Chapter 13
Revisiting an Old Theme in the Measurement of Inequality and Poverty

S. Subramanian

13.1 Introduction

Precisely how we choose to quantitatively assess the phenomena of inequality and poverty must necessarily serve as an important guide to our diagnosis of the gravity of these phenomena in the society under review, and therefore, to the nature and urgency of the public policy measures that are initiated to address these problems. This proposition is starkly in evidence in certain old, but unfortunately somewhat neglected, debates on the merits of relative and absolute indices of inequality and poverty. This paper offers a compact treatment of these debates which have been explored more thoroughly and elaborately elsewhere by the present author Subramanian (2018).

Most extant measures of poverty and inequality are ones which are normalized with respect to both the mean income and population size, that this, they are income- and population-*relative* measures. It will be maintained in this paper that relative measures, however, are as arbitrary and unreasonable, in their way, as are wholly absolute measures. This would pave the way for more 'moderate' *intermediate* measures that mitigate the problems of logical coherence and ethical appeal which tend to afflict comprehensively relative and comprehensively absolute measures. The paper illustrates, by means of a couple of simple empirical examples, how our view of inequality and poverty is a variable function of how we choose to measure these phenomena.

S. Subramanian (✉)
Madras Institute of Development Studies, 36-H, North Parade Road, Street Thomas' Mount, Chennai 600016, Tamil Nadu, India
e-mail: ssubramanianecon@gmail.com

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13.2 On Intermediate Measures of Inequality and Poverty

13.2.1 Inequality

Two problems confronted by distributional analysts are what might be called the problem of variable size and the problem of variable populations. The first problem poses the question of how to compare welfare, inequality and poverty between distributions of the same population but different mean incomes. The second problem poses the question of how to compare welfare, inequality and poverty between two distributions with the same mean income but different population sizes. The convention has been to locate the answers in two well-known 'Invariance' properties. The property of *Scale Invariance* says that when all incomes in a distribution are raised or lowered equi-proportionately, inequality must be deemed to remain the same. (In poverty comparisons, Scale Invariance would require poverty to remain unchanged when the poverty line and all incomes are changed in the same proportion.) The property of *Replication Invariance* states that when each income level in a distribution is replicated k times over (where k is any positive integer), inequality must be deemed to remain the same. (Replication Invariance is defined analogously for poverty comparisons). As one can easily see, Scale Invariance will certify that inequality is unchanged if the ratio of each person's income to the mean income remains unchanged. Replication Invariance certifies that inequality remains unchanged if the relative frequency of each income in a distribution remains unchanged. Scale-invariant inequality measures are thus wholly 'income-relative' measures, while replication-invariant measures are wholly 'population-relative' measures. The bulk of the theoretical and applied research in measurement favours a *relative* view of inequality and poverty. A very commonly employed relative measure of inequality is the (relative) Gini coefficient, G_R.

Scale Invariance is an unexceptionable property, on the face of it. However, as far back as the 1920s, Hugh Dalton expressed reservations about the unqualified appeal of relative inequality measures, as did Serge-Christophe Kolm, in the mid-1970s (see Dalton 1924; Kolm 1976a, b). This is because while an equi-proportionate increase in all incomes will leave relative inequality unchanged, it will, however, increase the absolute difference between incomes. Consider the two-person ordered income distributions x = (10, 20) and y = (20, 40). It is easy to see that y is derived from x by doubling each person's income: a relative measure of inequality will remain unchanged in going from x to y. However, the absolute difference between the two persons' incomes rises from 10 in distribution x to 20 in distribution y. From this absolute perspective, inequality must be deemed to have *increased*. This immediately presents the case for a rival to the Scale Invariance property, a property which Kolm called *Translation Invariance*, and which requires that inequality should remain unchanged with an equal addition to (or subtraction from) each person's income. In this view, it is not equi-proportionate changes in income, but rather equal absolute changes, under which measured inequality should remain unchanged. Patrick Moyes (1987) advanced an absolute version G_{A1} of the Gini coefficient, which is given simply by the product of the mean income m and the relative Gini: $G_{A1} = mG_R$.

