

Chapter 10

Multi-Objective Optimization for Secure Full-Duplex Wireless Communication Systems



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Abstract In traditional half-duplex (HD) communication systems, the HD base station (BS) can transmit artificial noise (AN) to jam the eavesdroppers for securing the downlink (DL) communication. However, guaranteeing uplink (UL) transmission is not possible with an HD BS because HD BSs cannot jam the eavesdroppers during UL transmission. In this chapter, we investigate the resource allocation algorithm design for secure multiuser systems employing a full-duplex (FD) BS for serving multiple HD DL and UL users simultaneously. In particular, the FD BS transmits AN to guarantee the concurrent DL and UL communication security. We propose a multi-objective optimization framework to study two conflicting yet desirable design objectives, namely total DL transmit power minimization and total UL transmit power minimization. To this end, the weighted Tchebycheff method is adopted to formulate the resource allocation algorithm design as a multi-objective optimization problem (MOOP). The considered MOOP takes into account the quality-of-service (QoS) requirements of all legitimate users for guaranteeing secure DL and UL transmission in the presence of potential eavesdroppers. Thereby, secure UL transmission is enabled by the FD BS, which would not be possible with an HD BS. Although the considered MOOP is non-convex, we solve it optimally by semidefinite programming (SDP) relaxation. Simulation results not only unveil the trade-off between the total DL transmit power and the total UL transmit power, but also confirm that the proposed secure FD system can guarantee concurrent secure DL and UL transmission and provide substantial power savings over a baseline system.

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10.1 Introduction

In the past two decades, the telecommunication industry has developed rapidly thanks to the advance of signal processing. The success of the third-generation (3G) and the fourth-generation (4G) wireless communication systems has promoted numerous innovative mobile applications and has led to an explosive and continuing growth in mobile data traffic [23]. According to the Cisco visual network index report [10], the global mobile data traffic will increase sevenfold between 2016 and 2021. As a result, it is foreseen that the demand for data traffic will exceed the capacity of the existing wireless communication systems in the near future. Besides, the exponential growth in high-data rate communications has triggered a tremendous demand for radio resources such as bandwidth and energy [22–24]. An important technique for reducing the energy and bandwidth consumption of wireless systems while satisfying quality-of-service (QoS) requirements is multiple-input multiple-output (MIMO), as it offers extra spatial degrees of freedom (DoF) facilitating the design of efficient resource allocation. However, the MIMO gain may be difficult to achieve in practice due to the high computational complexity of MIMO receivers. As an alternative, multiuser MIMO (MU-MIMO) has been proposed as an effective technique for realizing MIMO performance gains. In particular, in MU-MIMO systems, a transmitter equipped with multiple antennas (e.g., a base station (BS)) serves multiple single-antenna users, which shifts the computational complexity from the receivers to the transmitter [8]. Yet, the spectral resource is still underutilized even if MU-MIMO is employed as long as the BS adopts the traditional half-duplex (HD) protocol, where uplink (UL) and downlink (DL) communication are separated orthogonally in either time or frequency which leads to a significant waste of in-system resources.

On the other hand, security is a crucial issue for wireless communication due to the broadcast nature of the wireless medium. Traditionally, secure communication is achieved by cryptographic encryption performed at the application layer and is based on the assumption of limited computational capabilities of the eavesdroppers [4]. However, new computing technologies (e.g., quantum computers) may make this assumption invalid which results in a potential vulnerability of traditional approaches to secure communication. The pioneering work in [21] proposed an alternative approach for providing perfectly secure communication by utilizing the nature of the channel in the physical layer. Specifically, Wyner [21] revealed that secure communication can be achieved whenever the information receiver enjoys better channel conditions than the eavesdropper. Motivated by this finding, physical layer security has received significant attention for preventing eavesdropping in wireless communication systems [4, 14, 17, 26]. An important technique to ensure communication security via physical layer security is multiple-antenna transmission which utilizes the spatial DoF for degrading the quality of the eavesdroppers' channels. In particular, artificial noise (AN) transmission is an effective approach to deliberately impair the information reception at the eavesdroppers. For instance, in [17], a power allocation algorithm was designed for maximizing the secrecy

outage capacity via AN generation. In [26], the authors investigated the secrecy performance of DL massive MIMO systems and derived a lower bound on the achievable ergodic secrecy rate of the users. The authors of [14] proposed a robust resource allocation algorithm for guaranteeing secure multiuser communication with energy harvesting receivers. However, the aforementioned works focused on guaranteeing DL communication security between a HD BS and associated DL users. The obtained results may not be applicable for securing UL transmission. In fact, guaranteeing UL transmission is not possible with an HD BS. Particularly, HD BSs cannot jam the eavesdroppers during UL transmission because they can either transmit or receive in a given time instant but not both.

To overcome these issues, an intuitive concept for improving the spectral efficiency is to employ full-duplex (FD) transceivers which can transmit and receive signals at the same time and in the frequency band. More importantly, an FD BS enables simultaneous secure DL and UL communication by transmitting AN in the DL to interfere potential eavesdroppers [25]. However, deploying wireless FD nodes has been generally considered impractical for a long time since the signal reception is severely impaired by the self-interference (SI) caused by the signal leakage due to the simultaneous transmission at the same node [20]. Recently, the authors of [2] developed a single-antenna FD transceiver prototype which achieves 110 dB SI cancellation offering a substantial system throughput improvement compared to HD transceivers. This has attracted significant attention from both academia and industry. Several FD prototypes using different SI cancellation techniques have been built and they demonstrate that FD operation can achieve higher throughput than HD for various system settings [1, 6, 11]. For instance, in [6], the authors designed an FD prototype equipped with multiple antennas. In [1], it could be shown that FD transmission can double the ergodic capacity of MIMO systems. Although [2, 11] reported that SI can be partially cancelled through analog circuits and digital signal processing, the residual SI still severely degrades the performance of FD systems if it is not properly controlled. Besides, in multiuser communication systems, co-channel interference (CCI) caused by the UL transmission impairs the DL transmission [18]. Moreover, the CCI becomes more severe if there are multiple DL and UL users in the communication system. Therefore, careful resource allocation is necessary and critical to fully exploit the potential performance gains enabled by FD communications. In fact, the unique challenges introduced by FD networks do not exist in HD networks, and thus, the conventional designs for HD networks cannot be directly applied to FD networks. Hence, in this chapter, we focus on resource allocation algorithm design for guaranteeing concurrent secure DL and UL transmission in multiuser FD systems.

The remainder of this chapter is organized as follows. In Sect. 10.2, we introduce the adopted FD system model. In Sect. 10.3, the resource allocation algorithm design for guaranteeing secure communication in FD systems is formulated as a non-convex optimization problem. The formulated problem is solved optimally in Sect. 10.4, and simulation results are provided in Sect. 10.5. In Sect. 10.6, we conclude with a brief summary of this chapter.

Notation We use boldface capital and lower case letters to denote matrices and vectors, respectively. \mathbf{A}^H , $\text{Tr}(\mathbf{A})$, $\text{Rank}(\mathbf{A})$, and $\det(\mathbf{A})$ denote the Hermitian transpose, trace, rank, and determinant of matrix \mathbf{A} , respectively; \mathbf{A}^{-1} and \mathbf{A}^\dagger represent the inverse and Moore–Penrose pseudoinverse of matrix \mathbf{A} , respectively; $\mathbf{A} \succeq \mathbf{0}$, $\mathbf{A} \succ \mathbf{0}$, and $\mathbf{A} \preceq \mathbf{0}$ indicate that \mathbf{A} is a positive semidefinite, a positive definite, and a negative semidefinite matrix, respectively; \mathbf{I}_N is the $N \times N$ identity matrix; $\mathbb{C}^{N \times M}$ denotes the set of all $N \times M$ matrices with complex entries; \mathbb{H}^N denotes the set of all $N \times N$ Hermitian matrices; $|\cdot|$ and $\|\cdot\|$ denote the absolute value of a complex scalar and the Euclidean vector norm, respectively; $\mathcal{E}\{\cdot\}$ denotes statistical expectation; $\text{diag}(\mathbf{X})$ returns a diagonal matrix having the main diagonal elements of \mathbf{X} on its main diagonal. $[x]^+$ stands for $\max\{0, x\}$; the circularly symmetric complex Gaussian distribution with mean μ and variance σ^2 is denoted by $\mathcal{CN}(\mu, \sigma^2)$; and \sim stands for “distributed as.”

10.2 System Model

In this section, we present the considered MU-MIMO FD wireless communication system model.

