A Micromechanical Study of Fibre-Reinforced Composites with Uncertainty Quantification and Statically Equivalent Random Fibre Distribution

S. Koley, P. M. Mohite and C. S. Upadhyay

Abstract The effects of manufacturing defects creates uncertainty on the mechanical response of unidirectional fibre-reinforced composites. These are studied through modelling of three-dimensional Representative Volume Element (RVE). In this study, an algorithm is developed to generate the microstructure of unidirectional fibre reinforced composite with both regular and random fibre distribution and then analyzed using mathematical theory of homogenization to estimate the homogenized or effective material properties. Here, the RVEs are modelled with random fibre distribution but increasing the number of fibres gradually and also randomly varying their positions. This variation in the randomness of the fibre distribution affects the effective properties. Also, the effect of different loading conditions is investigated. The significance of this structural distribution of heterogeneities on the overall effective behaviour is discussed for random structures and uncertainties that occur in composite materials. The variations in the predicted elastic properties for the given volume fraction of the above mentioned scenarios are compared with the experimental values and good agreement is achieved. There is a significant percentage change in transverse shear moduli, G_{23} and ν_{23}, which are 21.87 and 35.20% with respect to the experimental results.

Keywords Composite materials · Microstructure · Randomness · Effective properties

1 Introduction

Composite materials have become an advisable choice in aerospace and other industries due to their high strength to weight ratio and high modulus [1]. In fibre reinforced composites, fibres are the main load-carrying members. Therefore, fibre

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volume fraction and fibre distribution morphology have a crucial influence on the strength and stiffness properties of composite material. To study these effects, many researchers addressed micromechanics of composite material. The issue concerning the micromechanical study is the generation of RVE with the desired dimensions and spatial distribution of reinforcements in RVE, which highly depends upon the manufacturing process. Many methods have been developed for the generation of RVE model with a random distribution of fibres using Poisson point distribution [2]. In this microscopic scale approach, the constituents are employed in conjunction with homogenization to predict the composite behaviour. Here, the average mechanical characteristics of a lamina have to be estimated from the known characteristics of the fibre and matrix materials taking into account the fibre volume fraction and fibre packing arrangement. The current study in micromechanics is to estimate the effective elastic properties of the RVE of the material with an equivalent random distribution of fibres [3, 4]. Effective elastic properties of composite material depend on the shape, properties and spatial distribution of the fibre. Among the various uncertainties present, the following uncertainties have been considered for the analysis: (a) effects of volume fraction (b) effect of the randomness in the fibre arrangements (c) effect of the uncertainty in the geometry of the fibre cross section.

In the current study, a micromechanical study [5] is proposed to calculate mechanical properties of composite material with regular fibre array and random fibre arrangement in a unit cell model [6]. In this case, an image of the cross section of a certain unidirectional ply is taken and transformed into a computer recognizable format by using binary image processing technique. From the binary image, RVEs are generated with single fibre and random fibre distribution by defining different scatter conditions and also to incorporate randomness in the RVE. The evaluation of the effect of fibre arrangements on the mechanical properties should be both qualitative and quantitative [7]. To achieve this goal, a comprehensive study is carried out with RVEs having 1, 12, 20 and 50 fibres. The overall properties are calculated as an average over the respective volumes of the constituents. Mathematical theory of homogenization [8] is used to obtain a suitable constitutive model at the macroscopic level. In the next study, we considered the RVEs with fibres randomly distributed in a periodic unit cell but with different fibre cross sections, i.e. circular, distorted and elliptical cross sections. Then, we studied how these different fibre cross sections affect the mechanical properties.

2 Characterization of Uncertainties in Composites

Due to the complex manufacturing process, the composite renders randomness in the material parameters. The instability in properties such as elastic modulus, fibre distribution and volume fraction measurements result in variability in the response of composite materials. Uncertainty can be addressed by means of material, geometric and structural considerations [2].