Kolm has suggested that in the presence of income growth, relative inequality measures tend to display 'rightist' values, while absolute measures display 'leftist' values; this characterization is switched around in the presence of income contraction. For notice that in moving from x = (10, 20) to y = (20, 40), a relative measure takes no account of the increase in the absolute difference between the two persons' incomes, which is certainly not reflective of a 'radical' perspective on inequality. By the same token, in moving from y = (20, 40) to z = (0, 20), an absolute measure takes no account of the fact that the share of the poorer person's income in total income has declined from one-third to zero—which again is certainly not reflective of a 'radical' perspective on inequality. Briefly, neither a wholly relative nor a wholly absolute conception of inequality is entirely satisfactory. This paves the way for what Kolm called an '*intermediate*' measure: in the context of the problem of variable size, an income-intermediate inequality measure would be one which satisfies the property of displaying an increase in value when all incomes in a distribution are raised by the same proportion and a reduction in value when all incomes in a distribution are raised by the same absolute amount. Such an income-intermediate version G_{I1} of the Gini coefficient would be given by the geometric mean of the income-relative and the income-absolute measures, parameterized by the quantity $\alpha \in [0, 1]$: $G_{I1}(\alpha) = (G_R)^{\alpha}(G_{A1})^{1-\alpha} = m^{1-\alpha}G_R$, where α is a measure of 'pro-absoluteness'. When α is exactly one-half, we have a '*properly centrist*' income-intermediate Gini measure $G^*_{I1} \equiv \sqrt{m}G_R$. We turn now to an analogous consideration of the problem of variable populations.

Given an n-person distribution x, suppose r is the number of individuals such that each of these individuals has at least one other person earning a higher income than herself. These are people who might be thought of as having a '*complaint*' (Temkin 1993) about inequality. Clearly, the maximum possible number of complainants is $n-1$. The proportion of complainants in x, then, is $r/(n-1)$. Suppose now that y is derived from x through a k-fold replication of the population at each income level in x. Then, the *number* of complainants in y will rise to kr, while the *proportion* of complainants will remain constant at $r/(n-1)$ $(= kr/k(n-1))$. A relative view of inequality is concerned only with the proportion of complainants, while an absolute view would take account of the number of complainants. Such an absolute view would renounce the Replication Invariance property in favour of one which we might call *Replication Scaling* (Subramanian 2002). Replication Scaling demands that a k-fold increase of the population at each income level should lead to a k-fold increase in measured inequality. A population-relative inequality measure is one which satisfies Replication Invariance, while a population-absolute measure is one which satisfies Replication Scaling. A population-absolute version G_{A2} of the relative Gini coefficient is given simply by the product of the total population n and the relative Gini: $G_{A2} = nG_R$. If we wish to avoid the 'extreme' values of both absolute and relative measures, then we would favour a *population-intermediate* measure, namely a measure which increases, but less than proportionately, with a k-fold increase in the population at each income level. Such a population-intermediate version G_{I2} of the Gini coefficient would be given by the geometric mean of the population-relative and the population-absolute measures, parameterized by the quantity $\beta \in [0, 1]$:

$G_{I2}(\beta) = (G_R)^\beta (G_{A2})^{1-\beta} = n^{1-\beta} G_R$, where β is a measure of 'pro-absoluteness'. When β is exactly one-half, we have a '*properly centrist*' population-intermediate Gini measure $G^*_{I2} \equiv \sqrt{n} G_R$.

A *comprehensively* absolute version of the relative Gini coefficient, that is, a version G_A, which is both income-absolute and population-absolute, would be given simply by the product of aggregate income mn and the relative Gini: $G_A = mnG_R$. And a *comprehensively intermediate* version of the Gini G_I, namely one which is both income-intermediate and population-intermediate, would be given by the geometric mean of the comprehensively relative and the comprehensively absolute measures, parameterized by the quantity $\gamma \in [0, 1]$: $G_I(\gamma) = (G_R)^\gamma (G_A)^{1-\gamma} = (nm)^{1-\gamma} G_R$, where γ is a measure of 'pro-absoluteness'. When γ is exactly one-half, we have a '*properly centrist*' income-intermediate Gini measure:

$$G^*_I \equiv \sqrt{nm} G_R. \tag{13.1}$$

For all the reasons discussed earlier, there is a strong case for employing a comprehensively centrist measure of inequality such as G^*_I in Eq. (13.1). The dominant tradition in applied work is to employ the comprehensively relative Gini coefficient. It is on the strength of the time trend of this latter measure of inequality in the distribution of consumption expenditure (especially in Rural India) that many commentators have inferred, effectively, that economic inequality in the country is not a seriously threatening issue (see, for example, Ahluwalia 2011; Bhagwati 2011; Bhalla 2011; Srinivasan 2017). The charts comparing the trends in the relative and comprehensively intermediate Gini coefficients, featured in Fig. 13.1a–d, separately for Rural and Urban India from 1987–88 to 2011–12, and based on data on the distribution of consumption expenditure in various rounds of the National Sample Survey, speak for themselves.

