A Method of Combined Orbit Determination of Multi-source Data with Modified Helmert Variance Component Estimation

Laiping Feng[1,2](✉), Rengui Ruan[1,2], and Anmin Zeng[1,2]

[1] National Key Laboratory of Geographic Information Engineering, Xi'an, China
fenglaiping@163.com
[2] Xi'an Research Institute of Surveying and Mapping, Xi'an, China

Abstract. The application of inter-satellite link technology, the rise of low-orbit satellite enhancement constellation and the maturity of on-borne GNSS orbit determination technology provide a new method for precise orbit determination different from the traditional ground monitoring station. The method can use a variety of observation data to achieve precise orbit determination under the condition of regional monitoring station. Due to the differences among inter-satellite link, low-orbit satellite-borne GNSS and ground monitoring station data, the reasonable weighting of all kinds of data in joint orbit determination is conducive to improving the accuracy of orbit determination. So this paper proposed a multi-source weighted method of orbit determination data based on Helmert variance component estimation. Meanwhile, it proposed a data processing method of joint orbit determination parameter classification, to solve the limitation of parameter quantity constraint in the process of orbit determination by the rigorous variance component estimation method. The simulation experiment shows that, compared with the weighting strategy based on different observation data accuracy, the Helmert variance component estimation method increases the average RMS of orbital accuracy from 0.073 m to 0.058 m, about 21% up, 31.8%, 13.6%, and 20.4% in R, T, and N directions respectively, by reasonably adjusting the weight allocation of three types of orbital data.

Keywords: Inter-satellite link · Ground monitoring station · Space-based monitoring station · Combined orbit determination · Helmert variance component estimation

1 Introduction

In order to improve the accuracy of orbit determination, the ground monitoring stations of the BDS navigation satellite system need global distribution. But in fact, it is hard to carry out due to geographical factors. Space-based monitoring stations provide a new method to enhance the orbit determination of regional monitoring stations [1, 2], based on LEO-low Earth Orbit satellites and inter-satellite links. It includes space-based monitoring stations augmentation and inter satellite link orbit determination

J. Sun et al. (Eds.): CSNC 2020, LNEE 651, pp. 47–57, 2020.
https://doi.org/10.1007/978-981-15-3711-0_5

augmentation. In the former, the LEO satellites are regarded as moving monitoring stations, obtained the high precision pseudo-distance and carrier observation data by its on-board receiver, together with the ground monitoring station data, it can calculate the precise orbit of the navigation satellite [3–8]. The latter is a method to obtain the precise orbit of a satellite by the combined processing of two-way pseudo-range observation and ground monitoring station data [9]. Therefore, all kinds of observation data from ground, space-based monitoring station and inter-satellite link constitute the basic observation quantity of joint orbit determination. However, the three types of data employed to jointly determine the orbit are quite different. For example, It causes the difference between ground and space-based data because of the different receivers and signal propagation paths, and inter-satellite data difference for its way of frequency, signal modulation and measuring systems. Due to the different equipment delays, it will influence the joint results on orbit determination in the form of system errors [10, 11]. It's a concern of combined orbit determination. Therefore, this paper proposed a multi-source weighted method of orbit determination data based on Helmert variance component estimation. And proposed a data processing method of joint orbit determination parameter classification, breaking through the parameter quantity constraint in the process of orbit determination by rigorous variance component estimation method.

2 Summary of Problems in Determining the Weights of Observation for Orbit Determination

2.1 Mathematical Model to Determine the Orbit

The observation model of the combined orbit determination, using ground monitoring station data, LEO satellite onboard data and inter satellite link data, can be simply expressed as follows.