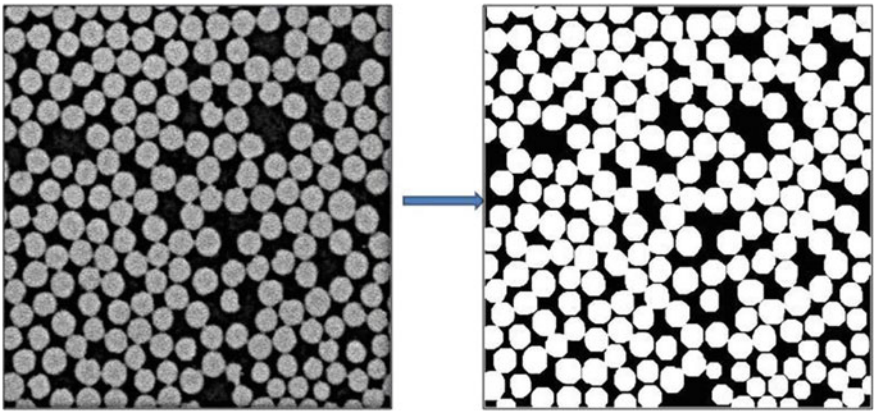


Fig. 1 Greyscale image to binary image

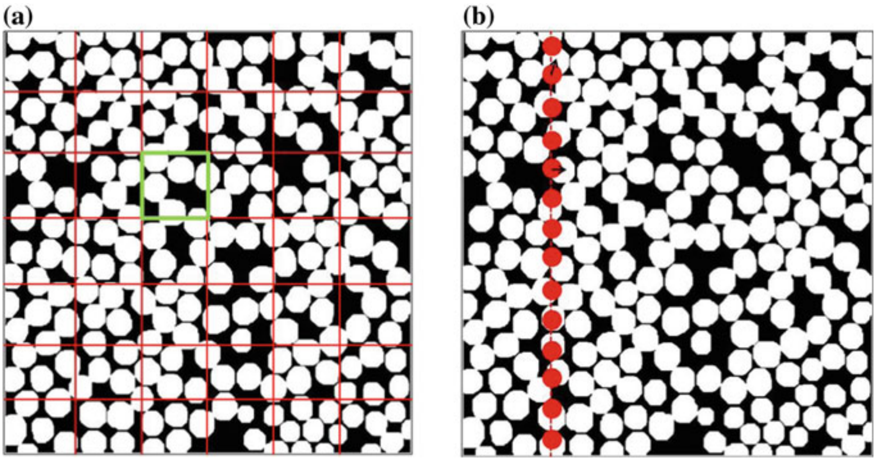


Fig. 2 Random fibre distribution. **a** Moving window technique used to calculate RVE, **b** Deviation of fibre from their ideal distribution

Initially, scatter has been estimated in the fibre distribution from digital image analysis. Using these data, information about the location of centre of the fibres, distribution of fibre radius and distance between neighbouring fibres are obtained. Then, the obtained statistical parameters are utilized to construct a statistically equivalent RVE based on certain numerical algorithms [9]. Figure 1 is a micrograph image of composite microstructure of size 706 × 678 pixels (Fig. 2).

From the image, for an ideal distribution of fibre, four random fibres are chosen along the breadth. Then, ideal positions are calculated from the deviation of centre of fibres in the original micrograph. Figure 3 also shows centres of ideal fibre distribution. After extracting the actual coordinates of the fibre centres, we estimate the

