A Parametric Study of Functionally Graded Variable Thickness Longitudinal Fin Under Fully Wet Condition

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Abstract Thermal analysis and comparison of the functionally graded longitudinal fin, having a different profile with an insulated tip in fully wet condition, are reported in the present work. In many air-conditioning and refrigeration equipments, the performance of the cooling coil is affected due to vapor condensation on its surface. In this work, the thermal conductivity of the longitudinal fin is varied with exponential law. For analysis and comparison of a fin having different profiles, their weight is assumed to be constant. With the help of the psychometric chart, a nonlinear cubic polynomial relationship is established between specific humidity and corresponding fin surface temperature. Considering a volume element of fin under steady state, energy balance concept is used to derive nonlinear differential heat transfer equation. This differential equation is solved using ***bvp4c*** command in MATLAB®. This technique is very useful to solve boundary value problems by collocation method. Further for a different combination of grading parameters, geometry parameters, and relative humidity, a differential equation is solved and results are shown in graphical form. The formulation is verified with standard results, and relative error obtained between these two results is negligible. These results give a better understanding of the thermal performance of functionally graded longitudinal wet fin, and generated data can be used for design purpose.

Keywords Functionally graded · Longitudinal fin · Fully wet

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1 Introduction

To increase the heat transfer rate in the heat exchanger, finned tubes are used. By attaching fin at tube surface, overall surface area increases and consequently heat transfer rate also increases. In many refrigeration, air-conditioning, and chemical processing industrial applications, generally the cooling coil surface temperature is below the dew point temperature of surrounding air. Moisture present in the air condenses on the tube, due to this performance of cooling coil gets affected. Many researchers have taken an interest in this problem and presented their work. To improve the thermal performance along with optimizing the design parameters, the variation in material properties can also be done. In functionally graded material, properties vary gradually with dimension. This property of functionally graded material is used here to improve the thermal performance of fin.

A comprehensive study of the literature on thermal performance analysis of fully wet fin has been performed. Kuehn et al. [1] assumed enthalpy difference affects the simultaneous heat and mass transfer and derived an analytical expression to find overall efficiency. McQuiston [2] derived a governing equation and solved it for efficiency under simultaneous heat and mass transfer conditions. It was considered that the driving force for mass transfer process was the difference between the specific humidity of surrounding moist air and saturated moist air on the surface. Chilton and Colburn [3] proposed a relationship between heat transfer coefficient and mass transfer coefficient, called Chilton–Colburn analogy. Elmahdy and Biggs [4] proposed an algorithm to calculate the efficiency of a longitudinal circular fin having uniform thickness under simultaneous heat and mass transfer. Provided results showed a decrease in efficiency with an increase in relative efficiency. Convey et al. [5] investigated the performance of vertical longitudinal fin in moist airflow and concluded that condensate film thickness and temperature gradient along the fin length depend on airflow and fin effectiveness which decreases due to the thermal resistance of condensate layer. Second-degree polynomial relationship was assumed between specific humidity of saturated air and fin surface temperature. Wu and Bong [6] took a linear relationship between the specific humidity at the fin surface and fin surface temperature. Both partially and fully wet condition fin efficiency were examined. Lin et al. [7] experimentally investigated the thermal performance of rectangular fin in wet condition. For fully wet condition, there was a very slight decrement in efficiency with an increase in relative efficiency. Xu et al. [8] proposed a modified McQuiston model to calculate fin efficiency. Effect of motion of condensate film on fin surface was also considered. Assuming linear relationship between humidity ratio on fin surface and corresponding surface temperature of the fin, Kundu [9] analyzed the performance and optimum design of longitudinal and pin fins. A very small change in efficiency with an increase in relative efficiency was found. Later Kundu [10] considered the polynomial relationship between humidity ratio and dry bulb temperature. With the help of ADM method, an approximate analytical solution for thermal performance of fin was proposed. Sharqawya and Zubair [11] mainly focused on rectangular fin in dry, partially and full wet condition assuming a linear relationship between specific humidity and corresponding fin base temperature. The overall fin efficiency depends on the condition of the fin surface, whether it is dry, partially wet, or fully wet, and also on atmospheric pressure. It increased with an increase in atmospheric pressure. Gaba et al. [12] examine the performance of functionally graded annular parabolic fins. For different grading and geometry parameters, the thermal performance was analyzed and their comparison was done. Udupa et al. [13] represented an overview of basic concepts, properties, classification, preparation methods, and applications of functionally graded composite materials. A case study on CNT reinforced aluminum matrix was discussed. By varying the weight percentage of CNT composition, its effect on grain boundaries and structure was reported. Bhavar et al. [14] provided brief information about functionally graded material. Its properties, classifications, method of manufacturing, and application have been reported. Subramaniam et al. [15] proposed the temperature distribution in functionally graded longitudinal fins of varying geometry for insulated tip condition. The temperature distribution over the fin length of constant thickness longitudinal fin was plotted for different grading parameters. It was concluded that the fin performance could be enhanced either by varying geometry or by varying the thermal conductivity.

