# Chapter 9
# Numerical Simulation of Stochastic Point Kinetics Equation in the Dynamical System of Nuclear Reactor


## 9.1 Introduction

In nuclear reactor dynamics, the point kinetics equations are the coupled differential equations for the neutron density and for the delayed neutron precursor concentrations. The point kinetics equations are the most vital model in nuclear engineering, and these equations model the time-dependent behavior of a nuclear reactor [1–4]. The time-dependent parameters in this system are the reactivity function and neutron source term. The dynamical process described by the point kinetics equations is stochastic in nature, and the neutron density and delayed neutron precursor concentrations vary randomly with time. At high power levels, random behavior is negligible. But at low power levels, such as at the beginning, random fluctuation in the neutron density and neutron precursor concentrations can be significant.

The point kinetics equations model a system of interacting populations, specifically the populations of neutrons and delayed neutron precursors. In this chapter, the physical dynamical system identified as a population process and the point kinetics equations have been analyzed to transform into a stochastic differential equation system that accurately models the random behavior of the process.

In the present chapter, the Euler–Maruyama method and Taylor 1.5 strong order approximation method have been applied efficiently and conveniently for the solution of stochastic point kinetics equation. The resulting systems of stochastic differential equations are solved over each time-step size in the partition. In the present investigation, the main attractive advantage, of these computational numerical methods, is their elegant applicability for solving stochastic point kinetics equations in a simple and efficient way.



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## 9.2 Outline of the Present Study

In the present chapter, the numerical approximation methods, applied to efficiently calculate the solution for stochastic point kinetics equations [1, 3] in nuclear reactor dynamics, are investigated. A system of Itô stochastic differential equations has been analyzed to model the neutron density and the delayed neutron precursors in a point nuclear reactor. The resulting system of Itô stochastic differential equations is solved over each time-step size. The methods are verified by considering different initial conditions, experimental data, and over-constant reactivities. The computational results indicate that the methods are simple and worthy for solving stochastic point kinetics equations. In this work, a numerical investigation is made in order to observe the random oscillations in neutron and precursor population dynamics in subcritical and critical reactors.

## 9.3 Strong and Weak Convergence

In this section, a brief discussion on strong convergence and week convergence has been presented.

### *9.3.1 Strong Convergence*

A discrete-time approximation method is said to converge strongly to the solution $X(t)$ at time $t$ if

$$\lim_{\Delta t -> 0} E\left|X(t) - \widehat{X}(t)\right| = 0 \tag{9.1}$$

where $\widehat{X}(t)$ is the approximate solution computed with constant step size $\Delta t$ and $E$ denotes expected value.

A SDE method converges strongly with order $\alpha$ if the expected value of the error is of $\alpha$th order in the step size, i.e., if for any time $t$

$$E\left|X(t) - \widehat{X}(t)\right| = \mathrm{O}((\Delta t)^{\alpha}) \tag{9.2}$$

for sufficiently small step size $\Delta t$ [5].

### 9.3.2 Weak Convergence

A discrete-time approximation $\widehat{X}(t)$ with constant step size $\Delta t$ is said to converge weakly to the solution *X(t)* at time $t$ if

$$\lim_{\Delta t- > 0}\left|E(f(X(t))) - E\left(f(\widehat{X}(t))\right)\right| = 0 \tag{9.3}$$

for all smooth functions $f$ in some class.

A SDE method converges weakly with order $\alpha$ if the error in the moments is of $\alpha$th order in the step size

$$\left|E(f(X(t))) - E\left(f(\widehat{X}(t))\right)\right| = \mathrm{O}((\Delta t)^{\alpha}) \tag{9.4}$$

for sufficiently small step size $\Delta t$ [5].

In other words, for a given time discretization $t_0 < t_1 < \ldots < t_n = T$,

- A method is said to have strong order of convergence $\alpha$ if there is a constant $K > 0$ such that

$$\sup_{t_k} E\left|X_{t_k} - \widehat{X}_{t_k}\right| < K(\Delta t_k)^{\alpha}$$

- A method is said to have weak order of convergence $\alpha$ if there is a constant $K > 0$ such that

$$\sup_{t_k}\left|E[X_{t_k}] - E[\widehat{X}_{t_k}]\right| < K(\Delta t_k)^{\alpha},$$

where $\Delta t_k = t_k - t_{k-1}$, $X_{t_k}$ and $\widehat{X}_{t_k}$ represents the exact solution and approximate solution at time $t_k$.