$$\begin{cases} \boldsymbol{L}_{sta} = \boldsymbol{G}(\boldsymbol{x}_s, \boldsymbol{x}_{sta}, \boldsymbol{x}_o, \boldsymbol{t}) + \xi_{sta}, \xi_{sta} \in \left(0, \boldsymbol{P}_{sta}^{-1}\right) \\ \boldsymbol{L}_{leo} = \boldsymbol{F}(\boldsymbol{x}_s, \boldsymbol{x}_{leo}, \boldsymbol{x}_o, \boldsymbol{t}) + \xi_{leo}, \xi_{leo} \in \left(0, \boldsymbol{P}_{leo}^{-1}\right) \\ \boldsymbol{L}_{isl} = \boldsymbol{R}(\boldsymbol{x}_s, \boldsymbol{x}_o, \boldsymbol{t}) + \xi_{isl}, \xi_{isl} \in \left(0, \boldsymbol{P}_{isl}^{-1}\right) \end{cases} \tag{1}$$

Where, $\boldsymbol{L}_{sta}$, $\boldsymbol{L}_{leo}$ respectively represent the observation data (BDS) from ground station and on-borne GNSS, $\boldsymbol{L}_{isl}$ (isl: inter-satellite link) represents the inter-satellite link data, $\xi_{sta}, \xi_{leo}, \xi_{isl}$ are the corresponding measurement errors, $\boldsymbol{P}_{sta}, \boldsymbol{P}_{leo}, \boldsymbol{P}_{isl}$ are the corresponding weight matrix, $\mathbf{t}$ is the time parameter. $\boldsymbol{x}_s$ and $\boldsymbol{x}_{leo}$ are the set of orbit parameters for GNSS satellite and LEO satellite, $\boldsymbol{x}_{sta}$ is a set of the monitoring station parametes, such as the station's coordinates, tropospheric delay etc., and $\boldsymbol{x}_o$ corresponds to the measurement parameters, such as carrier phase ambiguity, clock error, equipment delay of inter-satellite link, etc. [12].

The above observation equation is linearized and transformed into the following form.

$$\underbrace{\begin{bmatrix} \boldsymbol{v}_{sta} \\ \boldsymbol{v}_{leo} \\ \boldsymbol{v}_{isl} \end{bmatrix}}_{\boldsymbol{v}} = \underbrace{\begin{bmatrix} \frac{\partial \boldsymbol{G}}{\partial \boldsymbol{x}_s} & \boldsymbol{0} & \frac{\partial G}{\partial \boldsymbol{x}_{sta}} & \frac{\partial \boldsymbol{G}}{\partial \boldsymbol{x}_o} \\ \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{x}_s} & \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{x}_{leo}} & \boldsymbol{0} & \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{x}_o} \\ \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{x}_s} & \boldsymbol{0} & \boldsymbol{0} & \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{x}_o} \end{bmatrix}}_{\boldsymbol{A}} \underbrace{\begin{bmatrix} \delta \boldsymbol{x}_s \\ \delta \boldsymbol{x}_{leo} \\ \delta \boldsymbol{x}_{sta} \\ \delta \boldsymbol{x}_o \end{bmatrix}}_{\delta \boldsymbol{x}} - \underbrace{\begin{bmatrix} \boldsymbol{l}_{sta} \\ \boldsymbol{l}_{leo} \\ \boldsymbol{l}_{isl} \end{bmatrix}}_{\boldsymbol{l}} \tag{2}$$

Where

$$\begin{cases} \boldsymbol{l}_{sta} = \boldsymbol{L}_{sta} - \boldsymbol{G}\left(\boldsymbol{x}_s^0, \boldsymbol{x}_{sta}^0, \boldsymbol{x}_o^0, \boldsymbol{t}\right) \\ \boldsymbol{l}_{leo} = \boldsymbol{L}_{leo} - \boldsymbol{F}\left(\boldsymbol{x}_s^0, \boldsymbol{x}_{leo}^0, \boldsymbol{x}_o^0, \boldsymbol{t}\right) \\ \boldsymbol{l}_{isl} = \boldsymbol{L}_{isl} - \boldsymbol{R}\left(\boldsymbol{x}_s^0, \boldsymbol{x}_o^0, \boldsymbol{t}\right) \end{cases}$$