10.2.1 Multiuser System Model

We consider a multiuser communication system [19]. The system consists of an FD BS, K legitimate DL users, J legitimate UL users, and a roaming user, cf. Fig. 10.1. The FD BS is equipped with $N_T > 1$ antennas to facilitate simultaneous DL transmission and UL reception in the same frequency band.¹ The $K + J$ legitimate users are single-antenna HD mobile communication devices to ensure low hardware complexity. The number of antennas at the FD BS is assumed to be larger than the number of UL users to facilitate reliable UL signal detection, i.e., $N_T \geq J$. Besides, the DL and the UL users are scheduled for simultaneous UL and DL transmission. Unlike the local legitimate signal-antenna users, the roaming user is a traveling wireless device belonging to another communication system and is equipped with $N_R > 1$ antennas. The multiple-antenna roaming user is searching for access to local wireless services. However, it is possible that the roaming user deliberately intercepts the information signal intended for the legitimate users if they are in the same service area. As a result, the roaming user is a potential eavesdropper which has to be taken into account for resource allocation algorithm design to guarantee communication security. In this chapter, we refer to the roaming user as a potential

¹We note that transmitting and receiving signals simultaneously via the same antenna is feasible by exploiting a circulator [2].

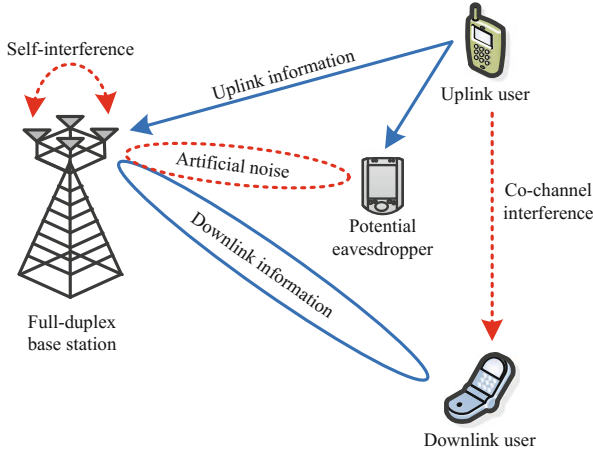


Fig. 10.1 A multiuser communication system with an FD BS, $K = 1$ HD DL user, $J = 1$ HD UL user, and one HD potential eavesdropper

eavesdropper and we assume $N_T > N_R$ for studying resource allocation algorithm design. Besides, in order to study the upper bound performance of the considered system, we assume that the FD BS has perfect channel state information (CSI) for resource allocation.

10.2.2 Channel Model

We focus on a frequency flat fading channel. In each scheduling time slot, the FD BS transmits K independent signal streams simultaneously at the same frequency to the K DL users. In particular, the information signal to DL user $k \in \{1, \dots, K\}$ can be expressed as $\mathbf{x}_k = \mathbf{w}_k d_k^{\text{DL}}$, where $d_k^{\text{DL}} \in \mathbb{C}$ and $\mathbf{w}_k \in \mathbb{C}^{N_T \times 1}$ are the information bearing signal for DL user k and the corresponding beamforming vector, respectively. Without loss of generality, we assume $\mathcal{E}\{|d_k^{\text{DL}}|^2\} = 1, \forall k \in \{1, \dots, K\}$.

However, the signal intended for the desired user may be eavesdropped by the roaming user. Hence, in order to ensure secure communication, the FD BS also transmits AN to interfere the reception of the roaming user (potential eavesdropper). Therefore, the transmit signal vector, $\mathbf{x} \in \mathbb{C}^{N_T \times 1}$, comprising K information streams and AN, is given by $\mathbf{x} = \sum_{k=1}^K \mathbf{x}_k + \mathbf{z}_{\text{AN}}$, where $\mathbf{z}_{\text{AN}} \in \mathbb{C}^{N_T \times 1}$ represents the AN vector generated by the FD BS to degrade the channel quality of the potential eavesdropper. In particular, \mathbf{z}_{AN} is modeled as a complex Gaussian random vector with $\mathbf{z}_{\text{AN}} \sim \mathcal{CN}(\mathbf{0}, \mathbf{Z}_{\text{AN}})$, where $\mathbf{Z}_{\text{AN}} \in \mathbb{H}^{N_T}$, $\mathbf{Z}_{\text{AN}} \succeq \mathbf{0}$, denotes the covariance matrix of the AN. Therefore, the received signals at DL user $k \in \{1, \dots, K\}$, the

FD BS, and the potential eavesdropper are given by

$$\mathbf{y}_{\text{DL}_k} = \mathbf{h}_k^H \mathbf{x}_k + \underbrace{\sum_{i \neq k}^K \mathbf{h}_k^H \mathbf{x}_i}_{\text{multiuser interference}} + \underbrace{\mathbf{h}_k^H \mathbf{z}_{\text{AN}}}_{\text{artificial noise}} + \underbrace{\sum_{j=1}^J \sqrt{P_j} f_{j,k} d_j^{\text{UL}}}_{\text{co-channel interference}} + n_{\text{DL}_k}, \quad (10.1)$$

$$\mathbf{y}_{\text{BS}} = \sum_{j=1}^J \sqrt{P_j} \mathbf{g}_j d_j^{\text{UL}} + \underbrace{\mathbf{H}_{\text{SI}} \sum_{k=1}^K \mathbf{x}_k}_{\text{self-interference}} + \underbrace{\mathbf{H}_{\text{SI}} \mathbf{z}_{\text{AN}}}_{\text{artificial noise}} + \mathbf{n}_{\text{BS}}, \quad \text{and} \quad (10.2)$$

$$\mathbf{y}_{\text{E}} = \underbrace{\sum_{k=1}^K \mathbf{L}^H \mathbf{x}_k}_{\text{DL signals}} + \underbrace{\sum_{j=1}^J \sqrt{P_j} \mathbf{e}_j d_j^{\text{UL}}}_{\text{UL signals}} + \underbrace{\mathbf{L}^H \mathbf{z}_{\text{AN}}}_{\text{artificial noise}} + \mathbf{n}_{\text{E}}, \quad (10.3)$$

respectively. The DL channel between the FD BS and user k is denoted by $\mathbf{h}_k \in \mathbb{C}^{N_{\text{T}} \times 1}$ and $f_{j,k} \in \mathbb{C}$ represents the channel between UL user j and DL user k . Variables d_j^{UL} , $\mathcal{E}\{|d_j^{\text{UL}}|^2\} = 1$, and P_j are the data and transmit power sent from UL user j to the FD BS, respectively. Vector $\mathbf{g}_j \in \mathbb{C}^{N_{\text{T}} \times 1}$ denotes the channel between UL user j and the FD BS. Matrix $\mathbf{H}_{\text{SI}} \in \mathbb{C}^{N_{\text{T}} \times N_{\text{T}}}$ denotes the SI channel of the FD BS. Matrix $\mathbf{L} \in \mathbb{C}^{N_{\text{T}} \times N_{\text{R}}}$ denotes the channel between the FD BS and the potential eavesdropper. Vector $\mathbf{e}_j \in \mathbb{C}^{N_{\text{R}} \times 1}$ denotes the channel between UL user j and the potential eavesdropper. Variables \mathbf{h}_k , $f_{j,k}$, \mathbf{g}_j , \mathbf{H}_{SI} , \mathbf{L} , and \mathbf{e}_j capture the joint effect of path loss and small scale fading. $\mathbf{n}_{\text{BS}} \sim \mathcal{CN}(\mathbf{0}, \sigma_{\text{BS}}^2 \mathbf{I}_{N_{\text{T}}})$, $n_{\text{DL}_k} \sim \mathcal{CN}(0, \sigma_k^2)$, and $\mathbf{n}_{\text{E}} \sim \mathcal{CN}(\mathbf{0}, \sigma_{\text{E}}^2 \mathbf{I}_{N_{\text{R}}})$ represent the additive white Gaussian noise (AWGN) at the FD BS, DL user k , and the potential eavesdropper, respectively. In (10.1), the term $\sum_{j=1}^J \sqrt{P_j} f_{j,k} d_j^{\text{UL}}$ denotes the aggregated CCI caused by the UL users to DL user k . In (10.2), the term $\mathbf{H}_{\text{SI}} \sum_{k=1}^K \mathbf{x}_k$ represents the SI.

10.3 Resource Allocation Problem Formulation

In this section, we first define the adopted performance metrics for the considered multiuser communication system. Then, we formulate the resource allocation problems for DL and UL transmit power minimization, respectively. For the sake of notational simplicity, we define the following variables: $\mathbf{H}_k = \mathbf{h}_k \mathbf{h}_k^H$, $k \in \{1, \dots, K\}$, and $\mathbf{G}_j = \mathbf{g}_j \mathbf{g}_j^H$, $j \in \{1, \dots, J\}$.

