabling and extending prompts into the design of the tasks has the potential to optimise cognitive challenge. Their study examined teacher perceptions of the outcomes of student engagement in complex and challenging tasks. The researcher and teacher acted as co-teachers and this allowed space for the teachers to observe the high levels of engagement, autonomy and substantial persistence the students demonstrated. This was attributed to a number of factors including a classroom culture which promoted cognitive struggle, high expectations, and consistent routines. This also supports recent views of Australian middle years students ($n = 3500$) who have identified that they wish for classrooms that provide challenging and interesting work which provide opportunities to for collaboration (Wilkie & Sullivan, 2018).

In another study, Russo and Hopkins (2017d) explored student perceptions when engaged in mathematics lessons using complex, challenging and cognitively engaging tasks. Overall the students identified positive enjoyment, effort and persistence and as they embraced the challenge indicated that they were prepared to struggle and engage productively to learn meaningful mathematics. However, Russo and Hopkins (2017a, 2017b, 2017c, 2017d) juxtapose the lesson format when considering the use of challenging tasks with young students. These researchers drew on cognitive load theory and the use of teacher and student voice to analyse results. They argue that considering both a process of teach first followed by a challenging task, or the presentation of the task first serve different purposes and outcomes. They describe a seven step approach in which the cognitive load for individual students is optimised. Students were divided in their perceptions about which form of learning they preferred.

Recent research has examined the notion of online challenging mathematical tasks for pre-service teachers (Geiger et al., 2018; Wells et al., 2019). While teacher education has moved to blended and online methods of delivery, there has been little research conducted to examine the effectiveness of this mode for developing mathematical knowledge for initial teacher educators. In addition, there are also few studies that have examined the notion of challenge in these contexts. Wells et al. (2019) focus on designing online challenging tasks to strengthen the personal mathematical capabilities of initial teacher education students in Australia and Germany. Seven principles of designing tasks were developed and trialled, which included: fit to circumstance, quality of the learning environment, engagement, transparency, accessible yet challenging, challenge, feedforward/feedback. Drawing on motivational theory (Expectancy-Value Theory), researchers identified that while pre-service teachers largely expected to achieve success on the challenging tasks (expectancy), their motivation to engage with the tasks varied considerably according to their own perceptions of mathematics and mathematical learning (subjective task valuing). Similarly, Geiger et al. (2018) found that online modules based on real world problems enhanced pre-service teachers understanding of mathematics.

3.2 *Drawing on Real and Meaningful Task Contexts*

The use of meaningful contexts which relate to students' experiences can be a way to engage them in tasks and to support them to solve challenging mathematical problems. Marshman (2018), advocates for authentic real-life tasks embedded in community issues, which in turn provides students with the opportunity to engage more creatively with mathematics and confidently communicate their mathematical ideas. For example, Bicknell et al. (2016) describe an intervention in which they worked with a teacher to design multiplication and division tasks for children aged five as drawing on both out of school contexts and others related to schooling experiences. In another study with secondary students by Muir, Beswick, Callingham, and Jade (2016), the students described the importance of relevance and interest when reflecting on important elements of learning quantitative reasoning (QR) through project-based learning. Similarly, Sawatzki (2016) reports on an intervention where tasks involving financial dilemmas were used. Teacher participants reported that the success of the lesson was dependent on whether students could access and relate to the context which was posed. For some of the tasks designed by the researcher, the context was not a strong fit with the students and this created extra literacy demands. Teachers reported using specific strategies such as role-play to launch the tasks and make them more accessible. Aligned with this study, Blue et al. (2018) also advocate for a compassionate approach to financial literacy education highlighting that classroom practices should include opportunities for positive and collaborative learning opportunities, making ethical, social and mathematical connections within the task, and considering the impact of financial decisions on others.

Other research investigates the development of contextualised rich mathematical tasks through the use of a social justice perspective. Anderson and Kriesler (2018) examined Year 7 students' engagement through the use of rich tasks that were underpinned by Gutstein and Peterson's (2013) 'Rethinking Mathematics', to support higher-ordered thinking through intellectually challenging tasks connected to students' lives, promoting classroom discussion and collaboration, and foregrounding students' voices. Analysis of data from field notes of classroom observations, surveys and teacher interviews revealed that engaging Year 7 students from low socio-economic schools with social justice mathematics tasks, required teachers to: (a) ensure the mathematics is relevant to students; (b) assist students to recognise that mathematics can be used to understand the world; (c) provide opportunities to use mathematics to critique social practices; and, (d) provide tasks that are appropriately intellectually challenging for students. The use of social justice tasks provided opportunity for these students to share their own experiences through classroom discussion, pose questions and become emotionally invested in the task and their learning of mathematics.

Contexts involved within a STEM perspective have gained increased attention over recent years (see Chap. 3, this Volume). Anderson and Katrak (2017) used a small-scale study in two classrooms of Year 7 and 8 students to show how the teachers' use of STEM context-based tasks increased their students' engagement. The tasks were tied to student interest and were designed as challenging inquiry tasks which required longer and more in-depth attention and persistence. A significant number of students in both classrooms reported how the open-ended tasks connected them to mathematics as a lived subject. However, there is evidence that mathematics concepts are not always meaningfully integrated with STEM context-based tasks. To avoid such pitfalls, English and King (2019) incorporated engineering, mathematics, and science within a set of bridge building activities for Year 6 students in which "engineering design served to both link and scaffold students' disciplinary knowledge and application" (p. 880). The set of problem activities provided a platform for elementary school students to demonstrate their justification for decisions made during the planning, designing, reflecting on, and redesigning of paper bridges. Their study suggests the importance of providing scaffolding activities within STEM-context tasks to facilitate students' applications of mathematics concepts.

A number of researchers (e.g., Brown & Redmond, 2017; Hunter, Hunter, & Bills, 2019) note the importance of the context of the task in terms of what mathematical competencies students are able to develop. Brown and Redmond argue the importance of students being able to apply the mathematics within their everyday world while at the same time having opportunities for 'mathematisation'. When collective argumentation (Brown, 2017) is used as a pedagogical tool within what Brown and Redmond (2017) describe as a problem centred curriculum which draws on the use of real-world type tasks of interest to the students, they describe many positive outcomes. These include a shift in role of the teacher from teller to facilitator of mathematical argumentation and increased student agency. Drawing on known contexts helps students to be able to recognise relevant information and select what mathematical knowledge and procedures best support completion of the task. Hunter and her colleagues concur, but they also promote the need for the tasks to not only have real world application but to also be culturally relevant in order to engage and give all students opportunities to learn mathematically. They draw on classroom episodes to illustrate how teachers can embed tasks in cultural contexts which support students to maintain their cultural identity while developing a strong mathematical disposition. Averill (2018) extends this thinking further in contending that this can only be achieved when there is strong teacher-family or community partnerships. She also calls for comprehensive teacher knowledge and understandings of the lived world and language of the Indigenous students (in this case Māori students).

3.3 Enhancing the Use of Typical Problems

While acknowledging the importance of mathematically-rich tasks in the teaching and learning of mathematics, Choy and Dindyal (2017a, 2017b, 2018) also contend that there is a need to investigate how innovative and powerful pedagogies can be developed within contexts where text-book type questions are used. In their study, they describe how two teachers demonstrated an innovative and pedagogically powerful use of a typical problem—standard examination questions or textbook type questions—for orchestrating a productive mathematics discussion. Their case studies present an interesting proposition: teachers can carefully select, craft, modify, and use typical problems to develop *conceptual understanding* beyond their usual purpose of developing procedural fluency. In some ways, their study may explain how experienced teachers in an examination-driven system, such as Singapore, could maintain a high level of mathematical engagement on a day-to-day basis using what many would deem routine exercises. Their findings correspond to those by Leong, Cheng, Toh, Kaur, and Toh (2019), where an experienced teacher had made the mathematical point of a lesson explicit through a deliberate use of instructional materials. Typical problems, which are omni-present in many classrooms, may provide a way to mitigate the "lack of time" issue faced by many teachers when implementing challenging tasks. Whether, and how, the use of typical problems can be further enhanced in other contexts, is an important area for further exploration.

3.4 Teacher Noticing and Responding

Parallel with the use of challenging and complex tasks in the mathematics classrooms is a focus on teacher noticing of actions, and connections students make to the big mathematical ideas as they work to sense-make. Teacher planning and being able to respond in the moment is an important aspect of teacher noticing. However, Sullivan and his colleagues (2016) caution that often teachers only see the superficial aspects of a task when planning. Cheeseman, Downton, and Livy (2017) suggest the use of enabling and extending prompts which are planned prior to lessons have the potential to support teachers to notice student reasoning and how they communicate this. This includes teacher planning of tasks (Choy, 2016). Choy and Dindyal (2017b) also highlight the importance of teacher noticing both prior to, and during the use of a problem. These researchers illustrate how a proficient teacher can take a typical textbook problem and 'in the moment' through careful planning and their noticing and responding change it to a rich and extended problem. In another study, Choy and Dindyal (2017a) illustrate how a teacher, through noticing, recognised the affordances of what they describe as a typical problem. Through her noticing the teacher was then able to extend and develop the problem in the classroom to facilitate a whole class interactive rich mathematical discussion.

However, it is not trivial for teachers to notice students' mathematical reasoning before, during, and after the lesson (Jazby & Widjaja, 2019; Lee & Choy, 2017). For example, Lee and Choy (2017) highlight that even experienced teachers may not notice the subtleties of important mathematical concepts. Similarly, it is very difficult for teachers to notice students' reasoning in-the-moment (Jazby & Widjaja, 2019). One way to address this challenge might be to keep in mind the attentional limits of people when designing mathematics tasks, and think of ways to make mathematical reasoning actions more explicit so that teachers could have a higher likelihood of noticing them when monitoring different groups of students (Jazby & Widjaja, 2019). This idea builds on the concept of preparing to notice (Mason, 2002) and positions noticing as a critical component of teaching expertise necessary for task design (Choy, 2016; Choy, Thomas, & Yoon, 2017).

Teacher noticing of mathematical ideas and their ability to ensure adequate coverage of the traditional mathematics curriculum when using project-based learning was a concern held by teachers noted in the study by Muir et al. (2016). The advisors in the study identified the difficulties of ensuring that student selected projects had the parameters to ensure curriculum coverage. The advisors were able to readily identify the opportunities for mathematics that they had noticed within student projects, however the researchers noted that within this there was little reference made to higher order mathematics such as generality. Overall, the researchers describe the tension between addressing relevant mathematics through project-based work and what to do if some aspects of the curriculum do not fit within this.

3.5 Representations

The use of representations within mathematics classrooms provide students with opportunities to model their thinking and reasoning. Additionally, representations such as number sentences can be used to progress student thinking to more sophisticated solution strategies. For example, in a study by Bicknell et al. (2016) the teacher encouraged her young students to work like mathematicians by providing them with exposure to symbols and equation structures to develop their internal representations. In another study by Wilkie and Clarke (2016), students were asked to reflect their perception of a geometric growing pattern by colouring it in. This supported the students to both notice the general relationship and to communicate this with their peers. These studies bring out the importance of skilful questioning that empowers students to look for deeper structures, which underpin such pattern tasks.

Similarly, Patahuddin, Usman, and Ramful (2018) highlight the instructional nuances that are necessary for making affordances for representations, when trying to build student understanding of the measure meaning of fractions. In a study with 7th grade students, four lessons were conducted to determine the affordances for using the representation of a number line, as a mathematical object, when teaching fraction concepts. Analysis of the video data revealed that at times when dissonance arose between the teacher's intentions and the students' interpretations, the teacher then used the area and measurement model of fractions. This resulted in instructional changes including modifying tasks, questioning, and prompting; followed by teacher actions to make the connection for students between the mathematical representation and the concept more apparent.

Other studies provide evidence of how representations can be used in unconventional physical ways to advance student learning. Ginns, Hu, Byrne, and Bobis (2016) report on a study where students used worked examples in an unconventional way tracing the corresponding elements of a diagram while reading each solution step. The experimental design included students engaging both with a geometry and number task involving the order of operations and using dynamic hand movements against a surface. Similarly, Mildenhall and Sherrif (2018) examined the use of multimodal learning, in particular the use of metaphor and modalities as an instructional approach in a Year 2 classroom with a focus on building computational strategies. This approach appeared to not only impact on planned experiences in the classroom but supported students' understanding of abstract mathematics.

Warren and Miller (2016) conducted a longitudinal study with the purpose of examining the effectiveness of the mathematics program RoleM focusing on representations, oral language and engagement in mathematics, delivered to teachers and Indigenous education officers through professional learning. There was a large data set drawn from teachers (n = 154), Indigenous education officers (n = 19), and students (n = 1738) across 22 schools in Queensland, Australia to present a comprehensive picture of the teaching and learning of mathematics to young students (Foundation—Year 3). Results of pre-post testing indicate that an approach focusing on multiple representations, in conjunction with developing language in the mathematics classroom, can have significant effects on student achievement in mathematics for students in Foundation—Year 3 of schooling.

Besides its effects on student achievement in mathematics, representations, both mathematical and non-mathematical, are also useful for gaining insights into students' mathematical thinking, behaviours, and affective responses. For example, MacDonald and Murphy (2019) have used the drawing-telling approach to elicit young children's understandings about measurement within their first few weeks of starting primary school. More importantly, they have found children's drawings have the potential to reveal how children understand the mathematical ideas, and why these ideas matter to them. These insights can provide a means to personalize learning experiences in the future. In order to enhance researchers' interpretations of children's representations, Way and Thom (2019) explored the use of a digital pen as a data gathering tool, which captured both the drawings and the verbalisation surrounding the creation of the drawing. In addition to drawings and verbalisations, researchers can also use students' written responses, interviews, and classroom observations to surface evidence of young children's affective responses towards mathematics (Quane, Chinnappan, & Trenholm, 2019). However, as Cheeseman and McDonough (2019) have demonstrated, while the use of representations to elicit students' ideas may be innovative and potentially important, the process of coding is not straight-forward and may require further research to hone the methods of analysis.

4 Where Are We at? Where Do We Need to Go?

In the final section, we focus on the current and ongoing challenges related to the implementation of powerful and innovative pedagogies in mathematics classrooms. We review challenges including teacher perceptions, use of grouping, power and status of students, and student attitudes towards innovative pedagogies. Within this, we also address potential solutions to the challenges offered by research studies and highlight the areas for further research.

4.1 Deficit Perceptions

A number of research studies (e.g., Bragg et al., 2016; Cheeseman, 2018; Russo & Hopkins, 2019) report on deficit perceptions from teachers towards differing aspects of students' engagement and learning in mathematics. For example, Bragg and colleagues (2016) reported on a tendency towards a teacher focus on the limitations of students' use of language in relation to reasoning while observing demonstration lessons. This was then expressed as a reason to not use reasoning-based pedagogy in the mathematics classroom.