$\boldsymbol{x}(\bullet)^0$ represents the initial value of the parameter, $\delta(\bullet)$ represents correction to the parameter. The corresponding least squares solution can be expressed as:

$$\begin{aligned} \delta\hat{\boldsymbol{x}} &= \left(\boldsymbol{A}^T\boldsymbol{P}\boldsymbol{A}\right)^{-1}\boldsymbol{A}^T\boldsymbol{P}\boldsymbol{l} \\ \hat{\mathbf{Q}}_{\boldsymbol{x}} &= \left(\boldsymbol{A}^T\boldsymbol{P}\boldsymbol{A}\right)^{-1} = \left(\boldsymbol{A}_{sta}^T\boldsymbol{P}_{sta}\boldsymbol{A}_{sta} + \boldsymbol{A}_{leo}^T\boldsymbol{P}_{leo}\boldsymbol{A}_{leo} + \boldsymbol{A}_{isl}^T\boldsymbol{P}_{isl}\boldsymbol{A}_{isl}\right)^{-1} \end{aligned} \tag{3}$$

Where

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{P}_{sta} & & \\ & \boldsymbol{P}_{leo} & \\ & & \boldsymbol{P}_{isl} \end{bmatrix}$$

$\boldsymbol{A}_{sta}$, $\boldsymbol{A}_{leo}$ and $\boldsymbol{A}_{isl}$ are the first, second and third rows of matrix $\boldsymbol{A}$ in formula (2). $\hat{\boldsymbol{Q}}_{\boldsymbol{x}}$ is the covariance matrix for estimating unknowns. Obviously, compared to the simplistic technique for determining the orbit parameters of ground monitoring stations alone, it's conducive to improving the accuracy of parameter estimation by adding low-satellite and inter-satellite link data.

2.2 Weight Determination Method for Observation Data

There are usually three ways of determining the weight ratio of any kind of observation data [13]. In the first, weight is determined based on the empirical model for processing long-term observation data [14]. The second is to determine the weight ratio from standard deviation of known observation data. The third is to adjust the weight ratio by using variance component estimation and posterior residual analysis. The standard deviation of the observed values is just the internal coincidence precision. Empirical weighting can not objectively reflect the actual deviation of observed data. In case of multi-source data with a large amount of complex-type observations, improper weight ratio determination will lead to a decline in processing accuracy of data. Helmert variance component estimation is widely used in the field of data processing, because it can reweight observation values according contribution to global solution, by utilizing adaptive iteration in the calculation process for posterior variance component estimation.

2.3 Helmert Variance Component Estimation

It is assumed that the linear system containing k kinds of observation data can be expressed as [15]:

$$\begin{aligned} & L_i = \boldsymbol{A}_i \boldsymbol{X}_i + \boldsymbol{\Delta}_i \\ & \sum\nolimits_{\boldsymbol{\Delta}_i} = \sum\nolimits_{\boldsymbol{L}_i} = \sigma_{0,i}^2 \boldsymbol{Q}_{\boldsymbol{\Delta}_i} = \sigma_{0,i}^2 \boldsymbol{P}_i^{-1} \end{aligned} \tag{4}$$

Where $\boldsymbol{L}_i$ is the type i observation value of dimension $m_i \times 1$, $\boldsymbol{A}_i$ is a coefficient matrix of $m_i \times n$, $\boldsymbol{X}$ is the coefficient matrix of $n \times 1$, $\boldsymbol{\Delta}_i$ is the random error vector of $m_i \times 1$, $\sigma_{0,i}^2$ is the unit weight variance of type i observations, $\sum_i$ is the covariance matrix of $\boldsymbol{\Delta}_i$, and $\boldsymbol{P}_i$ is the weight matrix of $\boldsymbol{\Delta}_i$.

Its normal equation is:

$$\left(\sum_{i=1}^{k} \boldsymbol{N}_i \right) \hat{X} = \sum_{i=1}^{k} U_i \tag{5}$$

Where $\boldsymbol{N}_i = \boldsymbol{A}_i^T \boldsymbol{P}_i \boldsymbol{A}_i$, $U_i = \boldsymbol{A}_i^T \boldsymbol{P}_i L_i$.