10.3.1 Achievable Rate and Secrecy Rate

The achievable rate (bit/s/Hz) of DL user k is given by

$$R_{\text{DL}_k} = \log_2(1 + \Gamma_k^{\text{DL}}) \text{ with} \quad (10.4)$$

$$\Gamma_k^{\text{DL}} = \frac{|\mathbf{h}_k^H \mathbf{w}_k|^2}{\sum_{r \neq k}^K |\mathbf{h}_k^H \mathbf{w}_r|^2 + \sum_{j=1}^J P_j |f_{j,k}|^2 + \text{Tr}(\mathbf{H}_k \mathbf{Z}_{\text{AN}}) + \sigma_k^2}, \quad (10.5)$$

where Γ_k^{DL} is the receive signal-to-interference-plus-noise ratio (SINR) at DL user k . Besides, the achievable rate of UL user j is given by

$$R_{\text{UL}_j} = \log_2(1 + \Gamma_j^{\text{UL}}) \text{ with} \quad (10.6)$$

$$\Gamma_j^{\text{UL}} = \frac{P_j |\mathbf{g}_j^H \mathbf{v}_j|^2}{\sum_{n \neq j}^J P_n |\mathbf{g}_n^H \mathbf{v}_j|^2 + S_{\text{SI}_j} + \sigma_{\text{BS}}^2 \|\mathbf{v}_j\|^2}, \quad \text{and} \quad (10.7)$$

$$S_{\text{SI}_j} = \text{Tr} \left(\rho \mathbf{V}_j \text{diag} \left(\mathbf{H}_{\text{SI}} \mathbf{Z}_{\text{AN}} \mathbf{H}_{\text{SI}}^H + \sum_{k=1}^K \mathbf{H}_{\text{SI}} \mathbf{w}_k \mathbf{w}_k^H \mathbf{H}_{\text{SI}}^H \right) \right), \quad (10.8)$$

where Γ_j^{UL} is the receive SINR of UL user j at the FD BS. The variable $\mathbf{v}_j \in \mathbb{C}^{N_{\text{r}} \times 1}$ is the receive beamforming vector for decoding the information received from UL user j and we define $\mathbf{V}_j = \mathbf{v}_j \mathbf{v}_j^H$, $j \in \{1, \dots, J\}$. In this chapter, zero-forcing receive beamforming (ZF-BF) is adopted. In this context, we note that ZF-BF closely approaches the performance of optimal minimum mean square error beamforming (MMSE-BF) when the noise term is not significant² [20] or the number of antennas is sufficiently large [16]. Besides, ZF-BF facilitates the design of a computationally efficient resource allocation algorithm. Hence, the receive beamformer for UL user j is chosen as $\mathbf{v}_j = (\mathbf{u}_j \mathbf{Q}^\dagger)^H$, where $\mathbf{u}_j = [\underbrace{0, \dots, 0}_{(j-1)}, 1, \underbrace{0, \dots, 0}_{(J-j)}]$, $\mathbf{Q}^\dagger = (\mathbf{Q}^H \mathbf{Q})^{-1} \mathbf{Q}^H$, and $\mathbf{Q} = [\mathbf{g}_1, \dots, \mathbf{g}_J]$. Since SI

cannot be cancelled perfectly in FD systems due to the limited dynamic range of the receiver even if the SI channel is perfectly known [5], we model the residual SI after cancellation at each receive antenna as an independent zero-mean Gaussian distortion noise whose variance is proportional to the received power of the antenna, i.e., the term $S_{\text{SI}_j} = \text{Tr} \left(\rho \mathbf{V}_j \text{diag} \left(\mathbf{H}_{\text{SI}} \mathbf{Z}_{\text{AN}} \mathbf{H}_{\text{SI}}^H + \sum_{k=1}^K \mathbf{H}_{\text{SI}} \mathbf{w}_k \mathbf{w}_k^H \mathbf{H}_{\text{SI}}^H \right) \right)$ in (10.7) and (10.8). We note that this SI model was first proposed in [5]. Besides, it was

²We note that the noise power at the BS is not expected to be the dominating factor for the system performance since BSs are usually equipped with a high quality low-noise amplifier (LNA).

shown in [13] that the adopted model accurately captures the combined effects of additive automatic gain control noise, non-linearities in the analog-to-digital converters and the gain control, and oscillator phase noise which are present in practical FD hardware. We refer to [5, Eq. (4)] for a more detailed discussion of the adopted SI model.

As discussed before, for guaranteeing communication security, the roaming user is treated as a potential eavesdropper who eavesdrops the information signals desired for all DL and UL users. Thereby, we design the resource allocation algorithm under a worst-case assumption for guaranteeing communication secrecy. In particular, we assume that the potential eavesdropper can cancel all multiuser interference before decoding the information of the desired user. Thus, under this assumption, the channel capacity between the FD BS and the potential eavesdropper for eavesdropping desired DL user k and the channel capacity between the UL user j and the potential eavesdropper for overhearing UL user j can be written as

$$C_{DL_k} = \log_2 \det(\mathbf{I}_{N_R} + \mathbf{X}^{-1} \mathbf{L}^H \mathbf{w}_k \mathbf{w}_k^H \mathbf{L}), \forall k, \quad \text{and} \quad (10.9)$$

$$C_{UL_j} = \log_2 \det(\mathbf{I}_{N_R} + P_j \mathbf{X}^{-1} \mathbf{e}_j \mathbf{e}_j^H), \forall j, \quad (10.10)$$

respectively, where $\mathbf{X} = \mathbf{L}^H \mathbf{Z}_{AN} \mathbf{L} + \sigma_E^2 \mathbf{I}_{N_R}$ denotes the interference-plus-noise covariance matrix for the potential eavesdropper. We emphasize that, unlike an HD BS, the FD BS can guarantee both DL security and UL security simultaneously via AN transmission. The achievable secrecy rates between the FD BS and DL user k and UL user j are given by

$$R_{DL_k}^{\text{Sec}} = \left[R_{DL_k} - C_{DL_k} \right]^+, \forall k, \quad \text{and} \quad R_{UL_j}^{\text{Sec}} = \left[R_{UL_j} - C_{UL_j} \right]^+, \forall j, \quad (10.11)$$

respectively.

10.3.2 Optimization Problem Formulation

In the following, we first introduce two individual system design optimization problems with conflicting design objectives. Then, we investigate the two system design objectives jointly under a multi-objective optimization framework [12, 15, 19]. In this chapter, we focus on the minimization of the total DL transmit power at the BS and the minimization of the total UL transmit power. In particular, since wireless BSs consume a considerable amount of energy, the associated energy cost has become a financial burden to the service providers. Hence, to reduce the energy consumption and its cost, it is necessary to design power efficient resource allocation schemes for reducing the transmit power of the BS. Thus, in this chapter, we aim to minimize the transmit power under secrecy and QoS constraints. In particular, for the secrecy constraints, the maximum information leakage to the

potential eavesdroppers is constrained. In fact, the secrecy constraints imposing the maximum information leakage provide flexibility in controlling the security level of communication for different practical applications. For example, services involving confidential information, e.g., online banking, have more stringent requirements on low information leakage than ordinary services, e.g., video streaming. On the other hand, multimedia services, e.g., video streaming, have a higher information leakage tolerance than plain text services, e.g., E-mail and short message service (SMS), since it is hard to recover high quality multimedia content from limited eavesdropped information. Therefore, the considered problem formulation with secrecy constraints takes into account the different required security levels for facilitating more flexible resource allocation. In particular, the first considered objective is the minimization of the total DL transmit power at the FD BS and is given by

Problem 1 (Total DL Transmit Power Minimization)

$$\begin{aligned}
 & \underset{\mathbf{Z}_{\text{AN}} \in \mathbb{H}^{N_{\text{T}}}, \mathbf{w}_k, P_j}{\text{minimize}} && \sum_{k=1}^K \|\mathbf{w}_k\|^2 + \text{Tr}(\mathbf{Z}_{\text{AN}}) \\
 \text{s.t. C1:} & && \frac{|\mathbf{h}_k^H \mathbf{w}_k|^2}{\sum_{r \neq k}^K |\mathbf{h}_k^H \mathbf{w}_r|^2 + \sum_{j=1}^J P_j |f_{j,k}|^2 + \text{Tr}(\mathbf{H}_k \mathbf{Z}_{\text{AN}}) + \sigma_k^2} \geq \Gamma_{\text{req}_k}^{\text{DL}}, \forall k, j, \\
 \text{C2:} & && \frac{P_j |\mathbf{g}_j^H \mathbf{v}_j|^2}{\sum_{n \neq j}^J P_n |\mathbf{g}_n^H \mathbf{v}_j|^2 + S_{\text{SI}_j} + \sigma_{\text{BS}}^2 \|\mathbf{v}_j\|^2} \geq \Gamma_{\text{req}_j}^{\text{UL}}, \forall j, \\
 \text{C3:} & && \log_2 \det(\mathbf{I}_{N_{\text{R}}} + \mathbf{X}^{-1} \mathbf{L}^H \mathbf{w}_k \mathbf{w}_k^H \mathbf{L}) \leq R_{\text{tol}_k}^{\text{DL}}, \forall k, \\
 \text{C4:} & && \log_2 \det(\mathbf{I}_{N_{\text{R}}} + P_j \mathbf{X}^{-1} \mathbf{e}_j \mathbf{e}_j^H) \leq R_{\text{tol}_j}^{\text{UL}}, \forall j, \\
 \text{C5:} & && P_j \geq 0, \forall j, \quad \text{C6: } \mathbf{Z}_{\text{AN}} \geq \mathbf{0}. \tag{10.12}
 \end{aligned}$$