13.2.2 Poverty

A widely employed family of relative poverty indices is the P^R_η family due to Foster et al. (1984), where, if z is the poverty line, x_i is the income of the ith poorest person in a community of n individuals of whom q are poor (i.e. have incomes lower than the poverty line), and $\eta(\geq 0)$ is a parameter reflecting aversion to inequality in the distribution of poor incomes, then

$$P^R_\eta = (1/nz^\eta) \sum_{i=1}^{q} (z - x_i)^\eta, \eta \geq 0. \tag{13.2}$$

As is well known, P^R_0 is just the headcount ratio of poverty, P^R_1 is the per capita income-gap ratio (or the product of the headcount ratio and the proportionate shortfall of the average income of the poor from the poverty line) and P^R_2 (the 'squared

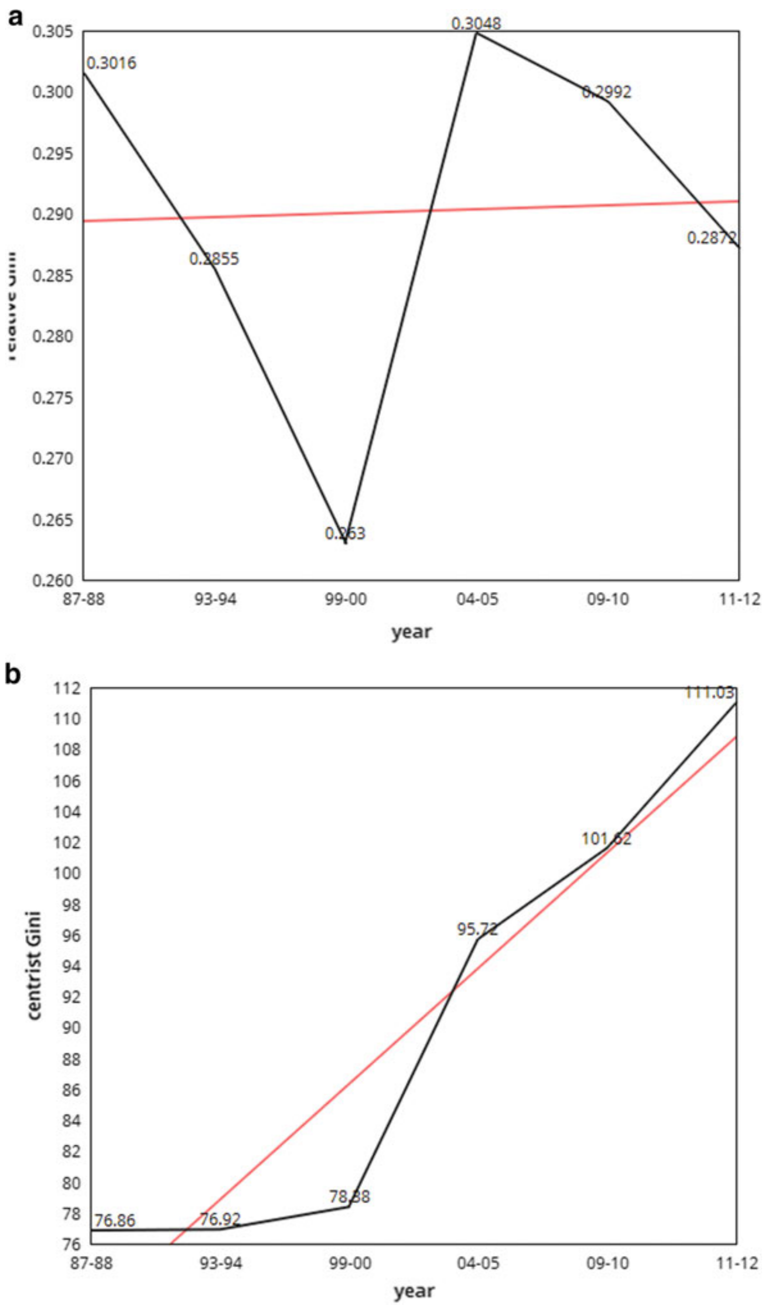