While there is an advocacy for problem-solving approaches to teaching mathematics both in curriculum documents and research studies (e.g., Australian Curriculum, Assessment and Reporting Authority, 2015; Marshman, 2018), there are indications that teachers perceive obstacles to adopting this pedagogical approach. Cheeseman (2018) examined the perceptions of 22 teachers in the early years of schooling (Foundation—Year 1) with regards to implementing a problem-solving approach in their classroom. Survey data revealed that teachers perceived obstacles such as children's readiness for this approach (skills, still requiring explicit teaching), planning; resources; tasks; and, time. Cheeseman reveals that the data is potentially indicating that there is a transmissive theory of learning for young children. That is, the teachers hold the view that young children need to be taught what to do in mathematics rather than adopting an approach that would provide young children the opportunity to struggle, persist, and make independent decisions.

The findings of these studies highlight a continuing need for the mathematics education research community to investigate how to productively support teachers to take a strengths-based perspective. This includes reflecting on how to disrupt teacher perceptions of the capability of students and come to view all students as capable of being active learners and deep mathematical thinkers.

4.2 Difficulties in Using Challenging Tasks

A study by Russo and Hopkins (2019) highlights the continued reluctance of teachers to use challenging tasks. They suggest the key reasons for the reluctance include negative student responses, a lack of time and resources and poor teacher knowledge coupled with their self-perceived lack of ability to cope with using complex tasks. Despite working alongside another teacher educator in a co-teacher role enacting lessons using challenging tasks and noting the many positive outcomes, the three teachers in the study still held differing views about which students' needs were met. One teacher perceived that all students' needs could be met by the use of challenging tasks, a second thought that they only met some students' needs and the third teacher espoused views that these were only suitable for high ability students. One aspect which was evident was the lack of agreement over what constituted a successful learning experience. These researchers suggest that teachers need to be able to see the positive outcomes and rich understandings students develop as they engage in different aspects of a task rather than the progress they make towards completion. All of the teachers described the difficulties they had in planning complex tasks. These researchers suggest that a way forward might be that teacher educators develop units of work which incorporate challenging tasks. To this end, Bobis and her colleagues (2019) developed a research-based professional development programme to explore the potential of carefully sequenced challenging tasks and found that both students and teachers were "gradually accepting these tasks as the norm in mathematics lessons" (p. 112) with an increased emphasis on student talk and questioning.

Moreover, as pointed out by Bobis and Tregoning (2019), it is important for mathematics educators to support teachers' shifts in identities as teachers of mathematics, beyond changes in teacher knowledge and practices, so as to enable transformative changes in teachers through these professional learning opportunities.

Another possible direction is to consider other ways in which meaningful learning experiences can be enacted in mathematics classrooms on a day-to-day basis. For example, Choy and Dindyal (2017a, 2017b, 2018) highlight how two teachers in Singapore modified and adapted typical problems for the purpose of teaching towards conceptual understanding, beyond the usual use of such problems to develop procedural fluency. Along a similar vein, Leong et al. (2019) also highlight how an experienced teacher made 'explicit' the mathematical structures through the design of lesson materials used on a day-to-day basis. These practices, together with the use of mathematically-rich tasks, may provide an alternative way to envision how mathematics tasks can be used differently to engage learners in an often time-stripped educational system.

4.3 Use of Ability Grouping

The use of ability grouping in mathematics classrooms continues to encourage teachers to see achievement as tied to student attributes rather than as factors within the classroom practices. Anthony and Hunter (2016) surveyed 102 mathematics support teachers in different schools across New Zealand in relation to their perspectives and use of ability grouping. Despite recognition of the inequitable opportunities afforded different students caused by ability grouping, the majority of teachers reported it as a key practice in their school. They reported their discomfort with the practice but explained their own uncertainties towards alternative grouping arrangements. In contrast, Anthony Hunter, and Hunter (2018) highlight the influence of teacher perceptions on their student capabilities, and illustrate how these perceptions can be challenged when teachers are supported to employ inquiry practices, challenging problems, and heterogeneous grouping through professional development. As these researchers caution, we need to consider carefully how teachers frame students' source of struggle.

Besides explicit ability grouping in mathematics classrooms, ability grouping may also be implicitly practiced in classrooms that practise differentiation. Although the practice of differentiation is aimed at supporting diverse students in our classrooms, its actual implementation may promote "pseudo within-class ability grouping practices" (Anthony, Hunter, & Hunter, 2019, p. 120). In their review and critique of the wide-ranging practices of differentiated instruction, they argue that differentiation in the mathematics classroom needs to move away from a divisive labelling perspective that focuses on performance outcomes to one that is reframed in terms of a social justice perspective, towards a focus on student well-being and productive mathematical disposition. This shift positions flexible and purposeful grouping as a means to empower "students to work with a range of peers to focus on specific mathematical skill development and a range of valued outcomes—including student voice and agency, pro-social skills, mathematical dispositions, and valuing of the mathematics within the home and cultural context" (p. 121).

Another similar and promising shift away from a deficit-view of ability grouping is the notion of strength-based grouping (Leach, 2019), which focuses on identifying and harnessing the strengths of individual students through purposeful grouping according to the capabilities that each child brings to the classrooms. How this notion of strength-based grouping can be further developed and implemented, especially in the context of differentiated instruction (Anthony et al., 2019), will be an important line of research to promote a more equitable, high-quality and culturally-responsive mathematics education.

4.4 Power and Status Structures Within the Mathematics Classroom

The power and status structures that exist in mathematics classrooms continue to impact on how students participate and share their knowledge. Gibbs and Hunter (2018) present a case study of two 10- year old students, to examine the students' participatory practices, evaluate their learning opportunities, and investigate the factors that promote or inhibit opportunities to learn. Students in this classroom are from culturally diverse backgrounds including Maori and Pasifika heritage. The two students who were selected had been identified by their classroom teacher as having challenges in mathematics. Findings from the study reveal that the barriers the students faced in class negatively impacted on their dispositions toward mathematics. In order to change the existing power and status structures, teachers need to facilitate patterns of participation to reposition students' status and provide opportunity to participate in class. Importantly, the notion that non-participation is a result of a student being shy or reluctant does not capture the complexity of the issue. Implications for teachers include the need to intervene to shift the status quo and create safe and equitable learning environments, which can lead to positive dispositions, a valuing of mathematics, and improved participation and achievement for culturally diverse students who are often considered to be underachieving in mathematics (Gibbs & Hunter, 2018).

4.5 Student Attitudes Towards Mathematics

Besides developing students' competencies in mathematical reasoning, research suggests that it is also important to focus on developing students' non-academic orientations such as their perceived value of a given subject, their beliefs in their ability to learn from successes and mistakes, as well as their resilience to persist through difficulty (Yeager & Dweck, 2012). In the Australasian context, there are a growing number of studies focusing on developing these values. For example, Muir et al. (2016) focused on student and advisor perceptions in a secondary school teaching quantitative reasoning (QR) through project-based learning. The students had been involved in this type of teaching for less than a year. A majority of students reported that they found doing QR problems stressful although responses indicated neutrality in terms of whether doing QR problems caused a sense of unease. The researchers suggest that these responses may be due to a pre-existing negative attitude towards mathematics with the neutral attitudes reflecting a willingness by students to suspend judgement given the only early experiences with this form of learning mathematics. Similarly, the advisors in the study referred to student tendency for mathematics avoidance and the negative association they had with the subject. However, other studies, such as Kalogeropoulos, Russo, Sullivan, Klooger, and Gunningham (2019) have highlighted the possibility of re-engaging mathematically-alienated students by developing a growth mindset through intervention. Through their Getting Ready in Numeracy (G.R.I.N.) programme, they were able to improve low performing students' affective, behavioural, and cognitive engagement.

In another study, Moala and Hunter (2019) explored the notion of resilience amongst 101 students from three low socio-economic, high poverty, and urban schools in New Zealand. Specifically, they focused on these students' responses to questions pertaining to learning mathematics in classrooms that involve participation in Developing Mathematical Inquiry Communities (DMIC). In these classrooms, teachers designed culturally and socially meaningful tasks, and implemented instructional practices that support respectful and mathematically-productive interactions. Their findings suggest that while students may have adopted a growth mindset in these classrooms, they did not like the hard parts of mathematics and were still developing in their capability to ask for specific help when stuck. These findings have two important implications for mathematics educators as they seek to develop resilience in their students. First, it is important that students should expect mathematics to be hard and they need to embrace challenges and ambiguities when learning mathematics. Second, they may need explicit instruction on how they can seek specific help when they are stuck. Given the importance of developing resilience in our mathematics learners, more needs to be studied and learned about the development of such learners.

5 Conclusion

The purpose of this chapter was to critically review Australasian research studies related to the development of innovative and powerful pedagogical practices in mathematics education between 2016 and 2019. Through the critique and synthesis of research literature exploring powerful pedagogical practices, this chapter provides a vision of what mathematics teaching and learning for the 21st century and beyond could encompass.

A key element threaded through this chapter is the need for systemic change in mathematics classrooms and wider schooling structures while maintaining a focus on social justice and equity (see Chap. 8, this Volume). One aspect that is evident through research studies related to the development of innovative learning environments for *all* students is the need for the facilitation of productive reasoned discourse. As demonstrated in the synthesis within this chapter, providing opportunities for students to engage in mathematical reasoning requires a shift of role for both educators and students. Teachers require support through the use of scaffolds, frameworks, and effective professional development to support their growth in understanding and attention to students' mathematical reasoning. There is also a need for careful consideration of how students can be facilitated to participate and engage in discourse in equitable ways. The chapter provides exemplars from the Australasian context of how this potentially can be achieved. The next step in this field of research is to develop scaled up reform approaches.

Developing innovative learning environments in mathematics education also requires attention to how educators can equip and empower students with 21st century skills. We identify this as a connective thread across research studies and highlight the importance of developing specific skills including creativity, critical thinking, and intellectual risk-taking for success in inquiry or problem-based learning environments. This also requires the reframing of mathematics as a process of creativity, inquiry and exploration. We also note as critical elements powerful, innovative pedagogical practices, and the need to foster both engagement and disposition of students (see Chap. 7, this Volume).

Tools are an essential component of the development of innovative and powerful pedagogies. Task design takes a significant role and this includes both the challenge of tasks and the contextual basis. Many researchers, both in this chapter review and in earlier work, note the need for cognitively challenging tasks. The use of meaningful contexts related to student experiences can be a way of engaging students and providing them with an entry point to a challenging task. A further view of the task context is that this can also be aligned with the development of students' competencies. Aligning the design of tasks to the development of students' competencies is non-trivial. To this end, teacher noticing has been demonstrated to be a critical pedagogical action when designing and using complex, challenging tasks. Ongoing research in this area highlights the complexity of developing teacher noticing of student thinking. Both planning of tasks and the development of enabling and extending prompts are highlighted as key ways to develop teacher noticing. Finally, representations are identified as key tools for students to model and represent their mathematical thinking. Of interest is the potential for representations along with skilful teacher questioning to position students to notice mathematical relationships and communicate these. Both the use of multiple representations and representations used in unconventional physical ways are shown to support student understanding of mathematics.

The final section of this chapter focused on the current and ongoing challenges of transforming mathematics classrooms into sites of innovation and powerful pedagogies. The challenges identified highlight the need for the mathematics education research community to continue to strengthen the translation of research into best practice within mathematics classrooms. Of importance is work that focuses on shifting teacher perspectives to strength and asset-based views. We call for studies which challenge the status quo of practices such as ability-grouping but equally importantly we note the need for studies which take a holistic approach to innovative and powerful pedagogies in mathematics classrooms rather than focusing on one aspect such as task design or student views. Additionally, we note the need for investigation of scaled up approaches to reform in Australasian contexts.

We end the chapter with a provocation in the form of an important question for the mathematics education research community of Australasia: how do we know that these practices are transferrable from one context to another? There is a critical distinction to make as we attempt to answer this question. That is, the "difference between knowledge that something can work and knowledge of how to actually make it work reliably over diverse contexts and populations" (Bryk, 2015, p. 469). Embracing this distinction will require us, as a community of mathematics educators, to explore and investigate ways to make these innovations work across different contexts. More importantly, we need to understand more deeply why these innovative practices work. Doing this may require us to adopt a different research paradigm. How this can be done will certainly be a fruitful line of research.

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Chapter 13
Teaching and Learning Mathematics with Digital Technologies

Catherine Attard, Nigel Calder, Kathryn Holmes, Kevin Larkin, and Sven Trenholm

Abstract This chapter provides a critical synthesis of research on technology-related classroom practice from the early years of schooling through to tertiary and initial teacher mathematics education. The synthesis considers ways in which research has explored the use of digital technologies through the three dimensions identified by Pierce and Stacey's map of pedagogical opportunity: mathematics content (subject), in classroom interactions (classroom), and through task design (task) (2010). The chapter also provides a synthesis of the research methodologies undertaken within the included studies. The review concludes with a discussion of emerging themes and a range of future directions for research into technology-related mathematics education.

Keywords Digital technologies · Digital tools · ICT · Mathematics · Tablets

1 Introduction

The use of digital technologies in the teaching and learning of mathematics continues to evolve, along with the increasing number and range of devices being used

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in mathematics education contexts and beyond formal education institutions in students' lives. Developments in digital technologies have led to a blurring of the boundaries between school and home learning. Digital technologies and blended learning approaches in schools and tertiary institutions have the potential to transform learning, and Australasian research in this field continues to emerge. Research investigating the influence of digital technology on learning, teaching, educational outcomes, and the delivery of curriculum content continues to be critical to ensure educators gain the maximum benefit of any potential affordances of digital technology.

In the previous Research in Mathematics Education in Australasia (RiMEA) chapter on digital technologies Geiger, et al., (2016) future directions for research were suggested, including a need for:

- Principles to assist teachers in discerning the quality of applications/software;
- Provision of insight in participation and management of interactions and relationships in online environments;
- Exploration of the potential of virtual worlds and technologies;
- Investigation of new forms of instruction such as blended and flipped learning approaches;
- Documentation of the role of digital technologies in pre- and in-service teacher education;
- Exploration of the notion of *futures* in terms of new digital tools; and
- Investigation of teaching approaches that leverage off the affordances of digital tools.

This chapter is a critical review of Australasian research conducted between 2016 and 2019 in relation to the use of digital technologies in mathematics education from early childhood though to tertiary education, including initial teacher education (ITE). Although some of the research directions suggested in the previous review have been undertaken during the current period, others have not. For example, there is only one study in this review that provides evidence of research exploring the potential of virtual worlds (Marshman, Woolcott, & Doyle, 2017) and there has also been little focus on technologies or investigations into participation and management of interactions and relationships in online environments. However, much of the research cited in the previous review has been built upon and expanded, such as the use of screencasting technologies to gain insight into students' mathematical reasoning and understanding (e.g., Murphy & Calder, 2017; Prescott & Maher, 2018), implying a move away from simply using digital technologies to build understandings of mathematical content as well as a shift away from students consuming content authored by others to students authoring their own content.

This review also sees the continued development of theoretical frameworks. Work by Lowrie and Larkin (2019) provides a heuristic for early years STEM learning that provides early years educators with guidance for using digital technologies. Research by Larkin (2016a) resulted in a proposed framework to assist teachers in determining the pedagogical potential of concrete and virtual manipulatives.

