Topology Optimization Using Strain Energy Distribution for 2D Structures

Srinivasan Bairy, Rafaque Ahmad and Hari K. Voruganti

Abstract Optimization is carried out to achieve the best out of given resources while satisfying constraints on performance, state variables, and resources thus avoiding the excessive use of resources and decrease the cost associated. Structural systems need to be designed for a minimum of weight, compliance, displacement, frequency, etc., to save cost and get optimal performance. For this, structural optimization is carried out. Topology optimization is one type of structural optimization in which topology of the structure is changed. Generally, topology optimization is performed using methods like solid isotropic material with penalization (SIMP), level set-based methods, phase field method, evolutionary structural optimization (ESO), and bidirectional evolutionary structural optimization (BESO). In the present work, a modified evolutionary algorithm is proposed for structural optimization with consideration to strain energy distribution. Addition of material is performed on a partially void space instead of material removal. As the final optimum structure bears only a fraction of initial structure, the method of structure growth using addition approach is better for computational efficiency. This method initially takes a void input design domain but to make numerical computation easy, negligible density is assumed. The objective is to achieve critical strain energy per unit volume which is less than the modulus of resilience according to the maximum strain energy criterion. According to the maximum strain energy theory, a safe structure should have strain energy per unit volume less than the modulus of resilience. Hence, the objective is to find a structure satisfying the above criterion with minimum weight. The main focus of the work is to find optimum topology. Effect of multiple loads, rate of material addition, and effect of the magnitude of loads are also considered for structural optimization. The results are close to the results reported in the literature.

Keywords Topology optimization · Strain energy · Finite elemental analysis

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1 Introduction

The world is facing the scarcity of resources and there is a demand for only the best products in a very high-technological competitive market. To solve this problem and hence minimize the wastage of resources, there is a need to find the best solution among all the solutions of a design problem in such a way that that functionality of the design is not hampered. Structural optimization aims to utilize structural resources most efficiently. Today, optimization of structures has become mandatory for different applications. Structural optimization is much found in the aerospace and automotive industries where weight minimization is the key criteria for design. Structural optimization can be classified into size optimization, shape optimization, and topology optimization. Size optimization is carried out to determine the optimal size, dimensions of cross-sectional areas, thickness, moment of inertia, etc. Shape optimization helps in identifying the optimal geometry of boundaries for the structures carrying loads. Among these three, topology optimization is the most general and gives more design freedom to engineers by expanding the design space to look for the optimal solution, and hence, it is most rewarding economically.

Bendsoe and Sigmund [1] mentioned that topology optimization is carried out to get the best distribution of the material in a design domain. Topology optimization determines the best connectivity, shape, and location of voids in a design domain. Woon et al. [2] carried out shape optimization by moving the boundary and internal nodes of the structure. Optimal coordinates are found using genetic algorithm and also mirroring option helps in reducing the computational time of symmetrical problem to half. Ding [3] mentions that boundary of the physical problem can be represented by using the nodes, piecewise polynomials, and B-spline models. Smooth boundaries are required to avoid stress concentration and to make the manufacturing process easy. Bendsoe [4] described topology optimization as a process of removing the material from the places of negligible contribution to the load carrying capacity of the structural member. It is carried for minimizing the compliance of the structure by the method of solid isotropic material with penalization (SIMP). In topology optimization, material density is chosen as a continuous variable varying from zero to one within an element of finite element mesh. Optimal density is obtained through an optimization algorithm by choosing the densities of each element as variable as explained by Bendsoe [4, 5]. Solid isotropic material with penalization considers the density of element as a design variable and penalizes density based on this power law model such that less contributing elements have minimum effect on the final design, and penalization is applied as mentioned by Duysinx and Bendsøe [6].

