# Control of End-Effector of a Multi-link Robot with Joint and Link Flexibility

**E. Madhusudan Raju, L. Siva Rama Krishna and Mohamed Abbas**

**Abstract** Flexibility in manipulators/robots is due to both joint and link flexibility that makes up the system. Flexible robots are preferred over conventional rigid robots in applications like invasive surgeries, space applications, and industries due to their prompt response, low energy requirement, faster operational speeds, and low weight to power ratio. Due to inherent flexibility, accurate positioning of end-effector in required path is difficult. Moreover flexibility of link makes it an infinite degree freedom system and mathematics is very involved. To simplify the problem and get reasonable results, flexible links are modeled based on Euler–Bernoulli beam theory and Assumed mode method is implemented. Joint flexibility is because of small clearances that are inherently present in the joint, because of both manufacturing and assembling constraints, these clearances cause sudden impacts between the joining parts (journal and bearing) resulting in impact force generation as the joints are manipulated. Resulting impact (hertzian contact) forces increase the overall input torque required to manipulate the end-effector according to our wish. This paper's objective is to build a dynamic model of a two-link RR type planar manipulator with link and joint flexibility, and determine the maximum error of tip position between a robot with/without flexibility, as the end effect or travels in required vertical path with payload. Further, apply orthodox control strategies (PD, PI, and PID) to reduce the error. The end-effector carries a payload equals its links mass. Using MSC Adams and MATLAB softwares, a co-simulation approach is developed. Both the controllers (PI, PID) radically reduced error through several iterations, PID control strategy

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G. S. V. L. Narasimham et al. (eds.), *Recent Trends in Mechanical Engineering*,
Lecture Notes in Mechanical Engineering,
https://doi.org/10.1007/978-981-15-1124-0_53

achieved better results than PI controller and by both approaches, more than 60% of the positional error is reduced.

**Keywords** Link flexibility · Joint clearance · Assumed mode method · Euler–Bernoulli's beam theory · PI and PID control techniques

## 1 Introduction

Robots/serial manipulators are widely used in repetitive, tiresome and hazardous jobs. These robotic manipulators are designed, built and fabricated with maximum stiffness to limit end-effector vibration for better positional accuracy, resulting in bulky industrial manipulators.

The merits of using a flexible manipulator are high-speed manipulation, low requirement of power and less material for fabrication resulting in a smaller actuator. By reducing manipulator weight, flexibility of the robot increases and the mathematical modeling becomes involved and tedious.

Flexible manipulators, core difficulty is limiting vibrations due to flexibility, it is resolved by including the effects of flexibility and applying various control strategies to accomplish minimum steady-state error and desired transient response. Space research provided the required impetus to study control of flexible manipulators, as they have lightweight reducing the launch cost and responding to issues in weight and space. The joint flexibility results in reduced rigidity of drive, shaft, and gear teeth deformation. Link flexibility results in transverse direction deformation owing to shearing and rotary inertial effects.

A one link flexible robotic system given in Fig. 1 would have rotated by an angle $\theta(t)$ when actuated, if it was a rigid link robot, but owing to its structural flexibility, it oscillates about a mean position based on the damping of the system, this is seen as a deviation of $y\ (x, t)$ from undeformed position from Fig. 1. Therefore, $y\ (x, t)$ motion depends on both joint angle $\theta$ and deformations of the link and joint.

The link flexibility is solved using Assumed mode method. This method is suitable for Euler–Bernoulli links, i.e. (maximum deformation is less than one-tenth of the

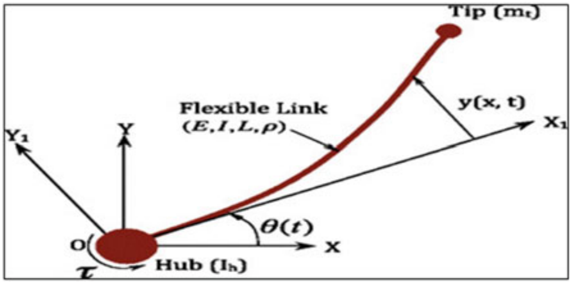

**Fig. 1** Flexible beam vibration

link length). Based on this an infinite degree freedom system is reduced to a problem of few prevailing mode shapes solved by Assumed mode method.