To build on the 2012–2015 RiMEA digital technology chapter Geiger, et al., (2016), we again consider the literature through elements of Pierce and Stacey's map of pedagogical opportunities (2010). However, in this review, we separate each stage of education into early years (Ages 0–8), school (primary and secondary—Year 3–12), tertiary, and ITE to allow us to compare and contrast the research being conducted within each stage of school and identify any gaps in the current research. Within each stage of school and ITE education we specifically explore the dimensions of *Tasks*, *Classroom Interactions*, and *Subject.* Through each of these dimensions we then interrogate recent research to seek themes relating to learners and learning, teachers and teaching, and classrooms. In our analysis of research conducted in tertiary mathematics education we take a slightly different approach in recognition of the differences between school and tertiary level education. To do this we briefly summarise research reviews found in our search. We then consider a range of studies directed at either improving pedagogical approaches or the creation and use of student resources. We then provide a critical synthesis of the research methodologies utilised within the studies cited in this chapter. The chapter concludes with an analysis of emerging themes and suggestions for future research.

2 Technology in Early Years Education

In this section we critique research that utilises digital technologies to develop particular mathematical content areas. According to Pierce and Stacey's pedagogical map, digital technologies can be useful in three sub-domains; however, the research conducted in this sub-strand in relation to early years mathematics only focusses on one i.e., rebalance emphasis on skill, concepts, and applications.

2.1 Subject

A strong emphasis of research involving mathematical subject areas in this period has concerned, as an umbrella term, spatial reasoning. Dindyal (2015) provides an important overview for the framing of research into spatial reasoning and makes the observation that, given children's exposure to a large number of technological devices, it is now normal and appropriate for teachers to use technology in teaching young children. This technology can be leveraged to "develop subtle ways of dealing with geometrical concepts and spatial reasoning at large" (p. 524). Dindyal also sounds two areas for caution: firstly, equity of access for various groups of young learners; and secondly to remind educators of the importance of effective planning in relation to the use of technology to support geometry learning.

The intersection of spatial reasoning and digital technologies in early years STEM is critiqued by Lowrie, Logan, and Larkin (2017), in a symposia publication. The three papers focussed respectively on: a) developing a learning program to promote children's engagement in STEM (Logan, Lowrie, & Bateup, 2017); b) the place of spatial reasoning in the early years (Lowrie, Logan, & Larkin, 2017); and c) using a design based approach to develop a mapping app (Larkin & Kinny-Lewis, 2017). In each of the papers the role of digital technology, in this case using custom designed apps for tablets, was prominent.

The symposia papers indicate that a focus on STEM practices, that include STEM ideas, methods and values, rather than developing integrated content-based learning experiences derived from the respective disciplines, is one that enhances student engagement with STEM, supported by digital technologies. Their pedagogical approach suggests that it is more important that young children actively use the iPads to develop mathematical understanding rather than passively using the iPads to achieve goals set by app developers.

As opposed to a stronger focus on the use of Interactive White Board (IWB) as a preferred technology reported in Geiger et al. (2016), and as an exception from the focus on spatial reasoning, Muir, Callingham, and Beswick (2016) investigate the impact of the use of an IWB on the learning of mental computation by students in Year One, and also on the pedagogy used by their teacher to assist students in their learning. Their findings indicate that the teacher demonstrated a high level of TPACK. This TPACK understanding resulted in learning experiences for students that were purposeful, individualised, and conceptually focussed. The learning was supported via the technology because it supported the teacher's content knowledge (via online resources) and subsequent levels of personal confidence, as well as encouraging a sequential set of instructions that were flexible and responsive to student learning needs.

2.2 Classroom Interaction

This section addresses research related to the impact of digital technologies on Early Years learning environments. In terms of the two sub-dimensions of the Pierce and Stacey pedagogical map i.e., social dynamics and didactic contract, the emphasis of these articles leans more heavily towards the social dynamics element; however, there are some implications in terms of how tablets also become tools that change the didactic contract. Pierce and Stacey provide a full description of social dynamics and didactic contract, for our purposes, social dynamics refers to changes in the classroom environment –e.g., paired work on an iPad or group work on a digital table and didactic contract refers to the role of technology in becoming, alongside the teacher, a new authority for learning and teaching –e.g., tutor type software.

A body of research investigated the impact of the use of digital technologies, in this case iPads, in impacting classroom interactions to enhance student learning. Calder (2017) suggests a number of consistent outcomes from research conducted across a number of projects regarding student led, data collection and interpretation activities. Firstly, the iPads could collect data in formats not possible via pen and paper—visual, audio etc. Secondly, the mobility of the devices allowed for immediate, in situ data collection and early interpretation. Thirdly, the devices facilitated the communication of findings to peers and teachers. Calder (2017) also reports of improved attitudes towards mathematics more broadly as a consequence of using the tablets. Furthermore, iPad screencasting was found to enable students to represent their solutions to a problem involving division with remainder (Murphy & Calder, 2017). The authors report that the screen casting app enabled the recording of multiple modes of communication (drawings, downloaded images, mathematical symbols, spoken and written language), and thus assisted the students to clarify their thinking and allow teachers to gain further insight into students' thinking and identification of misconceptions.

In a largely conceptual article, a heuristic—Experience, Represent and Apply (ERA) was used to propose a new way of using digital technology in preschool to support the learning of STEM (Lowrie & Larkin, 2019). This includes early number and spatial reasoning experiences including sorting, patterning, position and location language, perspective taking, encoding and decoding, debugging, and classifying. They suggest that Experience [E] is what children already know about a STEM concept, based on their existing social and language experiences; Represent [R] occurs when children use apps to engage with, and then represent, various STEM concepts. These representations include creating images, interpreting pictures, visualising and using symbols; Apply [A] activities occur where children build on their learning through a range of off-app activities, guided by their educators and their families.

The conceptual work of this paper forms the underpinning of a 2017–2019 project where the ERA framework was deployed in the context of STEM practices and provide data from approximately 400 educators and 4000 children, in over 100 centres Australia wide (Lowrie, Logan, & Larkin, 2019). The data indicates that the ERA heuristic was instrumental in assisting educators to embed STEM in play-based learning environments (Lowrie et al., 2019).

2.3 Tasks

The use of coding to support the development of digital representations of spatial concepts was the focus of research by Miller and Larkin (2017). Somewhat novelly for school based research, both intervention and control groups were established in a six-week coding and robotics teaching experiment with Year 2 students. Their aim was to explore how students developed mathematical knowledge and thinking as they participated in lessons using Scratch Jnr on desktop computers. The authors were seeking to determine how coding in primary school classrooms could support, or provide opportunity for, the learning of mathematics and in particular develop the proficiencies of problem solving and reasoning. Miller and Larkin indicate that limited research has been conducted on whether coding and robotics provides opportunities to develop early algebraic thinking. Following a small-scale intervention, their findings indicated that some students demonstrated higher levels of mathematical thinking than "required" in Year 2 by: working with 90 degree turns; demonstrating perspective taking abilities; and deducing a repeating pattern to provide a generalised code for making a square. Miller and Larkin claim that these findings are an early indication that coding and robotics may support children to identify and deduce patterns, an established precursor for more sophisticated algebraic thinking.

3 Primary and Secondary Education

This section reviews the research literature related to technology use in the primary and secondary school years. Most literature included in this section focussed on the primary school setting, revealing a distinct lack of research attentive to the specific needs of the secondary mathematics classroom. In terms of mathematical content, the majority of studies interrogated technology as it is used to promote geometric understanding however some dealt with other mathematics topics. Pedagogically, the key role of the teacher as an informed decision maker was emphasised, with greater emphasis placed on *how* teachers decided to use technological tools, rather than *which* tools they chose. The power of technology to make students' thinking observable was also noted, along with its potential to foster collaborative practices among students. In terms of how classroom tasks were impacted by technology, the notion that technology adds 'fun' elements to learning was critiqued and the need for teachers to have guidance in choosing technological applications was examined.

3.1 Subject

The technology used for teaching and learning in mathematics can vary according to the mathematical content under consideration. In line with previous reviews Geiger, et al., (2016); Geiger, Forgasz, Tan, Calder, & Hill, (2012) we examine research focussed on technology used to promote learning across a range of mathematical content areas. Although we find examples of research studies on a range of mathematics topics typically covered in the primary and secondary school years, there is a greater proportion of studies examining the role of technology for promoting geometric or spatial understanding.

A number of studies focussed on the pedagogical value of iPads in the primary classroom, particularly in relation to the teaching of geometry. Larkin, Kortenkamp, Ladel, and Etzold (2019) draw on Artifact Centric Activity Theory (ACAT) to describe an evaluative technique which enables teachers to evaluate the pedagogical potential of apps. The evaluative method described is applicable to both primary and secondary teachers and considers teaching as a network involving students, mathematical content, apps, the mathematical content within the apps, and the classroom context. The pedagogical value of any particular app is therefore dependent on a range of factors, not least the specific mathematical content under consideration. Given the large number of geometry apps available, Larkin (2016b) proposes a process for evaluating their educational potential for the primary classroom based on Dick's (2008) three measures of fidelity (cognitive, mathematical and pedagogical) and in relation to the *Australian Curriculum: Mathematics* finding that app quality is highly variable, with many apps only enabling student to 'trace' geometric shapes, rather than consider their geometrical properties.

Although most studies focussed on technology use for the teaching of geometry, Gorman and Way's (2018) study of six Year 4 students investigated the potential of technology to illuminate concepts related to number, and specifically decimal fractions. In this study a 'zoomable' number line was employed as the basis for student exploration of decimal fractions. The students were interviewed post-exploration and encouraged to share their thinking as they undertook the lesson. The students revealed knowledge of decimal density, whole number thinking, and the role of place value in understanding decimals. Their interviews exposed the cognitive conflict experienced during the investigation with the dynamic number line and the resulting 'self-correction' in reasoning which ultimately occurred. The authors point out that many 'teachable' moments occurred during the investigation highlighting the potential of this interactive tool for whole class instruction.

Finally, the links between computational thinking and mathematics have been explored. Calder (2018) examined the ways in which 10 year-olds engaged with mathematical ideas using *Scratch* for coding. There was evidence that the students improved in their spatial awareness, understanding of angles and positioning of coordinates. On a broader level, the links between computational thinking and mathematics were explored by Hickmott, Prieto-Rodriguez, and Holmes (2018). In a comprehensive scoping review they examined the literature base in relation to how computer programing can foster mathematics learning. They found that the research in this space was generally conducted by computer science academics rather than researchers with an education background, and that it was generally small in scale, focussing more on programming than the mathematics concepts involved. They also reported a relative lack of studies involving statistics, probability, functions and measurement, in comparison to number and algebra.

Although there is a clear focus on geometrical and spatial content in the research examined for this review, it is also clear that much emphasis is placed on the centrality of the teacher as the key decision maker in relation to the choice of technology, mathematical content and pedagogical approach. These decisions were the focus of one study (Loong & Herbert, 2018) which examined how two teachers made such decisions in their primary school classrooms. In this study, the authors examined the teachers' decisions using the SAMR (Puentedura, 2006) and TPACK (Mishra & Koehler, 2006) frameworks. They examined the teaching activities chosen by the teachers through the SAMR lens, leading to conclusions about the teachers' TPACK developmental stage. They surmise that teachers need advanced levels of TPACK to move to the enhanced or transformative levels described by the SAMR framework.

3.2 Classroom Interaction

The research reviewed in this section focuses on how the use of digital technologies plays out in primary and secondary classroom learning environments. As with research presented in the previous section, most of the research focussing on classroom practices involving technology was conducted in primary schools, signalling a need for more studies in secondary schools. Several studies conducted in primary school investigated the potential for digital technologies to enhance engagement with mathematics through a variety of mechanisms.

Across a series of papers Calder and Murphy (2017, 2018a, 2018b) explore how apps can be used by teachers in the primary classroom to promote mathematical understanding and engagement. They highlight that the affordances of particular apps are less important than the pedagogical decisions that the teacher makes when employing the app in the classroom (Calder & Murphy, 2018a), however, they do find that some app affordances are effective for promoting engagement and mathematical understanding. Apps that facilitate simultaneous screencasting and voice-recording created by the students themselves were seen to provide a new dynamic learning environment increasing student engagement (Calder & Murphy, 2018b). This work was further extended through the introduction of the idea of an assemblage as a means to understand the interplay between social and technical entities as apps are employed in the classroom (Calder & Murphy, 2018c). In this study the use of screencasting was viewed as a means of encouraging collaborative ways of working in the mathematics classroom, stimulating the contestation and validation of mathematical ideas and processes. Prescott and Maher (2018) also examined the use of screen casting, in particular focussing on *Explain Everything* and *Educreations*, as a means of allowing students to think mathematically. The study revealed the capacity for screen casting to facilitate teachers' formative assessment strategies by making students' thinking visible and by allowing students the opportunity to critique each other's work. This type of collaborative work within the mathematics classroom was found to increase student engagement. Similarly, in a study of 11 primary and secondary mathematics teachers, Ingram, Williamson-Leadley, and Pratt (2016) found high student engagement when using a 'Show and tell' app. The authors surmise that the act of making 'mathematical thinking visible' resulted in rich mathematical discussions about problem solving.

The concept of student engagement in mathematics through technology use in primary schools was directly addressed in three publications (Attard, 2018; Hilton, 2018; Orlando & Attard, 2016). Attard (2018) synthesises the results from three qualitative studies examining mobile technologies employing classroom observations, student focus groups and teacher interviews. Using the Framework for Engagement in Mathematics (FEM) (Attard, 2014) as a lens, she concluded that mobile technologies such as iPads do have the potential to improve student engagement in mathematics. However, this is not universally the case. The degree to which engagement is improved depends heavily on the pedagogical practices that embed their use, rather than the technological devices themselves. In a larger study in Queensland, Hilton

(2018) examined the impact of iPad use in Years 2–6 including the use of control group classes without iPad*s*. The study, which also employed Attard's (2014) FEM, found evidence that the use of iPads in mathematics had a positive influence on student engagement, enjoyment and self-perceptions, particularly for boys.

In addition to studies focussing on student engagement, Orlando and Attard (2016) conducted a study focussed on early career teachers and their use of mobile technologies for teaching mathematics. This paper reported on three related case studies of teachers in both primary and secondary school settings. The study questioned the untested but widely accepted myth that early career teachers should be well-versed in emerging technologies and may even be able to act as technology leaders in schools. The study also concluded that the tendency for early career teachers to be viewed in this way can be detrimental to their development as teachers.

Two studies (Willacy & Calder, 2017; Willacy, West, Murphy, & Calder, 2017) focussed on students in primary and lower secondary schools. Using a case study approach with four students age 11–13 years, the researchers found that three of the four students did experience increased engagement when apps were introduced into their mathematics classroom (Willacy & Calder, 2017). Taking a broad view of the factors influencing technology use, the study revealed three inter-related themes, which need to be considered to enable positive engagement through the use of technology: individual student aspects; pedagogical aspects; and societal aspects. The potential for mobile technologies to enable personalised approaches to learning was examined in the second study with upper primary school students (Willacy et al., 2017). As with other studies examined in this chapter, the key role of the teacher, rather than the technology itself, was emphasised. The degree to which personalised learning was achieved depended heavily on teacher decisions with regard to the extent of teacher direction, customisation features, work places and student-led learning.