An evolution algorithm is proposed by Xie and Steven [7] using a rejection ratio to remove the elements of lower stress. In optimization, constraints play a greater role in the outcome of the algorithm. Generally, in structural optimization, constraints are the volume, weight, and also sustaining the external loads. Xie and Grant [8] described a method for obtaining the optimal structure using an evolutionary structural optimization (ESO) method which works on the local stress of the individual elements. In ESO algorithm, the material is removed from very low stressed locations in every iteration using a rejection ratio. Lagaros et al. [9] presented a hybrid method to reduce the computational cost combining genetical algorithm method with the SQP method. This hybrid method first uses the GA to get the nearest solution to optimum solution, and then in the second half, the SQP method is used to achieve convergence to optimal quickly. Chu et al. [10] developed an optimization algorithm with the objective as stiffness. Lower sensitivity elements of FEA are removed using stiffness sensitivity which is measured by the change in the strain energy.

Abolbashari and Shadi [11] examined the effect of mesh size, rejection ratio, and type of element on the optimal shape and found that the optimum shape is dependent on these parameters. Huang and Xie [12] developed a bidirectional algorithm to increase the rate of convergence of the optimal solution. This algorithm adds the material at critical locations and removes the material from low stressed areas. Tanskanen [13] studied the theoretical aspects of ESO method and found that this method is equivalent to the compliance minimization problem. Li et al. [14] worked on a reference factor for suppressing checkerboard pattern in the optimal solution. Li et al. [15] mentioned that Vonmises stress criteria and stiffness criteria are equivalent in the evolutionary structural optimization problems. Kwok et al. [16] proposed a topology optimization method based on the principal stress lines.

Genetic algorithms also have been used in structural topology optimization. Rajeev and Krishnamoorthy [17] used the genetic algorithm to optimize truss problems. Deb and Surendra [18] also used GA for optimizing the location of nodes for a truss member. Members having negligible area assumed to be absent in the final solutions. Balamurugan et al. [19] claimed that a two-stage adaptive genetic algorithm is converging quickly to obtain a global solution. Deepak et al. [20] studied and compared different formulations for the topology optimization of compliant mechanisms. Mutual strain energy formulation found to be better for compliant mechanisms. Jog [21] proposed topology optimization for reducing structural vibrations. Dynamic compliance is used to optimize the structures vibration levels. This ensures that natural frequency is away from the dynamic frequency of the structures. Nandy et al. [22] optimized the structures for reducing the radiated noise.

Querin et al. [23] applied the bidirectional evolutionary algorithm (BESO) for a fully stressed design using Vonmises stress criterion by removing elements where stress is low and adding elements to void regions near high-stress locations in the design domain. BESO is another method in evolutionary optimization techniques which has provisions of both adding and removing material simultaneously from design domain from less critical regions, i.e., places in design domain where sensitivity number is low and add material to void regions, where material required will be more based on high sensitivity number of void elements. The number of elements to be added and deleted is based on heuristic criteria, not on sensitivity of void elements which causes this method to fail without producing optimal topology and also causing numerical problems like mesh dependency problems and checkerboard. From the literature, the points are noted. (1) Many algorithms remove the material from the initial domain [1]. In this method, material addition method for topology optimization is proposed. (2) In the existing algorithms, strain energy distribution as a criterion for structure topology design based on element addition approach is not presented. (3) Available algorithms did not consider the effect of change in the magnitude of loads, multiple loads on the structures. In our work, the addition of material is adopted instead of removal. The strain energy distribution is considered for optimization. We found the elements which are contributing more to the final solution so that they can be strengthened by adding material. Finite element analysis (FEA) is carried out to know these high contributing elements. As the entire process happens iterationwise, it is enough to check the given structure is safe or not according to the failure criteria chosen. This helps to stop the algorithm at safe design.

2 FEA Formulation

In our work, finite element analysis is used to know the critical locations in the structural component. FEA preprocessing divides the design domain into small finite elements and generates a mesh. Variation of field variables is approximated with shape functions. Rate of change of field variables variation gives the strain at required locations on the element. FEA analysis is carried out using the four-noded quadrilateral element. Isoparametric FEA method is used to approximate the field variables and geometric variables on the element at any location. Field variables are interpolated using nodal values (notation of nodal displacements at jth node are u_j and v_j in *X-direction* and *Y*-direction, respectively) and shape functions (notation of ith shape function is N_i).