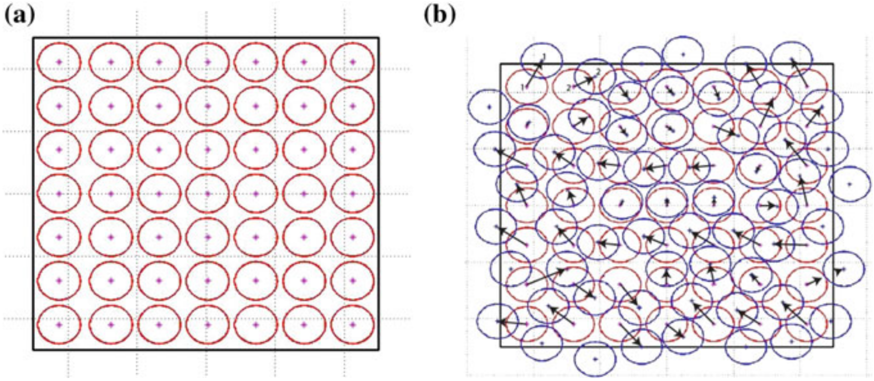


Fig. 3 RVE with regular and random fibre distribution. **a** Regular fibre arrangements in a square domain, **b** Modelling of RVE with random fibre distribution

ideal coordinates and then, the deviation from the real to the ideal coordinates are calculated. By doing this process, scatter has been estimated and with reference to this, an RVE is modelled with the random distribution of fibres.

3 Generation of RVE

In the estimation of effective properties of composite materials, the generation of RVE plays a vital role. A new algorithm has been proposed to generate the RVE achieving fibre volume fraction nearly 60% and the fibres are evenly distributed throughout the RVE. According to Hill [10] and Kanit et al. [11], representative volume should have two main properties: Its structure is "entirely typical" for the composite and it should contain a "sufficient number" of microstructure elements so that boundary conditions at the surface do not affect its effective properties.

Considering the above characteristics of generating the RVE, it has been generated with a random arrangement of fibres with reference to the scatter from the above micrograph. The RVEs with ideal/regular arrangements of fibres are shown in Fig. 3 by maintaining the fibre volume fraction of 0.6.

Figure 3 shows the scatter of the fibres from the original position to the random position but maintaining the minimum distance so that the fibres do not intersect each other. In Fig. 3, the fibres are displaced randomly both in x and y directions and maintaining the periodicity at the edges. The RVE possesses very similar microstructural features as in the original micrograph, such as fibre aggregation zones and also maintaining the geometric periodicity, i.e. periodicity across faces, edges and corners.

4 Mathematical Theory of Homogenization

This theory establishes mathematical relations between micro and macro-fields [8], using a multi-scale perturbation method, the effective properties emerge as a consequence of these relations. The local displacement field in the cell is given as

$$u(x; y) = u_0(x) + \varepsilon u_1(y) \tag{1}$$

where x is the actual coordinate, y is the scaled unit cell coordinate, $u_0(x)$ is the macro response and $u_1(y)$ is the periodic micro correction, ε denotes the ration of the RVE size to the global structural dimension. $u_1(y)$ can be obtained from each of the six fundamental macro-strains e_x^{ij}, by solving the periodic cell problem with applied unit macro strain.

$$-\frac{\partial}{\partial y_j}(C^{\varepsilon}_{ijkl} e^{y}_{kl}(\chi^{rs})) = \frac{\partial}{\partial y_j}\left(C_{ijrs}(y)\right) \tag{2}$$

where χ^{rs} is y periodic. From the periodic solution, the total strain $e_{ij}(u) \approx e^x_{ij}(u_0) + e^y_{ij}(u_1) \approx (1 + M_{ijkl})e^x_{kl}(u_0)$, where M_{ijkl} are the pointwise influence functions and they depend on v_f at the level of the cell.

5 Results and Discussion

In the present study, an in-house finite element code is developed to determine the homogenized properties of composite materials using the mathematical theory of homogenization. Polymeric matrix such as 3501-6 epoxy and AS4 carbon fibre material is considered in this study. These properties are given in Soden et al. [12]. The focus of the analysis is to study both the longitudinal, transverse and shear behaviour of the material at the microscale.

5.1 Prediction of the Elastic Constants of RVE with Single Fibre (Circular Cross Section)

RVE is modelled with single fibre keeping the fibre volume fraction 0.6. The RVE model is meshed with three-dimensional four-node tetrahedral elements. Effective properties for this RVE are estimated and the results are tabulated in Table 1.