The system design objective in (10.12) is to minimize the total DL transmit power which is comprised of the DL signal power and the AN power. Constants $\Gamma_{\text{req}_k}^{\text{DL}} > 0$ and $\Gamma_{\text{req}_j}^{\text{UL}} > 0$ in constraints C1 and C2 in (10.12) are the minimum required SINR for DL users $k \in \{1, \dots, K\}$ and UL users $j \in \{1, \dots, J\}$, respectively. $R_{\text{tol}_k}^{\text{DL}}$ and $R_{\text{tol}_j}^{\text{UL}}$ in C3 and C4, respectively, are pre-defined system parameters representing the maximum tolerable data rates at the potential eavesdropper for decoding the information of DL user k and UL user j , respectively. In fact, DL and UL security is guaranteed by constraints C3 and C4. In particular, if the above optimization problem is feasible, the proposed problem formulation guarantees that the secrecy

rate for DL user k is bounded below by $R_{DL_k}^{\text{Sec}} \geq \log_2(1 + \Gamma_{\text{req}_k}^{\text{DL}}) - R_{\text{tol}_k}^{\text{DL}}$ and the secrecy rate for UL user j is bounded below by $R_{UL_j}^{\text{Sec}} \geq \log_2(1 + \Gamma_{\text{req}_j}^{\text{UL}}) - R_{\text{tol}_j}^{\text{UL}}$. We note that the maximization of the system secrecy throughput for the considered multiuser systems is generally NP-hard³ [27]. Hence, we focus on the minimization of the total DL transmit power under secrecy constraints to obtain a tractable resource allocation design. Constraint C5 is the non-negative power constraint for UL user j . Constraint C6 and $\mathbf{Z}_{\text{AN}} \in \mathbb{H}^{N_T}$ are imposed since covariance matrix \mathbf{Z}_{AN} has to be a Hermitian positive semidefinite matrix. We note that the objective of Problem 1 is to minimize the total DL transmit power under constraints C1–C6 without regard for the consumed UL transmit powers.

Besides, as mobile devices are powered by batteries with limited energy storage capacity, minimizing the UL transmit power can prolong the lifetime of mobile devices. Therefore, the second system design objective is the minimization of total UL transmit power and can be mathematically formulated as

Problem 2 (Total UL Transmit Power Minimization)

$$\begin{aligned} & \underset{\mathbf{Z}_{\text{AN}} \in \mathbb{H}^{N_T}, \mathbf{w}_k, P_j}{\text{minimize}} && \sum_{j=1}^J P_j \\ & \text{s.t.} && \text{C1 – C6.} \end{aligned} \tag{10.13}$$

Problem 2 targets only the minimization of the total UL transmit power under constraints C1–C6 without taking into account the total consumed DL transmit power.

The objectives of Problems 1 and 2 are desirable for the system operator and the users, respectively. However, in secure FD wireless communication systems, these objectives conflict with each other. On the one hand, the DL information and AN transmission cause significant SI which impairs the UL signal reception. Hence, the UL users have to transmit with a higher power to compensate this interference to satisfy the minimum required receive SINR of the UL users at the FD BS. On the other hand, a high UL transmit power results in a strong CCI for DL signal reception and a higher risk of information leakage to the potential eavesdropper. Hence, the FD BS has to transmit both the DL information and the AN with higher power to ensure the QoS requirements of the DL users and the security requirements of the DL and UL users. However, this in turn causes high SI and gives rise to an escalating increase in transmit power for both UL and DL transmission. To overcome this problem, we resort to multi-objective optimization [12, 15]. In the literature, multi-

³Note that the maximization of the secrecy rate is also one possible system design objective. Yet, such a formulation may lead to exceedingly large energy consumption.

objective optimization is often adopted to study the trade-off between conflicting system design objectives via the concept of Pareto optimality [12, 15]. To facilitate our presentation, we denote the objective function of Problem i as $Q_i(\mathbf{w}_k, \mathbf{Z}_{AN}, P_j)$. The Pareto optimality of a resource allocation policy is defined in the following:

Definition 1 (Pareto Optimal [12]) A resource allocation policy, $\{\mathbf{w}_k, \mathbf{Z}_{AN}, P_j\}$, is Pareto optimal if and only if there does not exist any $\{\tilde{\mathbf{w}}_k, \tilde{\mathbf{Z}}_{AN}, \tilde{P}_j\}$ with

$$Q_i(\tilde{\mathbf{w}}_k, \tilde{\mathbf{Z}}_{AN}, \tilde{P}_j) < Q_i(\mathbf{w}_k, \mathbf{Z}_{AN}, P_j), \forall i \in \{1, 2\}. \quad (10.14)$$

In other words, a resource allocation policy is Pareto optimal if there is no other policy that improves at least one of the objectives without detriment to the other objective. In order to capture the complete Pareto optimal set, we formulate a third optimization problem to investigate the trade-off between Problems 1 and 2 by using the weighted Tchebycheff method [12]. The third problem formulation is given as

Problem 3 (Multi-Objective Optimization)

$$\begin{aligned} & \underset{\mathbf{Z}_{AN} \in \mathbb{H}^{N_T}, \mathbf{w}_k, P_j}{\text{minimize}} && \max_{i=1,2} \left\{ \varrho_i \left(Q_i(\mathbf{w}_k, \mathbf{Z}_{AN}, P_j) - Q_i^* \right) \right\} \\ & \text{s.t.} && \text{C1 - C6}, \end{aligned} \quad (10.15)$$

where $Q_1(\mathbf{w}_k, \mathbf{Z}_{AN}, P_j) = \sum_{k=1}^K \|\mathbf{w}_k\|^2 + \text{Tr}(\mathbf{Z}_{AN})$ and $Q_2(\mathbf{w}_k, \mathbf{Z}_{AN}, P_j) = \sum_{j=1}^J P_j$. Q_i^* is the optimal objective value of the i -th problem and is treated as a constant for Problem 3. Variable $\varrho_i \geq 0$, $\sum_i \varrho_i = 1$, specifies the priority of the i -th objective compared to the other objectives and reflects the preference of the system operator. By varying ϱ_i , a complete Pareto optimal set corresponding to a set of resource allocation policies can be obtained when (10.15) is solved. Thus, the operator can select a proper resource allocation policy from the set of available policies. Compared to other formulations for handling MOOPs in the literature (e.g., the weighted product method, the exponentially weighted criterion, and the ϵ -constraint method [12]), the weighted Tchebycheff method can achieve the complete Pareto optimal set with a lower computational complexity, despite the non-convexity (if any) of the considered problem. It is noted that Problem 3 is equivalent to Problem i when $\varrho_i = 1$ and $\varrho_j = 0, \forall j \neq i$. Here, equivalence means that both problems have the same optimal solution.

10.4 Solution of the Optimization Problem

Problems 1–3 are non-convex problems due to the non-convex constraints C3 and C4. To solve these problems efficiently, we first transform C3 and C4 into equivalent linear matrix inequality (LMI) constraints. Then, Problems 1–3 are solved optimally by semidefinite programming (SDP) relaxation.