The lateral deformation '*y*' of the link at a section at a specified time provides the length of position vector. Further, a payload 'MP' at the tip of the link is also attached. The governing equation to represent the vibration of link can be written as given in Eq. (1).

$$E\,I\frac{\partial^4 y(x,t)}{\partial x^4} + \rho\,A\frac{\partial^2 y(x,t)}{\partial t^2} = 0 \tag{1}$$

Boundary Conditions: As Eq. 1 involves a second-order derivative in time and a fourth-order derivative in '*x*', two initial conditions and four boundary conditions are required for unique solution of $y\ (x,\ t)$ as given by Eqs. 2–7.

$$y(x,\ t = 0) = y_i \tag{2}$$

$$\frac{\partial y(x,t=0)}{\partial t} = \left.\frac{\partial y(x,t)}{\partial t}\right|_{t=0} = y_i^{'} \tag{3}$$

$$y(x,t)|_{x=0} = 0 \tag{4}$$

$$\left.\frac{\partial y(x,t)}{\partial x}\right|_{x=0} = 0 \tag{5}$$

$$EI\left.\frac{\partial^2 y(x,t)}{\partial x^2}\right|_{x=L} = -J_L\frac{\partial^2}{\partial t^2}\left(\left.\frac{\partial y(x,t)}{\partial x}\right|_{x=L}\right) \tag{6}$$

$$EI\left.\frac{\partial^3 y(x,t)}{\partial x^3}\right|_{x=L} = -M_L\frac{\partial^2}{\partial t^2}\left(y(x,t)|_{x=L}\right) \tag{7}$$

The result of Eq. 1 is given as follows:

$$y(x,t) = \sum_{j=1}^{n} C_{1,\ j}\ \sin(\omega_j t)\left\{\left(\cos(\beta_j x) - \cosh(\beta_j x)\right) - \alpha\left(\sin(\beta_j x) - \sinh(\beta_j x)\right)\right\} \tag{8}$$

where

$$\alpha = \frac{-\beta_j^3\cos(\beta_j L) - \beta_j^3\cosh(\beta_j L) + \frac{M_L}{\rho}\beta_j^4\sin(\beta_j L) - \frac{M_L}{\rho}\beta_j^4\sinh(\beta_j L)}{-\beta_j^3\sin(\beta_j L) - \beta_j^3\sinh(\beta_j L) - \frac{M_L}{\rho}\beta_j^4\cos(\beta_j L) + \frac{M_L}{\rho}\beta_j^4\cosh(\beta_j L)} \tag{9}$$

The calculated $y\ (x,\ t)$ is for only single-link and calculations of velocities and accelerations of multi-link manipulators built on Eq. 9 are difficult. Therefore, alternative solutions were investigated and zeroed on MSC Adams software.

## 2 Literature Review

M. Moallem [1] presented a co-simulation method using MscADAMS and MATLAB for solving complex mechanical systems and obtained satisfactory results proving that co-simulation can be used to solve complex dynamic systems. R.M. Mahamood et al. [2] in their work provided a method to solve kinematic and dynamic problems of Stäubli TX40 robot using co-simulation method. D. Zhang and S. Zhou[3] studied Co-simulation using ADAMS and MATLAB for Active Vibration Control of Flexible beam with Piezoelectric Stack Actuator. The virtual prototype of flexible beam with piezoelectric actuator is created in MSC ADAMS. The controller based on FXLMS algorithm is established in MATLAB. A.M. Abdullahi et al. [4] studied the vibration effects on input tracking control of Flexible manipulator using LQR with Non-collocated PID Controller. Jerzy et al. [5] proposed adaptive control method for single-link flexible manipulator.