The Euler–Maruyama method has strong convergence of order $\alpha = 1/2$, which is poorer of the convergence for the Euler method in the deterministic case, which is order $\alpha = 1$. However, the Euler–Maruyama method has week convergence of order $\alpha = 1$.

## 9.4 Evolution of Stochastic Neutron Point Kinetics Model

It is the most vital part of nuclear reactor dynamics, to derive the point kinetics equations in order to separate the birth and death process of neutron population. It will help us to form a stochastic model. The deterministic time-dependent equations satisfied by the neutron density and the delayed neutron precursors are as follows [1]

$$\frac{\partial N}{\partial t} = Dv\nabla^2 N - (\Sigma_a - \Sigma_f)vN + [(1-\beta)k_\infty \Sigma_a - \Sigma_f]vN + \sum_i \lambda_i C_i + S_0, \quad (9.5)$$

$$\frac{\partial C_i}{\partial t} = \beta_i k_\infty \Sigma_a vN - \lambda_i C_i, \quad i = 1, 2, \ldots, m, \quad (9.6)$$

where $N(r,t)$ is the neutron density at a point $r$ at time $t$. The coefficients $D$, $v$, $\Sigma_a$ *and* $\Sigma_f$ are, respectively, diffusion constants, the neutron speed, the macroscopic neutron absorption, and fission cross sections. The capture cross section is $\Sigma_a - \Sigma_f$. If $\beta = \sum_{i=1}^{m} \beta_i$ is the delayed neutron fraction, the prompt neutron contribution to the source is $[(1-\beta)k_\infty \Sigma_a - \Sigma_f]vN$ and the prompt neutron fraction is $(1-\beta)$. The number of neutrons produced per neutrons absorbed is $k_\infty$ (also called infinite-medium reproduction factor). The rate of transformations from neutron precursors to the neutron population is $\sum_{i=1}^{m} \lambda_i C_i$ where the delayed constant is $\lambda_i$ and $C_i(r,t)$ is the density of the $i$th type of precursor for $i = 1, 2, \ldots, m$. Sources of neutrons extraneous to the fission process are represented by $S_0(r,t)$.

In the present analysis, captures (or leakages) of neutrons are considered as deaths. The fission process is considered a pure birth process where $v(1-\beta) - 1$ neutrons are born in each fission along with precursor fraction $v\beta$.

Let us assume that $N = f(r)n(t)$ and $C_i = g_i(r)c_i(t)$ are separable in time and space where $n(t)$ and $c_i(t)$ are the total number of neutrons and precursors of the $i$th type at time $t$, respectively.

Using these, Hetrick [1] and Hayes et al. [4] derived the deterministic point kinetics equation as

$$\frac{dn}{dt} = -\left[\frac{-\rho + 1 - \alpha}{l}\right] n + \left[\frac{1-\alpha-\beta}{l}\right] n + \sum_{i=1}^{m} \lambda_i c_i + q, \quad (9.7a)$$

$$\frac{dc_i}{dt} = \frac{\beta_i}{l} n - \lambda_i c_i, \quad i = 1, 2, \ldots, m, \quad (9.7b)$$

where $q(t) = \frac{S_0(r,t)}{f(r)}$, $\rho$ is reactivity, neutron generation time $l = \frac{1}{k_\infty v \Sigma_a}$, $\alpha$ is defined as $\alpha = \frac{\Sigma_f}{\Sigma_a k_\infty} \approx \frac{1}{v}$, and $v$ is the average number of neutrons per fission. Here, $n(t)$ is the population size of neutrons and $c_i(t)$ is the population size of the $i$th neutron precursor. The neutron reactions can be separated into three terms as follows:

$$\frac{dn}{dt} = \underbrace{-\left[\frac{-\rho + 1 - \alpha}{l}\right]}_{\text{deaths}} n + \underbrace{\left[\frac{1-\alpha-\beta}{l}\right]}_{\text{births}} n + \underbrace{\sum_{i=1}^{m} \lambda_i c_i}_{\text{transformations}} + q,$$

$$\frac{dc_i}{dt} = \frac{\beta_i}{l} n - \lambda_i c_i, \quad i = 1, 2, \ldots, m.$$