From the above-mentioned literature review, it can be said that in wet fins, considering the polynomial relationship between fin surface temperature and corresponding specific humidity, the effect of variation of thermal conductivity with dimensions is not reported by any researchers. This gap gives the scope for further research in this area, and hence, an analytical treatment of functionally graded variable thickness longitudinal fin under fully wet condition has been reported in the present work.

2 Mathematical Formulation

A longitudinal fin of length $\boldsymbol{L}$, base thickness $\boldsymbol{\delta_o}$, width $\boldsymbol{w}$, and constant base temperature $\boldsymbol{T_b}$ are considered as shown in Fig. 1. During the thermal analysis, wet fin is assumed to be steady, one dimensional, with moist air near the condensate film being saturated and effect of temperature variation on thermal conductivity is neglected as the fin is of functionally graded material with spatial variation of thermal properties only as it is supposed to work in narrow temperature range. A cubic relationship between specific humidity and corresponding fin temperature is assumed.

$$w = A + B * T + C * T^2 + D * T^3. \tag{1}$$

A, B, C, D are constants whose values are given in Table 1, calculated with the help of psychrometric chart and regression analysis for the temperature range 0–30 °C [10].

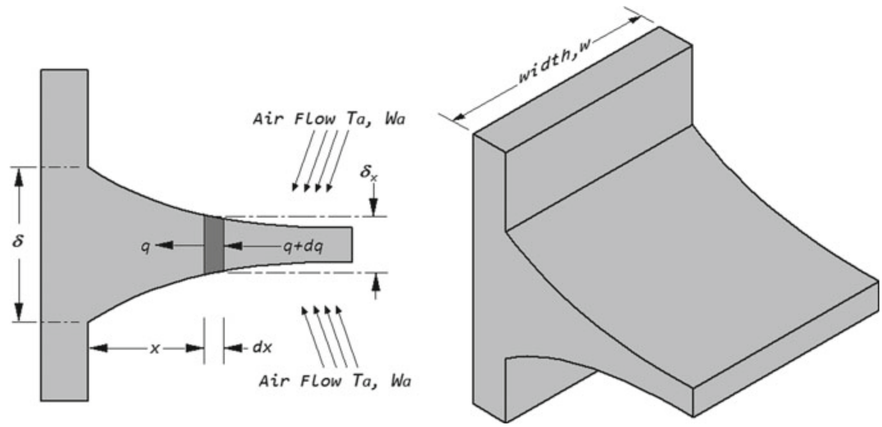

Fig. 1 Schematic diagram of a completely wet straight longitudinal fin

Table 1 Values of constant *A, B, C, D*

A	*B*	*C*	*D*
0.0037444	0.0003078	0.0000046	0.0000004

Thermal conductivity ***K*** is taken as the exponential function of ***X*** as

$$K = ae^{(-bX)}. \quad \text{Where } X = \frac{x}{L}. \tag{2}$$

where ***a*** is the conductivity coefficient and has the dimension of W/m K.

The thickness of fin is assumed the parabolic function of length co-ordinate given as

$$\delta = \delta_o(1 - n * X^m). \tag{3}$$

where ***m*** and ***n*** are dimensionless constants.

Applying the energy balance on a small element dx, governing equation of wet fin is derived as follows:

$$(q + \mathrm{d}q) + h * P * \mathrm{d}x * (T_a - T) + h_m * P * \mathrm{d}x * (w_a - w) - q = 0. \tag{4}$$

Equation (4) is a boundary value problem and is given for following boundary conditions:

$$T(x)_{x=0} = T_b. \tag{5}$$

$$\frac{\mathrm{d}T}{\mathrm{d}x_{x=L}} = 0. \tag{6}$$

Relationship between heat transfer $\boldsymbol{h}$ and mass transfer $\boldsymbol{h_m}$ is taken from [3] and is given by

$$\frac{h}{h_m} = C_p * Le^{\frac{2}{3}}. \tag{7}$$

where $\boldsymbol{Le}$ is the Lewis coefficient and $\boldsymbol{C_p}$ specific heat of moist air.