Mechanical properties for the composite laminae were considered from experiment studies carried out in Soden et al. [12].

Predicted effective elastic constants are given in Table 1, which are close to the experimental results (see Table 2). The typical percentage differences of the effective

Table 1 Effective properties of RVE model with single fibre

V_f	E_1 (GPa)	E_2 (GPa)	E_3 (GPa)	G_{12} (GPa)	G_{13} (GPa)	G_{23} (GPa)	ν_{12}	ν_{13}	ν_{23}
0.60	139.58	9.55	9.55	4.70	4.70	3.06	0.25	0.25	0.26

Table 2 Effective properties of T-300/3501-6 epoxy composite (experimental results) [12]

V_f	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	G_{23} (GPa)	ν_{12}	ν_{23}
0.60	138	11.00	5.50	3.93	0.28	0.40

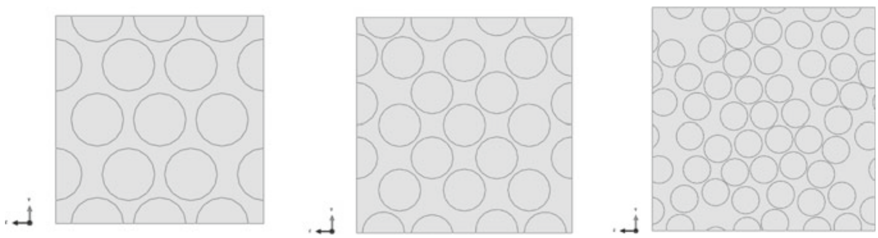
elastic constants are –1.14% for E_1, 13.21% for E_2 and E_3, 14.45% for G_{12} and G_{13} and 21.87% for G_{23} with respect to the experimental results.

5.2 Prediction of the Elastic Constants of RVE with 12, 20 and 50 Fibres

RVEs are generated with 12, 20 and 50 fibres maintaining the volume fraction of 0.6. Here, the fibres are randomly distributed as shown in Fig. 4. Three cases are considered: Case 1: RVE with 12 fibres, Case 2: RVE with 20 fibres and Case 3: RVE with 50 fibres. For Case 1, three RVE models are generated, for Case 2, four RVE models are generated and for Case 3, eight RVE models are generated. In Fig. 4, sample RVEs of each case are shown but the results are given for all the models.

Observations are made for the prediction of the effective elastic constants of the RVEs with randomly distributed fibres (Table 3).

Case 1: Three RVE models are considered, i.e. RVE 1, RVE 2 and RVE 3. Axial modulus, E_1 predicted for all the three RVEs with different fibre arrangements has percentage difference less than 1% with respect to the effective properties of RVE



(a) Case 1 (b) Case 2 (c) Case 3

Fig. 4 Micrograph of RVEs with 12, 20 and 50 fibres

Table 3 Effective properties of RVE model with 12 fibres

Model	E_1 (GPa)	E_2 (GPa)	E_3 (GPa)	G_{12} (GPa)	G_{13} (GPa)	G_{23} (GPa)	ν_{12}	ν_{13}	ν_{23}
RVE 1	138.19	8.77	8.88	4.51	4.81	3.57	0.25	0.25	0.26
RVE 2	138.70	8.96	8.88	4.91	4.67	3.52	0.25	0.25	0.31
RVE 3	138.54	9.06	9.01	5.06	4.86	3.50	0.25	0.25	0.30
Average	138.48	8.93	8.93	4.83	4.78	3.53	0.25	0.25	0.29
SD	0.26	0.15	0.07	0.28	0.10	0.03	0.00	0.00	0.03
% error	0.19	1.68	0.78	5.80	2.09	0.85	0.00	0.00	10.34

Table 4 Percentage change in the effective properties of RVEs with 12 fibres with respect to RVE model with single fibre