The neutron birth rate due to fission is $b = \frac{1-\alpha-\beta}{l(-1+(1-\beta)v)}$, where the denominator has the term $(-1+(1-\beta)v)$ which represents the number of neutrons (newborn) produced in each fission process. The neutron death rate due to captures or leakage is $d = \frac{-\rho+1-\alpha}{l}$. The transformation rate $\lambda_i c_i$ represents the rate that the $i$th precursor is transformed into neutrons and $q$ represents the rate that source neutrons are produced.

To derive the stochastic dynamical system, we consider for simplicity only one precursor, i.e., $\beta = \beta_1$, where $\beta$ is the total delayed neutron fraction for one precursor.

The point kinetics equations for one precursor are as follows

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \left[\frac{-\rho+1-\alpha}{l}\right]n + \left[\frac{1-\alpha-\beta}{l}\right]n + \lambda_1 c_1 + q, \quad \frac{\mathrm{d}c_1}{\mathrm{d}t} = \frac{\beta_1}{l}n - \lambda_1 c_1.$$

Now, we consider in the small duration of time interval $\Delta t$ where probability of more than one occurred event is small. There are four different possibilities for an event at this small time $\Delta t$. Let $[\Delta n, \Delta c_1]^{\mathrm{T}}$ be the change of $n$ and $c_1$ in time $\Delta t$ where the changes are assumed approximately normally distributed. The four possibilities for $[\Delta n, \Delta c_1]^{\mathrm{T}}$ are

$$E_1 = \begin{bmatrix} \Delta n \\ \Delta c_1 \end{bmatrix}_1 \equiv \begin{bmatrix} -1 \\ 0 \end{bmatrix},$$

$$E_2 = \begin{bmatrix} \Delta n \\ \Delta c_1 \end{bmatrix}_2 \equiv \begin{bmatrix} -1+(1-\beta)v \\ \beta_1 v \end{bmatrix},$$

$$E_3 = \begin{bmatrix} \Delta n \\ \Delta c_1 \end{bmatrix}_3 \equiv \begin{bmatrix} 1 \\ -1 \end{bmatrix},$$

$$E_4 = \begin{bmatrix} \Delta n \\ \Delta c_1 \end{bmatrix}_4 \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

where the first event $E_1$ denotes a death, the second event $E_2$ represents birth of $(-1+(1-\beta)v)$ neutrons and $\beta_1 v$ delayed neutron precursors produced in the fission process, the third event $E_3$ represents a transformation of a delayed neutron precursor to a neutron, and the last one $E_4$ event represents a neutron source. The respective probabilities of these events are

$$P(E_1) = n\Delta t d,$$

$$P(E_2) = n\Delta t b = \frac{1}{vl} n\Delta t\,, \quad \text{since } b = \frac{1-\alpha-\beta}{l(-1+(1-\beta)v)} \text{ and } \alpha = \frac{\Sigma_f}{\Sigma_a k_\infty} \approx \frac{1}{v}$$

$$P(E_3) = c_1 \Delta t \lambda_1,$$

$$P(E_4) = q\Delta t.$$

In this present analysis, it is assumed that the extraneous source randomly produces neutrons following Poisson process with intensity $q$.

According to our earlier assumption, the changes in neutron population and precursor concentration are approximately normally distributed with mean $E\left(\begin{bmatrix}\Delta n\\ \Delta c_1\end{bmatrix}\right)$ and variance $\mathrm{Var}\left(\begin{bmatrix}\Delta n\\ \Delta c_1\end{bmatrix}\right)$.