Solving Eq. (4) with the help of Eqs. (1)–(3) and (5)–(7) and reducing it into normalized form with the help of Eqs. (8) and (9) given below.

$$\text{Normalized Length } X = \frac{x}{L}. \tag{8}$$

$$\text{Normalized Temperature } \theta = \frac{T_a - T}{T_a - T_b}. \tag{9}$$

Differential equation becomes,

$$\frac{d^2\theta}{dX^2} + \left[-b + \frac{-n * m * X^{(m-1)}}{1 - n * X^m}\right] \frac{d\theta}{dX} \frac{2 * h * L^2 * (\delta + W)}{K * W * \delta o * (1 - n * X^m)} \left[K_1 + K_2\theta - K_3\theta^2 + K_4\theta^3\right]. \tag{10}$$

where

$$K_1 = \xi\left[\frac{w_a - A - B * T_a - C * T_a^2 - D * T_a^3}{T_a - T_b}\right]. \tag{11}$$

$$K_2 = 1 + \xi\left[B + 2 * C * T_a + 3 * D * T_a^2\right]. \tag{12}$$

$$K_3 = \xi[(C + 3 * D * T_a)(T_a - T_b)]. \tag{13}$$

$$K_4 = D * \xi(T_a - T_b)^2. \tag{14}$$

$$\xi = \frac{h_{fg}}{C_p * Le^{\left(\frac{2}{3}\right)}}. \tag{15}$$

where $\boldsymbol{K_1}$, $\boldsymbol{K_2}$, $\boldsymbol{K_3}$, $\boldsymbol{K_4}$ given in Eqs. (11)–(14) are constants, and $\boldsymbol{\xi}$ is a dimensionless latent heat parameter. The above differential Eq. (10) has been solved using ***bvp4c*** function in MATLAB® software.

3 Results and Discussion

The temperature distribution of functionally graded longitudinal fin having a different profile with an insulated tip in fully wet condition is obtained by solving second-order partial differential equation, using subroutine ***bvp4c*** in MATLAB® [16]. From the literature survey, it is found that study related to thermal performance analysis of longitudinal fin under simultaneous heat and mass transfer, considering functionally grading material as fin material is not reported yet. The formulation is validated for rectangular longitudinal fin under simultaneous heat and mass transfer with Sharqawy and Zubair [11] in Fig. 2, and a good agreement between these two results are obtained for various relative humidity conditions.

Aluminum has been used as a fin base material. Fin base temperature 7 °C, surrounding air temperature 27 °C, fin length 0.2 m, air heat transfer coefficient 36 W/m^2K, fin parameter (***mL***) 0.8, and different relative humidity 60, 80, 100% are considered to solve Eq. (10), using ***bvp4c*** in MATLAB®, for different grading and geometry parameters. These numerical results and a variation of dimensionless temperature along with dimensionless co-ordinates are plotted and shown in Figs. 3 and 4.

For geometry parameters ***m*** = 0.5, ***n*** = 0.5 and relative humidity 60%, the effect of grading parameter ***b*** is shown in Fig. 3a. With a decrease in value of ***b***, normalized temperature increases at a particular position of the fin. Consequently, fin surface temperature decreases at that particular position, which in turn results in greater temperature difference with decreasing value of ***b***, causing more heat transfer. For the small length of the fin, the effect of grading parameter ***b*** can be neglected.

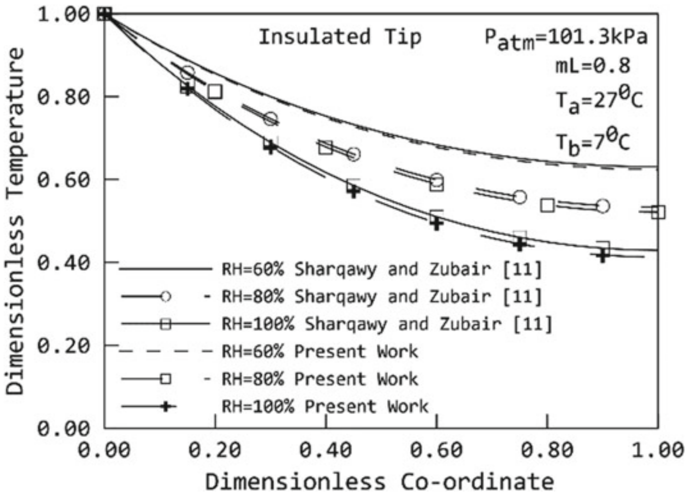

Fig. 2 Comparison of normalized temperature distribution over fin length with that of obtained by Sharqawy and Zubair [11]