In an element, geometric variables are approximated using the following relations with four known geometric coordinates. The four shape functions for the quadratic element are shown below:

$$N_1 = \frac{(1-r)(1-s)}{4};\ N_2 = \frac{(1+r)(1-s)}{4};$$
$$N_3 = \frac{(1+r)(1+s)}{4};\ N_4 = \frac{(1-r)(1+s)}{4} \tag{1}$$

In an element, geometric variables are approximated by using the following relation with four known geometric coordinates.

$$\begin{Bmatrix} x \\ y \end{Bmatrix} = \sum_{i=1}^{4} N_i \begin{Bmatrix} x_i \\ y_i \end{Bmatrix} \tag{2}$$

Displacement of field variables is approximated within the element with known displacements at the four nodes of the element.

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \sum_{i=1}^{4} N_i \begin{Bmatrix} u_i \\ v_i \end{Bmatrix} \tag{3}$$

In FEA, the physical element in (x, y) coordinate is mapped to master element in (r, s) coordinate. Transformation matrix J is used to map the special derivatives available in (r, s) coordinate back into (x, y) coordinate. Strain vector ϵ is determined using $[\epsilon] = [B] \times [\mathrm{U}]$. $[B]$ is the strain–displacement matrix and $[U]$ is the displacement matrix. Stiffness for an element is calculated using the following relation. The thickness of the element is assumed as 1 unit.

$$[\boldsymbol{K}] = \iint_{-1}^{1} [\boldsymbol{B}]^{\boldsymbol{T}} [\boldsymbol{D}] \boldsymbol{B} |\boldsymbol{J}| \mathbf{d}\boldsymbol{r} \mathbf{d}\boldsymbol{s} \tag{4}$$

where $[\boldsymbol{D}]$ is material constitutive matrix.

3 Proposed Method

A method is proposed with consideration to strain energy distribution. Initially, 2D design domain is divided into a certain number of finite elements having a partially dense material equivalent to a void space. The void space is ensured by multiplying regular density stiffness matrix with negligible density factor of 10–8 similar to SIMP methodology. Loading and boundary conditions are selected on nodes according to the physical problem. The field variable such as displacement and their derivatives such as strains and stresses are obtained with finite element analysis. Then critical elements (elements with high strain energy value) are identified among all elements in the design domain. The critical elements are strengthened at each iteration by adjusting the density factor to 1 for those elements. This procedure is continued until the maximum energy in any element is just below the critical level, i.e., the structure becomes safe so the density of all elements in optimum topology is one, and this gives distinctly sharp boundaries of the final design. The entire algorithm is implemented using MATLAB. The flowchart of the algorithm is shown in Fig. 1.

4 Results and Discussions

In this study, four case studies are considered. In each case study, the design domain is divided into 5000 finite elements and then assigned a density factor of 10–8. The properties of material considered are Young's modulus $(E) = 200$ GPa and Poisson's ratio $= 0.3$. Plane stress conditions are considered. The final solution consists of elements whose density factor is 1.