3.3 Tasks

In this section we examine how technology can impact on the tasks that teachers use for teaching and learning. Kawka and Larkin (2018) question the notion of 'edutainment' in relation to apps developed to engage young students in mathematics. They examine the integration of popular culture, fantasy and 'fun' elements into apps designed to educate children about mathematics. Interestingly, they reveal that many of the fictional contexts within the apps have little meaningful connection to the mathematics that the children are meant to learn. The authors question the notion of 'fun' as a useful construct for learning mathematics and caution against choosing apps for their 'fun' value, which can be construed as a significant distraction from the mathematics content.

Ratnayake, Oates, and Thomas (2016) investigated how 12 teachers, working in groups of three, used digital technology to develop and implement algebra tasks for secondary mathematics classrooms. They determined that a range of factors assisted the teachers as they worked collaboratively to design suitable tasks, including having a clear focus on the mathematics content, proficiency with digital technologies, clearly set goals for the lessons and considered student thinking and potential difficulties.

In summary, the research on digital technology use for mathematics teaching and learning in primary and secondary classrooms is heavily skewed towards primary school settings, indicating a gap in ongoing knowledge development in secondary schools. This is potentially concerning as there is a significant cost for parents and schools in providing access to digital technologies for learning in secondary schools and yet the educational benefits have not yet been widely evaluated by researchers. Also, there is evidence that many mathematics topic areas are not being taught with technology, and that there is a clear preference for geometrical and spatial content. While this is not necessarily of concern, it does highlight that digital technologies are possibly under-utilised in the mathematics classroom and that their potential is yet to be fully realised. In general, there was little research on 'task' development and use. While pre-developed apps were popular in primary settings, there was only one study demonstrating how secondary teachers might collaborate to develop digital tasks for secondary students. Interestingly this study highlighted the importance of teachers having well-developed technology skills, emphasising the role of TPACK as an enabler for effective use of digital technologies for teaching and learning mathematics.

4 Initial Teacher Education

As was the case in Geiger, et al. (2016), there continues to be a lack of research on particular subject areas of mathematics in ITE. Although a number of the research articles discussed here include mathematical content e.g., Fractions and Division (Handal, Campbell, Cavanagh, & Petocz, 2016); data collection and interpretation, (Geiger, et al., 2016); or Geometry (Larkin 2016b)—this content is the vehicle supporting changes to either Classroom Interactions or Tasks in mathematics education—rather than being the focus of the research.

Almost all of the research conducted in this review period has focussed on the Classroom Interaction aspect of the pedagogical map. As in Geiger, et al. (2016), we take a broad perspective of what constitutes a "classroom". Given the rise of blended and fully online mathematics education in tertiary institutions, this is appropriate and timely; indeed, each of the research projects discussed in this section incorporate online learning experiences of one kind or another. The articles in this section are classified as to whether the primary focus was on how digital technologies change the didactic contract or the changed social dynamics that resulted from their use.

4.1 Classroom—Didactic Contract

A major focus within this theme concerns the pedagogical opportunities that arise when digital technology changes the didactic contract between lecturer and students.

The use of Mathcasts, defined here as screen recordings of explanations of mathematics concepts, was trialled by Galligan, Hobohm, and Peake (2017) to determine their usefulness in supporting pre-service teachers (PSTs) to learn mathematics. In this project, using a framework designed by the authors, the PSTs created their own mathcast demonstrating how they intended to teach a mathematics concept to primary school students. Galligan et al. (2017) report a number of interesting findings. Firstly, the PSTs found the framework useful as it helped them focus on purpose and context, structural elements such as visual quality, clarity and fluency of delivery, and their own developing PCK. Secondly, the creation of a mathematics artefact engaged students in the learning process and consequently improved their personal understanding of mathematics (CK). Finally, the mathcasts become a resource that PSTs could use in their future mathematics classroom practice.

Although using a different technology, in this case online modules, an argument is made by Geiger, et al. (2016) that mathematics should be taught as it is practised, i.e., as a dynamic inquiry into the nature of real-world phenomena. The conference paper is part of a broader project that sought to enable undergraduate PSTs to experience mathematics and science in a similar manner to how mathematicians and scientists practise it beyond the confines of the classroom. Here the authors provide insight into the process of developing one online module—*Modelling the present: Predicting the future*—to better understand how collaboration between mathematicians, scientists, mathematics educators and science educators can be utilised in designing an online learning module with a focus on mathematical modelling. The authors report that the creation of online modules provided opportunities for PSTs to contextualise mathematics in real world contexts, where mathematics is central to both understanding and solving the problem scenario. The use of digital tools (spreadsheets, online tools and resources and short explanatory videos prepared by mathematicians) was critical in this endeavour. As students were engaged in authentic mathematics activities using real contexts and real data (collected online), an important by product of this creative approach was enhanced positive dispositions towards mathematics.

(2016) Although still using iPads, and still concerning changes to the didactic contract, a different approach to technology use is critiqued by Galligan, Hobohm, and Peake (2017). Handal, Campbell, Cavanagh, and Petocz (2016) examine the role of mathematics apps in providing a "technology that is ubiquitous in nature, highly portable and endowed with multimedia capabilities offering a new dimension to curriculum making learning accessible 'anywhere, anytime'" (p. 200). However, the content validity of the apps is not always clear. These authors created a tool, which was used by PSTs, to measure the content validity of mathematics apps. The findings of this research were mixed: on one hand, students found the apps useful for their future teaching and could see that, once the instructional role for the apps was clarified, their use could create rich learning experiences; on the other hand, most

PSTs could not determine differences between four TPACK constructs—TPACK, PCK, TPK and TCK in relation to the apps and thus more work needs to be done in assisting PSTs to clearly understand the different pedagogical uses for apps.

4.2 Classroom—Social Dynamics

The next section of the review examines a series of articles (Larkin, 2016a, 2017a, 2017b) outlining ongoing development of two large, cross campus undergraduate mathematics courses offered in blended and face to face modes. Although each publication takes a slightly different perspective on the use of digital technologies to support mathematics learning, each focuses on classroom dynamics (in this case the classroom includes online interactions and face to face lectures, workshops and tutorials), and how these dynamics can be changed via the use of digital technologies. Thus, they will be considered here as one, extended research project.

The overarching conceptual framework for the series of research publication is Transactional Distance Theory (TDT). According to Larkin, TDT is a theory with its origins in the work of Moore (1993) in distance education, that suggests modifications to three core classroom elements (structure, dialogue and student autonomy) are critical in student learning, especially when this learning includes asynchronous interactions. In moving from a face to face to blended course, Larkin (2016a) made changes to the allowable 'three hours contact per week' using a $(1 + 1 + 1)$ model which consisted of a one hour online pre-recorded lecture with a focus on theories of mathematics learning; a one hour weekly workshop with a focus on demonstrations of appropriate language, materials and symbolic representations; and a one hour tutorial that specifically enacted MPK in various teaching scenarios. Larkin (2016a) reports that feedback from students indicated that they appreciated the flexibility of the online components that complemented, but did not replace, some of the face-to-face components of the course.

The $1 + 1 + 1$ was, by and large, a structural change but did not account in detail with the student experience of the model in terms of student engagement, supported by digital technology, in blended, online environments. In a subsequent article Larkin (2017a) argues that the integration of digital technologies into the existing university digital architecture is important for the uptake of these technologies as PSTs resist changing platforms to access content. The primary research contribution of this project was the choice of digital tool (i.e., Desktop Capture) for the delivery of content. Whilst studio-recording labs were available and encouraged by the university, Larkin (2017a), based on the research of Hibbert (2014) and Popova, Kirschner, and Joiner (2014) made the pedagogical decision to record the lectures using Desktop Capture in his office. Feedback from PSTs (Larkin, 2017a) indicated that they valued the sense of familiarity and the relaxed tone of the office-based desktop captured lectures, and this encouraged their engagement with them. In addition, the delivery of much of the theory in the online lectures had the added benefit of "freeing up" the face to face components to be much more interactive with small group activities and teaching demonstrations being the normal pattern of lecture delivery (Larkin, 2017a). The final component of the project was a study that sought to determine the impact of video or no video of the lecturer in the delivery of online lectures on the experience of PSTs. In this study (Larkin 2017b), the PSTs engaged with online lectures that were modified to include accompanying video of the lecturer for the entire lecture, only the start of the lecture, or no video at all. The small pilot study found that the PSTs overwhelming preferred to see the lecturer during the entire online lecture and that this deepened their engagement with the course.

4.3 Tasks

We conclude this section with a discussion regarding three research projects investigating the impact of modifying tasks to support ITE learning of mathematics. Two of the three relate to the task dimension of simulating real situations; however, one is from the perspective of teachers and their pedagogy and the other from the perspective of learners and their learning. The third relates to the use of online challenging tasks.

Digital Learning Objects (DLOs) are becoming more common as many university mathematics education courses are expanding their footprint in the online space. The reasoning of PSTs in relation to how and why they selected digital learning objects (DLOs) when planning to teach mathematics was investigated by Hawera, Sharma, and Wright (2017). These authors indicate that PSTs were positive in their intentions to use DLOs in their teaching of measurement as they found them likely to provide opportunities for children to access, construct, review and consolidate mathematical thinking and also help children to understand measurement concepts and/or formula. Hawera et al. (2017) suggest that one implication from their research is that tertiary mathematics education courses should provide PSTs with ample time to explore the use of the DLOs in their pedagogical practice, thereby supporting the development of student TPACK expertise.

A second technology beginning to be more broadly utilised in Higher Education is the use of simulations. Similar to the Hawera et al. (2017) paper, the research by Marshman, Woolcott, and Doyle (2017) focussesd on a task that simulates real experiences; however, the focus for these researchers was much more closely aligned to the perspective of learners and their learning (albeit the learners are PSTs). The paper investigated whether immersive technology (in this case CAVE2TM—a 3D, full body experience) could support their developing spatial thinking. In this study, learning experiences for PSTs that centred on spatial reasoning were provided; including an examination of both learning (as understandings) and perceptions. The findings from Marshman et al. (2017) were mixed. On the positive side, the immersive experience was an engaging one for PSTs and it encouraged increased collaboration with peers. Whilst the 3D spatial environment was initially confronting, PSTs generally found ways to utilise this unique resource and think about their personal spatial reasoning competence. On the negative side, some PSTs continued to have difficulty reconciling the 3D reality of objects and the various perspectives from which these objects could be viewed, perhaps because their previous experiences of shapes and objects were largely (un)developed based on 2D experience with school-based geometry. In addition, some PSTs expressed confusion regarding their spatial understanding of parallel lines and perspective—given that in the 3D representations the parallel lines appeared to meet. Overall, as spatial reasoning is malleable, Marshman et al. (2017) argue that it is vital PSTs are given opportunities to improve their spatial thinking and reasoning skills as part of their ITE and are encouraged to continue developing these skills.

The third of the research papers in the task component investigated, using Expectancy-Value Theory (EVT), how challenging tasks could be used to improve ITE students mathematics capability (Fielding-Wells et al., 2019). The authors used EVT as it provided them insight into the engagement potential of a task by identifying individual's motivational influences, according to two constructs—expectancy of success and valuing of task. Their findings suggest, in order for ITE to be competent and confident in teaching mathematics using challenging tasks, they need to overcome the notion that mathematics is only procedural, they need to have their own learning about challenging tasks scaffolded, and the value and utility of challenging tasks needs to be made more explicit.

5 Use of Digital Technologies in Tertiary Mathematics

In this section we review the research literature related to the use of digital technology at the tertiary level in non-ITE contexts. There is an established and growing body of literature that contrasts the different experiences, both in teaching and learning, of school versus tertiary level mathematics education (see Clark & Lovric, 2009). In partial recognition of these differences, we depart from the structure set out in the previous sections of this chapter. We instead divide the research into two broad sections: First, as a distinguishing characteristic of the current iteration of the quadrennial review, we briefly summarise five research reviews. Second, we consider a range of studies, most involving the use of video, directed at either improving pedagogical approaches or the creation and use of student resources. Overall, we found 19 studies involving the use of digital technologies in tertiary mathematics education, a significant drop from the last review where 35 were found. Most (13) of these studies were published in peer-reviewed journals with the remaining found in conference proceedings (4) and book chapters (2). Once again, the subject context for these studies reflect an ongoing focus on first-year undergraduate mathematics teaching and learning.

5.1 Research Reviews

Perhaps most noteworthy in this quadrennial review are the number of “stock takes”, to use Oates’ (2016) expression, related to the use of digital technology in tertiary mathematics. Just over one quarter of the papers examined the state of the field or a subfield, most systematically. Compared to past quadrennial reviews, this appeared to be somewhat of an anomaly which suggested to us a level of maturity in the field as several researchers reflect on past studies, gauging the state of various subfields of digital technology usage at the tertiary level. Compared to the last review, many of these studies continue to suggest, implicitly and explicitly, that a gulf continues to exist between the promise and actual benefits of using digital technologies (see Laborde & Sträßer, 2010).

There is a common theme concerning interventions proposed to bridge this gulf. Lake et al. (2017) and Thomas, Hong, and Oates (2017), for example, focus on ‘innovations’ enabled using technology. One theme Thomas et al. (2017) emphasise is teacher agency: teachers’ choices are critical to the successful implementation of digital technologies. Similarly, regarding the use of Computer Algebra Systems (CAS), Tobin and Weiss (2016) argue a new curriculum is needed to take advantage of CAS, rather than simply adding CAS to the current curriculum. An example is provided by Ponce Campuzano et al. (2019) in this review relating to teaching Vector Calculus with GeoGebra. Finally, Trenholm, Peschke, and Chinnappan (2019), investigating the state of fully online mathematics instruction through the lens of large-scale research, found this modality of instruction is not working well compared to either face-to-face instruction in mathematics or to fully online instruction occurring in other disciplines. They suggest more pedagogical (not just technological) innovations are needed, which they argue both face-to-face and fully online teaching may benefit from. Overall, these reviews draw attention to the human element associated with effective use of digital technologies as a tool for teaching and learning.

Related to the role of the teacher, a recurring issue identified in the research, which is challenging successful innovation, concerns how to communicate mathematically in digital technology-enabled mediums. Several researchers have raised concerns about the constraints these technologies place on the effective communication of mathematical language, syntax and symbolism, critical to interactions and, ultimately, successful task completion in mathematics (Maclaren, Wilson & Klymchuk, 2017; Tobin & Weiss, 2016; Trenholm, Alcock, & Robinson, 2016). For example, as input devices, the qwerty and mouse are used differently with different software packages and tool pallets. This increased cognitive load may add to an already challenging subject to learn and does not compare with the familiarity of freehand writing of mathematics on paper or chalk/whiteboards. Such challenges appear indicative of ongoing struggles to reorient the nature of learning mathematics (see Abrahamson & Sánchez-García, 2016).