Model	E_1	E_2	E_3	G_{12}	G_{13}	G_{23}	ν_{12}	ν_{13}	ν_{23}
RVE 1	0.98	8.13	6.95	4.01	–2.28	–16.36	–1.31	0.67	–19.91
RVE 2	0.62	6.11	6.88	–4.44	0.72	–14.74	0.52	–0.71	–18.71
RVE 3	0.74	5.06	5.59	–7.71	–3.46	–14.24	0.44	–0.40	–15.51

Table 5 Effective properties of RVE model with 20 fibre

Model	E_1 (GPa)	E_2 (GPa)	E_3 (GPa)	G_{12} (GPa)	G_{13} (GPa)	G_{23} (GPa)	ν_{12}	ν_{13}	ν_{23}
RVE 1	139.23	8.83	8.84	5.02	5.10	3.69	0.25	0.25	0.32
RVE 2	139.42	9.13	9.21	4.97	5.22	3.52	0.25	0.25	0.29
RVE 3	139.45	9.08	9.12	4.94	5.04	3.54	0.25	0.25	0.29
RVE 4	139.45	9.11	9.23	4.90	5.20	3.51	0.25	0.25	0.29
Average	139.39	9.04	9.11	4.96	5.17	3.57	0.25	0.25	0.30
SD	0.11	0.14	0.18	0.05	0.11	0.09	0.00	0.00	0.01
% error	0.08	1.55	1.98	1.01	2.13	2.52	0.00	0.00	3.33

with single fibre given in Table 4. The standard deviation for all the properties is less than 1 but the percentage error is more for G_{12} and ν_{23}, i.e. 5.8% and 10.34%, respectively and for the other properties, it is less than 2%.

But for the transverse shear modulus, G_{23} the percentage difference is much higher compared to axial shear modulus G_{12} and G_{13} and it is about 15%.

Case 2: Four RVE models are considered and observations are made for the predicted effective elastic constants. Axial modulus, E_1 for all the four RVEs are very close to the RVE with single fibre and also with the experimental value. The standard deviation for all the properties is less than 1 (Table 5).

Axial modulus, E_1 predicted for all the four RVEs with different fibre arrangements has a percentage difference less than 1% as given in Table 6. But for RVE 1,

Table 6 Percentage change in the effective properties of RVEs with 20 fibres with respect to RVE model with single fibre

Model	E_1	E_2	E_3	G_{12}	G_{13}	G_{23}	ν_{12}	ν_{13}	ν_{23}
RVE 1	0.25	7.49	7.32	–6.74	–8.54	–20.39	0.40	0.63	–21.57
RVE 2	0.12	4.28	3.47	–5.68	–11.07	–14.74	–0.32	0.91	–11.30
RVE 3	0.09	4.86	4.38	–5.16	–7.27	–15.49	–0.16	0.59	–13.43
RVE 4	0.09	4.47	3.29	–4.34	–12.26	–14.39	–0.63	1.19	–11.11

Table 7 Effective properties of RVE model with 50 fibre (circular cross section)

Model	E_1 (GPa)	E_2 (GPa)	E_3 (GPa)	G_{12} (GPa)	G_{13} (GPa)	G_{23} (GPa)	ν_{12}	ν_{13}	ν_{23}
RVE 1	136.19	9.20	9.28	4.93	5.20	3.53	0.25	0.25	0.28
RVE 2	136.01	9.10	9.14	5.01	5.16	3.61	0.25	0.25	0.29
RVE 3	136.04	9.20	9.24	4.24	5.05	3.57	0.25	0.25	0.28
RVE 4	136.73	9.27	9.26	5.45	5.41	3.60	0.25	0.25	0.28
RVE 5	136.64	9.25	9.28	5.41	5.44	3.59	0.25	0.25	0.28
RVE 6	136.47	9.22	9.21	5.43	5.40	3.59	0.25	0.25	0.29
RVE 7	136.44	9.21	9.16	5.36	5.29	3.60	0.25	0.25	0.29
RVE 8	136.60	9.31	9.26	5.45	5.29	3.56	0.25	0.25	0.28
Average	136.39	9.22	9.23	5.17	5.28	3.59	0.25	0.25	0.28
SD	0.28	0.06	0.05	0.43	0.14	0.02	0.00	0.00	0.01
% error	0.21	0.65	0.54	8.32	2.65	0.56	0.00	0.00	3.52

the percentage difference is more compared to the other three RVE models. This is because the number of fibres at the edges of this RVE is more.

