Chapter 24
Analysis of Rotation Capacity of RC Beams Over Formation of Plastic Hinges

R. Prabhakara, B. Nambiyanna, R. Nakul and Bharathi Ganesh

Abstract Rotation capacity of a joint in a structural member under constant moment depends on the characteristics of the formation of plastic hinge. The rotation capacity and plastic hinge formation in RC member depends on many parameters and therefore makes it different over the formation of plastic hinge in steel structural members. It has been observed from the literature that the rotation capacity in RC structural member is influenced by several parameters such as mechanical properties of concrete and steel, geometry of the member, external load and boundary conditions, thus makes it to be an interesting structural behaviour which has to be explored very well. Parametric analysis on the rotation capacity and formation plastic hinge was carried out by collecting more than hundred data from the available literature. Relevant graphs were plotted to demonstrate the variation of plastic hinge length (lp*)* over the influencing parameters such as cross-sectional dimensions of the beams, grade of the concrete, tension reinforcement and span-to-depth ratio. The length of plastic hinge was calculated based on existing analytical models given by the researchers. Owing to the advancement in the analysis of nonlinear behaviour of structures using sophisticated software, the characterization of plastic hinges has to incorporated in the analysis. These analytical models are very useful and provided more knowledge

R. Prabhakara (✉)
Structural Engineering Division, PGS VTU, Belagavi, Karnataka, India
e-mail: r.prabhakara@gmail.com

B. Nambiyanna
Civil Engineering Department, Ramaiah Institute of Technology, Bangalore, Karnataka, India
e-mail: nambiyanna1@gmail.com

R. Nakul
Civil Engineering Department, Presidency University, Ittgalpura, Rajankunte, Bangalore, Karnataka, India
e-mail: nakul@presidencyuniversity.in

B. Ganesh
Department of Civil Engineering, Nitte Meenaski Institute of Technology, BSF Campus Yalahanka, Bangalore, Karnataka, India
e-mail: bharathi.ganesh@nmit.ac.in

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on the behaviour of RC structures. In this study, existing analytical models are evaluated based on the different parameters. The plastic hinge length was determined by nonlinear regression analysis. It was found from the analysis that, large variation was found over the length of plastic hinge for the existing plastic hinge models and the proposed nonlinear regression analysis model yielded better results.

Notations

d	Effective depth of beam, mm
l	Effective length of the beam, mm
l_p	Plastic hinge length, mm
$l_{p.reg}$	Plastic hinge length by regression mm
z	Distance from critical section to point of contraflexure, mm
f'_c	Designed cylinder compressive strength in MPa
f_y	Specified yield strength of the reinforcement, MPa
ρ	Tension zone reinforcement in %
ρ	Compression zone reinforcement in %
θ_p	Plastic rotation (rad)

24.1 Introduction

Engineers are facing the challenging situation in the analysis of RC structures while providing safe and economical buildings to the society. Nonlinear analysis has become one of the important means of achieving the above goal. RC structures are made out of structural elements such as column, beam and slab; also their elastic and inelastic behaviour determines the load response behaviour of the structure. Rotation capacity and formation of plastic hinge in the structural members is one of the main issues in understanding the inelastic behaviour of the structure. More specifically in the earthquake design of structures, ductility plays a key role in the design criteria set by many countries' code provisions.

When the load applied is amplified, the hinge begins to form at locations where the moment is equal to the plastic moment, if further increase in the applied load then hinges transforming into plastic hinges and converting into mechanism followed with failure. Framed structures are designed to achieve the above by proportioning the beams and columns so that the majority of the plastic hinges are formed in the beams not in the column. For this reason, an important parameter is the plastic hinge, which controls the rotation capacity of the structural members. There are two different types of plastic hinges formed in RC structure, such as reversing plastic hinges and unidirectional plastic hinges. The reversing plastic hinge sustains both positive and negative inelastic rotations in the same region, and unidirectional plastic hinge sustains both in different regions of the beam element during earthquake [1]. It was observed that plastic hinge length l_p in RC beams influenced by a number of parameters such as cross-sectional dimensions, yield and ultimate curvature, material characteristics, tension and compression reinforcement ratios, support type, intensity and nature of loading [2, 3].