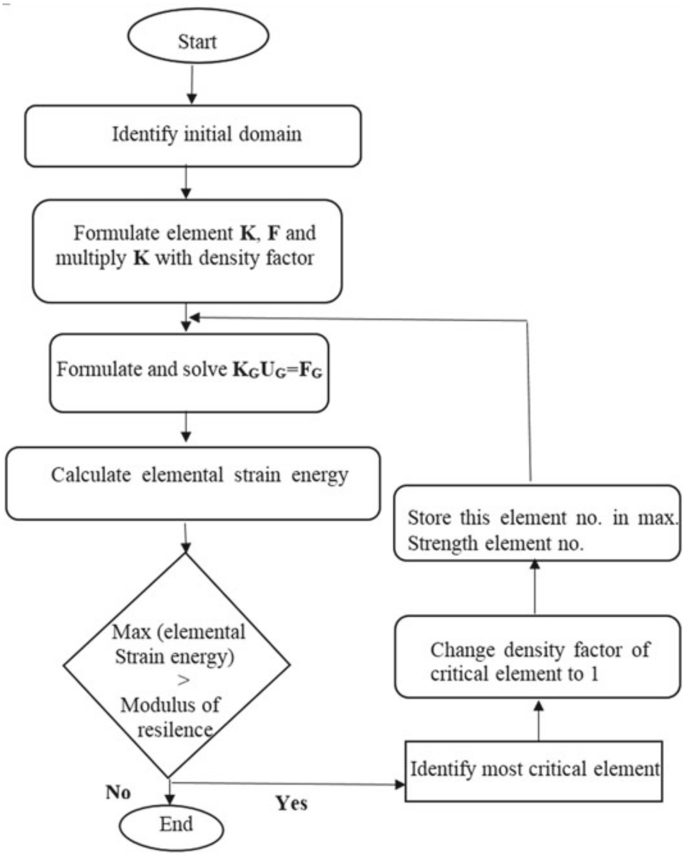


Fig. 1 Flowchart of the proposed algorithm

4.1 Case Study 1: Effect of Dimensions of the Initial Domain on the Final Optimized Result

Consider a simply supported beam of length 5 m and height 0.4 m with a load of 100 KN applied at the bottom of the mid-section and self-weight of the beam is not considered as shown in Fig. 2. The FEM formulation is based on 2D plane stress elements.

The structure is optimized using the proposed method and the final solution is consisting of 516 elements shown in Fig. 3. In each iteration, one element is added, and so, the total number of iterations is equal to the number of elements in the final solution. Initially, strain energy dropped significantly with the addition of the elements until connectivity between the load and the support is established. While establishing connectivity, the strain energy is almost constant. Again, it dropped while forming new connectivity. The strain energy plot with respect to the iteration

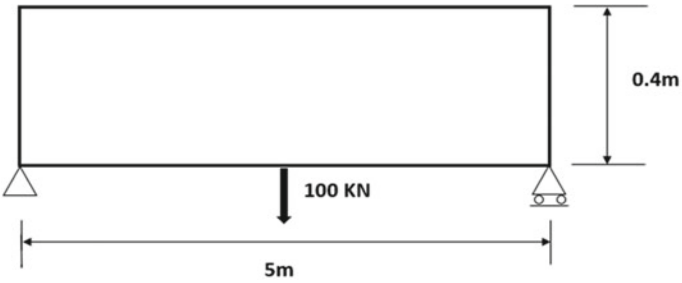


Fig. 2 Simply supported beam with applied load

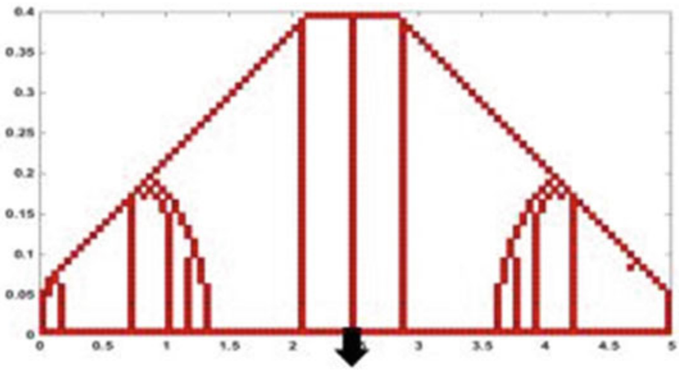


Fig. 3 Topology optimization of the simply supported beam

number is shown in Fig. 4. The percentage of final material with density factor 1 is 10.32 with respect to the initial domain.