## 3 Problem Statement

This paper's objectives are to determine the effects of flexibility on tip position variation of a two-link RR type planar robotic arm as the end-effector (tip) is displaced in a specified vertical motion with a specified payload.

Control methods like (PID and PI) were used to minimize the positional error between the Rigid and Flexible-rigid links robots. The key objective is to develop a control strategy so that the ensuing robot system possesses accuracy of a rigid system and swiftness of a flexible system. Depending on the results obtained, a good control strategy will be implemented for future usage. The dimensions and properties reflected in this paper are given in Table 1.

**Table 1** Link parameters

| | Length (mm) | Width (mm) | Depth (mm) | Mass (kg) | Density (kg/mm$^3$) |
|---|---|---|---|---|---|
| Link 1 | 300 | 40 | 20 | 2 | $7.8 \times 10^{-6}$ |
| Link 2 | 400 | 40 | 20 | 2.6 | $7.8 \times 10^{-6}$ |
| End base | 80 | 20 | 20 | 0.5 | $7.8 \times 10^{-6}$ |
| Gripper 1 | 50 | 10 | 20 | 0.045 | $7.8 \times 10^{-6}$ |
| Gripper 2 | 50 | 10 | 20 | 0.045 | $7.8 \times 10^{-6}$ |
| Payload | – | – | – | 5.1 | $7.8 \times 10^{-6}$ |

## 4 Research Methodology

The solution methodology adopted is as follows. The link dimensions were taken to replicate human arm dimensions. Operating MSC Adams software, a Revolute–Revolute-type two-link manipulator made of rigid and flexible (links and joints) are modeled so that end-effector moves in required vertical path with a payload of 5.1 kg (link mass). Torques (input) were applied at the joints (two revolute) triggering the links angular rotation generating end-effector linear motion. The two-link rigid manipulator's end-effector is forced to move only in identified vertical path as given in Fig. 2. The angular rotations of both links and end-effector motion are the outputs to be monitored. The end-effector moves for 10 s in 2000 steps. The end-effector motion, joint 1 & 2 angular rotations monitored for same 10 s are taken as reference values.

When the rigid links are substituted by flexible links and joints in RR type manipulator and identical input torques are applied at the both the joints. The resulting position of end-effector positions, joints 1 & 2angular rotations are measured again for same 10 s and in 2000 steps. Figure 3 displays a flexible link built-in MSC Adams. Flexible links are modeled based on Euler–Bernoulli beams, i.e., the maximum deflection should be less than one-tenth of the total length of the beam. MSC Adams solves this problem using Assumed mode method. Positional error is taken as variation between the two (Rigid–Rigid and Flexible link) end-effector tip positions and joint 1 & 2 angular difference for a time of 10 s in 2000 steps. When one link flexible manipulator is made to rotate with various angular speeds the variation between the assumed mode method using MATLAB and Msc Adams was negligibly small. Therefore, Msc Adams software is used for modeling multiple body flexible systems with ease. From the results obtained in Fig. 4 for only link flexibility, one can deduce that as the time and number of steps are increased the positional error