The main objective of the present study is to propose a soft computing method that can estimate the rotation capacity and length of plastic hinge by using the available data from experimental studies conducted on RC beams [4–9]. The regression analysis was carried out to find out the influencing parameters on which the rotation capacity and plastic hinge length and also to demonstrate the relationship between the plastic hinge length and rotation capacity.

24.2 Rotation Capacity and Plastic Hinge Length

In the present investigation, the rotation capacity is defined as the rotation of critical section of a beam from yield stage to the ultimate stage, and the plastic hinge length is defined as the length of inelastic zone over which bending moment will be larger than or equal to yield moment over the length of the beam. It is generally agreed that the inelastic rotations are concentrated over a length called "plastic hinge length" where $M \geq M_y$ as shown in Fig. 24.1. The length of plastic hinge is dependent on many parameters, such as the shape of the ultimate bending moment diagram, and is also affected by the length between zero moment points and distance between support, reinforcement ratio, the characteristic strength of concrete and depth of section [2]. The corresponding plastic hinge length is determined with respect to integration of the curvature distribution for distinctive members.

The correlation between the cross section, ductility and the length of plastic hinge needs to be accurately determined. It has been observed there is a large variation in the plastic hinge length formulae proposed by the researchers as shown in Table 24.1. Sawyer, Corley and Mattock, for example, considered only the member's length and depth. Priestley and Park consider the length and diameter of reinforcement of the beam.

Fig. 24.1 Length of plastic hinge

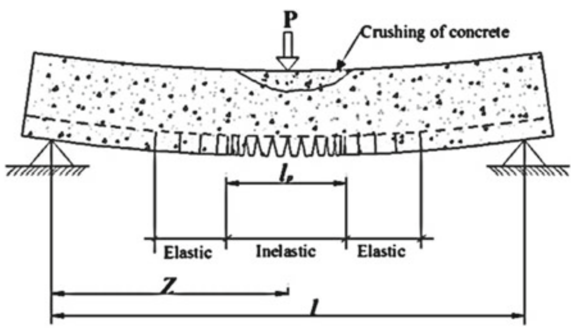

Table 24.1 Existing length of plastic hinge (l_p) models [8]

Reference	Length plastic hinge expression (l_p)
Baker (1956)	$k(z/d)^{1/4}\ d$
Sawyer (1964)	$0.25d + 0.075z$
Corley (1966)	$0.5d + 0.2\sqrt{d}\,(z/d)$
Mattock (1967)	$0.5d + 0.05z$
Priestley and Park (1987)	$0.08z + 6d_b$
Paulay and Priestley(1992)	$0.08z + 0.022d_b$
Sheikh and Khoury (1993)	1.0 h
Coleman and Spacone (2001)	$G_f^c/[0.6f'_c(\varepsilon_{20} - \varepsilon_c + 0.8 f'_c/E_c)]$
Panagiotakos and Fardis (2001)	$0.18z + 0.021d_b$

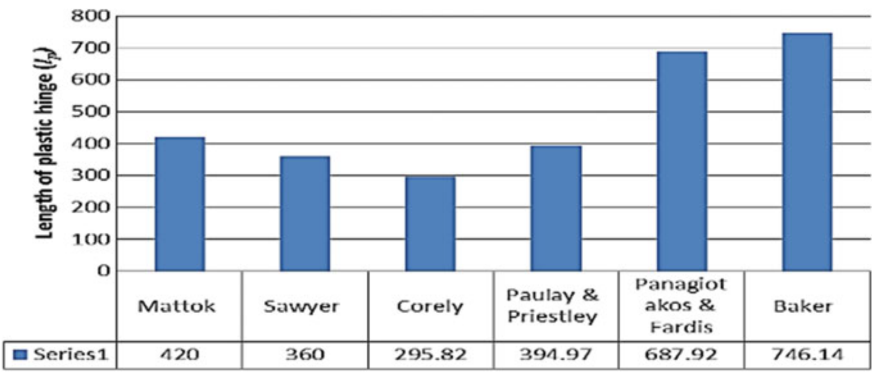

Fig. 24.2 Variation of plastic hinge length (l_p)

To be able to demonstrate the variations in the length of plastic hinge, the geometric and material properties were taken as an example from [6] $l = 6000$ mm, $b = 300$ mm, $d = 540$ mm, $f'_c = 30.9$ MPa, $z = 3000$ mm and $l/d = 11.1$. For the above data, plastic hinge lengths were obtained using the above equations and they are shown in Fig. 24.2. It was observed a large variation in the plastic hinge length which encouraged us to study on the parameters effect and regression analysis.