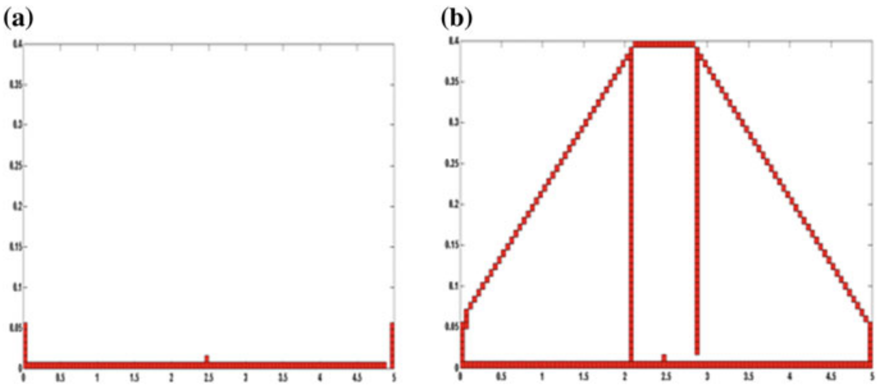


Fig. 4 **a** Iterative result at 109th iteration and **b** result at 300th iteration

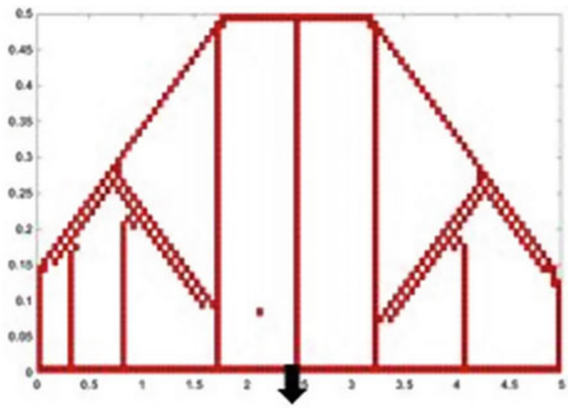


Fig. 5 Optimized result for different dimension (height = 0.5 m)

The representative intermediate results at iteration number 109 and at 300 where strain energy dropped considerably are shown in Fig. 4.

The dimensions of the beam are changed by increasing its height 0.5 m and the corresponding result is shown in Fig. 5. The percentage of final material is 10.4 with respect to the initial domain. This result shows that the optimal results will depend upon the dimensions of the initial domain.

4.2 Case Study 2: Effect of the Rate of Material Addition on the Optimization Result

In this case study, the effect of the rate of the amount of material addition on the final solution is studied. For this case, a simply supported beam with a length of 4 m and a height of 3 m is considered. A load of 50 KN is applied at the bottom of the middle section as shown in Fig. 6.

Rate of material addition has effect on final topology because if material added per iteration is more than it may cause inclusion of some elements which may be having

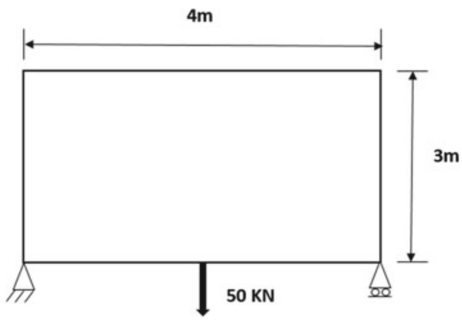


Fig. 6 Initial domain of simply supported beam

less strain energy in other subsequent iterations, therefore, optimal topology may not be captured but it reduces computational time with less number of elements added per iteration topology development is more detailed with more computational effort. Optimal solution with the addition of 15 elements at a time is having 912 elements (18.24% of the initial domain) in the final solution as shown in Fig. 7, whereas with the addition of 5 elements per iteration, the solution is having 330 elements as shown in Fig. 8. The solution resulted in the addition of a single element per iteration (66% of the initial domain) is shown in Fig. 9. So, for more refined final topology, the low rate of material addition is better.