Here, the mean change in the small interval of time $\Delta t$

$$E\left(\begin{bmatrix}\Delta n\\ \Delta c_1\end{bmatrix}\right) = \sum_{k=1}^{4} P_k \begin{bmatrix}\Delta n\\ \Delta c_1\end{bmatrix}_k = \begin{bmatrix}\frac{\rho-\beta}{l} n + \lambda_1 c_1 + q\\ \frac{\beta_1}{l} n - \lambda_1 c_1\end{bmatrix} \Delta t,$$

and the variance of change in small time $\Delta t$

$$\mathrm{Var}\left(\begin{bmatrix}\Delta n\\ \Delta c_1\end{bmatrix}\right) = E\left(\begin{bmatrix}\Delta n\\ \Delta c_1\end{bmatrix}[\Delta n \quad \Delta c_1]\right) - \left(E\left(\begin{bmatrix}\Delta n\\ \Delta c_1\end{bmatrix}\right)\right)^2$$

$$= \sum_{k=1}^{4} P_k \begin{bmatrix}\Delta n\\ \Delta c_1\end{bmatrix}_k [\Delta n \quad \Delta c_1]_k = \widehat{B}\Delta t,$$

where

$$\widehat{B} = \begin{bmatrix} \gamma n + \lambda_1 c_1 + q & \frac{\beta_1}{l}(-1+(1-\beta)v)n - \lambda_1 c_1 \\ \frac{\beta_1}{l}(-1+(1-\beta)v)n - \lambda_1 c_1 & \frac{\beta_1^2 v}{l} n + \lambda_1 c_1 \end{bmatrix},$$

where

$$\gamma = \frac{-1-\rho+2\beta+(1-\beta)^2 v}{l}.$$

Now, by central limit theorem, the random variate

$$\frac{\begin{bmatrix}\Delta n\\ \Delta c_1\end{bmatrix} - E\left(\begin{bmatrix}\Delta n\\ \Delta c_1\end{bmatrix}\right)}{\sqrt{\mathrm{Var}\left(\begin{bmatrix}\Delta n\\ \Delta c_1\end{bmatrix}\right)}}$$

follows standard normal distribution. The above result implies

$$\begin{bmatrix}\Delta n\\ \Delta c_1\end{bmatrix} = E\left(\begin{bmatrix}\Delta n\\ \Delta c_1\end{bmatrix}\right) + \sqrt{\mathrm{Var}\left(\begin{bmatrix}\Delta n\\ \Delta c_1\end{bmatrix}\right)}\begin{bmatrix}\eta_1\\ \eta_2\end{bmatrix}, \quad \text{where } \eta_1, \eta_2 \sim N(0,1) \tag{9.8}$$

Thus, we have

$$\begin{bmatrix}n(t+\Delta t)\\ c_1(t+\Delta t)\end{bmatrix} = \begin{bmatrix}n(t)\\ c_1(t)\end{bmatrix} + \begin{bmatrix}\frac{\rho-\beta}{l}n + \lambda_1 c_1\\ \frac{\beta_1}{l}n + \lambda_1 c_1\end{bmatrix}\Delta t + \begin{bmatrix}q\\ 0\end{bmatrix}\Delta t + \widehat{B}^{1/2}\sqrt{\Delta t}\begin{bmatrix}\eta_1\\ \eta_2\end{bmatrix}, \tag{9.9}$$

where $\widehat{B}^{1/2}$ is the square root of the matrix $\widehat{B}$. Dividing both sides of Eq. (9.9) by $\Delta t$ and then taking limit $\Delta t \to 0$, we achieve the following Itô stochastic differential equation system

$$\frac{d}{\mathrm{d}t}\begin{bmatrix}n\\ c_1\end{bmatrix} = \widehat{A}\begin{bmatrix}n\\ c_1\end{bmatrix} + \begin{bmatrix}q\\ 0\end{bmatrix} + \widehat{B}^{1/2}\frac{\mathrm{d}\overrightarrow{W}}{\mathrm{d}t}, \tag{9.10}$$

where

$$\widehat{A} = \begin{bmatrix}\frac{\rho-\beta}{l} & \lambda_1\\ \frac{\beta_1}{l} & -\lambda_1\end{bmatrix},$$