5.2 Teacher Pedagogy and Student Resources

The one technological tool that continues to challenge the current boundaries of educational practice in tertiary mathematics (and more broadly) is video. The growing accessibility and ubiquity of video technology have provided teachers and students with the means of producing and editing quality videos for use in teaching and learning. Not counting reviews, just over half (8) of the studies focussed on some aspect of video use. Of these studies, three focussed on tasks using student-created videos, suggesting a growing area of pedagogical innovation and a need for further research (Dunn, Loch, & Scott, 2018; Galligan, Hobohm, & Peake, 2017; Loch & Lamborn, 2016).

Most studies of video use relate to the delivery of direct instruction. Studies on the use of screencasts (Dunn, Loch, & Scott, 2018; McLoughlin & Loch, 2016) and recorded lecture videos (Tisdell & Loch, 2017; Trenholm, Hajek, Robinson, Chinnappan, Albrecht, and Ashman, 2019) reflect a continuing interest in exploiting this technology for the delivery of tertiary mathematics instruction. Although most research suggests students are generally satisfied with this form of instruction (either as a supplemental resource or a replacement for live lectures; e.g., Trenholm, Alcock, & Robinson, 2012), Trenholm, et al. (2019) found 'regular' recorded lecture video use associated with increased measures of surface approaches to learning. This quasi-experimental pre- and post-test study design, using validated scale measures, provided some needed insight into learning processes around the use of video in teaching and learning tertiary mathematics. Further directions for research include measuring the effect of interactive activities placed at specified points in recorded lecture videos.

As identified in the last quadrennial review, more insight is needed into these processes as they relate to the use of digital technologies. Currently dominant are qualitative methodological approaches using, generally, thematic analysis, with surveys by far the favoured research instrument. Like the last review, most questions interrogated students' perceptions of their learning experience. Without diminishing the important contribution of these efforts (or the use of qualitative research approaches), future research might consider more quantitative approaches.

At the tertiary level, some of this research may be done using log and administrative data, such as "click" data which may be culled from Learning Management Systems. These approaches, now commonly falling under the umbrella of learning analytics research, were evident in our review. Of all the quantitative research approaches we found, most (4) used administrative and/or log data in combination with other data (Johnston, 2017; Quinn, Hajek, & Aarão, 2017; Tisdell & Loch, 2017; Trenholm, et al., 2019). The relatively small number of related studies suggests this research approach is still in its infancy in this field. Notwithstanding current challenges around consent and ethics (e.g., Slade & Prinsloo, 2013), this is an area for potential further development, not least given the capacity for data production and collection associated with the use of digital technologies. Alongside current dominant qualitative approaches, this work may help to clarify some of the complexities associated with the use of various digital technology tools in tertiary education.

Our review of literature related to tertiary mathematics has highlighted a number of strengths and gaps in the research. For example, research into the use of video continues to be an area of strength, though many questions remain. The first-year experience, particularly related to engineering mathematics, remains a steady focus, though little research appears to be targeting mathematics subjects taught in later years or even at the graduate level. Of particular note, more than one quarter of the studies we found were reviews, which suggested to us a level of maturity in this area of research in Australasia.

6 Current Methodological Approaches in Technology Research

In a move away from previous chapters reviewing digital technologies, we now shift our focus to explore the methodologies undertaken in the research reviewed above. An understanding of methodological approaches will provide further insight into future directions for research and the ways in which the research might be conducted.

Although digital technologies and their corresponding use in mathematics education are evolving, some degree of constancy and rigour is beginning to settle over the methodologies utilised to examine the field. The range of lenses used is still eclectic with each distinctive in nature, yet within each of these distinctive methodologies, most are consolidating their approach, enhancing validity as more studies use them. Some are hinged to elements that might centre predominantly on a curriculum area or application. For instance, instrumental orchestration (Trouche, 2004) is frequently used in CAS and dynamic geometry research; however, only one of the recent Australasian studies have used it, i.e., Thomas, Hong, and Oates (2017) in their examination of first year mathematics students' use of digital technologies. Other methodologies relatively prevalent in the field of using digital technologies in mathematics education are: design-based; socio-cultural, such as semiotic mediation; interpretative phenomenology and action research.

This range of methodologies enables us to examine the field in a more critical way. If comparable processes, affective aspects, and/or conceptual thinking are situated in similar contexts, but examined through the varying lenses of differing methodologies, they might open up varying perspectives and insights. Having this range of perspectives and insights allows critical comparison of consistencies or tensions between the studies. Likewise, critical analysis can be applied to studies that are situated in different contexts but use the same methodology to examine the research questions.

Importantly, it is the research questions themselves that predominantly drive the selection of the methodology and research design. Although usually considered as research designs, strong arguments have been made for mixed methods (e.g., Creswell & Creswell, 2018) and case studies (e.g., Yin, 2014) to also be considered as methodologies. Hence, these are included in this synthesis. As well, the on-going development of digital technologies has opened up opportunities for new ways to generate data to answer research questions in the field, and new methodologies have accompanied these emerging approaches. The consideration of each methodology will be illustrated with examples from the associated literature already discussed in this chapter.

A case study is an in-depth examination of a particular case, with its focus often a contemporary phenomenon within a real-life context (Yin, 2014). Case studies can be employed to investigate phenomena that are collaboratively designed by multidisciplinary teams. For instance, the processes utilised between mathematicians, scientists, mathematics educators and science educators in designing an online learning module for mathematical modelling (Geiger et al., 2016). Their process included three phases: initial case study development; case study review; and the linking of case studies. Reporting on a case study from within a larger action research project, Muir, Callingham, and Beswick (2016) examined a case where the aim was for teachers to enhance their teaching practice through the use of IWBs. The single lesson case gave particular nuanced understanding of how to make effective use of the features of the technology to engage students and maintain their interest, encourage participation, and demonstrate particular mathematical strategies and skills.

In a case study of a Regional Health Schools outpatients' engagement in mathematics learning, Willacy and Calder (2017) reported on four teenage students' use of apps in their learning, with the case study methodology revealing insights into ways to keep students engaged when working in situ at home. Comparative case studies can reveal more fine-grained insights as the differences and similarities between the cases are compared. An example of such a comparison highlighted the complexities of primary-school teachers' use of digital technology through two case studies (Loong & Herbert, 2018). This allowed them to investigate their research questions with the rich and varied generation of data around particular situations. As well, comparative case studies were undertaken with: Pre-service teachers' (PSTs') using a blended-learning approach (Larkin, 2016a, 2017a) in which he reported the impact changes to delivery mode (online, blended, and face-to-face) had on experiences of the course; the comparison of three studies to answer questions related to student engagement when using digital technologies in mathematics teaching and learning (Attard, 2018); the examination of whether using apps motivated reluctant learners in three different locations (Calder & Campbell, 2016); and the analysis of early-career teachers' experiences of using digital technology to teach mathematics (Orlando & Attard, 2016) which indicated across varying contexts, that teaching with technology is different to using technology.

While examining the case of a primary school that used screencasting apps on mobile technologies to produce "create-alouds" Prescott and Maher (2018) indicated that the approach provided school-wide insights into the opportunities for the teachers to explore collaborative tasks and formative assessments. In a similar way, Galligan and Hobohm (2018) used a case study to examine tertiary students' use of an evaluative tool to develop effective maths-casts. Meanwhile, a sequential case study was used to consider a flipped classroom approach with a numerical methods course (Johnston, 2017). As well as the depth of insight that the data revealed, the sequential approach enabled an ongoing development of instruments, each informed by the previous.

Many of the studies in this chapter used variations of sociocultural research methodologies. Underpinned by Vygotsky's (1978) participatory theories, the sociocultural lens is concerned with perceptions and interpretations that are imbued with the cultural and historical discourses from which they emerge. The concern for participants' connections and perceptions of their lived-in worlds, e.g., classrooms, has led to their manifestation in various forms of educational research. Several studies reported research viewed through a sociocultural lens: in a study of the ways primary-school teachers use "show-and-tell" apps in their mathematics teaching (Ingram, Williamson-Leadley, & Pratt, 2016); with the examination of six primary children using decimals on an interactive number line (Gorman & Way, 2018); and with examining the video-recordings of seven-year-olds using a screencasting app to explain their solutions of a simple division problem (Murphy & Calder, 2017). An interpretative methodology was also used to derive a version of socio-technical assemblage from the collaborative analysis of teachers using a range of creative apps, materials and the associated social elements (Calder & Murphy, 2017); and with Willacy, West, Murphy, and Calder (2017) in their investigation of personalisation and differentiation when using MT. Using a VR "e-cave" Marshman, Woolcott, and Dole (2017) investigated pre-service teachers' experiences with immersive technology and whether their reflections on their 3D thinking and reasoning abilities supported spatial thinking when developing learning activities. These, and others such as Oates' (2016) personal reflection, were able to analyze participants' reasons and motivations and gain fine-grained insights within the specificity of a particular situation.

The aim of mixed methods methodology is to use multiple methods, data sources, and analytic approaches to better capture the breadth and depth of complex phenomena and enhance understanding, with one data strand intersecting with, adding to, and making meaning for the other (Creswell & Plano Clark, 2011). In the examination of the influence of teaching and learning mathematics with iPads on students' engagement and attitudes to mathematics, Hilton (2018) combined surveys of over 400 participants in the first year, pre and post-intervention surveys in the second year, and semi-structured interviews with small focus groups, in the third year. The weaving of these methods enabled in-depth analysis of the complex phenomena, with each method opening up space to enhance the understandings of the other. Survey and focus groups were undertaken in a similar approach to examine Stage II mathematics students' perspectives of a flipped-classroom approach to lectures (Novak, Kensington-Miller, & Evans, 2017). Likewise, research such as Hawera, Sharma, and Wright's (2017) study of how PST's how can best be supported to use digital technologies for mathematics teaching also used a mixed methods approach, employing pre and post-intervention testing and video-recorded observation to explore how students developed mathematical knowledge and thinking as they participated in coding and robotics lessons. In another mixed methods study undertaken in engineering mathematics tertiary classes, McLoughlin and Loch (2016) explored the role of screencasting in scaffolding flexible learning and engagement, while in order to analyse complex phenomena or settings Maclaren, Wilson, and Klymchuk (2017) used mixed methods to examine the place of gesture and annotation in teaching STEM subjects using pen-enabled Tablet PCs. This also included some lecturers of pure mathematics and statistics classes.

Other studies adapted or cultivated methodological approaches to best explore their research, such as the development of an instrument that integrated the semantic items of three related scales aimed at characterising the perceived worth of mathematics-education apps (Handal, Campbell, Cavanagh, & Petocz, 2016). While this collected a mixture of numerical and written data, the analysis was qualitative and they considered it effective in establishing content worth. In another novel approach, Kawka and Larkin (2018) aimed to disrupt the notion of using apps to project mathematics learning as a fun experience. They created a digital artwork, *Arithmomania* that challenges how users interact with education apps by employing the aesthetic of the glitch, characterised by reifying disorder and malfunction. Glitching enabled the divergence of the educational component from the fun component, with the mathematical element portrayed as varying layers of colour and sound.

Several studies undertook systematic literature reviews to gain insight and analysis of particular phenomena e.g., Lake et al. (2017); Tobin and Weiss (2016); and Trenholm et al. (2019b). There were also a number of papers that engaged a contemporary hermeneutic methodology, where the data were analyzed through iterations of interpretation, shifts in researcher perspective and then re-engagement from fresh perspectives. This methodology enabled the layering of interpretations of data which when done collaboratively with teacher co-researchers allowed rich insights and understandings to evolve. This methodology was used to examine a number of questions related to primary children using apps for learning (Calder & Murphy, 2018a, 2018b, 2018c) and in research where primary children used the coding app *Scratch* to design number games for their younger buddy class (Calder, 2018).

Design-based research involves iterations of the review and design process, with the intervention practice followed by the review and modify stages of the cycle. Through this process, the intention is to incrementally improve an artifact or process within its situated context, with the aim to enhance practice (Anderson & Shattuck, 2012). While employing a design-based methodology to develop a mapping app for developing spatial reasoning in early years learners, Larkin and Kinny-Lewis (2017) used the four-stage agile design principles: Discovery, Alpha, Beta, and Live, as they underwent iterations of design and feedback. They incorporated user feedback from the early years' learners through observations of the children's behaviours (e.g., smiling, looking confused) as key indicators of their level of engagement with the app. Also utilizing design iterations of interactions between the user (Subject), app (Artifact) and mathematics content (Object), Larkin et al. (2019) further developed the ACAT framework to analyze the ways that two apps were used with classes (Group). They also considered how they behave when used (Rules) and contend that

ACAT is highly useful for evaluating any mathematics app. A quasi-experimental design study was utilised to investigate the use of recorded lecture videos (RLVs) in undergraduate mathematics instruction (Trenholm et al., 2019a).

Also, utilizing iterative design elements in the development of an instrument to investigate the quality of mathematical apps, Larkin and Milford (2018a, 2018b) initially used an integrated framework evolved from three existing measures: the Haugland Scale, Productive Pedagogies, and Gee's Principles. The framework went through several design and trial iterations, before being coupled with a statistical tool, cluster analysis, to revisit the earlier evaluation. This combination of design cycles, that included measures of perception and review hinged to mechanistic statistical analysis, is perhaps indicative of potential methodologies or research designs that integrate both socio-cultural and machine-driven lenses through which to generate and analyze data, with the consequent unpacking revealing better understanding of a range of digitally-enhanced realities.

Action research follows similar principles and an iterative process as does design-based research, but has a deliberately situated reflection stage in the cycle, rather than the reflection being ongoing, while also not having a distinctive collaborative design stage (Anderson & Shattuck, 2012). Also creating a purpose-designed instrument, Galligan, Hobohm, and Peake's (2017) paper describes an action research process of developing and refining a tool for the creation and evaluation of quality student-produced mathscasts. The study then analyses its effectiveness in relation to pedagogy and mathematical understanding. Other studies used action-research iterations to initiate change; Loch and Lamborn (2016) when aiming to make mathematics relevant to first-year engineering students, and Quinn, Hajek, and Aarão (2017) with their intention to optimize the blending of online and face-to-face teaching and learning for first-year engineering students.

7 Conclusions and Future Directions for Research

This chapter presented an analysis of Australasian research conducted between 2016 and 2019 pertaining to the use of digital technologies across all stages of education. In doing so we interrogated the research to understand the current foci of research, and if, and what, changes have occurred since the last review (Geiger, et al., 2016). As stated in the introduction, areas of research have been built upon and expanded, and as in the last review, we found a broad research agenda and a broad range of methodologies employed in research related to the use of digital technologies.

The authors of the last review made several suggestions for future research into technology enhanced mathematics education. This current review has revealed many of the suggestions have not yet been realized, particularly in relation to the management of interactions and relationships that occur in online environments. Given the increasing use of such environments, this is of some concern, as is the lack of research into the influence of social media on mathematics teaching and learning. Related to this is the limited exploration of virtual worlds and technologies that promote student design within online environments. However, this use of technology is not yet common in schools and this may account for the lack of emerging research in this area.