Fig. 13.1 **a** Relative Gini for consumption distribution: Rural India 1987–88 to 2011–12, **b** Centrist Gini for consumption distribution: Rural India 1987–88 to 2011–12, **c** Relative Gini for consumption distribution: Urban India 1987–88 to 2011–12, **d** Centrist Gini for consumption distribution: Urban India 1987–88 to 2011–12

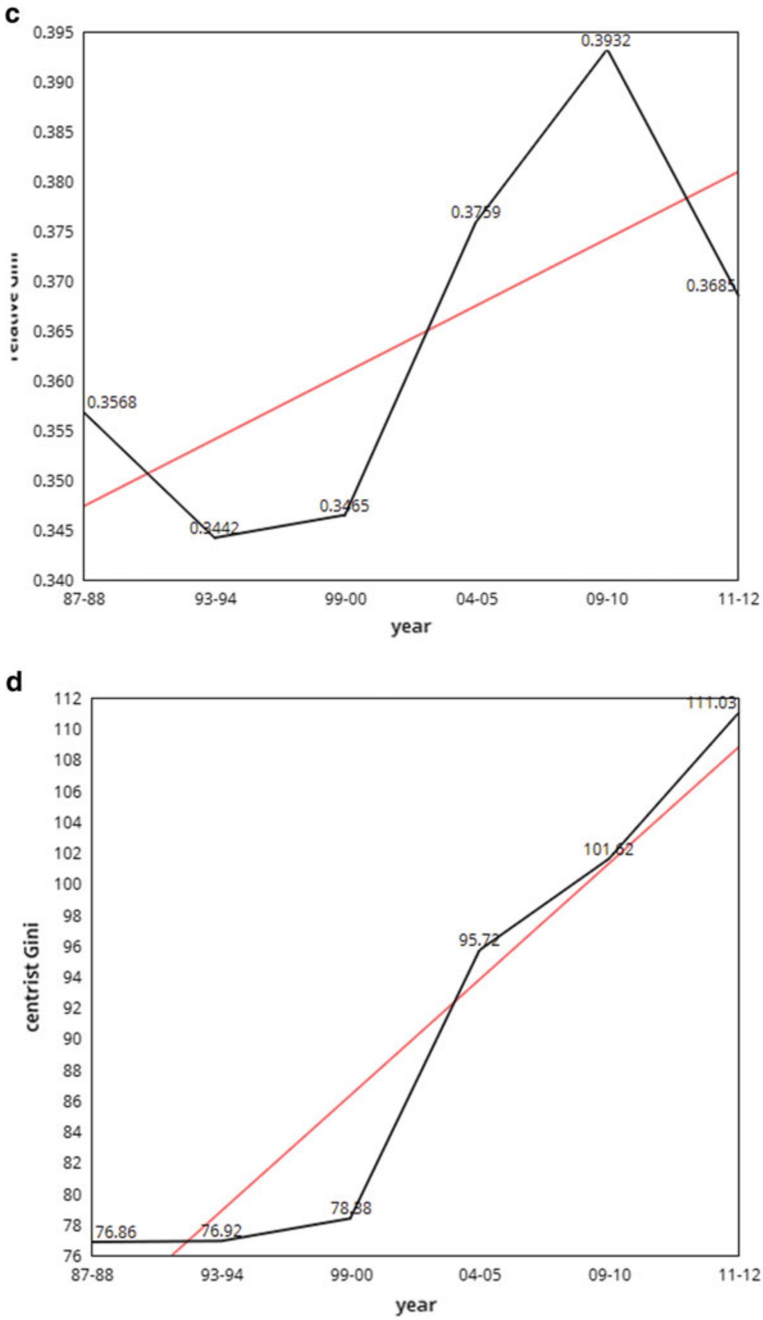

Fig. 13.1 (continued)

poverty-gap' index) additionally incorporates information on the squared coefficient of variation in the distribution of poor incomes, that is, is sensitive to inequality among the poor. The comprehensively absolute counterparts of the relative Foster–Greer–Thorbecke poverty indices are obtained by simply desisting from normalizing the indices with respect to population size (n) and the poverty line (z), and are given by

$$P_\eta^A = \sum_{i=1}^{q} (z - x_i)^\eta \equiv n z^\eta P_\eta^R, \eta \geq 0. \tag{13.3}$$