$$\widehat{B} = \begin{bmatrix}\gamma n + \lambda_1 c_1 + q & \frac{\beta_1}{l}(-1+(1-\beta)v)n - \lambda_1 c_1\\ \frac{\beta_1}{l}(-1+(1-\beta)v)n - \lambda_1 c_1 & \frac{\beta_1^2}{l}n + \lambda_1 c_1\end{bmatrix},$$

and

$$\overrightarrow{W}(t) = \begin{bmatrix}W_1(t)\\ W_2(t)\end{bmatrix},$$

where $W_1(t)$ and $W_2(t)$ are Wiener processes. Equation (9.10) represents the stochastic point kinetics equations for one precursor. Now generalizing the above argument to $m$ precursors, we can obtain the following Itô stochastic differential equation system for $m$ precursors

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} n \\ c_1 \\ c_2 \\ \vdots \\ c_m \end{bmatrix} = \widehat{A}\begin{bmatrix} n \\ c_1 \\ c_2 \\ \vdots \\ c_m \end{bmatrix} + \begin{bmatrix} q \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \widehat{B}^{1/2}\frac{\mathrm{d}\overrightarrow{W}}{\mathrm{d}t}. \tag{9.11}$$

In Eq. (9.11), $\widehat{A}$ and $\widehat{B}$ are as follows

$$\widehat{A} = \begin{bmatrix} \frac{\rho-\beta}{l} & \lambda_1 & \lambda_2 & \cdots & \lambda_m \\ \frac{\beta_1}{l} & -\lambda_1 & 0 & \cdots & 0 \\ \frac{\beta_2}{l} & 0 & -\lambda_2 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ \frac{\beta_m}{l} & 0 & \cdots & 0 & -\lambda_m \end{bmatrix}, \tag{9.12}$$

$$\widehat{B} = \begin{bmatrix} \zeta & a_1 & a_2 & \cdots & a_m \\ a_1 & r_1 & b_{2,3} & \cdots & b_{2,m+1} \\ a_2 & b_{3,2} & r_2 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & b_{m,m+1} \\ a_m & b_{m+1,2} & \cdots & b_{m+1,m} & r_m \end{bmatrix}, \tag{9.13}$$

where

$$\zeta = \gamma n + \sum_{j=1}^{m} \lambda_j c_j + q,$$

$$\gamma = \frac{-1-\rho+2\beta+(1-\beta)^2 v}{l},$$

$$a_j = \frac{\beta_j}{l}(-1+(1-\beta)v)n - \lambda_j c_j,$$

$$b_{i,j} = \frac{\beta_{i-1}\beta_{j-1}v}{l}n,$$

and

$$r_i = \frac{\beta_i^2 v}{l} n + \lambda_i c_i.$$

Equation (9.11) represents the generalization of the standard point kinetics model since for $\widehat{B} = 0$, it reduces to the standard deterministic point kinetics model [3].

## 9.5 Application of Euler–Maruyama Method and Strong Order 1.5 Taylor Method for the Solution of Stochastic Point Kinetics Model

The stochastic point kinetics equations for $m$ delayed groups are as follows

$$\frac{\mathrm{d}\overrightarrow{x}}{\mathrm{d}t} = A\overrightarrow{x} + B(t)\overrightarrow{x} + \overrightarrow{F}(t) + \widehat{B}^{1/2}\frac{\mathrm{d}\overrightarrow{W}}{\mathrm{d}t}, \tag{9.14}$$

where $\hat{B}$ is given in Eq. (9.13),

$$\overrightarrow{x} = \begin{bmatrix} n \\ c_1 \\ c_2 \\ \vdots \\ c_m \end{bmatrix}, \tag{9.15}$$

$$A = \begin{bmatrix} \frac{-\beta}{l} & \lambda_1 & \lambda_2 & \cdots & \lambda_m \\ \frac{\beta_1}{l} & -\lambda_1 & 0 & \cdots & 0 \\ \frac{\beta_2}{l} & 0 & -\lambda_2 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ \frac{\beta_m}{l} & 0 & \cdots & 0 & -\lambda_m \end{bmatrix}, \tag{9.16}$$

$$B(t) = \begin{bmatrix} \frac{\rho(t)}{l} & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}, \tag{9.17}$$

and

$$\overrightarrow{F}(t) = \begin{bmatrix} q(t) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{9.18}$$

It can be noticed that $\widehat{A} = A + B(t)$.