Several themes emerged from our review that address suggestions made in the previous review. Research on the use of screencasting across the school and tertiary sectors, including ITE, appears to have increased and is a dominant feature in this review. This increase implies some developments in task design and a shift away from the use of consumer focussed applications such as drill and practice apps and apps that are buried in 'fun' contexts such as those described in the paper by Kawka and Larkin (2018). Task design has been suggested by Tsai and Chai (2012) as a possible third-order barrier to effective technology integration. They posit that such knowledge "lies in the dynamic creation of knowledge and practice by teachers when they are confronted with the advancement of ICT and its associated pedagogical affordances" (p. 1058), believing the capacity for 'design thinking' is the new barrier to technology use in education.

Although the increased use of screencasting provides us with some evidence of technology redefining pedagogical practices, there is little research reporting on other and more innovative task design using emerging technologies such as virtual reality and aritifical intelligence. We also found evidence the third order barrier may be limiting the ways in which students interact with digital technologies due to the level of teacher decision-making and its influence on how task design and technology is used. Throughout the research reviewed it is evident that teachers remain in control of how technology is used and in some cases, where learning is 'flipped' and live lectures or lessons are replaced by video-recordings, it appears that rather than transforming learning, practices have reverted to teacher-centred approaches. This leads us to question how 'anywhere, anytime' learning influences the ways teachers interact with their students as discussed in the work by Trenholm, et al. (2019).

This current review period revealed a narrow emphasis of research on the use of digital technology to teach mathematical content. Although studies that include, what could be termed Geometry, appear to dominate in the early childhood, school, and tertiary sectors, there is no evidence of digital technology being used in ITE to develop content knowledge. This is of some concern in early childhood and primary ITE given the current concerns in some countries about the mathematical content knowledge held by generalist early childhood and primary teachers. However, of note with regard to mathematics content is an increased level of research investigating mathematics teaching and learning within the context of STEM education in the early and primary years.

A final theme that has emerged more strongly in this review is that of student engagement. Although there were several studies that specifically focussed on digital technology and its influence on engagement, others reported on student engagement as a by-product of technology use across all levels of education. Given that the Attard study (2018) indicated engagement is largely dependent on the teacher's technology-related pedagogical practices, we find strong links across each of the themes emerging from this review and make the following recommendations for future research directions:

- Investigation of how digital technologies are being used to develop mathematical content knowledge in pre-service early childhood and primary teachers, including those undertaking specialisations in mathematics;
- Develop deeper understandings of how technology can be used to position students to have more voice and control in mathematics classrooms and promote rich, two-way interaction;
- Exploration of the relationship between teacher technology-related decision-making and teacher engagement in the planning of technology-related practice;
- Focus on innovative task-design with current and 'over the horizon' technologies
- Further investigation into the use of digital technologies to teach the breadth of content knowledge across all levels of education; and
- Investigation into the effects of learning management tools on the ways students access teaching resources (including videos) beyond the classroom and the ways these tool provide teachers access to evidence of student learning.

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Chapter 14
Changing Landscapes

Glenda Anthony

Abstract This chapter considers how the research reviewed in *Research into Mathematics Education in Australasia-10* is situated and responsive to the 'changing landscape' of education; and importantly in terms of impact, how this research contributes towards 'changing the landscape' of mathematics/education. In particular, research that is situated in the changing educational landscape concerns 21st century educational outcomes, numeracy, STEM, and technology. In addressing impact, primacy is given to (i) equity and social justice; (ii) teaching practices that involve dialogic teaching and communities of inquiry; (iii) professional learning; and (iv) curriculum innovation. Cognisant of the ever-changing influences on mathematics education, the chapter concludes with a discussion of further research priorities that may inform change going forward. Drawing on specific chapter recommendations, I focus on three areas: (i) assessment; (ii) 21st century learning; (iii) and the use of voice within research endeavours.

Keywords Education landscape · Equity · Ambitious pedagogy · 21st-century education · Curriculum

1 Introduction

Collectively, the chapters in *Research into Mathematics Education in Australasia-10 [RiMEA-10]* signify the scope and depth of the research field in Australasia during the time period of 2016–2019. For each chapter, teams of experts have provided a comprehensive critique of their respective field, including a review of current contributions, actual and potential impact, and research opportunities going forward. In comparison with earlier reviews my sense is that the research in this period has been particularly responsive to the changing socio-political and educational agendas. Indeed, many of the reported research projects are closely embedded in global education reforms, frequently referred to as the 21st century education movement (Ananiadou & Claro, 2009). Moving from an education landscape characterised by

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curricula based on distinct and separate disciplines, where pedagogical emphasis focused on individual cognitive competency, outcomes for learners in a remodeled landscape include a focus on social, affective, physical, and self-management competencies within a culture of sustainable wellbeing (Berryman, Lawrence, & Lamont, 2018; OECD, 2019). In this chapter, I use a metaphor of changing landscapes—with *changing* regarded both as an adjective and as a verb—to highlight how mathematics education research within the review period:

- is situated and responsive to the ‘changing landscape’ of education; and
- contributes towards ‘changing the landscape’ of education.

I conclude with an overview of key research foci that may be beneficial in ensuring that the mathematics education landscape continues to change and adapt in a productive way. In each of the sections that follows reference to specific research studies are used as exemplars to warrant claims, rather than as a sense of an exhaustive list.

2 Working Within the Changing Landscape of Mathematics Education

The space and nature of mathematics education in the educational landscape is increasingly contested. As noted in several chapters in *RiMEA-10*, mathematics and mathematics education can no longer be viewed as isolated from other disciplines. The role of mathematics education, in our progressively interconnected global society, challenged by unparalleled social, economic, ecological, and political crises, is constantly being redefined. Building on D’Ambrosio’s (2004) contention that “mathematics is powerful enough to help us build a civilization with dignity for all, in which inequity, arrogance, violence and bigotry have no place, and in which threatening life, in any form, is rejected” (p. ix), the landscape is, at least in pockets, changing. The new landscape aims to reflect a more socially just and responsible education, where doing mathematics involves engaging in “processes of understanding and fulfilling our civic and moral responsibilities (Furuto, 2019, p. 105).

The most obvious signs of this changing educational landscape can be seen in interdisciplinary curriculum changes and increasing attention to equity and well-being. Curriculum foci, defined in the *OECD Learning Compass 2030* (OECD, 2019), centre on three core foundations: (1) cognitive foundations, which include literacy and numeracy; (2) health foundations; and (3) social and emotional foundations. Collectively, these represent the “fundamental conditions and core skills, knowledge, attitudes and values that are prerequisites for further learning across the entire curriculum”. Research reviewed in *RiMEA-10* aligns with these core foundations—most notably in terms of increased numeracy, technology, and STEM research. Moreover, as will be discussed in the next section, research is making significant impact on issues of equity and well-being.

An analysis of past *RiMEA*s by Maker, Dole, Visnovska, Goos, Bennison, and Fry (see Chap. 2, this Volume) highlights a shift in the landscape of mathematics education research from a focus on single mathematical domains (e.g., geometry, statistics) towards mathematics as activity (e.g., modelling and reasoning, STEM, and numeracy). According to Maker et al. this shift signifies a recognition of the need for research to address the "problems of *how and why* mathematics might be created and sustained by collectives of problem-solvers" (p. x). Indeed, *RiMEA-10* features, for the first time, a chapter focused on Science, Technology, Engineering and Mathematics (STEM) research (Anderson, English, Fitzallen, & Symons, Chap. 3, this Volume). The prominence of STEM, foreshadowed by the concluding chapter of *RiMEA-9* (English, 2016), has prompted new research funding priorities (e.g., *Unlocking Curious Minds*, Ministry of Business, Innovation & Employment [MBIE]), new educational policy and centres that highlight technological innovation and access for marginalised learners (e.g., Matthews, 2015), and new business partnerships that aim to bridge the gap between academic analysis and industry requirements. Fuelled by concerns about senior student performance in STEM related subjects (Office of the Chief Scientist, 2017), the active promotion of STEM career requirements and pathways by professional organisations (e.g., Engineering Australia, 2017) has become a key driver in the new look landscape.

Anderson et al. (Chap. 3, this Volume) highlight the emergent way that researchers are engaging in this changing landscape; a landscape that challenges the reasons for learning mathematics, the nature of the mathematics curriculum, and participation rates and associated equity issues. While their review provides evidence of initiatives that encourage connections between schools and community, most notably scientists and local business and industry, several studies (e.g., Doig & Williams, 2019) affirm English's (2016) earlier concern that mathematics could potentially be overlooked with STEM developments.

According to Anderson et al. (Chap. 3), current assessment practices present a significant barrier within the changing STEM landscape. Despite the fact that generic skills such as innovative problem finding and solving, visualizing, collaborating, and communicating are common across areas such as engineering and mathematics, Anderson et al. claim that the current reliance on high-stakes subject-based assessment at senior school level continue to hinder efforts to include integrated STEM learning experiences. Barriers also referenced teachers' capacity and support for change in pedagogies. As noted by Anderson et al., structural changes to the learning landscape require changes in teaching—termed "inspired teaching" (Office of the Chief Scientist, 2012). Efforts to recruit more high performing STEM graduates and commit funding to developing digital literacy capabilities, and principals' capacities to lead STEM initiatives in their schools, alongside investment in curriculum innovations (e.g., the *Integrated STEM Project*, Ward, Lyden, Fitzallen, & Panton, 2018) are examples of strategies designed to remove or minimise barriers.

Despite these ongoing concerns, Anderson et al.'s review (Chap. 3, this Volume) also showcases the emergence of pockets of innovation within the landscape. For example, the *Maths Inside* project (Coupland et al., 2017) was noted as a valuable project that linked mathematics and science in a way that helped teachers and students appreciate how mathematics is applied ‘inside’ science and other areas. Examples of project that supported the development of students’ disciplinary understanding included Chalmers and Nason’s (2017) project that used robotics contexts and the *Modelling with Data*: *Advancing STEM in the Primary Curriculum* project (English, Watson, & Fitzallen, 2017) that supported students’ understandings of the big ideas of variation and expectation.

Within the changing landscape, impact of digital devices and software was also related, but not exclusive, to the increase of STEM activities. Attard, Calder, Holmes, Larkin, and Trenholm (Chap. 13, this Volume) claim that the increased reliance on digital technology has led to a blurring of the boundaries between school and home learning and research focused on supporting tasks, classroom interactions, and subject integration. For example, Miller and Larkin’s (2017) task-based research illustrated how coding and robotics can support children’s development of algebraic thinking; Calder and Murphy (2018) described how student screencasting and voice-recoding can be used to support a new dynamic learning environment; and Gorman and Way’s (2018) study illustrated how a ‘zoomable’ number line supported students’ exploration of decimal fractions.

A defining feature of the technology supported landscape—be it early-years, primary and secondary, or tertiary setting—is the shift away from learners being consumers of content authored by others towards learners authoring their own content. Likewise, this shift in how learners interact within the learning landscape is a feature of Geiger, Yasukawa, Fielding-Wells, Bennison, and Sawatzki’s chapter (Chap. 4, this Volume). Describing numeracy as a social practice, Geiger et al. argue that numeracy has become more than specific practices related to the use of mathematics; it is tied to “ways of thinking, modes of reasoning and means of knowledge generation within communities that are defined by distinct social or cultural types of activity” (p. x). Acceptance that numeracy involves more than the mastery of basic mathematical skills, gives recognition to the changing socio-political landscape that positions numeracy as part of reflective, discriminating, and responsible citizenship.

In combination, the chapters on STEM, technology, and numeracy evidence the breath of learning outcomes associated with 21st Century education (OECD, 2019). As such, Geiger et al. (Chap. 4, this Volume) welcome the addition of research studies that are designing numeracy provisions, and also educational provisions, more broadly. For example, in Furness, Robertson, Hunter, Hodgetts, and Nikora’s (2017) study of adult literacy and numeracy programs, success was measured in terms of difference to the lives of Māori and Pacific people through a re-interpretation of ‘wellbeing’ that included the physical, spiritual, the mind and emotions, and harmonious relationship between individuals and their (socio-material) environment. Other studies within the broadened landscape examined numeracy practices and skills of adults who were homeless or at risk of homelessness, vocational teachers, carpentry and automotive technology workers, stone-wall builders, urban waste collectors, and orchard managers.

Under the influence of governments, policymakers, and the finance industry, financial literacy emerged as another key feature of the numeracy landscape. Similarly, with STEM, research in financial literacy is transdisciplinary in nature—inclusive of economics, education, finance, psychology and sociology. Likewise, financial literary research in mathematics classrooms focused on 21st century learning outcomes. Sawatzki's (2017) research, for example, found that financial problem contexts had the potential to broaden students' horizons and better prepare them for life-wide economic participation.

Across *RiMEA-10* chapters, it was apparent that the role of the teacher continues to maintain a dominant position in the changing landscape. Accountability for performance outcomes, and concerns about quality of teaching are linked to large-scale international assessments (Bahr & Mellor, 2016) and compliance policies (The New Zealand Government, 2018). And as in *RiMEA-9*, accountability changes in initial teacher education abound. For example, in New Zealand, new requirements governing the approval, monitoring, and review of initial teacher education programs (Teaching Council of Aotearoa, 2019) set expectations that providers demonstrate how programs enable pre-service teachers to meet graduating standards "in a supported environment". To ensure graduating teachers develop culturally responsive and inclusive teaching practices that address the learning needs of students from Māori and Pasifika backgrounds, programs must demonstrate explicit consideration of Tātaiako cultural competencies (Ministry of Education, 2011) and Tapasā cultural 'compass' (Ministry of Education, 2019). Assessment regimes in Australia now include the *Literacy and Numeracy Test for Initial Teacher Education*, either as an entry or graduation requirement, across all initial teacher education programs, and, as described by Way et al. (Chap. 5, this Volume), the quasi-graduation requirement of the *Teaching Performance Assessment*. In addition, Geiger et al. (Chap. 4, this Volume) note curriculum pressures for initial teacher education to support students to understand not only the mathematics they need as teachers "but also the political knowledge, the politics of mathematical knowledge and how they could support others to develop agency through the use of critical mathematical thinking" (p. x).

Reference to critical mathematical thinking within a social space is featured in several *RiMEA-10* chapters. Simultaneously focused on the co-learning space, studies reflect a view that learning mathematics is positioned as socially situated rather than individualistic. According to Vale, Averill, Hall, Forgasz, and Leder (Chap. 8, this Volume), those studies that involved explorations of the teaching/learning nexus in authentic settings—be they informal and formal, or educational or home/workplace—have been particularly important in enabling researchers to dig deep into the landscape to explore issues of equity and inclusiveness. However, with reference to practicing teachers, Bobis, Kaur, Cartwright, and Darragh (Chap. 6, this Volume) note that teacher learning in the authentic contexts of blogs, twitter feeds, and Facebook platforms remains under-researched.