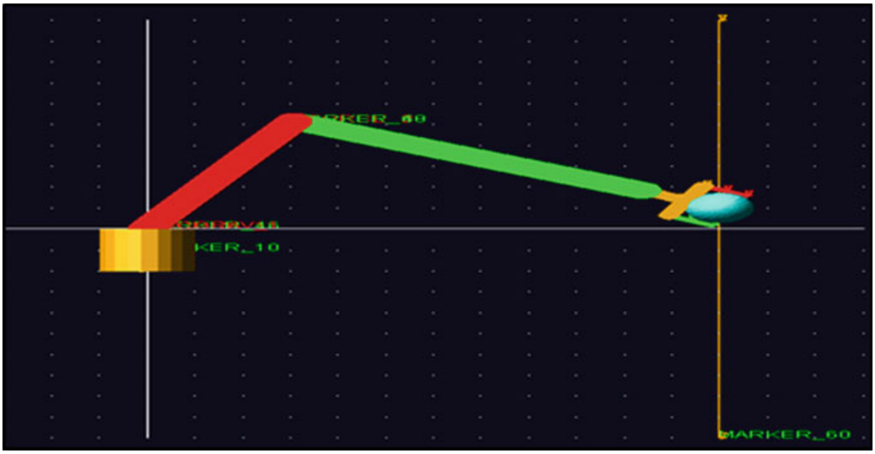

**Fig. 2** Model with vertical constraint

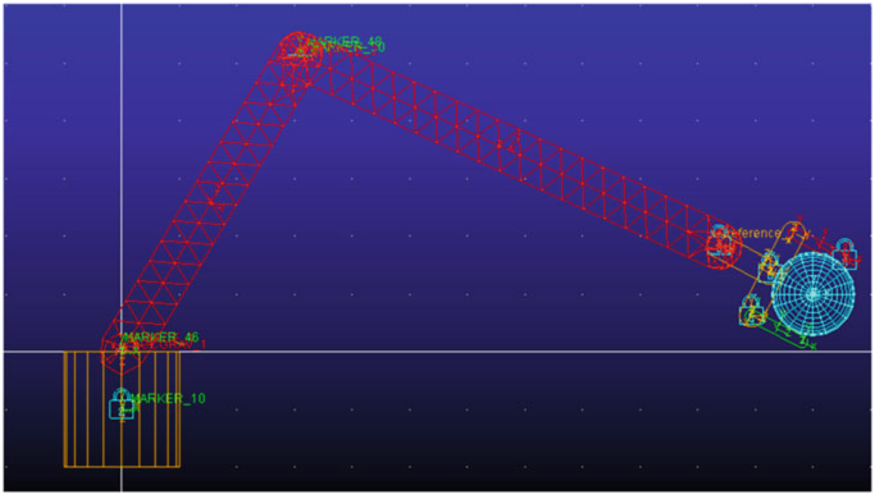

**Fig. 3** Building of Flexible model in MSC Adams software

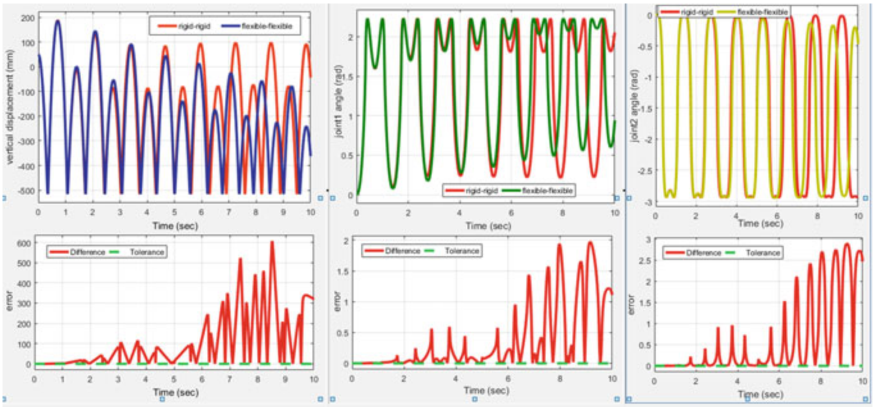

**Fig. 4** Errors at end-effector, joint one and two due to link flexibility only

increased. Moreover, the end-effector with a payload of 5.1 kg (mass of two-links) is also affecting the values of positional errors.

When only link flexibility is provided the tip position's maximum error is equal to 561 mm, maximum error at first and second joints is about 1.88 radians and 2.79 radians respectively. The variation of these values with time is provided in Fig. 4.