It is easy to see from Eq. (13.3) that if P_0^R is the headcount ratio, then P_0^A is the aggregate headcount. The aggregate headcount, unlike the headcount ratio, violates what one might call a '*Likelihood Principle*', namely the principle that a poverty measure should convey some information on the probability of encountering a poor person in any community. The headcount ratio, unlike the aggregate headcount, violates a '*Population Focus Principle*', namely that a poverty measure should not be sensitive to increases in the *non*-poor population. In general, both comprehensively relative poverty measures (i.e. measures that are relative with respect to both income and population) and comprehensively absolute poverty measures (i.e. measures which are absolute with respect to both income and population) are predicated on 'extreme values', and the case for 'intermediate' poverty measures is as persuasive as is the case for intermediate inequality measures. The comprehensively intermediate family of Foster–Greer–Thorbecke (FGT) poverty indices is given by the geometric mean of the class of relative FGT measures and absolute FGT measures, in terms of a parameter $\delta \in [0, 1]$ and given by

$$P_\eta^I(\delta) = (P_\eta^R)^\delta (P_\eta^A)^{1-\delta} = (n z^\eta)^{1-\delta} P_\eta^R, \delta \in [0, 1]. \tag{13.4}$$

As δ in Eq. (13.4) increases from 0 to 1, the poverty measure becomes less and less absolute and more and more relative. A 'properly centrist' intermediate measure P_η^{I*} is one which gives equal weight to both the absolute and the relative conceptions of poverty and is realized when δ in Eq. (13.4) is set at one-half:

$$P_\eta^{I*} = \sqrt{n z^\eta} P_\eta^R. \tag{13.5}$$

While the overwhelmingly popular convention in the measurement literature is to employ purely relative poverty measures, it is our contention that properly centrist measures such as P_η^{I*} mitigate the extreme outcomes to which the values underlying relative and absolute measures are prone. (In this connection, the reader is referred to the works of, among others, Zheng 2007; Subramanian 2018.)

Here is an empirical example, involving urban poverty estimates for India based on National Sample Survey data on the distribution of consumption expenditure in 2004–05 and 2011–12, of how our diagnosis of money-metric poverty can change when we relax some of the customary assumptions underlying the 'identification'

Table 13.1 Relative poverty for a fixed poverty line in Urban India: 2004–05 and 2011–12

Year	Poverty line at 2001 prices (Rupees)		P_0^R	P_1^R	P_2^R
2004–05	505.27		0.2674	0.0634	0.0204
2011–12	505.27		0.1344	0.0252	0.0071
Terminal year poverty as a percentage of base year poverty	*	*	50.26%	39.48%	34.80%

Source Estimates based on the figures in Tables 1 and 2 of Subramanian (2018), themselves computed from the 61st and 68th Rounds of the National Sample Survey on distribution of consumption expenditure

and 'aggregation' exercises of standard poverty measurement (to the extent that such measurement is meaningful). In Table 13.1, we present information on the headcount ratio, the per capita income-gap ratio and the squared poverty-gap index—*each in its customarily purely relative form*—for a poverty line that is unvarying in real terms over the two years involved in the poverty comparison: following the Tendulkar Committee's (Planning Commission 2009) recommendation, the poverty line is pegged at Rs. 505.27 at 2001 prices (the price deflator employed being the Consumer Price Index for Industrial Workers (CPIIW)). By this reckoning, poverty in 2011–12 is just between a third and a half of poverty in 2004–05, depending on which relative poverty measured is employed. In Table 13.2, we defer to the view that the poverty line should be continuously adapted and augmented with time, such as has been advocated by commentators like Peter Townsend (1979). One way of doing this is to allow the poverty line of Rs.505.27 in 2004–05 to increase at the arbitrary, but modest, compound rate of growth of one per cent per annum, so that, in 2011–12 the line becomes Rs. 563.71 at 2001 prices. Further, we relax the norm of relativity in the aggregation exercise to allow for properly centrist measures. In such an event, the poverty level in 2011–12 as a proportion of its level in 2004–05 rises from about a