The percentage difference for transverse shear modulus, G_{23} is much more for RVE 1, i.e 20% compared to other three RVE models. It is observed that the average values of the elastic properties of RVE models with 20 fibres is higher comparable to RVE with 12 fibres.

Case 3: Eight RVE models are considered, i.e. RVE 1 to RVE 8 and it is observed that there is a reduction in effective axial modulus E_1 compared with the RVEs with 12 and 20 fibres. The standard deviation is less than 1 for all the properties but the percentage error is also less than 1% except for G_{12}, G_{13} and ν_{23}. The percentage error for G_{12}, G_{13} and ν_{23} are 8, 2 and 3%, respectively. But if we notice the percentage change with respect to RVE with single fibre, for the shear, i.e the axial shear modulus G_{13} and transverse shear modulus G_{23} are more than 9% (Tables 7,8).

In the next study, RVEs with uncertainty in fibre cross section are considered. Here, three cases are considered, Case 1: RVE with circular cross section, Case 2: RVE with a distorted cross section and Case 3: RVE with an elliptical cross section. For all the three cases, eight RVE models are generated.

Table 8 Percentage change in the effective properties of RVEs with 50 fibres with respect to RVE model with single fibre

Model	E_1	E_2	E_3	G_{12}	G_{13}	G_{23}	ν_{12}	ν_{13}	ν_{23}
RVE 1	2.42	3.60	2.77	–4.93	–10.52	–15.34	–0.44	0.79	–9.03
RVE 2	2.56	4.66	4.22	–6.68	–9.66	–17.75	–0.16	0.52	–12.81
RVE 3	2.53	3.60	3.16	9.78	–7.46	–16.58	–0.16	0.52	–9.76
RVE 4	2.04	2.86	2.93	–16.01	–15.03	–17.46	0.52	0.39	–8.76
RVE 5	2.10	3.02	2.74	–15.10	–15.78	–17.19	0.36	0.52	–8.45
RVE 6	2.22	3.32	3.43	–15.57	–14.79	–17.03	0.44	0.28	–10.19
RVE 7	2.25	3.51	4.01	–14.03	–12.43	–17.31	0.59	–0.04	–11.11
RVE 8	2.13	2.40	2.96	–16.01	–12.49	–16.02	0.91	–0.24	–8.37

Table 9 Effective properties of RVE model with 50 fibre (distorted cross section)

Model	E_1 (GPa)	E_2 (GPa)	E_3 (GPa)	G_{12} (GPa)	G_{13} (GPa)	G_{23} (GPa)	ν_{12}	ν_{13}	ν_{23}
RVE 1	132.18	9.12	9.07	5.41	5.28	3.50	0.25	0.25	0.29
RVE 2	131.63	8.96	8.89	5.24	5.11	3.53	0.25	0.26	0.30
RVE 3	131.48	8.97	9.00	5.02	5.07	3.47	0.25	0.25	0.29
RVE 4	130.91	8.99	8.98	5.22	5.17	3.46	0.25	0.25	0.29
RVE 5	130.11	8.95	8.95	5.21	5.15	3.43	0.25	0.26	0.29
RVE 6	131.48	8.99	8.99	5.33	5.33	3.47	0.25	0.25	0.29
RVE 7	130.65	8.95	8.91	5.30	5.21	3.47	0.25	0.26	0.30
RVE 8	131.79	9.13	9.08	5.55	5.41	3.49	0.25	0.25	0.29
Average	131.28	9.01	8.99	5.29	5.22	3.48	0.25	0.25	0.29
SD	0.68	0.07	0.07	0.16	0.12	0.03	0.00	0.00	0.00
% error	0.52	0.78	0.78	3.02	2.30	0.86	0.00	0.00	0.00

RVE with circular fibre cross section has been discussed in the previous section. Now, RVE with distorted fibre cross section are considered. Observations are made for the distorted fibre cross section, where the effective properties for all the models are much less than the values compared with the RVE with single fibre. This is mainly due to reduction of fibre volume fraction (Table 9).