Figure 5 shows the introduction of clearance of 0.2 mm at the first joint between link one and the base. The joint flexibility because of the clearance gap between the mating parts at the joint (i.e. Journal and the bearing). The clearance at the joints 1 is varied from 0.2 mm to 1 mm in steps of 0.2 mm and 2nd joint is rigid. This variation is shown in Fig. 6. The results show large impact forces are generated at clearance of

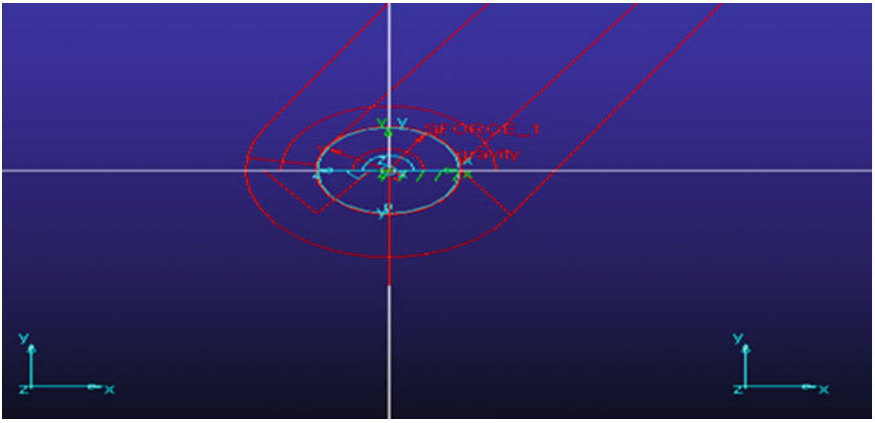

**Fig. 5** Creation of clearance of 0.2 mm at joint 1

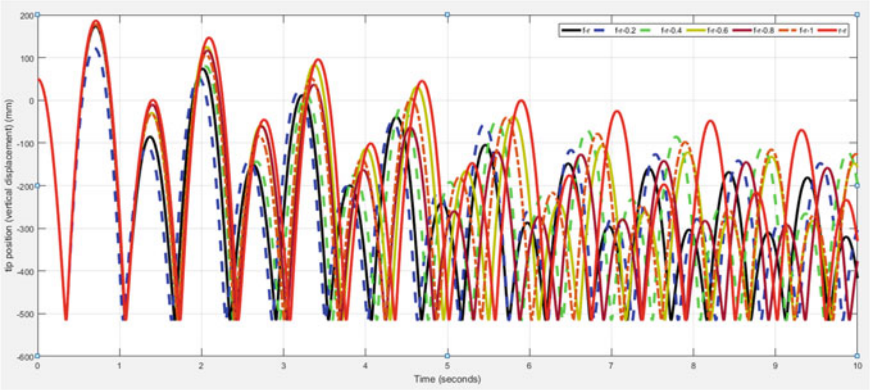

**Fig. 6** Variation of end-effector position as clearance is varied from 0.2 mm to 1 mm with flexible links

0.2 mm. If only joint flexibility was present as clearance increased, the impact forces would have increased. However, as the robot system had both link and joint flexibility, and then largest deviation was produced for a clearance of 0.2 mm as shown in Fig. 6. As Hertzian contact forces take into consideration both flexibility and damping of the component into consideration, the results obtained are more realistic compared to other methods. These contacts stresses resulted in higher torques resulting in a bigger and heavier motor/actuator for the same system. Figure 6 shows the results of the variation of three outputs (end-effector motion, angular motions of joint one and two) as the clearance is varied from 0.2 mm to 1 mm. Maximum impact forces were generated for clearance of 0.2 mm. Therefore the same clearance is provided to the flexible link system. Figure 6 shows the error variation for all three outputs.

When both link and joint flexibility are considered, the tip position's maximum error was 574.28 mm; maximum error at first and second joints is about 1.93 radians and 2.82 radians respectively.

To reduce these positional errors for the three outputs (end-effector, joints 1 & 2) to a least value, a control method is required. From the Literature, two conventional controls strategies, i.e., Proportional-Integral Derivate (PID) and Proportional-Integral (PI) control schemes are selected.