This effect is also studied with another example of a cantilever beam of 1.5 m length and 3 m height. It is loaded at the middle with 100 KN as shown in Fig. 10. Change in rate of addition gave the same shape structure but the number of elements in the final optimum solution is different as shown in Fig. 11. The optimal structure for the addition of single element per iteration is having 105 elements (2.5% of the

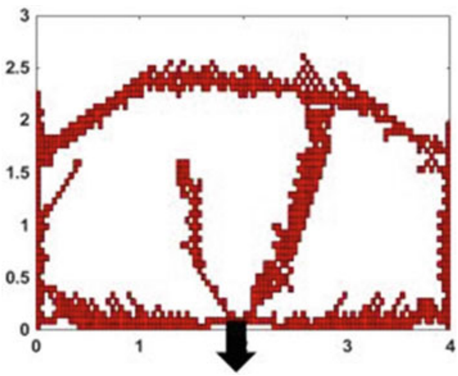


Fig. 7 Optimized result for material addition rate of 15

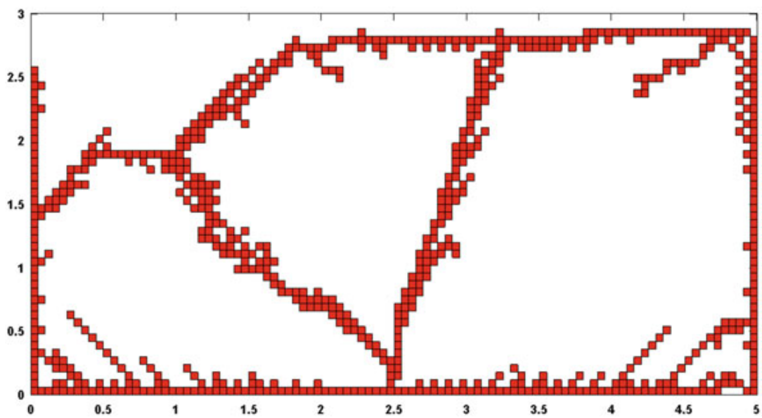


Fig. 8 Optimized result for material addition rate of 5

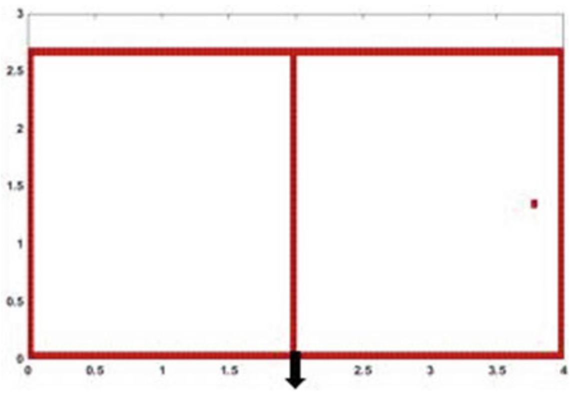


Fig. 9 Optimized result for material addition rate of 1

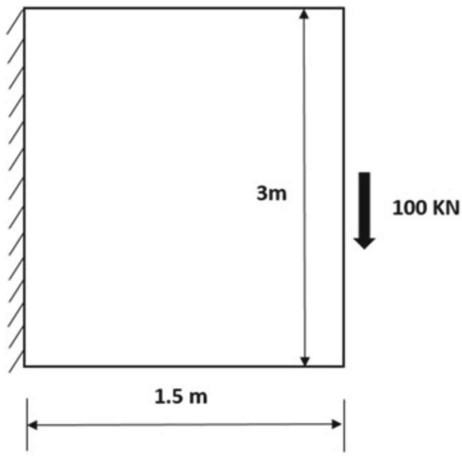


Fig. 10 Initial domain for a cantilever beam

initial domain) and the optimal structure for the addition of 2 elements per iteration is having 156 elements (3.12% of the initial domain).