The above formula is based on the same variance factor. Normally variance factors of different types of observations are not the same and can be adjusted in accordance with the variance component. The basic principle of the Helmert variance component estimation is to estimate the posterior unit weight $\hat{\sigma}_{0,i}^2$ of the class parameter, based on the correction vector $V_i = L_i - A_i \hat{\boldsymbol{X}}_i$ obtained by adjustment.

2.3.1 Rigorous Formula of Helmert Variance Component Estimation

The rigorous formula of Helmert variance component estimation is:

$$\underset{k \times k}{\boldsymbol{S}} \cdot \underset{k \times 1}{\hat{\boldsymbol{\sigma}}^2} = \underset{k \times 1}{\boldsymbol{W}} \tag{6}$$

Where,

$$\boldsymbol{S} = \begin{bmatrix} s_{1,1} & s_{1,2} & \cdots & s_{1,k} \\ s_{1,2} & s_{2,2} & \cdots & s_{2,k} \\ \vdots & \vdots & \ddots & \vdots \\ s_{1,k} & s_{2,k} & \cdots & s_{k,k} \end{bmatrix} \tag{7}$$

$$\hat{\boldsymbol{\sigma}}^2 = \left[\hat{\sigma}_{0,1}^2, \quad \hat{\sigma}_{0,2}^2, \quad \cdots, \quad \hat{\sigma}_{0,k}^2 \right]^T \tag{8}$$

$$\boldsymbol{W} = \left[\boldsymbol{V}_1^T \boldsymbol{P}_1 \boldsymbol{V}_1, \quad \boldsymbol{V}_2^T \boldsymbol{P}_2 \boldsymbol{V}_2, \quad \cdots, \quad \boldsymbol{V}_k^T \boldsymbol{P}_k \boldsymbol{V}_k \right]^T \tag{9}$$

$$\begin{aligned} & s_{i,i} = m_i - 2tr\left(\boldsymbol{N}^{-1} \boldsymbol{N}_i\right) + tr\left(\boldsymbol{N}^{-1} \boldsymbol{N}_i\right)^2 \\ & s_{i,j} = tr\left(\boldsymbol{N}^{-1} \boldsymbol{N}_i \boldsymbol{N}^{-1} \boldsymbol{N}_j\right) \end{aligned} \tag{10}$$

Solution of (6):

$$\hat{\boldsymbol{\sigma}}^2 = \boldsymbol{S}^{-1}\boldsymbol{W} \tag{11}$$

In data processing of the joint orbit determination on various observations, there are large data volume from ground monitoring stations, inter-satellite link and on-borne BDS. On this condition, the estimation with rigorous formula would be a heavy work. The following is an improved method for faster calculation.

2.3.2 Simplified Formula of Helmert Variance Component Estimation

In (8), let

$$\hat{\sigma}^2_{0,1} = \hat{\sigma}^2_{0,2} = \cdots = \hat{\sigma}^2_{0,k} = \hat{\sigma}^2_{0,i}(\neq \hat{\sigma}^2_0) \tag{12}$$

We have:

$$\begin{aligned} \boldsymbol{V}_i^T\boldsymbol{P}_i\boldsymbol{V}_i &= \left(m_i - 2tr\left(\boldsymbol{N}^{-1}\boldsymbol{N}_i\right) + tr\left(\boldsymbol{N}^{-1}\boldsymbol{N}_i\boldsymbol{N}^{-1}\sum_{j=1}^{k}\boldsymbol{N}_j\right)\right)\hat{\sigma}^2_{0,i} \\ &= \left(m_i - 2tr\left(\boldsymbol{N}^{-1}\boldsymbol{N}_i\right) + tr\left(\boldsymbol{N}^{-1}\boldsymbol{N}_i\boldsymbol{N}^{-1}\boldsymbol{N}^{-1}\right)\right)\hat{\sigma}^2_{0,i} \\ &= \left(m_i - tr\left(\boldsymbol{N}^{-1}\boldsymbol{N}_i\right)\right)\hat{\sigma}^2_{0,i} \end{aligned} \tag{13}$$

The above formula is a simplified version of the Helmert variance component estimation. At the start of the iteration, formula (12) is not valid, the valuation being biased. After several iterations; though, the formula will prove satisfactory, the final result would be unbiased.