While we live in an increasingly technology enriched society, the impact on the teaching learning landscape is mixed. According to Galligan, Coupland, Dunn, Oates, and Hernandez Martinez (Chap. 11, this Volume) and Attard et al. (Chap. 13, this Volume) the use of digital technology has not consistently improved learning opportunities. While studies exploring the likes of screencasting in schools provided us with some evidence of technology redefining pedagogical practices, Attard et al. concluded that for many of the reviewed studies it was evident that teachers remain in control of how technology is utilised. Likewise, in the tertiary context, research suggested that pedagogical adaptations of technology have not as yet delivered the hoped-for outcomes. For example, Hoogland and Tout (2018) found that computer-based mathematics testing encouraged lower-order thinking and for other studies where live lectures or lessons were replaced by video-recordings, it appeared that rather than transforming learning, practices remained teacher-centred.

In addressing persistent concerns around equity and underachievement of marginalised groups, it is particularly heartening that research has moved from an exclusive focus of highlighting inequity to showcasing 'practices' that serve to disrupt patterns of systemic alienation and underachievement. Vale et al. (Chap. 8, this Volume) highlight how established groups of researchers are working differently with the resources within the landscape; embracing the principle that creating, supporting, and sustaining a culture of access and equity requires "being responsive to students' backgrounds, experiences, cultural perspectives, traditions, and knowledge when designing and implementing a mathematics program" (National Council of Teachers of Mathematics, 2014, p. 1)" (p. x). For example, Miller, Warren, and Armour's (2018) exploration of the cultural discourse that occurred at the boundary between Indigenous and Western knowledge systems credited two teaching and learning actions that created space for mathematical discourse: (i) acknowledging cultural ontology and (ii) acknowledging semiotic systems associated with the oral traditions of their culture. Likewise, Averill (2018) noted how singing, story-telling, metaphor, and dance associated with students' heritage cultures could enhance mathematical understanding and well-being. However, ongoing efforts to support teachers to implement culturally relevant practices through a critical analysis of curriculum resources remains challenging, with researchers (e.g., Edmonds-Wathen, 2017; Hunter & Hunter, 2019) arguing for an urgent focus on how students' prior knowledge might be used as a resource.

Issues of well-being, while ever present, have for more clearly surfaced as a feature of the current learning landscape. Compared to the average student across OECD countries, the latest PISA study (PISA, 2018) noted that both Australian and New Zealand students reported being bullied more frequently, felt more afraid of failing, and were more likely to have skipped school and feel lonely at school. For example, data from New Zealand, suggested that 15% of students skipped school at least once in the two weeks prior to the 2018 PISA testing. In Australia, Bennison, Goos, and Belinski's (2018) study that used the voices of mathematics teachers, guidance counsellors, and students to identify factors that contributed to student participation in mathematics, highlighted the complexity of educational practice. Factors supporting positive student outcomes, included the likes of curriculum organisation across year levels, and staffing of mathematics classes, the culture of the Mathematics Department, and teachers holding high expectations of the students. It is important in this changing landscape, where issues of mental health and student disengagement are to the fore (Willis, Hyde, & Black, 2019) that we have access to studies of exemplary practice. The following section highlights studies that provide a way forward such as that of Cheeseman (2018) which showed how teacher's deficit perceptions of readiness impacted young learners in terms of opportunities to engage productively with mathematical practices.

The recruitment of mathematics teachers remains another concern within the changing landscape. As noted by both Vale et al. (Chap. 8, this Volume) and Way et al. (Chap. 5, this Volume), Australasian efforts to recruit teacher candidates, and support both in-field and out-of-field mathematics teachers in rural and remote communities remains critical. This appears to contrast the landscape for teachers in Singapore were recruitment is strong and professional learning is supported by a culture of life-long learning.

3 Changing the Landscape of Mathematics Education

The previous section drew on studies that were clearly situated in, and responding to, changes in the broader educational landscape. This section discusses how research within the *RiMEA-10* review period is impacting to *change* the landscape of mathematics education. While there is inevitably some overlap in the changes noted in the current and previous section, discussion in this section is organised according to the following themes: (i) equity and social justice; (ii) teaching practices that involve dialogic teaching and communities of inquiry; (iii) professional learning; and (iv) curriculum innovation.

In addressing equity concerns, research across multiple chapters challenge the inevitability of inequitable student outcome through notions of reframing who and how learners can participate in mathematics education. Showcasing research that involves enactment of ambitious teaching practices that are both inclusive and culturally responsive (e.g., Hunter, Hunter, Anthony, McChesney, 2018; Jorgenson, 2018) reviewed studies affirm that improvements to learning outcomes require significant change in both teacher practice and knowledge. A significant advancement in these intervention studies is the inclusion of teacher knowledge previously considered outside of the domain of mathematics teachers: that is, knowledge of the community, the whānau, and the culturally lived lives of one's students (Jorgensen, 2017; Warren & Miller, 2016).

Importantly, reviewed studies demonstrate that in order to enact socially-just practices in teaching, responsive pedagogies must attend to constructing productive and compassionate relationships in the classroom as well as respectful relationships with the school community. That is, changing the landscape requires a shift in the way partnerships are formed and power is distributed and this includes substantive attention to the utilisation of the languages and cultures of marginalised communities. For example, Hunter et al. (2016) provided evidence of improved equitable outcomes when educators relate to Pasifika students "as culturally located people with rich funds of knowledge to contribute" (p. 208).

However, in acknowledging the importance of the first language, the role of pedagogical practices of Indigenous knowledge systems, and genuine engagement with community, Vale et al. (Chap. 8, this Volume) caution that progress towards enhancing equitable access to mathematics learning remains problematic; particularly so, in spaces where tensions abound regarding who holds power in policy making, mathematics teaching, and schools. Edmonds-Wathen (2017) and Te Maro (2018) suggest that these tensions are exasperated by the different status held by Western and Indigenous knowledge systems within English-medium formal education. While research using decolonial theories and methodologies has potential for reshaping the educational landscape—for example, in New Zealand where mathematics education occurs both in a bilingual and full immersion te reo context—researchers (e.g., Trinick, 2019; Tweed, 2016) contend that the development of mathematics education within the Māori space represents an ongoing and challenging project. To avoid mathematics education promoting an impoverished view of mātauranga (Māori knowledge), Te Maro (2018) calls for consideration of "the possibility for maths education to be developed by informed and conscientised communities" (p. 245)—those that are aware of the social and political conditions that impact on their ability to make emancipatory decisions and take transformative action through their own contexts, languages and practices. According to Te Maro, only then will kaupapa and mātauranga mathematics (more than curriculum maths) "be equitably privileged in terms of space, time and activity with kaupapa and mātauranga Māori" (p. 245).

Intertwined with equity, Hunter, Miller, Choy, and Hunter (Chap. 12, this Volume) contend that research advocating ambitious pedagogies suggests significant shifts in the framing of opportunities to learn mathematics. Research focused on learners' active co-participation in a community of learning and associated dialogical pedagogies (Attard, Edwards-Groves, & Grootenboer, 2018) opens new transformative spaces within the learning landscape. For example, in New Zealand, the ongoing implementation of *Developing Mathematical Inquiry Communities* [DMIC] professional development program supports schools to shift their practice towards more collaborative heterogenous groupwork (Hunter et al., 2018). As a consequence, this project has served to disrupt the previously Numeracy Stage based grouping allocation of students (see MoE, 2008) within the mathematics classroom.

Accompanying the development of communities of inquiry, research exploring productive approaches to classroom interactions (e.g., Jazby, 2019) has built new understandings of students and teachers as active learners and decision makers. Drawing on a practice perspective, Grootenboer and Edwards-Groves (2019) proposed that one's mathematical identity is "formed within and through being stirred into mathematical practices—both the substantive practices of mathematics *and* learning practices of mathematics" (p. 442). In research classrooms where learners have been supported to develop practices of mathematical discourse and argumentation, studies have found possibilities for increased student agency, and productive dispositions (Attard et al., 2018; Sawatzki, 2017). Moreover, when teachers were supported to develop such spaces where learners' thinking is valued as a resource, studies report increased levels of mathematical confidence and enjoyment for both teachers and students (Civil, Hunter, & Crespo, 2019).

Central to the research on ambitious pedagogies, reviewed research in *RiMEA-10* featured a sustained program of research, from early years through to tertiary learning, concerning opportunities to learn through engagement with challenging tasks. As noted by Downton, MacDonald, Cheeseman, Russo, and McChesney (Chap. 9, this Volume), research in early-years settings reflected a "strong thread of the use of investigative play, mathematical inquiry and investigations, challenging tasks, problem solving throughout with teachers adopting ambitious pedagogical approaches when working with young children" (p. 236). Building on the seminal work of Sullivan and colleagues, research (e.g., Livy, Muir, & Sullivan, 2018) has promoted the normalisation of cognitive struggle and high teacher expectations in the design and implementation of challenging tasks. The focus on tasks design supported a stream of research concerning the role of the meaningful contexts—contexts that embraced STEM, financial well-being, and culturally relevant contexts, frequently connected to a social justice agenda.

In positioning challenging tasks within the landscape, researchers also argued that classroom differentiation practices needs to move away from a divisive labelling perspective that focuses on high versus low performance outcomes towards a focus on student well-being and productive mathematical disposition. Research projects aimed to disrupt the high levels of deficit teacher expectations, most notably associated with Māori, Pacific and Indigenous learners (e.g., Hunter & Hunter, 2017). In-school research interventions (e.g., Peterson, Rubie-Davies, Osborne, & Silbey, 2016) found that successful adaptations to teacher practice required shifts to flexible grouping, enhancement of the class climate, culturally responsive differentiation, and supporting students' goal setting. In particular, researchers argued that more flexible and purposeful grouping within mathematics classrooms would support students to work with a range of peers "to focus on specific mathematical skill development and a range of valued outcomes—including student voice and agency, pro-social skills, mathematical dispositions, and valuing of the mathematics within the home and cultural context" (Anthony, Hunter, & Hunter, 2019, p. 121).

As noted in the previous section, changes to the learning landscape require professional support for teachers. Influenced by research that draws on social learning theories (Prodromou, Robutti, & Panero, 2018), research reviewed by Bobis et al. (Chap. 6, this Volume) signal a shift in the professional learning landscape away from "university-based, 'supply-side', 'off-line' forms of knowledge production conducted by university researchers for teachers towards an emergent school-based, demand-side, on-line, in situ forms of knowledge production by teachers with support from university scholars" (Wong & Kaur, 2018, p. 427). As noted by Bobis et al. (Chap. 6, this Volume), successful responses to teachers' individual or institutional needs within their local context was marked by collaborations between mathematics education researchers and teachers. As such, for professional learning within Indigenous communities, Vale et al. (Chap. 8, this Volume) points to the critical importance of active participation of students, Indigenous education officers, teacher aides, and whānau in recognising and enabling Indigenous ontology to contribute to teacher learning.

Working across and within schools and community, with extended mentorship for at least three years, *Developing Mathematical Inquiry Communities* [DMIC] (Hunter et al., 2018) offered an exemplar of a new form of professional learning partnership that uses multiple interactive "research pathways" that according to Cai, Morris, Hohensee, Hwang, Robison, and Hiebert (2019) are likely to have a more direct impact on practice than traditional linear research pathways. In reference to DMIC, Civil et al. (2019) argue that the combination of leadership, policy stewardship, rich understanding of the influence of local contexts are critical mediators of fidelity and sustainability of reform, alongside the in-class mentoring that attends to teachers' instructional practices/problems in the moment, providing appropriate grain-size information that teachers need to improve their practice. For DMIC, changes in the learning landscape have resulted in improved student outcomes as assessed across multiple attributes including prosocial skills, reduction in bullying in the playground, increased parental partnerships, and productive mathematical dispositions and identities.

Within *RiMEA-10*, research impact in changing the initial teacher education landscape focused on supports for newly qualified teachers to act as positive change agents for 21st Century education reforms. Proactive in changing the landscape, some providers in Australia worked with the Teacher Education Ministerial Advisory Group [TEMAG] (2014) resulting in recommendations to create *Primary Mathematics Specialisation* in their programs. As well as creating new programs, Way et al. (Chap. 5, this Volume) noted a groundswell of research informing pedagogical change within mathematics methods courses in terms of opportunities to learn. For example, research on practice-based pedagogies included practice-based rehearsals, role plays, video analysis, the use of authentic samples of school-student work, and enhanced relationships with in-school mentors. Studies also explored changing the way that pre-service teachers could enhance their own personal mathematics knowledge through authentic, personalised, and collaborative learning experiences using mobile technologies. For example, Marshman, Woolcott, and Dole (2017) explored pre-service teachers' development of spatial reasoning within a 3D virtual environment.

Way et al. (Chap. 5, this Volume) and Vale et al. (Chap. 8, this Volume) also noted the impact on provisions for equity reforms. Possibly due to the explicitness of Teacher Council documents (2018) and policy documents (e.g., *Tātaiako: Cultural Competencies for Teachers of Māori Learners*, MoE, 2011; *Tapasā: Cultural competencies framework for teachers of Pacific learners*, MoE, 2018) the bulk of this research was New Zealand based. Changes affirmed that mathematics education courses are legitimate sites for pre-service teachers to learn how to implement culturally responsive pedagogies and plan for diversity and inclusion. For example, Wilson, McChesney, and Brown (2017) demonstrated how a mathematics methods course could model the inclusion of active partnership between learners, Māori language, as well as Māori pedagogies, contexts, beliefs, philosophies, protocols, and values.

A noted feature of research supporting changes to initial teacher education was a move from single institutional-based studies towards more collaborative partnerships—be it teacher educators collaborating with others across institutions, discipline boundaries, and specialist areas of teacher education, or sectors of education. Moreover, research in *RiMEA-10* indicated the need for a stronger emphasis on teacher educator professional learning. Exemplars included Rawlins, Anthony, Averill, and Drake's (2019) depiction of how active listening to pre-service teachers during practice rehearsals, combined with the deliberate generation of student voice, contributed to teacher educators' expertise to prepare teachers for ambitious mathematics teaching. Using student voice indirectly, Sellings and Brandenburg (2018) advocated teacher educators use 'data praxis' to inform their work. Exploration outside of the boundaries of initial teacher education, including projects utilising interdisciplinary collaborations among mathematics and science educators, mathematicians and scientists, also were used to create adaptions to the landscape. Conceptualising the boundaries between disciplines as sociocultural difference, Goos and Bennison (2018) described them as generating new practices—and, therefore, new learning.