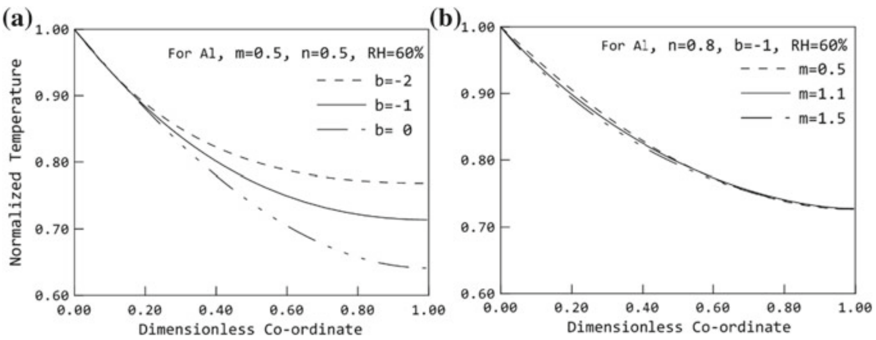

Fig. 3 Normalized temperature distribution along with dimensionless co-ordinate for different values of grading parameter

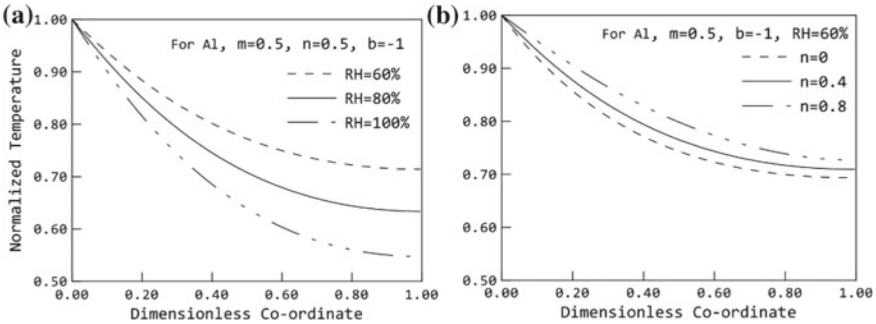

Fig. 4 Normalized temperature distribution along with dimensionless co-ordinate for different values of RH

Effect of variation of geometry parameter $\boldsymbol{m}$, for $\boldsymbol{n} = 0.8$, $\boldsymbol{b} = -1$, and relative humidity 60% is shown in Fig. 3b. Keeping $\boldsymbol{n}$ constant, the effect of variation of $\boldsymbol{m}$ on fin surface temperature is very less.

Figure 4a shows that an increase in relative humidity increases fin surface temperature at a particular location, which can be justified as follows. Due to condensation on the fin surface, fin surface temperature increases. As relative humidity increases, vapor present in air increases, so more vapor condenses on fin surface. As such, more heat of condensation liberates on the fin, surface causes an increase in its temperature.

Considering the effect of increment in the value of $\boldsymbol{n}$, taking $\boldsymbol{m} = 0.5$, relative humidity 60%, $\boldsymbol{b} = -1$, it is shown in Fig. 4b that with an increase in $\boldsymbol{n}$, the difference between fin surface temperature and air temperature increases at a particular location. For rectangular fin ($\boldsymbol{n} = 0$), this temperature difference is lower resulting in lesser heat transfer. Varying the value of m gives scope to enhance the heat transfer.

4 Conclusion

Parametric study of functionally graded variable geometry longitudinal wet fin is presented. Temperature distributions along the fin length for different values of relative humidity ***RH***, grading parameter ***b,*** and geometric parameters ***m*** and ***n*** are shown in Figs. 3 and 4. Increase in relative humidity reduces the temperature difference between saturated wet film temperature and corresponding fin surface temperature at a particular location, which results in a reduction of heat transfer. Fin performance decreases as relative humidity increases. It is remarked that a higher value of ***n*** and lower value of ***m*** gives better results for the same fin parameter. Fin performance can be increased by using negative grading parameter ***b***. Overall, by varying the material property along the fin length or varying the fin profile or combination of both, the performance of the wet fin can be enhanced.

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