Omit the trace section in Eq. (13), then

$$\hat{\sigma}^2_{0,i} = \frac{\boldsymbol{V}_i^T\boldsymbol{P}_i\boldsymbol{V}_i}{m_i} \tag{14}$$

The sum of (14) is known to have biased valuation, so is called the approximate formula of Helmert variance component estimation.

If $\boldsymbol{P}_i$ is true, then Eq. (12) is true, and

$$\hat{\sigma}^2_{0,i} = \hat{\sigma}^2_0 \tag{15}$$

So we know from (13) that:

$$\begin{aligned} \sum_{i=1}^{k}\boldsymbol{V}_i^T\boldsymbol{P}_i\boldsymbol{V}_i &= \sum_{i=1}^{k}\left(m_i - tr\left(\boldsymbol{N}^{-1}\boldsymbol{N}_i\right)\right)\hat{\sigma}^2_{0,i} \\ &= \left(m - tr\left(\boldsymbol{N}^{-1}\sum_{i=1}^{k}\boldsymbol{N}_i\right)\right)\hat{\sigma}^2_0 \\ &= (m-n)\hat{\sigma}^2_0 \end{aligned} \tag{16}$$

That is:

$$\hat{\sigma}_0^2 = \frac{\sum_{i=1}^{k} \boldsymbol{V}_i^T \boldsymbol{P}_i \boldsymbol{V}_i}{m - n} \tag{17}$$

Usually when given the weight, this formula can be used to calculate the unit weight variance valuation.

In adjustment calculations, the off-diagonal elements of the coefficient matrix $\boldsymbol{N}$ and $\boldsymbol{N}_i$ of the normal equation are usually smaller than the main diagonal elements, so a better approximation can be obtained by omitting them.

Let $\boldsymbol{N}_i$ from which remove the off-diagonal element be:

$$\boldsymbol{N}_i = \boldsymbol{A}_i^T \boldsymbol{P}_i \boldsymbol{A}_i = diag([p\boldsymbol{aa}]_i, \quad [p\boldsymbol{bb}]_i, \quad \cdots, \quad [p\boldsymbol{bb}]_i) \tag{18}$$

So:

$$\begin{aligned} tr\left(\boldsymbol{N}^{-1}\boldsymbol{N}_i\right) &= \sum_{i=1}^{k} \frac{\boldsymbol{N}_{f_i}}{\boldsymbol{N}_f}, f = a, b, c, \cdots, n \\ tr\left(\boldsymbol{N}^{-1}\boldsymbol{N}_i\boldsymbol{N}^{-1}\boldsymbol{N}_j\right)^2 &= \sum_{i=1}^{k} \frac{\boldsymbol{N}_{f_i}\boldsymbol{N}_{f_j}}{\boldsymbol{N}_f} \end{aligned} \tag{19}$$

By using formula (19) to obtain the trace, the calculation can be simplified and faster.

2.3.3 Bäumker Variance Component Estimation Formula

Estimation proposed by Bäumker in 1984 [16] is:

$$\hat{\sigma}_{0,i}^2 = \frac{\boldsymbol{V}_i^T \boldsymbol{P}_i \boldsymbol{V}_i}{g_i} \tag{20}$$

Where:

$$g_i = m_i - \frac{m_i}{m} t \tag{21}$$

Where m is the number of all observations, m_i is the number of observed values of class i, and n is the number of all unknown parameters. Compared with the original method, it improved the calculation efficiency greatly in multi-type and mass data processing, avoiding the complex matrix calculation.