### 4.1 Applying Proportional-Integral (PI) Controller at Both Joints

Applying PI and PID control strategies directly in MSC Adams generated problems and getting a solution proved difficult. Therefore, utilizing MSC Adams and MATLAB, a co-simulation method was envisaged. Both flexible and rigid links models were developed in MSC Adams and were imported to MATLAB. Utilizing Simulink, MscAdams model was assimilated in MATLAB environment as displayed in Fig. 7 and as shown in Figs. 6 and 8 block diagrams circuits were generated for both PID & PI control strategies. The tip position's erroris the output and the resultant rotations of the joints (angular) are the inputs for the Simulink model. The specific constant values $K_p$ and $K_i$ given in Table 2 are chosen such that the magnitudes of all the outputs are minimum resulting in stable behavior. Figure 6 displays the SIMULINK model, so developed in MATLAB. Figure 8 displays the effect of PI controller on the robot system.

When PI control strategy was applied, the tip position's maximum error reduced to 233.84 mm and Average RMS error of 46 mm the at joint 1 & 2 angular position error (maximum) is about 0.98 radians and 1.53 radians, respectively. Figure 7 displays

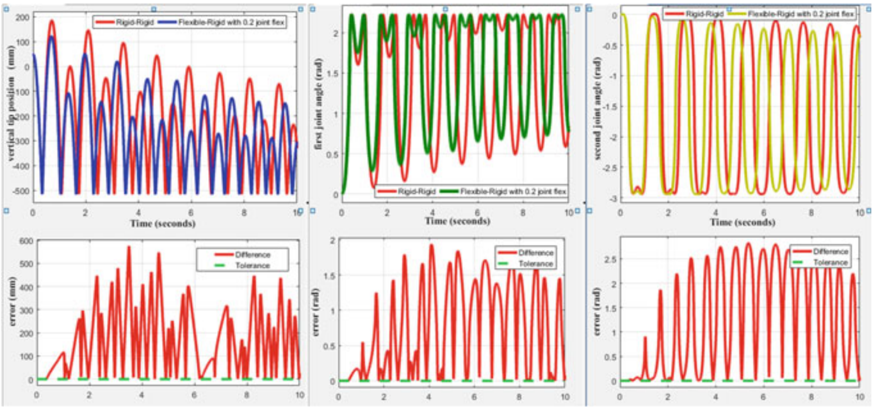

**Fig. 7** Errors at end-effector, joint one and two due to both link and joint flexibility

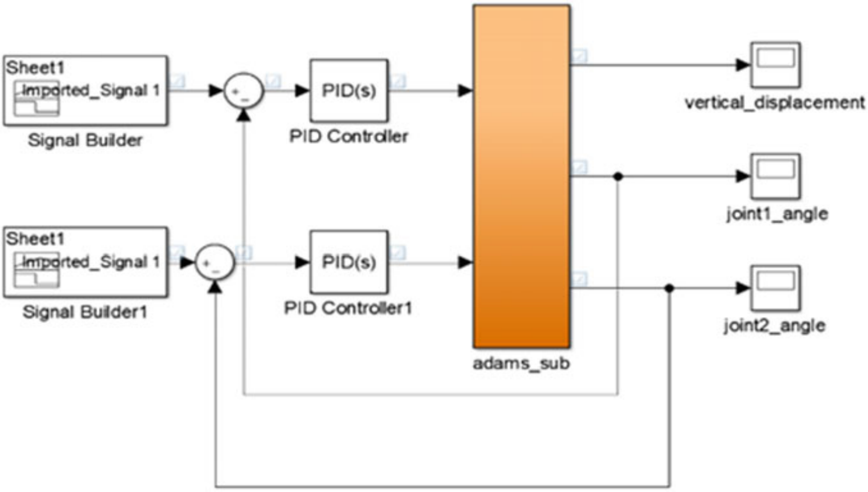

**Fig. 8** PID controller Simulink model

**Table 2** PI controller constants

| | P constant | I constant |
|---|---|---|
| Joint 1 | 2800 | 2400 |
| Joint 2 | 200 | 100 |