To facilitate the SDP relaxation, we define $\mathbf{W}_k = \mathbf{w}_k \mathbf{w}_k^H$ and rewrite Problems 1–3 in the following equivalent forms:

Equivalent Problem 1

$$\begin{aligned}
 & \text{minimize} && \sum_{k=1}^K \text{Tr}(\mathbf{W}_k) + \text{Tr}(\mathbf{Z}_{\text{AN}}) \\
 & \mathbf{W}_k, \mathbf{Z}_{\text{AN}} \in \mathbb{H}^{N_T}, P_j && \\
 \text{s.t. C1:} & && \frac{\text{Tr}(\mathbf{H}_k \mathbf{W}_k)}{\sum_{r \neq k}^K \text{Tr}(\mathbf{H}_k \mathbf{W}_r) + \sum_{j=1}^J P_j |f_{j,k}|^2 + \text{Tr}(\mathbf{H}_k \mathbf{Z}_{\text{AN}}) + \sigma_k^2} \geq \Gamma_{\text{req}_k}^{\text{DL}}, \quad \forall k, j, \\
 \text{C2:} & && \frac{P_j \text{Tr}(\mathbf{G}_j \mathbf{V}_j)}{\sum_{n \neq j}^J P_n \text{Tr}(\mathbf{G}_n \mathbf{V}_j) + I_{\text{SI}_j} + \sigma_{\text{BS}}^2 \text{Tr}(\mathbf{V}_j)} \geq \Gamma_{\text{req}_j}^{\text{UL}}, \quad \forall j, \\
 \text{C3:} & && \log_2 \det(\mathbf{I}_{N_R} + \mathbf{X}^{-1} \mathbf{L}^H \mathbf{W}_k \mathbf{L}) \leq R_{\text{tol}_k}^{\text{DL}}, \quad \forall k, \\
 \text{C4:} & && \log_2 \det(\mathbf{I}_{N_R} + P_j \mathbf{X}^{-1} \mathbf{e}_j \mathbf{e}_j^H) \leq R_{\text{tol}_j}^{\text{UL}}, \quad \forall j, \\
 \text{C5:} & && P_j \geq 0, \quad \forall j, & \text{C6: } \mathbf{Z}_{\text{AN}} \geq \mathbf{0}, \\
 \text{C7:} & && \mathbf{W}_k \geq \mathbf{0}, \quad \forall k, & \text{C8: Rank}(\mathbf{W}_k) \leq 1, \quad \forall k, \tag{10.16}
 \end{aligned}$$

where

$$I_{\text{SI}_j} = \text{Tr} \left(\rho \mathbf{V}_j \text{diag} \left(\mathbf{H}_{\text{SI}} \mathbf{Z}_{\text{AN}} \mathbf{H}_{\text{SI}}^H + \sum_{k=1}^K \mathbf{H}_{\text{SI}} \mathbf{W}_k \mathbf{H}_{\text{SI}}^H \right) \right). \tag{10.17}$$

$\mathbf{W}_k \succeq \mathbf{0}$, $\mathbf{W}_k \in \mathbb{H}^{N_T}$, and $\text{Rank}(\mathbf{W}_k) \leq 1$ in (10.16) are imposed to guarantee that $\mathbf{W}_k = \mathbf{w}_k \mathbf{w}_k^H$ holds after optimization. Similarly, Problems 2 and 3 can be transformed, respectively, into:

Equivalent Problem 2

$$\begin{aligned} & \underset{\mathbf{W}_k, \mathbf{Z}_{AN} \in \mathbb{H}^{N_T}, P_j}{\text{minimize}} && \sum_{j=1}^J P_j \\ & \text{s.t.} && \text{C1} - \text{C8}. \end{aligned} \quad (10.18)$$

and

Equivalent Problem 3

$$\begin{aligned} & \underset{\mathbf{W}_k, \mathbf{Z}_{AN} \in \mathbb{H}^{N_T}, P_j, \tau}{\text{minimize}} && \tau \\ & \text{s.t.} && \text{C1} - \text{C8}, \\ & && \text{C9: } Q_i(Q_i - Q_i^*) \leq \tau, \forall i \in \{1, 2\}. \end{aligned} \quad (10.19)$$

Note that in Equivalent Problem 3, τ is an auxiliary optimization variable and (10.19) is the equivalent epigraph representation of (10.15).

Since Problem 3 is a generalization of Problems 1 and 2, we focus on solving Problem 3. Now, we handle the non-convex constraints C3 and C4 by introducing the following proposition.

Proposition 1 For $R_{\text{tol}_k}^{\text{DL}} > 0$ and $R_{\text{tol}_j}^{\text{UL}} > 0$, we have the following implications for constraints C3 and C4 of equivalent Problems 1–3, respectively:

$$\text{C3} \Rightarrow \widetilde{\text{C3}}: \mathbf{L}^H \mathbf{W}_k \mathbf{L} \preceq \xi_k^{\text{DL}} \mathbf{X}, \quad \forall k, \quad (10.20)$$

$$\text{C4} \Leftrightarrow \widetilde{\text{C4}}: P_j \mathbf{e}_j \mathbf{e}_j^H \preceq \xi_j^{\text{UL}} \mathbf{X}, \quad \forall j, \quad (10.21)$$

where $\xi_k^{\text{DL}} = 2^{R_{\text{tol}_k}^{\text{DL}}} - 1$ and $\xi_j^{\text{UL}} = 2^{R_{\text{tol}_j}^{\text{UL}}} - 1$. We note that C3 and $\widetilde{\text{C3}}$ are equivalent, respectively, if $\text{Rank}(\mathbf{W}_k) \leq 1$. Besides, C4 and $\widetilde{\text{C4}}$ are always equivalent.

Proof Please refer to Appendix 1. □

Note that $\widetilde{\text{C3}}$ and $\widetilde{\text{C4}}$ are convex LMI constraints which can be handled easily. The remaining non-convex constraint in (10.19) is the rank-one constraint C8. Solving such a rank-constrained problem is generally NP-hard [9]. Hence, to obtain a tractable solution, we relax constraint C8: $\text{Rank}(\mathbf{W}_k) \leq 1$ by removing it from the problem formulation, such that the considered problem becomes a convex SDP given by

$$\begin{aligned}
 & \underset{\mathbf{W}_k, \mathbf{Z}_{\text{AN}} \in \mathbb{H}^{N_{\text{T}}}, P_j, \tau}{\text{minimize}} && \tau \\
 & \text{s.t. C1, C2, C5, C6, C7,} \\
 & \widetilde{\text{C3}}: \mathbf{L}^H \mathbf{W}_k \mathbf{L} \preceq \xi_k^{\text{DL}} \mathbf{X}, \quad \forall k, \\
 & \widetilde{\text{C4}}: P_j \mathbf{e}_j \mathbf{e}_j^H \preceq \xi_j^{\text{UL}} \mathbf{X}, \quad \forall j, \\
 & \text{C9: } \varrho_i(Q_i - Q_i^*) \leq \tau, \quad \forall i \in \{1, 2\}. \tag{10.22}
 \end{aligned}$$

The relaxed convex problem in (10.22) can be solved efficiently by standard convex program solvers such as CVX [7]. Besides, if the solution obtained for a relaxed SDP problem is a rank-one matrix, i.e., $\text{Rank}(\mathbf{W}_k) = 1$ for $\mathbf{W}_k \neq \mathbf{0}$, $\forall k$, then it is also the optimal solution of the original problem. Next, we verify the tightness of the adopted SDP relaxation in the following theorem.

Theorem 1 *Assuming the considered problem is feasible, for $\Gamma_{\text{req}_k}^{\text{DL}} > 0$, we can always obtain or construct a rank-one optimal matrix \mathbf{W}_k^* which is an optimal solution for (10.22).*

Proof Please refer to Appendix 2. □

By Theorem 1, the optimal beamforming vector \mathbf{w}_k^* can be recovered from \mathbf{W}_k^* by performing eigenvalue decomposition of \mathbf{W}_k^* and selection of the principle eigenvector as \mathbf{w}_k^* .

10.5 Simulation Results

In this section, we investigate the performance of the proposed multi-objective optimization based resource allocation scheme through simulations. The most important simulation parameters are specified in Table 10.1. There are $K = 3$ DL users, $J = 7$ UL users, and one potential eavesdropper in the considered cell. The users and the potential eavesdropper are randomly and uniformly distributed between the reference distance of 30 m and the maximum service distance of 600 m. The FD BS is located at the center of the cell and equipped with N_{T} antennas. The

Table 10.1 System parameters used in simulations

Carrier center frequency and system bandwidth	1.9 GHz and 1 MHz
Path loss exponent and SI cancellation constant, ρ	3.6 and -85 dB [2]
DL user noise power and UL BS noise power, σ_k^2 and σ_{BS}^2	-110 dBm
Potential eavesdropper noise power, σ_E^2 , and BS antenna gain	-110 dBm and 10 dBi
Maximum tolerable eavesdropping data rate for DL users, $R_{tol_k}^{DL}$	0.1 bit/s/Hz
Maximum tolerable eavesdropping data rate for UL users, $R_{tol_j}^{UL}$	0.1 bit/s/Hz

potential eavesdropper is equipped with $N_R = 2$ antennas. The small scale fading of the DL channels, UL channels, CCI channels, and eavesdropping channels is modeled as independent and identically distributed Rayleigh fading. The multipath fading coefficients of the SI channel are generated as independent and identically distributed Rician random variables with Rician factor 5 dB. In addition, we assume that all DL users and all UL users require the same minimum SINRs, respectively, i.e., $\Gamma_{req_k}^{DL} = \Gamma_{req}^{DL}$ and $\Gamma_{req_j}^{UL} = \Gamma_{req}^{UL}$.

Besides, we also consider the performance of a baseline scheme for comparison. For the baseline scheme, we adopt ZF-BF as DL transmission scheme such that the multiuser interference is avoided at the legitimate DL users. In particular, the direction of beamformer \mathbf{w}_k for legitimate user k is fixed and lies in the null space of the other legitimate DL users' channels. Then, we jointly optimize \mathbf{Z}_{AN} , P_j , and the power allocated to \mathbf{w}_k under the MOOP formulation subject to the same constraints as in (10.22) via SDP relaxation.