3 Improved Formula of Variance Component in Combined Orbit Determination

In formula (21), different types of observations correspond to the same batch of unknown parameters, while in the joint orbit determination of the three types of observation data, the corresponding parameters to be estimated are not the same. As shown in Table 1, the information of the parameters to be estimated in the ground receiver data, the satellite-borne receiver data and the inter-satellite link data are different.

Table 1. Parameter types of data processing in combined orbit determination

Parameter types	Ground receiver data	Satellite-borne data	Inter-satellite link data
Observation station coordinates, clock difference and troposphere parameter	√	–	–
Carrier phase ambiguity parameter	√	√	–
Low-orbit satellite orbit and clock difference parameters	–	√	–
Satellite navigation orbit and clock difference	√	√	√

The table above shows that the unknown parameters of different types of observed quantities are different. Data from satellite-borne or inter-satellite link is of no use for observation station coordinates, clock difference and troposphere parameter. So it is not adequate to calculate directly according to Eq. (21). An improved formula is proposed:

$$g_i = m_i - \sum_{j=1}^{n} \frac{m_i^j}{m^j} \tag{22}$$

Where m_i^j is the number of i-class observation quantities related to the parameter j, and m^j is the total number of observations related to the parameter j. This formula describes more finely the contribution of observation to unknowns.

4 Experimental Verification

4.1 Experimental Data

In order to verify the effectiveness of the improved Helmert variance component estimation method and combined orbit determination weight, three types of data including ground, low orbit and inter-satellite link are simulated, and 3GEO + 3 IGSO + 24MEO (walk24/3/2) [17] are adopted. The Ka bean angle range of inter-satellite link is 15–60°. The low orbit constellation is the Walker6/6/2 configuration.

The measurement accuracy of ground and satellite-borne receivers is 0.3 m in pseudorange, and 0.002 m in the carrier phase. The measurement accuracy of inter-satellite link data is 0.1 m and the system deviation is 0.1 m. Three schemes are adopted to conduct orbit determination experiments.

4.2 Experimental Schemes

The studies show that during the process of orbit determination, ground station and satellite-borne data do not need to undergo different processing [13]. While it's hard to assign weights to the inter-satellite link data for the system errors. To compare the influence of weight size on results, and that of variance component estimation on combined orbit determination, there are three methods listed in the scheme below, mainly based on the assignment of large and small weights to inter-satellite link data.

Scheme 1: Assign weights according to the magnitude of the noise of the observed values (single-frequency, pseudo-range, phase of ground station and space-borne data is 0.3 m and 0.2 m respectively, while inter-satellite data is 0.1 m).
Scheme 2: Utilize artificial methods to reduce or increase inter-satellite weight values, and assign them according to the multiple relationships of the original weights $p/5.0$, $p/10.0$, $5p$ and $10p$ (respectively corresponding to IIA, IIB, IIC, IID).
Scheme 3: Estimate posterior variance using the improved Helmert variance component estimation method, and improve weight ratio.

4.3 Results and Analysis

The weight correction factors of the three observed quantities, obtained by posterior variance component estimation, are respectively: 3.8:5.6:0.38. This means that on the basis of weighting in accordance with the accuracy of observation, the weight of ground monitoring station data and low-orbit satellite-borne data is increased, while the weight of inter-satellite link data is decreased. Table 1 shows the 3-dimensional position errors of the orbit determination results of each navigation satellite corresponding to the three schemes. It can be seen that in scheme 1, the average 3d orbit determination accuracy of the satellites is 0.073 m, obtained by weighing the accuracy of the observed values. In scheme 2, the orbit determination accuracy is 0.067 m, 0.075 m and 0.080 m respectively of the 4 kinds of weights, which indicates that weight assignment has a definite influence on orbit determination accuracy. In scheme 3, variance component estimation is used to determine the orbit, with the average accuracy of the satellite's 3-D orbit being 0.058 m, which is the highest accuracy in comparison with other strategies. This shows that variance component estimation is effective for the determination of combined orbits in this paper. Furthermore if comparing the results of schemes 3 and 2, we can see the results in scheme 3 are more consistent with the P/10/0. In fact, the weight ratio of all 3 types of data is 10:10:1, compared with other schemes, is much closer to posterior variance component estimation. That confirmed the effectiveness of the Helmert variance component estimation method. Compared with scheme 1, which relies on the traditional weighting strategy based on the accuracy of observed data, scheme 3 improves the accuracy from 0.075 m to 0.058 m, about 21% up.