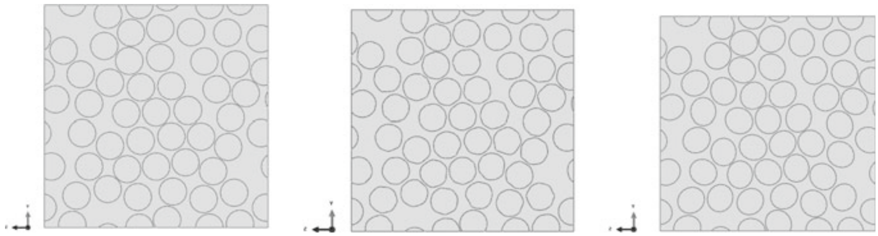
The standard deviation is less than 1 for all the properties but the percentage error is also less than 1% except for G_{12} and G_{13} (Table 10).

From these tables, it can be seen that the values of E_1, E_2 and E_3 show about 7% change. Similarly, this change for G_{12} is about 18% and for G_{13} and G_{23} is 15% when compared to that of RVE with single fibre. Further, there is a change in fibre volume fraction for each RVE as they contain fibres with distorted cross section. This distortion also affects the percentage change in the effective properties.

For the last case, RVEs are modelled with elliptical fibre cross section with + 0.5% of the radius of the circular fibre cross section as major axis and –0.5%

Table 10 Percentage change in the effective properties of RVEs with 50 fibres with respect to RVE model with single fibre

Model	E_1	E_2	E_3	G_{12}	G_{13}	G_{23}	ν_{12}	ν_{13}	ν_{23}
RVE 1	5.29	4.44	4.96	–15.06	–12.37	–14.09	–0.08	–0.83	–11.19
RVE 2	5.69	6.14	6.79	–11.47	–8.77	–15.27	–0.24	–1.07	–15.93
RVE 3	5.79	5.97	5.70	–6.75	–7.89	–13.38	–1.03	–0.67	–13.12
RVE 4	6.21	5.79	5.91	–10.99	–9.94	–13.03	–0.87	–0.99	–13.19
RVE 5	6.78	6.18	6.21	–10.85	–9.55	–12.06	–0.99	–1.19	–13.54
RVE 6	5.79	5.76	5.82	–13.44	–13.29	–13.26	–0.87	–0.87	–13.62
RVE 7	6.39	6.23	6.65	–12.77	–10.84	–13.33	–0.67	–1.35	–15.01
RVE 8	5.58	4.35	4.83	–18.08	–15.12	–13.86	0.12	–0.91	–11.19



(a) Case 1 (b) Case 2 (c) Case 3

Fig. 5 Micrograph of RVE with different fibre cross sections

of the circular fibre cross section as minor axis as shown in Fig. 5. The fibres are distributed throughout the window with random orientations, i.e. the major axis of the elliptical fibre cross section is placed randomly in the *y*–*z* plane.

The effective properties are shown in Table 11. The effective properties for E_1, E_2 and E_3 are less than the experimental values but the shear values are almost the same or more than the experimental values. The standard deviation for all the properties are less than 1 but the percentage change is greater than 1 for G_{12} and G_{13} (Table 12).

Observations are obtained for the fibres with elliptical fibre cross section, the percentage change for effective axial modulus E_1 is about 3% whereas for E_2 and E_3 it is about 5%. The percentage change for shear moduli is more than 18%.