### 9.5.1 Euler–Maruyama Method for the Solution of Stochastic Point Kinetics Model

This method is also known as order 0.5 strong Itô–Taylor approximation. By applying Euler–Maruyama method in Eq. (1.142) of Chap. 1 into Eq. (9.14), we obtain

$$\overrightarrow{x}_{i+1} = \overrightarrow{x}_i + (A + B_i)\overrightarrow{x}_i h + \overrightarrow{F}(t_i)h + B^{1/2}\sqrt{h}\,\overrightarrow{\eta}_i, \tag{9.19}$$

where $\mathrm{d}\overrightarrow{W}_i = \overrightarrow{W}_i - \overrightarrow{W}_{i-1} = \sqrt{h}\,\overrightarrow{\eta}_i$ and $h = t_{i+1} - t_i$. Here, $\overrightarrow{\eta}_i$ is a vector whose components are random numbers chosen from $N(0,1)$.

### 9.5.2 Strong Order 1.5 Taylor Method for the Solution of Stochastic Point Kinetics Model

We apply strong order 1.5 Taylor approximation method in Eq. (1.143) of Chap. 1 into Eq. (9.14) yielding

$$\overrightarrow{x}_{i+1} = \overrightarrow{x}_i + \left((A + B_i)\overrightarrow{x}_i + \overrightarrow{F}_i\right)h + \widehat{B}^{1/2}\sqrt{h}\,\overrightarrow{\eta}_i + (A + B_i)\widehat{B}^{1/2}\Delta Z_i + \frac{1}{2}\left((A + B_i)\overrightarrow{x}_i + \overrightarrow{F}_i\right)(A + B_i)h^2, \tag{9.20}$$

where $\Delta Z_i = \frac{1}{2}h(\Delta W_i + \Delta V_i/\sqrt{3})$ and $\Delta V_i = \sqrt{h}\,N(0,1)$.

### 9.5.3 Numerical Results and Discussion

In the present analysis, we consider the first example of nuclear reactor problem with the following parameters $\lambda_1 = 0.1, \quad \beta_1 = 0.05 = \beta, \quad v = 2.5, \quad v = 2.5,$ neutron source $q = 200, \quad l = 2/3$ and $\rho(t) = -1/3$ for $t \geq 0$. The initial condition is $\overrightarrow{x}(0) = [400 \quad 300]^{\mathrm{T}}$. We observe through 5000 trails, the good agreement between two methods with other available methods for 40 time intervals at time $t$ = 2 s. The means and standard deviation of $n(2)$ and $c_1(2)$ are presented in Table 9.1.

In the second example, we assume the initial condition as

$$\overrightarrow{x}(0) = 100 \begin{bmatrix} 1 \\ \frac{\beta_1}{\lambda_1 l} \\ \frac{\beta_2}{\lambda_2 l} \\ \vdots \\ \frac{\beta_m}{\lambda_m l} \end{bmatrix}.$$

The following parameters are used in this example [1, 3] $\beta = 0.007$, $v = 2.5$, $l = 0.00002$, $q = 0$, $\lambda_i = [0.0127, 0.0317, 0.115, 0.311, 1.4, 3.87]$ and $\beta_i = [0.000266, 0.001491, 0.001316, 0.002849, 0.000896, 0.000182]$ with $m = 6$ delayed groups. The computational results at $t = 0.1$ and $t = 0.001$ are given in Tables 9.2 and 9.3, respectively, for Monte Carlo, stochastic PCA [4], Euler–Maruyama, and Taylor 1.5 strong order. It can be seen that there exist approximately close agreements between the three approaches in consideration of different step reactivities $\rho = 0.003$ and $\rho = 0.007$. The mean neutron density and two individual neutron samples are cited in Fig. 9.1. The mean precursor density and two precursor sample paths are cited in Fig. 9.2. For these calculations, we used 5000 trials in both Euler–Maruyama and Taylor 1.5 strong order method.