Table 13.2 Centrist poverty for a variable poverty line in Urban India: 2004–05 and 2011–12

Year	Poverty line at 2001 prices (Rupees)		P_0^{I*} (Millions of Persons)	P_1^{I*} (Millions of Rupees)	P_2^{I*} (Millions of Rupees)
2004–05	505.27		3.68	25.47	184.22
2011–12	563.71		3.67	17.70	114.93
Terminal year poverty as a percentage of base year poverty	*	*	99.73%	69.49%	62.39%

Source Same as Table 13.1

third to about three-fifths for the FGT-2 index, from about two-fifths to about seven-tenths for the FGT-1 index, and from about one-half to about one hundred per cent for the FGT-0 index: the decline in poverty rates becomes altogether less dramatic!

13.3 Summary and Conclusion

Our response to the problems of disparity and deprivation is inevitably determined by our perception of the magnitudes of, and trends in, these phenomena. Our perception, in turn, is inevitably determined by the precise protocols of measurement we choose to employ in order to assess the quantitative significance of the phenomena in question. One must be a 'measurement-nihilist' to deny the truth of this proposition and does not have to be a 'measurement-fetishist' in order to affirm it. This is particularly in evidence in certain old debates on whether inequality and poverty are best measured in relative, in absolute, or in some intermediate form. The debates can be traced back to the pioneering work of Hugh Dalton in the 1920s and their revival by Serge-Christophe Kolm in the 1970s. Despite the profound importance of the issues of logical and ethical appeal involved in the debates, they have tended, unfortunately, to be largely neglected in the measurement literature, in favour of wholly relative measures of inequality and poverty. When we correct for this bias, we find that the problems of both inequality and poverty in India are more severe than results based on conventional measurement procedures will allow.

Measurement is far from being the only matter of concern when we deal with issues of disparity and deprivation. Equally, however, it is very far from being a matter of inconsequential concern.

References

Ahluwalia, M. S. (2011). Prospects and policy challenges in the twelfth plan. *Economic and Political Weekly, XLVI, 21,* 88–105.

Bhalla, S. S. (2011). Inclusion and growth in India: Some facts, some conclusions. LSE Asia Research Centre Working Paper 39.

Bhagwati, J. (2011). Indian reforms: Yesterday and today. In P. S. Mehta & B. Chatterjee (Eds.) *Growth and Poverty: The Great Debate.* Jaipur: Cuts International.

Dalton, H. (1924). The measurement of the inequality of incomes. *The Economic Journal, 30*(119), 348–361.

Foster, J. E., Greer, J., & Thorbecke, E. (1984). A class of decomposable poverty measures. *Econometrica, 52*(3), 761–766.

Kolm, S Ch. (1976a). Unequal inequalities I. *Journal of Economic Theory, 12*(3), 416–454.

Kolm, S Ch. (1976b). Unequal inequalities II. *Journal of Economic Theory, 13*(1), 82–111.

Moyes, P. (1987). A new concept of lorenz domination. *Economics Letters, 23*(2), 203–207.

Planning Commission. (2009).*Report of the expert group to review the methodology for estimation of poverty*. Government of India: New Delhi.

Srinivasan, T. N. (2017). Planning, poverty and political economy of reforms: A tribute to Suresh D. Tendulkar. In K. L. Krishna, V. Pandit, K. Sundaram & P. Dua (Eds.), *Perspectives on Economic Policy and Development in India: In Honour of Suresh Tendulkar*, Springer: Delhi.

Subramanian, S. (2002). Counting the poor: An elementary difficulty in the measurement of poverty. *Economics and Philosophy*, 277–285.

Subramanian, S. (2018). On Comprehensively intermediate measures of inequality and poverty, with an illustrative application to global data. *Journal of Globalization and Development*. https://doi.org/10.1515/jgd-2017-0027.

Subramanian, S., & Lalvani, M. (2018). Poverty, growth, inequality: Some general and India-specific considerations. *Indian Growth and Development Review, 11*(2), 136–151.

Temkin, L. (1993). *Inequality*. Clarendon: Oxford University Press.

Townsend, P. (1979). *The development of research on poverty', in department of health and social security: Social security research: The definition and measurement of poverty*. London: HMSO.

Zheng, B. (2007). Unit-consistent poverty indices. *Economic Theory, 31*(1), 113–142.