4.3 *Case Study 3: Effect of Magnitude of Applied Load on the Optimized Result*

Generally, if the magnitude of the load is changed, size optimization is carried out on the optimized topology so that structure can be strengthened at critical locations. Here, in our study, the effect of the magnitude of the load on optimized structure is shown without considering the size optimization. For this case, a cantilever beam of size is considered as shown in Fig. 12. The optimization process produces a solution with few members so the optimum result is truss-like structure when the beam is

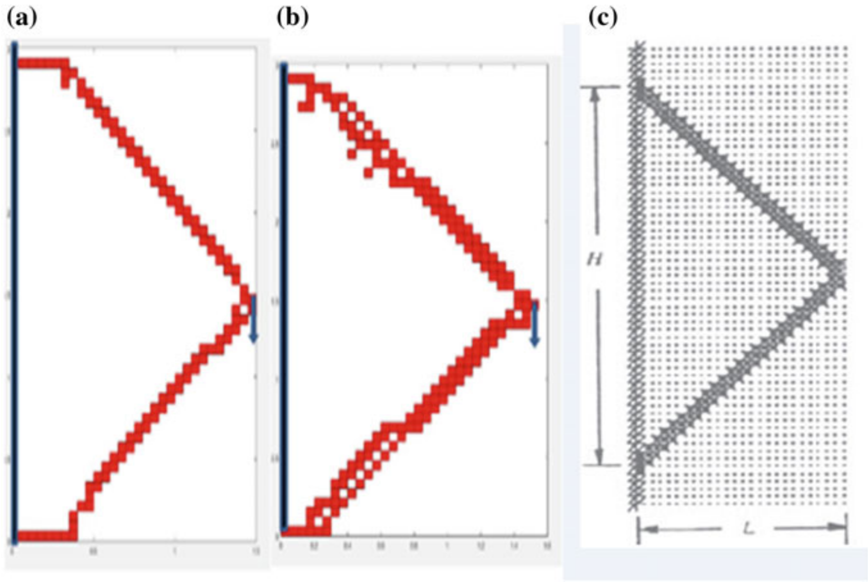
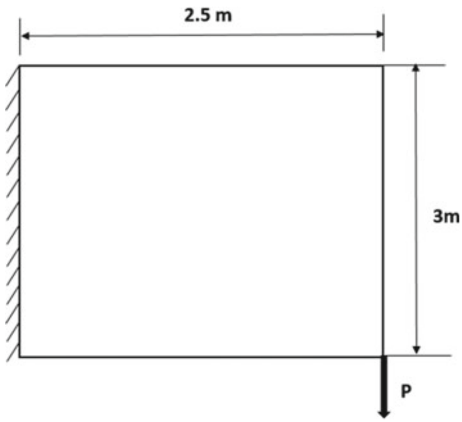


Fig. 11 **a** Optimal shape for 1 element addition, **b** optimal shape for 2 element addition, and **c** Optimum shape from literature (Xie and Steven [7])

Fig. 12 Initial domain for cantilever beam



subjected to smaller loads since for smaller loads, few members are sufficient. The optimized results for the cantilever beam with less load (150 KN) and a high load (1500 KN) are shown in Fig. 13.

Effect of increasing the load on the optimum solution causes the addition of more members in optimum topology. The percentage of material with 150 KN load is 3.8 and that with 150 KN is 7.2.

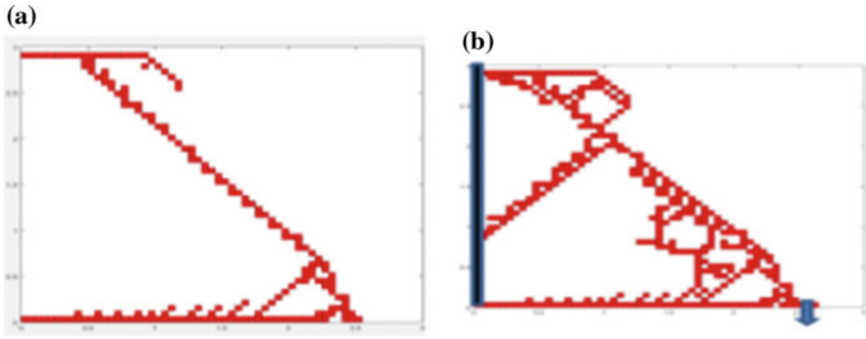


Fig. 13 **a** Solution for 150 KN and **b** solution for 1500 KN

4.4 Case Study 4: Effect of Multiple Loads

Consider a simply supported beam of length 5 m with a load of 150 KN as shown in Fig. 14. Two more loads of the same magnitude are added at 1.25 and 3.75 m as