In the preceding section, the impact of technology and STEM was noted as a feature of the changing landscape. Within this environment, reviewed research across several *RiMEA-10* chapters signals changes to curriculum. In the early-year sector, intervention studies featured in Downton et al.'s Chap. 9 (this Volume) provides evidence of the value for younger students engaging in more challenging mathematical problems and modelling activities. For example, Cheeseman, McDonough, and Golemac (2017) illustrated how "thinking conversations" within mathematical investigations supported young children to engage in big mathematical ideas such as equivalence and making mathematical generalisation. At the school level, Woolcott. Lowrie, Marshman, Logan, Ramful, and Whannell (Chap. 10, this Volume) argue that embracing a spatial reasoning focus beyond geometry may enable students to access complex ideas in non-traditional ways. Shifting the focus of the mathematical landscape from number knowledge to spatial reasoning—described as spatialising the mathematics curriculum was the focus of a project by Mulligan, Woolcott, Mitchelmore, and Davis (2018). A desired outcome of this project is to generate an "innovative knowledge framework based on spatial reasoning that identifies new pathways for mathematics learning, pedagogy and curriculum" (p. 77). Also prioritising spatial reasoning, Jorgensen and Lowrie's (2018) intervention focused on improving visuo-spatial skills and competencies of Indigenous students. Again, focusing on practices rather than content per se, Bragg, Herbert, Loong, Vale, and Widjaja (2016) and Herbert (2019) investigated the teaching and assessing of student reasoning. At the tertiary level, Galligan et al. (Chap. 11, this Volume) highlight research that advances our understanding of learning specific mathematical concepts such as complex numbers, functions, and graph theory. Moreover, for statistics education, Pfannkuch, Ben-Zvi, and Budget (2018) claimed that a paradigm shift toward a modelling perspective provides greater connection between statistics and probability; a connection that potentially would support the setting of new and more relevant goals for probability education in the 21st century.

In summary, the research reviewed in *RiMEA-10* offers both real and promising new understandings that are making significant changes to the mathematics education landscape in terms of equity and associated ambitious pedagogies, and curriculum innovation. Importantly, this research acknowledges both the value and the challenges involved in building and working within expanded partnerships when addressing innovation in key areas of learning/teaching mathematics, curriculum and tasks development, and professional learning.

4 Opportunities Going Forward Within the Changing Landscape

Given that the research practices of our MERGA community are driven by the participants, we can expect over time changes in our shared ways of behaving, our language, our habits, our values and our tool use. As discussed, changes in research foci concern the role of mathematics/education in society and reforms directed to ambitious pedagogical practices that emphasise a productive classroom culture that embraces higher-order thinking, collaborative inquiry, and dispositions that support productive struggle. Cognisant of the ever-changing influences on mathematics education, *RiMEA-10* chapters pointed to the need for further research to inform change. Drawing across these specific recommendations, I have chosen to focus on three areas: (i) assessment; (ii) 21st century learning; (iii) and the use of voice within research endeavours.

Interestingly, given the known impact of assessment on educational practice (Kloosterman & Burkhardt, 2017), there is a surprising paucity of assessment research as noted across several chapters. Indeed, the limited number of research outputs reflects a longer-term trend identified by Serow, Callingham, and Tout (2016) in *RiMEA-9*. As such, it appears that Serow et al.'s call for evidence-based research that informs the direction of assessment practices in mathematics education—inclusive of those at the classroom/school-based, national, and international level—remains largely unrealised. As noted by Geiger et al. (Chap. 4, this Volume) while national and international numeracy assessments continue to raise concerns over the quality of numeracy teaching and learning practices, reviewed studies into the impact of the likes of NAPLAN testing (e.g., Carmichael, Muir, & Callingham, 2017; Carter, Klenowski, & Chalmers, 2016) provide little detail concerning the impact on mathematics pedagogy, curriculum design and planning, student engagement, and attitudes towards numeracy. In the main, the focus of assessment research has been to highlight inequities in student achievement (e.g., Goss, Sonnemann, Chisholm, & Nelson, 2016) and to explore indicators of learners' future mathematical success (e.g., Moss, Bruce, & Bobis, 2015).

Driven by research highlighting marginalised students' underachievement on traditional measures (e.g., Jorgensen, 2017), research exploring the complexities of what matters as achievement outcomes will require significant adaptations to the assessment landscape and tools. Providing supports for teachers to enact and assess ambitious teaching that promotes mathematical practices such as collaborative argumentation and reasoning will require that we build new assessment tools. The few studies that have investigated shifts in assessment practice (e.g., Herbert's (2019) investigation of teachers' assessment of ambitious teaching practices that promote reasoning and Prescott and Maher's (2018) study of how screen casting may facilitate teachers' formative assessment strategies) suggest that professional learning support for teachers will important. Downton et al. (Chap. 9, this Volume) note that in New Zealand, the recent abolishment of the unpopular National Standards as the national assessment and reporting system (Ministry of Education, 2018) provides the opportunity for the adoption of more effective assessment processes. However, much of this policy work, including ongoing consultation with teachers, parents and communities, is in the emergent phase and lacking a sound research-base. Noting English's (2016) earlier concern about environments that "push teachers to increasingly structured assessments" (p. 1079), it is critical that we heed Downton et al.'s. call for research to explore the impact of assessment, particularly school entry assessment and structured assessment in the early years of schooling.

Way et al. (Chap. 5, this Volume) highlight the need for further research to inform policy changes related to assessment within initial teacher education. It is concerning that despite concerns about recent external accountability requirements in initial teacher education (Education Services Australia, 2018; Teaching Council, 2018), there is only one reviewed study on mathematics education assessment (Bragg & Lang, 2018). An ever-changing initial teacher education landscape means that research priorities must broaden from long-standing concerns related to the practice-theory divide towards seeking accountability and understandings about initial teacher education "and its relationships to teacher and student learning" (Ell et al. 2017, p. 345). The broader research program of Ell and colleagues (e.g., Chang et al., 2019; Ell et al., 2017) provide possibilities to map the influences of teacher learning and measure the complexity of teaching practice for equity; research that might well be applicable to mathematics education.

The lack of assessment research is also reflective of the increasing tensions within mathematics education, about what counts as desirable outcomes. Within the field of adult numeracy, for example, Geiger et al. (Chap. 4, this Volume) noted the emergence of tensions between traditional academic research and studies conducted by corporate researchers. By way of illustration, the OECD *Survey of Adult Skills* (SAS) measures how adults use literacy, numeracy and problem-solving skills in personal, civic and work life—outcomes linked to broader goals and gains than mathematical knowledge and skills. Indeed, Geiger et al. urge the MERGA community to exercise a "corporate critical orientation" to address the lack of secondary analysis of data from large-scale assessment data sets such as PISA. Within the emergent STEM research, we can see some beginnings of alternative types of assessment (e.g., English, 2018) that support knowledge-driven decisions. In the tertiary mathematics sector, current research is likewise questioning whether mathematics graduates are sufficiently equipped with 21st century skills. For example, from a transition to work perspective. King, Varsavsky, Belward, and Matthews (2017) found that less than

30% of their surveyed 144 final year mathematics students felt that ethical thinking, teamwork, and writing and communication skills were appropriately included or assessed in their programs.

This links to the second issue for discussion: the intersection of mathematics education and 21st century skills and knowledge. As noted from a Singaporean perspective (Cheng, 2017) attention to the changing purpose of education is essential for creating “a more equitable society”. Responding to the challenge to keep abreast of the social, technological, and economic change, and as yet unknown future careers and life will require that our young people develop both powerful reasoning and knowledge in mathematics alongside the capability to think critically, creatively, and flexibly. Given that mathematical competence “is someone’s insightful readiness to act appropriately in response to all kinds of mathematical challenges pertaining to given situations” (Niss & Højgaard, 2019, p. 12), research is urgently needed to inform mathematics curriculum changes that will support students to develop a broader set of knowledge, skills, values and dispositions that will enable young people to address the big issues, and global shifts, such as climate change, ecological sustainability, increasing urbanisation, and ageing population, to name a few. As indicated by Geiger et al. (Chap. 4, this Volume), Woolcott et al. (Chap. 10, this Volume), and Attard et al. (Chap. 13, this Volume), adaptations to the learning landscape must embrace connections to the multidisciplinary nature of students’ lives outside of school. Commending the need for researchers to be cognisant of the increasing opportunities for interdisciplinary links, Woolcott et al. extoll the value of studies in spatial reasoning that cross boundaries of mathematics, geography, design and technology, engineering and geosciences, digital design, and 3D printing. In advocating for radical changes in the landscape moulded by a new curriculum rather than an add-on, Attard et al. argue that much of the potential of technology is still to be realised. Importantly, Attard et al. signpost the need to develop deeper understandings of how technology, including over the horizon technologies, can be used to position students to have more voice and control in mathematics classrooms.

From a broader base, the changing purpose, reflected in the *OECD Learning Compass 2030* (OECD, 2019, p. 3), three core foundations—cognitive foundations, which include literacy and numeracy, upon which digital literacy and data literacy can be built; health foundations, including physical and mental health, and well-being; and social and emotional foundations, including moral and ethics—align with the aim of greater development of student agency and transformative competences and have significant implications for the role of mathematics education in student well-being outcomes. Research associated with notions of productive struggle, resilience (Moala & Hunter, 2019), and mindsets (Koch, 2018) are indicative in establishing early signposts in the changing classroom landscape.

Likewise, calls from the broader educational landscape to research alternative pedagogies such as Renshaw and Tooth’s (2017) ‘pedagogies of place’ that enable opportunities for learners to engage with problems that matter to them aligns with emergent research in mathematics education that promote greater awareness of learners’ and communities’ socio-cultural context (Hunter & Sawatzki, 2019). In statistics, for example, studies such as Watson (2017) demonstrate how open data sets can be used for authentic student explorations, arguing that this may well be a productive means of preparing students for the use of big data in their future lives.

Pedagogical adaptations that value learners' and communities' socio-cultural contexts and histories segue to the third future-focused research direction—'voice'. An emergent theme in *RiMERA-10* alludes to the potential of voice in co-constructing productive learning environments. Inclusive of valuing the child, learner, teacher/educator, and family in partnership, the use of voice within research shifts the frame from "research *on* to research *with* participants. For example, Ingram, Hatisaru, Grootenboer, and Beswick (Chap. 7, this Volume) contend that the landscape would benefit more site-based action research, where educators (and perhaps students) and researchers engage in collaborative research and development in response to local concerns. That is, "knowing what kinds of information are most useful for teachers, which in turn, requires knowing their most pressing instructional problems, which in turn, requires knowing key details of the context in which they work and knowing the grain size of information that will address their problems" (Jinfa, Anne, Charles, Stephen, Victoria, & James, 2019, p. 4) may increase the impact of research. Projects that involve significant partnerships with teachers; partnerships that enable their voice to be central to the problem setting, and the problem design (e.g., Goos & Bennison, 2018) suggest that voices across settings may be productive. Indeed, Downton et al.'s (Chap. 9, this Volume) concern that the alignment and fragility of transitions from prior-to-school settings to primary school and policy efforts to promote the complementarity of the early childhood and school curricula is unresolved is a case for collaborative voice research.

Another important form of voice is represented by the emergence of student voice in research. Here the inclusion of voice is premised on the conviction that young people have unique perspectives on learning, teaching, and schooling; that their insights warrant not only the attention but also the responses of adults; and that they should be afforded opportunities to actively shape their education (Cook-Sather, 2019). While student voice has been utilised in many ways the bulk of *RIMEA-10* studies position students as the object of study. Voice sought through interviews, video-stimulated reflective dialogue (King, 2018), photovoice (Hall, Robinson, Flegg, & Wilkinson, 2019), and video productions (Dunn, Loch, & Scott, 2018) provides data on attitudes, beliefs, values, expectations, and in some instances, learning opportunities. For example, Wilkie and Sullivan's (2018) investigation of middle school students' motivation revealed that many students desired opportunities to do more challenging or interesting mathematical work or to be allowed to work with others. Hunter and Hunter's (2017) use of student voice exposed the role of values such as reciprocity and collectivism as significant attributes of the learning landscape in classrooms where the focus has shifted from individuals towards communities. A smaller number of studies explored the role of voice in action with relation to status and positioning (e.g., Leach, 2019). However, reflecting on emergence of student voice research, Vale et al. (Chap. 8, this Volume) note the need to continue to build and value the voice of Indigenous, Māori, and Pasifika communities as part of opening up and changing the opportunities for students in these disadvantaged or marginalised communities.

Going forward, there is an opportunity for research to consider new roles for student voice—as co-researcher and policy informants (see Ab Kadir, 2019). As noted by Cook-Sather (2019), shifting from research *on* to research *with* students alters the power dynamics from the more typical hierarchical and distanced research relationships. Working with student voice would enable researchers to be more inclusive and participatory with young children, and with older students their revised roles, structures and processes, may support the development of the capacities they need to be research collaborators and leaders. For example, Renshaw's Radford address at the 2019 *Australian Association of Research in Education* conference included two student co-researchers. Drawing on an environmental research project, the young students provided compelling accounts that reflected their emotional and heightened "sense of caring for, being places within, and being responsive to the more-than-human work".

Another, particularly powerful form of engagement with student voice is the use of cogenerative dialogues (Beltramo, 2017). Saunders, Averill, and Mcrae's (2018) research used cogenerative dialogues with ākonga/students to shape how culturally responsive pedagogy was enacted in the researchers' own mathematics classroom. Likewise, to support the co-construction of a mathematical inquiry community, Rice (2019) described how the cogens helped her, as a teacher, "develop deeper relationships with students, resulting in more informed and honest conversations about teaching and learning" (p. 91). In confronting the tension between attention to student well-being concerns and academic performance improvement Rice reflected that "although a student's academic needs remained paramount, participating in cogens has resulted in a greater appreciation for students' social and emotional needs when working within groups" (p. 91).

Within the field of professional learning research, the teacher voice has been used to understand the landscape, but more recently to reshape the learning landscape. For example, studies that explore the lived experiences of pre-service teachers through student voice (e.g., Rawlins et al., 2019) illustrated how a deep understanding of these experiences (e.g., difficulties attempting to adopt a problem-solving approach in classrooms) resulted in adaptations to pedagogies and supports. Working with in-service teachers, Eden (2019) explored the role of teacher voice within collaborative inquiry to co-generate professional noticing practices as part of shifting towards communities of inquiry.

Going forward, the use of voice as an integral part of the research process will not only support more relevant and inclusive knowledge generation but will also alert researchers to questions and dilemmas that are as yet unknown. In traversing new landscapes, Proulx and Maheux (2019) argue that we need to take the "opportunity to reflect on how one's research questions are actually rooted in some "existing" needs, and how much they contribute to highlight new concerns" (p. 300). Indeed, Proulx and Maheux claim that participation in the wider advancement of the mathematics education landscape will require we formulate research questions that are not even yet thought of (in schools or elsewhere). As noted by Renshaw (2019), research on evidence-based practices (e.g., Hattie, 2012) looks backwards. In contrast, Renshaw conjectures that to move forward may require a yet undefined 'pedagogy'—in his area of environmental education, he suggested a 'pedagogy for enchantment' based on kinship and connection to the local. In building new research knowledge that impacts the mathematics education landscape for the better, a move to embrace and empower the increased use of student voice within our research may be needed.

In summary, *RiMEA-10* provides ample evidence that MERGA researchers are aware of and working within a changing educational landscape. Of particular note is the contribution to equity and participation issues within mathematics education, and associated shifts in advocacy and understanding of ambitious pedagogical practices. Reforms associated with 21st century learning outcomes, while slow to enact in practice, appear to be clearly on the radar of researchers who advocate critical social practices associated with mathematics education. More than just informing pedagogical change, research into the learning of mathematics is instrumental in informing exciting and potentially long-term new curriculum directions. Collectively, these studies contribute to MERGA researchers' substantial impact on the mathematics education landscape.

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