**Table 9.1** Comparison of numerical computational methods for one precursor

| | Monte Carlo | Stochastic PCA [4] | Euler–Maruyama approximation | Strong order 1.5 Taylor approximation |
|---|---|---|---|---|
| $E(n(2))$ | 400.03 | 395.32 | 412.23 | 412.10 |
| $\sigma(n(2))$ | 27.311 | 29.411 | 34.391 | 34.519 |
| $E(c_1(2))$ | 300.00 | 300.67 | 315.96 | 315.93 |
| $\sigma(c_1(2))$ | 7.8073 | 8.3564 | 8.2656 | 8.3158 |

**Table 9.2** Comparison for subcritical step reactivity $\rho = 0.003$

| | Monte Carlo | Stochastic PCA [4] | Euler–Maruyama | Taylor 1.5 strong order |
|---|---|---|---|---|
| $E(n(0.1))$ | 183.04 | 186.31 | 208.599 | 199.408 |
| $\sigma(n(0.1))$ | 168.79 | 164.16 | 255.954 | 168.547 |
| $E(c_1(0.1))$ | $4.478 \times 10^5$ | $4.491 \times 10^5$ | $4.498 \times 10^5$ | $4.497 \times 10^5$ |
| $\sigma(c_1(0.1))$ | 1495.7 | 1917.2 | 1233.38 | 1218.82 |

**Table 9.3** Comparison for critical step reactivity $\rho = 0.007$

| | Monte Carlo | Stochastic PCA [4] | Euler–Maruyama | Taylor 1.5 strong order |
|---|---|---|---|---|
| $E(n(0.001))$ | 135.67 | 134.55 | 139.568 | 139.569 |
| $\sigma(n(0.001))$ | 93.376 | 91.242 | 92.042 | 92.047 |
| $E(c_1(0.001))$ | $4.464 \times 10^5$ | $4.464 \times 10^5$ | $4.463 \times 10^5$ | $4.463 \times 10^5$ |
| $\sigma(c_1(0.001))$ | 16.226 | 19.444 | 6.071 | 18.337 |

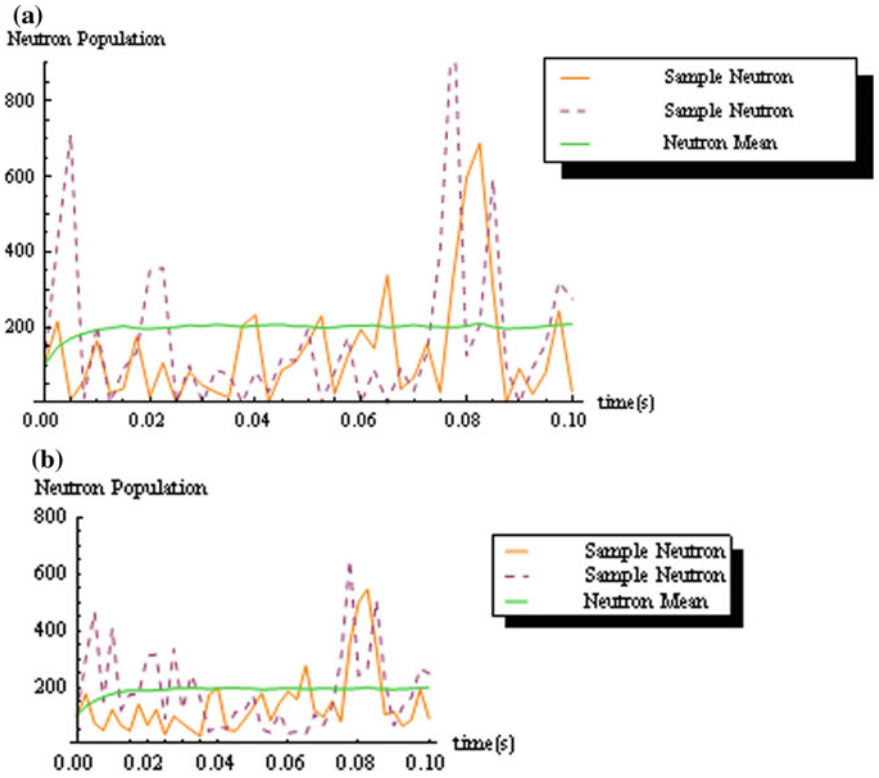


**Fig. 9.1** **a** Neutron density obtained by Euler–Maruyama method using a subcritical step reactivity $\rho = 0.003$ and **b** neutron density obtained by strong 1.5 order Taylor method using a subcritical step reactivity $\rho = 0.003$