Table 2 shows the orbital errors of the above schemes in radial (R), tangential (T) and normal (N) directions, and the results are basically consistent with previous ones. The Helmert variance component estimation was valid in R, T and N directions, when the variance components were weight adjusted, obtaining better orbit determination accuracy than other methods. Orbit determination accuracy in the R direction was 0.015 m, 0.038 m in T and 0.039 m in N, increasing by 31.8%, 13.6% and 20.4% respectively compared to scheme 1 (Table 3).

Table 2. 3D orbit determination accuracy of different weighted strategies according to satellite number statistics (unit: m)

PRN	Scheme 1	Scheme 2				Scheme 3
	p	IIA: $p/5.0$	IIB: $p/10.0$	IIC: $5p$	IID: $10p$	M-Helmert
01	0.044	0.041	0.053	0.035	0.035	0.050
02	0.083	0.045	0.054	0.044	0.047	0.052
03	0.164	0.085	0.090	0.085	0.089	0.084
04	0.067	0.159	0.130	0.176	0.181	0.115
05	0.096	0.053	0.033	0.071	0.075	0.029
06	0.064	0.058	0.041	0.075	0.080	0.035
07	0.035	0.089	0.061	0.106	0.109	0.052
08	0.049	0.058	0.048	0.069	0.071	0.043
09	0.096	0.040	0.054	0.030	0.030	0.051
10	0.055	0.042	0.033	0.056	0.062	0.032
11	0.075	0.091	0.073	0.103	0.107	0.066
12	0.081	0.045	0.020	0.067	0.075	0.018
13	0.037	0.068	0.052	0.086	0.093	0.046
14	0.039	0.079	0.074	0.086	0.089	0.068
15	0.044	0.040	0.053	0.040	0.045	0.050
16	0.075	0.045	0.060	0.035	0.037	0.057
17	0.065	0.038	0.021	0.051	0.053	0.020
18	0.069	0.069	0.051	0.083	0.087	0.044
19	0.050	0.062	0.056	0.070	0.075	0.052
20	0.093	0.070	0.075	0.076	0.083	0.072
21	0.046	0.053	0.064	0.053	0.057	0.061
22	0.063	0.090	0.082	0.094	0.096	0.074
23	0.170	0.043	0.042	0.049	0.051	0.039
24	0.056	0.058	0.041	0.067	0.067	0.035
25	0.047	0.162	0.138	0.184	0.194	0.134
26	0.067	0.055	0.069	0.062	0.068	0.067
27	0.040	0.052	0.079	0.050	0.057	0.077
28	0.111	0.067	0.078	0.078	0.086	0.076
29	0.132	0.034	0.041	0.056	0.063	0.039
30	0.074	0.109	0.105	0.119	0.126	0.098
Mean	0.073	0.067	0.062	0.075	0.080	0.058

Table 3. Statistics of orbit determination accuracy in the R/T/N direction with different weight methods (unit: m)

Scheme	Inter-satellite values	R	T	N
I	P	0.022	0.044	0.049
II-A	P/5.0	0.020	0.043	0.047
II-B	P/10.0	0.016	0.041	0.043
II-C	P*5.0	0.021	0.046	0.052
II-D	P*10.0	0.023	0.051	0.057
III	Helmert	0.015	0.038	0.039

5 Conclusion

In this study, the variance component estimation method is used in fusion weighted processing and orbit determination, on the data from ground, satellite-borne and inter-satellite link. The following conclusions were obtained:

1. On the data characteristics of large differences among the various types and system errors in inter-satellite data, this paper proposed a weighted Helmert variance component estimation method. With the limitations of strict variance component estimation, propose a data processing method on parameter classification in joint orbit determination.
2. Combined orbital determination was obtained by using simulated data on the Helmert variance component estimation. By comparing the results of schemes, an orbit determination strategy based on variance component estimation is the optimum solution. The weight correction factor obtained from the three observed quantities in posterior variance component estimation are more in line with actual joint processing, and the accuracy of the solutions is the best.
3. Compared with the traditional weighting strategy based on observation data accuracy, the Helmert variance component estimation reasonably adjusted the weight allocation of multiple-source tracking data. The average RMS orbit determination accuracy increased from 0.073 m to 0.058 m, increases of 31.8%, 13.6% and 20.4% respectively in R, T and N directions. The overall accuracy increased by 21%.

References

1. Feng, L., Mao, Y., Song, X., et al.: Analysis on the accuracy of Beidou combined orbit determination enhanced by LEO and ISL[J]. Acta Geod. et Cartogr. Sin. **45**(S2), 902–907 (2016)
2. Meng, Y., Bian, L., Wang, Y.: Global navigation augmentation system based on Hongyan satellite constellation. Space Int. **10**, 20–27 (2018)
3. Svehla, D., Rothacher, M.: Kinematic and reduced-dynamic precise orbit determination of low earth orbiters. Adv. Geosci. **2003**(1), 47–56 (2003)
4. Rothacher, M.: Precise orbit determination for low earth orbiters. In: Colloquium on Atmospheric Remote Sensing Using the Global Positioning System (2004)

5. Zhu, S., Reigber, C., Konig, R.: Integrated adjustment of CHAMP, GRACE, and GPS data. J. Geodesy **78**, 103–108 (2004)
6. Shen, D., Meng, Y., Bian, L., et al.: A global navigation augmentation system based on LEO communication constellation. J. Terahertz Sci. Electron. Inf. Technol. **17**(2), 209–215 (2019)
7. Kang, Z., Tapley, B., et al.: Precise orbit determination for the GRACE mission using only GPS data. J. Geodesy **80**, 322–331 (2006)
8. Geng, J., Shi, C., Zhao, Q., et al.: Precsise determination of GPS satellite orbit by combining ground and space-borne data. J. Wuhan Univ. Inf. Sci. Ed. **32**(10), 906–909 (2007)
9. Zhu, J.: Research on orbit determination and time synchronization method for navigation satellites based on inter-satellite link. University of National Defense Science and Technology, Changsha (2012)
10. Xiao, Y.: Research on key technology of autonomous navigation satellites, graduate school of the Chinese academy of sciences (Shanghai Institute of Technical Physics), Shanghai (2016)
11. Rui, G., Zhou, J.H., Hu, X.G., et al.: Precise orbit determination and rapid orbit recovery supported by time synchronization. Adv. Space Res. **55**(12), 2889–2898 (2015)
12. Guo, R., Chen, J., Zhu, L., et al.: Kinematic orbit determination method optimization and test analysis for BDS satellites with short-arc tracking data. Acta Geod. et Cartogr. Sin. **46**(4), 411–420 (2017). https://doi.org/10.11947/j.AGCS.2017.20160361
13. Qin, X..: Research on orbit determination theory and low-orbit satellite borne GPS method. University of Information Engineering (2009)
14. Xiao, G., Sui, L., Liu, C., et al.: Weight determination method for single point positioning of polaris navigation and positioning system. Acta Surv. Mapp **43**(9), 902–907 (2014)
15. Zu, A., Li, J., Wang, Y., et al. A BDS satellite orbit determination algorithm using variance component estimation. Surv. Mapp. Sci. (2009)
16. Hu, W.: Modern Adjustment Theory and its Application. PLA Press, Beijing (1992)
17. Wang, R., Ma, X., Li, M.: Optimal design of satellite clusters with regional coverage using genetic algorithm. Acta Astronaut. 23(3), 24–28 (2002)