24.3 Parametric Study

In the present study, the central point loading configuration and different equivalent length of beams were selected from the literature [4–9]. The range of the parameters such as characteristics strength of concrete and steel, width and depth of beams,

Table 24.2 Range of parameters used

Parameters	Parameter range
f'_c (MPa)	20–129.1
f_y (MPa)	250–678
l (mm)	1000–12,000
b (mm)	50–500
d (mm)	90–950
z (mm)	500–6000
ρ (%)	0.13–6.45
ρ' (%)	0–4.84
l/d	4.17–21.82

tension and compression steel percentages and all other essential parameters that affect the beam behaviour under central point loading is shown in Table 24.2.

24.4 Determination of Length of Plastic Hinge

The length of the plastic hinge l_p was determined based on different parameters obtained from the previous studies. The beam plastic rotation (θ_p) was calculated from Eq. (1) [10].

$$\theta_p = \frac{0.004}{(x_u/d)} \tag{1}$$

The plastic hinge length l_p was calculated based on Eq. (2) [2], and curvature at yielding moment (φ_y) and the curvature at ultimate moment (φ_u) and plastic hinge length values were analytically calculated.

$$l_p = \frac{\theta_p}{(\varphi_u - \varphi_y)} = \frac{\theta_p}{\varphi_p} \tag{2}$$

Illustrative graphs have been plotted to observe the variation of l_p against the parameters which are shown in Fig. 24.3a–f.

It has been observed from Fig. 24.3a–f that the length of plastic hinge has not shown much variation with grade of concrete, and however, reduction in l_p was observed with increase in reinforcement ratio and further it was observed that l_p increased when the depth of the beam increased, *l*/*d* ratio, *z* and *z/d* ratio.

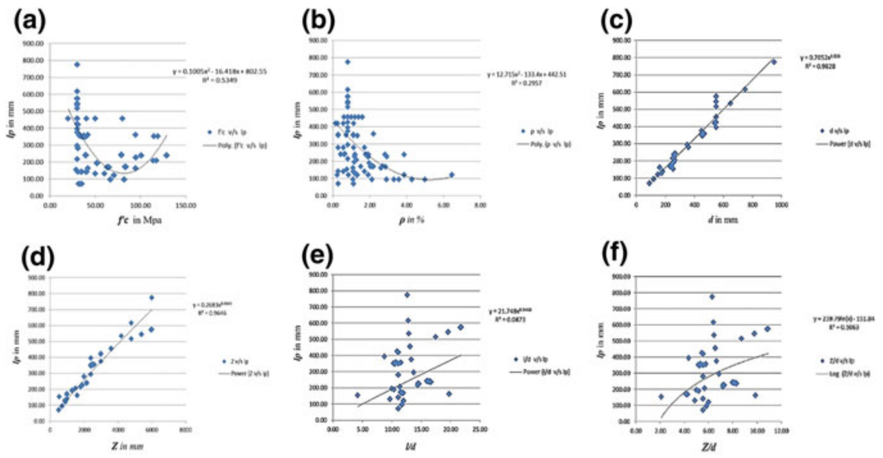

Fig. 24.3 Plastic hinge length (lp*)* versus various parameters

24.5 Nonlinear Regression Analysis

Nonlinear regression analysis is a method of mathematical modelling. In this study, cross-sectional dimensions, material characteristics, yielding and ultimate deflections, tension and compression reinforcement ratios were observed and parameters that affect on length of plastic hinge are analysed by using the nonlinear regression analysis. Formulated our regression equation for plastic hinge length (l_{p}) as follows.

$$l_p = 0.37 \times b^{0.00896} \times d^{0.667} \times f'c^{-0.0016} \times z^{0.3264} \times \rho^{-0.00207} \times \left(\frac{l}{d}\right)^{0.059266} \quad (3)$$

It has been observed from Fig. 24.4 that the comparison between regression plastic hinge length $\boldsymbol{l}_{\mathbf{p\ reg}}$ versus existing models $\boldsymbol{l}_{\mathbf{p}}$ trend lines shows that the proposed model is predicted well with Mattock, Sawyer, Corely and Baker's models and R^2 values are almost equal to unity.