10.5.1 Transmit Power Trade-off Region

In Fig. 10.2, we study the trade-off between the DL and the UL total transmit powers for different numbers of antennas at the FD BS. The trade-off region is obtained by solving (10.22) for different values of $0 \leq \varrho_i \leq 1, \forall i \in \{1, 2\}$, i.e., the ϱ_i are varied uniformly using a step size of 0.01 subject to $\sum_i \varrho_i = 1$. We assume a minimum required DL SINR of $\Gamma_{req}^{DL} = 10$ dB and a minimum required UL SINR of $\Gamma_{req}^{UL} = 5$ dB. It can be observed from Fig. 10.2 that the total UL transmit power is a monotonically decreasing function with respect to the total DL transmit power. In other words, minimizing the total UL power consumption leads to a higher power consumption in the DL and vice versa. This result confirms that the minimization of the total UL transmit power and the minimization of the total DL transmit power are conflicting design objectives. For the case of $N_T = 12$, 8 dB in UL transmit power can be saved by increasing the total DL transmit power by 5 dB. In addition, Fig. 10.2 also indicates that a significant amount of transmit power can be saved in the FD system by increasing the number of BS antennas. This is due to the fact that the extra degrees freedom offered by the additional antennas facilitate a more power efficient resource allocation. However, due to channel hardening, there is a

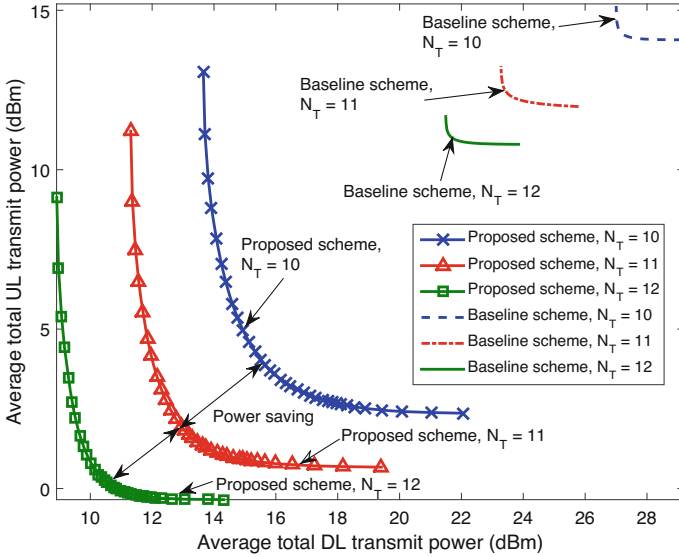


Fig. 10.2 Average system objective trade-off region achieved by the proposed resource allocation scheme. The double-sided arrows indicate the power saving due to additional antennas

diminishing return in the power saving as the number of antennas at the FD BS increases [20]. On the other hand, Fig. 10.2 also depicts the trade-off region for the baseline resource allocation scheme for comparison. As can be seen, the trade-off regions achieved by the baseline scheme are above the curves for the proposed optimal scheme. This indicates that the proposed resource allocation scheme is more power efficient than the baseline scheme for both DL and UL transmission. Indeed, the proposed resource allocation scheme can fully exploit the available degrees of freedom to perform globally optimal resource allocation. On the contrary, for the baseline scheme, the transmitter is incapable of fully utilizing the available degrees of freedom since the direction of the transmit beamformer \mathbf{w}_k is fixed. Specifically, the fixed beamformer \mathbf{w}_k can cause severe SI and increases the risk of eavesdropping which results in a high power consumption for UL transmission and the AN. Besides, the trade-off region for the baseline scheme is strictly smaller than that for the proposed optimal scheme. For instance, when $N_T = 12$, the baseline scheme can save only 1 dB of UL transmit power by increasing the total DL transmit power by 2.5 dB, due to the limited flexibility of the baseline scheme in handling the interference.

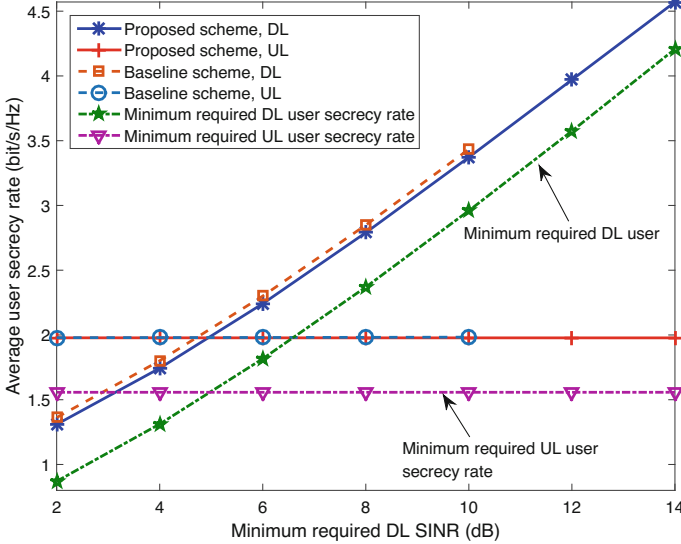


Fig. 10.3 Average user secrecy rate (bits/s/Hz) versus the minimum required DL SINR (dB), Γ_{req}^{DL}

10.5.2 Average User Secrecy Rate Versus Minimum Required SINR

Figure 10.3 depicts the average user secrecy rate of the DL and UL users versus the minimum required DL SINR, Γ_{req}^{DL} , for $R_{tol_k}^{DL} = R_{tol_j}^{UL} = 0.5$ bit/s/Hz and $\Gamma_{req}^{UL} = 5$ dB. The FD BS is equipped with $N_T = 10$ antennas. We select the resource allocation policy with $\varrho_1 = 0.1$ and $\varrho_2 = 0.9$.

The average user secrecy rates for the DL and UL users are calculated by averaging the total DL and the total UL secrecy rates, i.e., $\frac{\sum_{k=1}^K R_{DL_k}^{Sec}}{K}$ and $\frac{\sum_{j=1}^J R_{UL_j}^{Sec}}{J}$, respectively. As can be seen, that the average DL user secrecy rate increases with Γ_{req}^{DL} since the channel capacity between DL user k and the potential eavesdropper is limited to $C_{DL_k} = 0.5$ bit/s/Hz. Besides, the average UL user secrecy rate depends only weakly on Γ_{req}^{DL} for the proposed scheme. In addition, we compare the average DL and UL user secrecy rates of the proposed scheme with the minimum required DL and UL user secrecy rates, i.e., $\log_2(1 + \Gamma_{req}^{DL}) - R_{tol_k}^{DL}$ and $\log_2(1 + \Gamma_{req}^{UL}) - R_{tol_j}^{UL}$, respectively. As can be seen, the average user secrecy rate achieved by the proposed scheme fulfills the minimum required user secrecy rate in both DL and UL which confirms that the security of both links can be guaranteed simultaneously. This is due to the proposed optimization algorithm design. On the other hand, the baseline scheme achieves a slightly higher secrecy rate than the proposed scheme for Γ_{req}^{DL} ranges from 2 to 10 dB. However, to accomplish this, the baseline scheme requires exceedingly large transmit powers at both the FD BS and the UL users compared to

the proposed scheme, cf. Fig. 10.2. Besides, the superior performance of the baseline scheme in terms of secrecy rate also comes at the expense of an extremely high outage probability. In particular, the baseline scheme is always infeasible when $\Gamma_{\text{req}}^{\text{DL}}$ is larger than 10 dB.

10.6 Conclusions

In this chapter, we studied the power efficient resource allocation algorithm design for enabling secure MU-MIMO wireless communication with an FD BS. The algorithm design was formulated as a non-convex MOOP via the weighted Tchebycheff method. The proposed problem aimed at jointly minimizing the total DL and UL transmit powers for achieving simultaneous secure DL and UL transmission. The proposed MOOP was solved optimally by SDP relaxation. We proved that the globally optimal solution can always be obtained or constructed by solving at most two convex SDP optimization problems. Simulation results not only revealed the trade-off between the total DL and UL transmit power consumption, but also confirm that the proposed FD system provides substantial power savings over the baseline scheme. Furthermore, our results revealed that an FD BS can guarantee secure UL transmission which is not possible with an HD BS.