6 Conclusion

In the present study, a statistical representation of unidirectional fibre composite with random fibre distribution at microscale has been developed. A numerical tool developed using MATLAB generates the RVE for a volume fraction of 60%. Geometric periodicity is implemented while developing the RVE to ensure the continuity of the

Table 11 Effective properties of RVE model with 50 fibres (elliptical cross section)

Model	E_1 (GPa)	E_2 (GPa)	E_3 (GPa)	G_{12} (GPa)	G_{13} (GPa)	G_{23} (GPa)	ν_{12}	ν_{13}	ν_{23}
RVE 1	135.94	9.26	9.19	5.52	5.31	3.57	0.25	0.25	0.28
RVE 2	135.94	9.15	9.09	5.48	5.29	3.62	0.25	0.25	0.29
RVE 3	136.31	9.21	9.23	5.27	5.23	3.58	0.25	0.25	0.28
RVE 4	136.39	9.26	9.22	5.50	5.29	3.59	0.25	0.25	0.28
RVE 5	136.48	9.27	9.27	5.41	5.34	3.58	0.25	0.25	0.28
RVE 6	135.91	9.16	9.19	5.27	5.42	3.58	0.25	0.25	0.29
RVE 7	135.93	9.18	9.28	5.12	5.58	3.55	0.25	0.25	0.28
RVE 8	135.93	9.26	9.25	5.35	5.31	3.54	0.25	0.25	0.28
Average	136.11	9.22	9.22	5.37	5.35	3.58	0.25	0.25	0.28
SD	0.24	0.05	0.06	0.14	0.11	0.03	0.00	0.00	0.00
% error	0.18	0.54	0.65	2.61	2.06	0.84	0.00	0.00	0.00

Table 12 Percentage change in the effective properties of RVEs with 50 fibres with respect to RVE model with single fibre

Model	E_1	E_2	E_3	G_{12}	G_{13}	G_{23}	ν_{12}	ν_{13}	ν_{23}
RVE 1	2.60	2.91	3.69	–17.45	–12.88	–16.41	0.83	–0.48	–9.72
RVE 2	2.60	4.13	4.74	–16.59	–12.62	–18.13	0.63	–0.20	–12.92
RVE 3	2.33	3.46	3.26	–12.13	–11.17	–16.90	0.28	0.28	–9.57
RVE 4	2.28	2.89	3.41	–17.04	–12.51	–17.25	0.75	–0.04	–9.57
RVE 5	2.21	2.81	2.87	–15.11	–13.60	–16.77	0.52	0.28	–8.64
RVE 6	2.62	4.02	3.63	–12.15	–15.27	–16.86	–0.12	0.48	–10.73
RVE 7	2.61	3.79	2.78	–8.87	–18.69	–15.69	–0.59	0.99	–8.68
RVE 8	2.61	2.97	3.07	–13.72	–12.84	–15.49	0.36	–0.08	–8.60

fibres across the neighbouring RVEs. Mathematical theory of homogenization has been materialized for the prediction of effective stiffness. Eight RVEs are modelled for circular, distorted and elliptical fibre cross sections with random fibre distribution and studied how these uncertainties affect the effective properties.

The key conclusions that can be made from this study are listed as

1. For RVE with 12 fibres, the percentage error is more for G_{12} and ν_{23}, i.e. 5.80% and 10.34%, respectively and for the other properties, it is less than 2%.
2. In particular, the percentage change of predicated transverse shear moduli G_{23} from UD composites with fibre distributed at random over transverse cross section has higher values comparable to the experimental data. For the RVE with single fibre, the percentage change is 21.87% with respect to experimental result and for the other RVEs with more number of fibres, not only the transverse shear moduli

has higher value but there is also increase in percentage change values for axial shear between 6–17%.

3. The percentage change with respect to single fibre RVE, E_1 is about 2% for circular and elliptical fibre cross section whereas for distorted fibre cross section, it is about (5–6)%. The uncertainty of fibre cross section is studied and the effective properties are estimated. It is observed that E_1 is about 136 GPa for the models with circular fibre cross section but it is reduced to (130–131) GPa for distorted fibre cross section which is mainly due to reduction in fibre volume fraction.

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