Fig. 14 Simply supported beam with a single load

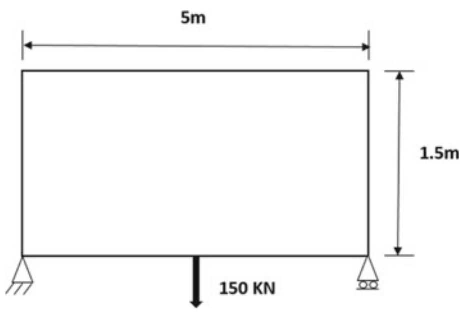
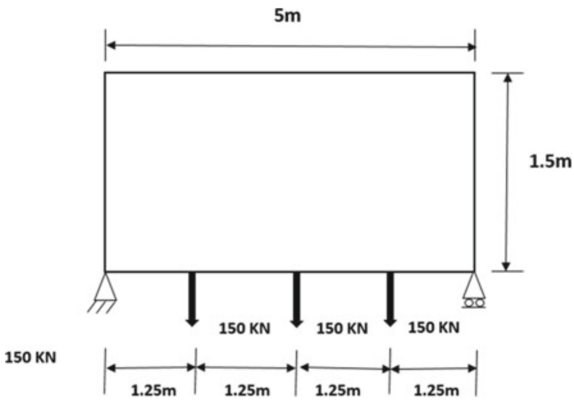


Fig. 15 Simply supported beam with multiple loads



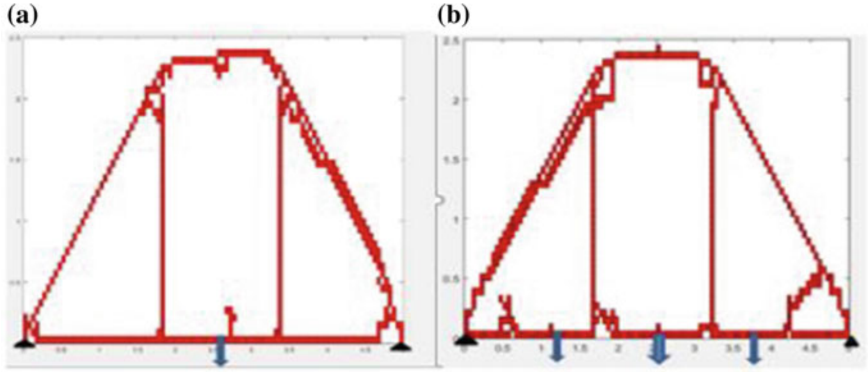


Fig. 16 **a** Simply supported beam with a single load and **b** simply supported beam with multiple loads

shown in Fig. 15. The optimum solution for single and multiple loads is shown in Fig. 16.

The multiple loads generate extra members between the point of application of load and support to give strength to structure. The percentage of final material with a single load is 7 and that of multiple loaded beams is 7.8.

5 Conclusion

In this paper, a modified evolutionary algorithm which is heuristic is proposed for topology optimization with strain energy distribution as a criterion. Optimum structures obtained using this method are close to the existing optimum solutions [8]. Proposed algorithm strengthens highly strained elements through increasing their density factor. This is a material addition process. Results depend on the chosen failure criteria. The algorithm ensures the optimum solution to be safe as per maximum strain energy criteria. From the case studies, it is observed that, if loads of a structure changes, the corresponding topology also changes. The multiple loaded beams have more intermediate members to bear extra loads than a single load case. The main focus of the work is to capture the topology well, and the results can be further improved so that the structures are manufacturable using the filter techniques [24]. The present work can be applied to other 2D structural domains and can be extended to 3D domains.

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