Appendix 1: Proof of Proposition 1

We start the proof by rewriting constraints C3 and C4 as follows:

$$\begin{aligned} \text{C3:} \quad & \det(\mathbf{I}_{N_R} + \mathbf{X}^{-1} \mathbf{L}^H \mathbf{W}_k \mathbf{L}) \leq 2^{R_{\text{tol}_k}^{\text{DL}}} \\ & \stackrel{(a)}{\iff} \det(\mathbf{I}_{N_R} + \mathbf{X}^{-1/2} \mathbf{L}^H \mathbf{W}_k \mathbf{L} \mathbf{X}^{-1/2}) \leq 2^{R_{\text{tol}_k}^{\text{DL}}}, \end{aligned} \quad (10.23)$$

$$\begin{aligned} \text{C4:} \quad & \det(\mathbf{I}_{N_R} + P_j \mathbf{X}^{-1} \mathbf{e}_j \mathbf{e}_j^H) \leq 2^{R_{\text{tol}_j}^{\text{UL}}} \\ & \stackrel{(b)}{\iff} \det(\mathbf{I}_{N_R} + P_j \mathbf{X}^{-1/2} \mathbf{e}_j \mathbf{e}_j^H \mathbf{X}^{-1/2}) \leq 2^{R_{\text{tol}_j}^{\text{UL}}}. \end{aligned} \quad (10.24)$$

(a) and (b) hold due to a basic matrix equality, namely $\det(\mathbf{I} + \mathbf{A}\mathbf{B}) = \det(\mathbf{I} + \mathbf{B}\mathbf{A})$. Then, we study a lower bound of (10.23) and (10.24) by applying the following Lemma.

Lemma 1 (Determinant Inequality [19]) *For any semidefinite matrix $\mathbf{A} \succeq \mathbf{0}$, the inequality $\det(\mathbf{I} + \mathbf{A}) \geq 1 + \text{Tr}(\mathbf{A})$ holds where equality holds if and only if $\text{Rank}(\mathbf{A}) \leq 1$.*

We note that $\mathbf{X}^{-1/2}\mathbf{L}^H\mathbf{W}_k\mathbf{LX}^{-1/2} \succeq \mathbf{0}$ holds in (10.23). Thus, applying Lemma 1 to (10.23) yields

$$\det(\mathbf{I}_{N_R} + \mathbf{X}^{-1/2}\mathbf{L}^H\mathbf{W}_k\mathbf{LX}^{-1/2}) \geq 1 + \text{Tr}(\mathbf{X}^{-1/2}\mathbf{L}^H\mathbf{W}_k\mathbf{LX}^{-1/2}). \quad (10.25)$$

As a result, by combining (10.23) and (10.25), we have the following implications:

$$\begin{aligned} \text{Tr}(\mathbf{X}^{-1/2}\mathbf{L}^H\mathbf{W}_k\mathbf{LX}^{-1/2}) &\leq \xi_k^{\text{DL}} \\ &\stackrel{(c)}{\implies} \lambda_{\max}(\mathbf{X}^{-1/2}\mathbf{L}^H\mathbf{W}_k\mathbf{LX}^{-1/2}) \leq \xi_k^{\text{DL}} \\ &\iff \mathbf{X}^{-1/2}\mathbf{L}^H\mathbf{W}_k\mathbf{LX}^{-1/2} \preceq \xi_k^{\text{DL}}\mathbf{I}_{N_R} \\ &\iff \mathbf{L}^H\mathbf{W}_k\mathbf{L} \preceq \xi_k^{\text{DL}}\mathbf{X}, \end{aligned} \quad (10.26)$$

where $\lambda_{\max}(\mathbf{A})$ denotes the maximum eigenvalue of matrix \mathbf{A} and (c) is due to the fact that $\text{Tr}(\mathbf{A}) \geq \lambda_{\max}(\mathbf{A})$ holds for any $\mathbf{A} \succeq \mathbf{0}$. Besides, if $\text{Rank}(\mathbf{W}_k) \leq 1$, we have

$$\begin{aligned} &\text{Rank}(\mathbf{X}^{-1/2}\mathbf{L}^H\mathbf{W}_k\mathbf{LX}^{-1/2}) \\ &\leq \min \left\{ \text{Rank}(\mathbf{X}^{-1/2}\mathbf{L}^H), \text{Rank}(\mathbf{W}_k\mathbf{LX}^{-1/2}) \right\} \\ &\leq \text{Rank}(\mathbf{W}_k\mathbf{LX}^{-1/2}) \leq 1. \end{aligned} \quad (10.27)$$

Then, equality holds in (10.25). Besides, in (10.26), $\text{Tr}(\mathbf{X}^{-1/2}\mathbf{L}^H\mathbf{W}_k\mathbf{LX}^{-1/2}) \leq \xi_k^{\text{DL}}$ is equivalent to $\lambda_{\max}(\mathbf{X}^{-1/2}\mathbf{L}^H\mathbf{W}_k\mathbf{LX}^{-1/2}) \leq \xi_k^{\text{DL}}$. Therefore, (10.23) and (10.26) are equivalent if $\text{Rank}(\mathbf{W}_k) \leq 1$.

As for constraint C4, we note that $\text{Rank}(P_j\mathbf{X}^{-1/2}\mathbf{e}_j\mathbf{e}_j^H\mathbf{X}^{-1/2}) \leq 1$ always holds. Therefore, by applying Lemma 1 to (10.24), we have

$$\det(\mathbf{I}_{N_R} + P_j\mathbf{X}^{-1/2}\mathbf{e}_j\mathbf{e}_j^H\mathbf{X}^{-1/2}) = 1 + \text{Tr}(P_j\mathbf{X}^{-1/2}\mathbf{e}_j\mathbf{e}_j^H\mathbf{X}^{-1/2}). \quad (10.28)$$

Then, by combining (10.24) and (10.28), we have the following implications:

$$\begin{aligned} \text{C4} &\iff \text{Tr}(P_j\mathbf{X}^{-1/2}\mathbf{e}_j\mathbf{e}_j^H\mathbf{X}^{-1/2}) \leq \xi_j^{\text{UL}} \\ &\iff \lambda_{\max}(P_j\mathbf{X}^{-1/2}\mathbf{e}_j\mathbf{e}_j^H\mathbf{X}^{-1/2}) \leq \xi_j^{\text{UL}} \\ &\iff P_j\mathbf{X}^{-1/2}\mathbf{e}_j\mathbf{e}_j^H\mathbf{X}^{-1/2} \preceq \xi_j^{\text{UL}}\mathbf{I}_{N_R} \\ &\iff P_j\mathbf{e}_j\mathbf{e}_j^H \preceq \xi_j^{\text{UL}}\mathbf{X}. \end{aligned} \quad (10.29)$$

Appendix 2: Proof of Theorem 1

The SDP relaxed version of equivalent Problem 3 in (10.22) is jointly convex with respect to the optimization variables and satisfies Slater's constraint qualification. Therefore, strong duality holds and solving the dual problem is equivalent to solving the primal problem [3]. For obtaining the dual problem, we first need the Lagrangian function of the primal problem in (10.22) which is given by

$$\begin{aligned} \mathcal{L} = & \varrho_1 \pi_1 \sum_{k=1}^K \text{Tr}(\mathbf{W}_k) - \sum_{k=1}^K \text{Tr}(\mathbf{W}_k \mathbf{Y}_k) + \sum_{j=1}^J \mu_j \sum_{k=1}^K \text{Tr}(\rho \mathbf{W}_k \mathbf{H}_{\text{SI}}^H \mathbf{V}_j \mathbf{H}_{\text{SI}}) \\ & - \sum_{k=1}^K \text{Tr} \left(\frac{\lambda_k \mathbf{H}_k \mathbf{W}_k}{\Gamma_{\text{req}_k}^{\text{DL}}} \right) + \sum_{k=1}^K \text{Tr}(\mathbf{L}^H \mathbf{W}_k \mathbf{L} \mathbf{D}_k) + \Lambda. \end{aligned} \quad (10.30)$$

Here, Λ denotes the collection of terms that only involve variables that are independent of \mathbf{W}_k . λ_k , μ_j , and π_i are the Lagrange multipliers associated with constraints C1, C2, and C9, respectively. Matrix $\mathbf{D}_k \in \mathbb{C}^{N_R \times N_R}$ is the Lagrange multiplier matrix for constraint $\widetilde{\text{C3}}$. Matrix $\mathbf{Y}_k \in \mathbb{C}^{N_T \times N_T}$ is the Lagrange multiplier matrix for the positive semidefinite constraint C7 on matrix \mathbf{W}_k . For notational simplicity, we define Ψ as the set of scalar Lagrange multipliers for constraints C1, C2, C5, and C9 and Φ as the set of matrix Lagrange multipliers for constraints $\widetilde{\text{C3}}$, $\widetilde{\text{C4}}$, C6, and C7. Thus, the dual problem for the SDP relaxed problem in (10.22) is given by

$$\begin{aligned} & \underset{\Psi \geq 0, \Phi \geq \mathbf{0}}{\text{maximize}} \quad \underset{\mathbf{W}_k, \mathbf{Z}_{\text{AN}} \in \mathbb{H}^{N_T}, P_j, \tau}{\text{minimize}} \quad \mathcal{L}(\mathbf{W}_k, \mathbf{Z}_{\text{AN}}, P_j, \Psi, \Phi) \\ & \text{s.t.} \quad \sum_{i=1}^2 \pi_i = 1. \end{aligned} \quad (10.31)$$