It has been observed from Fig. 24.5 that the comparison between regression plastic hinge length $\boldsymbol{l}_{\mathbf{p\ reg}}$ versus $\boldsymbol{\theta}_{\mathbf{p}}$ trend lines shows that there is no significant relationship between l_{p} and θ_{p}.

24.6 Conclusions

Based on the parametric study and proposed nonlinear regression analysis, the following conclusions are drawn.

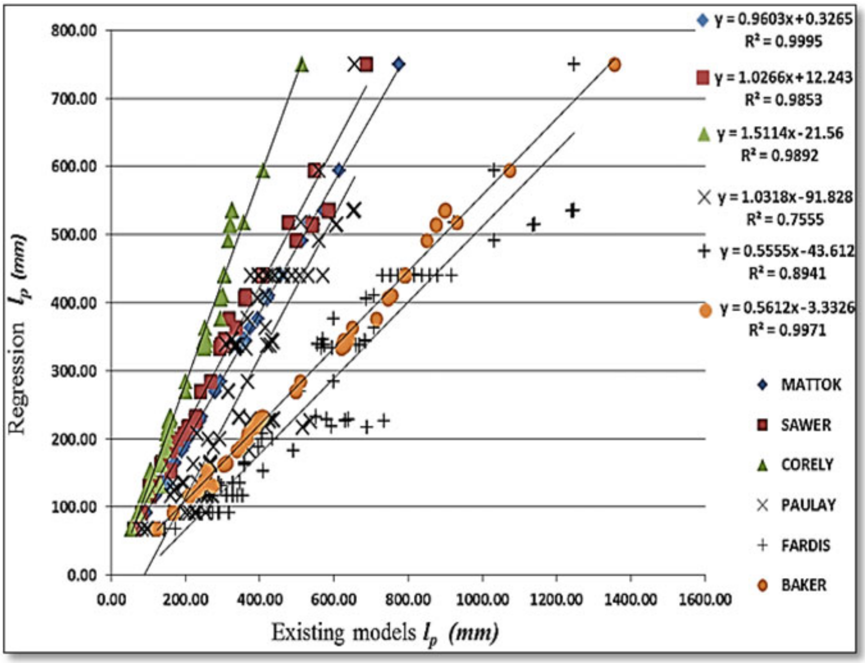

Fig. 24.4 Comparison between regression $l_{p\ reg}$ and existing models l_p

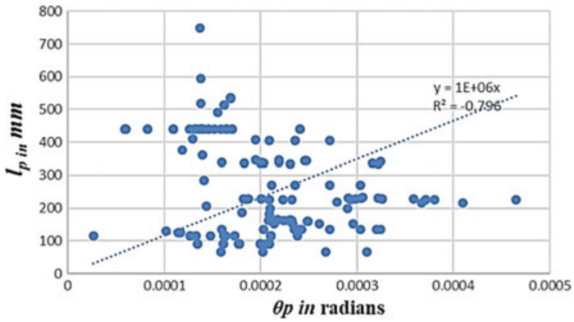

Fig. 24.5 Comparison between regression l_p and θ_p

- The plastic hinge length results are obtained by using plastic rotation and curvatures at ultimate and yield load (vide Eqs. 1 and 2).
- Demonstrative graphs have been plotted to show the variation of influencing parameters. It was observed from Fig. 24.3 that the length of plastic hinge has not shown much variation with grade of concrete, and however, reduction in l_p was observed with increase in reinforcement ratio.
- Further, it was observed that l_p amplified with increase in depth, *l/d* ratio, *z* and ***z/d*** *ratio.*

- Analytical expressions given in the literature indicated that only the cross-sectional dimensions are under the bending effect of beam and other influencing parameters were not considered.
- The plastic hinge length (l_p) obtained by expressions given by the researchers gave different values and are not comparable (Fig. 24.2).
- It has been observed that analytical plastic hinge expression and proposed nonlinear regression expression yielded results that are comparable.
- Based on the comparison between $\boldsymbol{l_p}$ calculated by existing model and $\boldsymbol{l_{p.reg}}$ calculated by nonlinear regression model, the results show that the nonlinear regression plastic hinge model ($\boldsymbol{l_{p.reg}}$) is versatile and can be used all types RC beams.
- Based on comparison between rotation capacity and length of plastic hinge, there was no significant relation between them and hence, further investigation has to be done over large number of data.

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