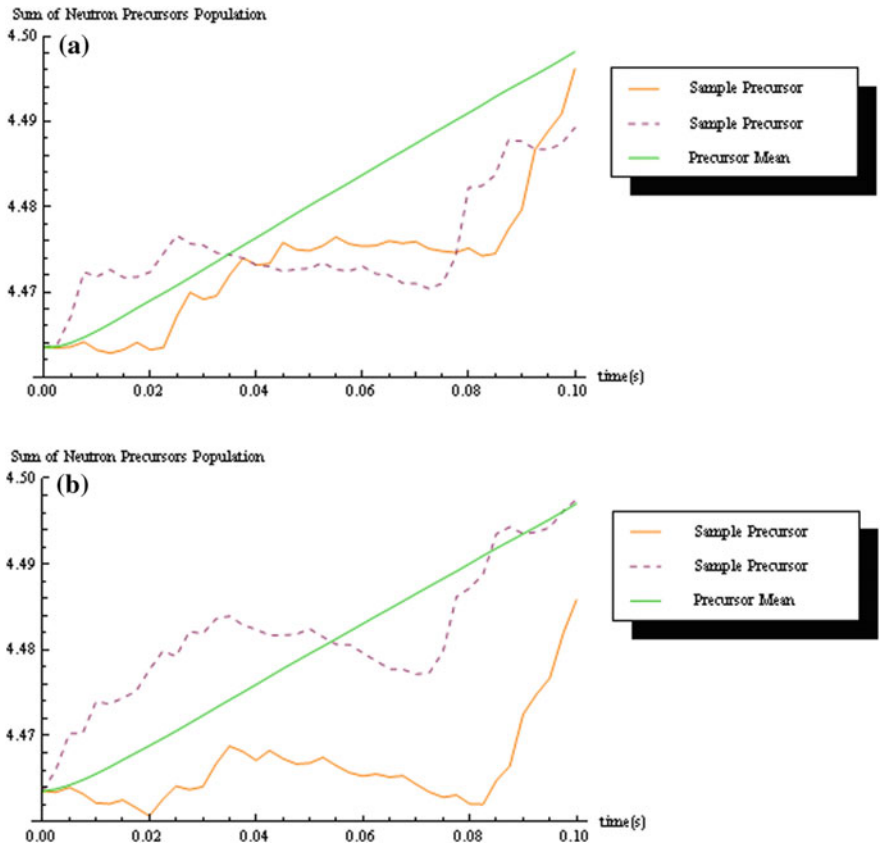


**Fig. 9.2** **a** Precursor density obtained by Euler–Maruyama method using a subcritical step reactivity $\rho = 0.003$ and **b** precursor density obtained by strong 1.5 order Taylor method using a subcritical step reactivity $\rho = 0.003$

## 9.6 Conclusion

In this present research work, the stochastic point kinetics equations have been solved by using Euler–Maruyama and strong order 1.5 Taylor numerical methods having easier and efficient calculation in comparison with stochastic PCA method. The methods, in this investigation, are clearly effective numerical methods for solving the stochastic point kinetics equations. The methods are simple, efficient to calculate, and accurate with fewer round-off error. The derivation of stochastic point kinetics equations may be complicated but numerical solutions obtained more conveniently. The behavior of the stochastic neutron and precursor distributions within a reactor can be explicitly described by the stochastic point kinetics equations. The obvious reason seems to be that the intrinsic stochastic dynamic phenomena in the reactor system can be properly treated with the stochastic point

kinetics equations. In this chapter, a numerical investigation was performed in order to observe the random fluctuations in neutron and precursor population dynamics in subcritical and critical reactors.

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