Constraint $\sum_{i=1}^2 \pi_i = 1$ is imposed to guarantee a bounded solution of the dual problem [3]. Then, we reveal the structure of the optimal \mathbf{W}_k of (10.22) by studying the Karush–Kuhn–Tucker (KKT) conditions. The KKT conditions for the optimal \mathbf{W}_k^* are given by

$$\mathbf{Y}_k^*, \mathbf{D}_k^* \succeq \mathbf{0}, \quad \lambda_k^*, \mu_j^*, \pi_i^* \geq 0, \quad (10.32)$$

$$\mathbf{Y}_k^* \mathbf{W}_k^* = \mathbf{0}, \quad (10.33)$$

$$\nabla_{\mathbf{W}_k^*} \mathcal{L} = \mathbf{0}, \quad (10.34)$$

where \mathbf{Y}_k^* , \mathbf{D}_k^* , λ_k^* , μ_j^* , and π_i^* are the optimal Lagrange multipliers for dual problem (10.31). $\nabla_{\mathbf{W}_k^*} \mathcal{L}$ denotes the gradient of Lagrangian function \mathcal{L} with respect to matrix \mathbf{W}_k^* . The KKT condition in (10.34) can be expressed as

$$\begin{aligned} \mathbf{Y}_k^* + \frac{\lambda_k^* \mathbf{H}_k}{\Gamma_{\text{req}_k}^{\text{DL}}} \\ = \varrho_1 \pi_1 \mathbf{I}_{N_T} + \sum_{j=1}^J \mu_j^* \rho \mathbf{H}_{\text{SI}}^H \text{diag}(\mathbf{V}_j) \mathbf{H}_{\text{SI}} + \mathbf{L} \mathbf{D}_k^* \mathbf{L}^H. \end{aligned} \quad (10.35)$$

Now, we divide the proof into two cases according to the value of ϱ_1 . First, for the case of $0 < \varrho_1 \leq 1$, we define

$$\mathbf{A}_k^* = \sum_{j=1}^J \mu_j^* \rho \mathbf{H}_{\text{SI}}^H \text{diag}(\mathbf{V}_j) \mathbf{H}_{\text{SI}} + \mathbf{L} \mathbf{D}_k^* \mathbf{L}^H, \quad (10.36)$$

$$\mathbf{\Pi}_k^* = \varrho_1 \pi_1 \mathbf{I}_{N_T} + \mathbf{A}_k^*, \quad (10.37)$$

for notational simplicity. Then, (10.35) implies

$$\mathbf{Y}^* = \mathbf{\Pi}_k^* - \frac{\lambda_k^* \mathbf{H}_k}{\Gamma_{\text{req}_k}^{\text{DL}}}. \quad (10.38)$$

Pre-multiplying both sides of (10.38) by \mathbf{W}_k^* , and utilizing (10.33), we have

$$\mathbf{W}_k^* \mathbf{\Pi}_k^* = \mathbf{W}_k^* \frac{\lambda_k^* \mathbf{H}_k}{\Gamma_{\text{req}_k}^{\text{DL}}}. \quad (10.39)$$

By applying basic inequalities for the rank of matrices, the following relation holds:

$$\begin{aligned} \text{Rank}(\mathbf{W}_k^*) &\stackrel{(a)}{=} \text{Rank}(\mathbf{W}_k^* \mathbf{\Pi}_k^*) = \text{Rank} \left(\mathbf{W}_k^* \frac{\lambda_k^* \mathbf{H}_k}{\Gamma_{\text{req}_k}^{\text{DL}}} \right) \\ &\stackrel{(b)}{\leq} \min \left\{ \text{Rank}(\mathbf{W}_k^*), \text{Rank} \left(\frac{\lambda_k^* \mathbf{H}_k}{\Gamma_{\text{req}_k}^{\text{DL}}} \right) \right\} \\ &\stackrel{(c)}{\leq} \text{Rank} \left(\frac{\lambda_k^* \mathbf{H}_k}{\Gamma_{\text{req}_k}^{\text{DL}}} \right) \leq 1, \end{aligned} \quad (10.40)$$

where (a) is valid because $\mathbf{\Pi}_k^* > \mathbf{0}$, (b) is due to the basic result $\text{Rank}(\mathbf{AB}) \leq \min \{ \text{Rank}(\mathbf{A}), \text{Rank}(\mathbf{B}) \}$, and (c) is due to the fact that $\min\{a, b\} \leq a$. We note that $\mathbf{W}_k^* \neq \mathbf{0}$ for $\Gamma_{\text{req}_k}^{\text{DL}} > 0$. Thus, $\text{Rank}(\mathbf{W}_k^*) = 1$.

Then, for the case of $\varrho_1 = 0$, we show that we can always construct a rank-one optimal solution \mathbf{W}_k^{**} . We note that the problem in (10.22) with $\varrho_1 = 0$ is equivalent to a total UL transmit power minimization problem which is given by

$$\begin{aligned} & \underset{\mathbf{W}_k, \mathbf{Z}_{\text{AN}} \in \mathbb{H}^{N_T}, P_j}{\text{minimize}} && \sum_{j=1}^J P_j \\ \text{s.t.} &&& \text{C1, C2, } \widetilde{\text{C3}}, \widetilde{\text{C4}}, \text{C5, C6, C7, C9.} \end{aligned} \quad (10.41)$$

We first solve the above convex optimization problem and obtain the UL transmit power P_j^{**} , the DL beamforming matrix \mathbf{W}_k^* , and the AN covariance matrix \mathbf{Z}_{AN}^* . If $\text{Rank}(\mathbf{W}_k^*) = 1, \forall k$, then the globally optimal solution of problem (10.19) for $\varrho_1 = 0$ is achieved. Otherwise, we substitute P_j^{**} and \mathbf{Z}_{AN}^* into the following auxiliary problem:

$$\begin{aligned} & \underset{\mathbf{W}_k \in \mathbb{H}^{N_T}}{\text{minimize}} && \sum_{k=1}^K \text{Tr}(\mathbf{W}_k) + \text{Tr}(\mathbf{Z}_{\text{AN}}^*) \\ \text{s.t.} &&& \text{C1, C2, } \widetilde{\text{C3}}, \widetilde{\text{C4}}, \text{C5, C6, C7, C9.} \end{aligned} \quad (10.42)$$

Since the problem in (10.42) shares the feasible set of problem (10.41), problem (10.42) is also feasible. Now, we claim that for a given P_j^{**} and \mathbf{Z}_{AN}^* in (10.42), the solution \mathbf{W}_k^{**} of (10.42) is a rank-one matrix. First, the gradient of the Lagrangian function for (10.42) with respect to \mathbf{W}_k^{**} can be expressed as

$$\mathbf{Y}^{**} = \mathbf{\Pi}_k^{**} - \frac{\lambda_k^{**} \mathbf{H}_k}{\Gamma_{\text{req}_k}^{\text{DL}}}, \quad (10.43)$$

where

$$\mathbf{\Pi}_k^{**} = \mathbf{I}_{N_T} + \mathbf{A}_k^{**} \quad \text{and} \quad (10.44)$$

$$\mathbf{A}_k^{**} = \sum_{j=1}^J \mu_j^{**} \rho \mathbf{H}_{\text{SI}}^H \text{diag}(\mathbf{V}_j) \mathbf{H}_{\text{SI}} + \mathbf{L} \mathbf{D}_k^{**} \mathbf{L}^H. \quad (10.45)$$

\mathbf{Y}_k^{**} , \mathbf{D}_k^{**} , λ_k^{**} , and μ_j^{**} are the optimal Lagrange multipliers for the dual problem of (10.42). Pre-multiplying both sides of (10.43) by the optimal solution \mathbf{W}_k^{**} , we have

$$\mathbf{W}_k^{**} \mathbf{\Pi}_k^{**} = \mathbf{W}_k^{**} \frac{\lambda_k^{**} \mathbf{H}_k}{\Gamma_{\text{req}_k}^{\text{DL}}}. \quad (10.46)$$

We note that $\mathbf{\Pi}_k^{**}$ is a full-rank matrix, i.e., $\mathbf{\Pi}_k^{**} > \mathbf{0}$, and (10.46) has the same form as (10.39). Thus, we can follow the same approach as for the case of $0 < \varrho_i \leq 1$ for showing that \mathbf{W}_k^{**} is a rank-one matrix. Also, since \mathbf{W}_k^{**} is a feasible solution of (10.41) for P_j^{**} , an optimal rank-one matrix \mathbf{W}_k^{**} for the case of $\varrho_1 = 0$ is constructed.

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