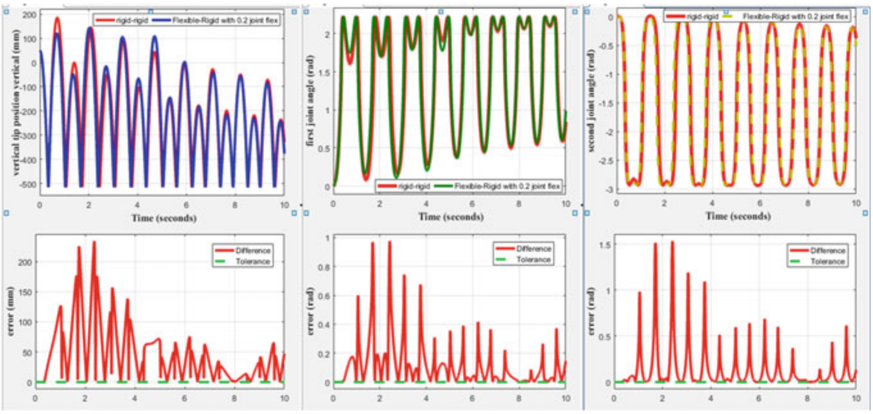

**Fig. 9** The errors in tip position, joint 1 & 2 angles with PI controller

these results and one can deduce that these errors are considerably reduced implying that PI control.

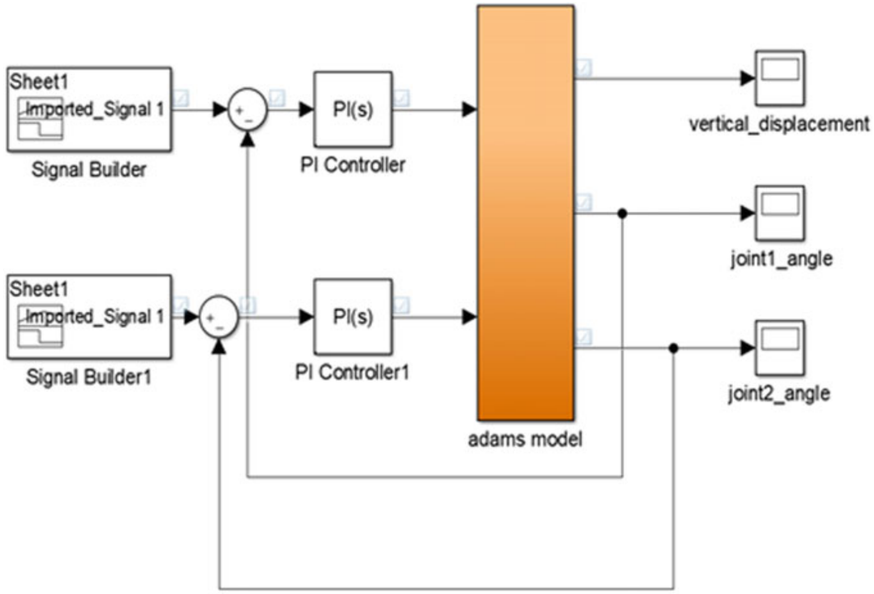

Fig. 10 PID controller Simulink model

## 4.2 Applying Proportional-Integral-Derivative(PID) Controller at the Joints

The constants (*Kp*, *Ki*, and *Kd*) in PI and PID controllers are chosen in a trial and error basis (given in Table 3), as the inputs are modified and the outputs are observed as end-effector moves in a vertical path till these error values have become minimum. The tip and joint angles with their corresponding errors in tip position, joint 1 & joint 2 angles with PID controller (Fig. 11).

When PID control strategy was applied, the tip position's maximum error reduced to 144.53 mm and Average RMS error of 33.2 mm the maximum angular position error at joint 1 & 2 is 0.69 radians and 1.11 radians respectively. Figure 9 shows these results and it can be said that these errors are considerably reduced implying that PID control is satisfactorily countering the effects of flexibility.

**Table 3** PID controller constants

| | P constant | I constant | D constant | Filter coefficient |
|---|---|---|---|---|
| Joint 1 | 2400 | 1000 | 1000.2 | 7.56 |
| Joint 2 | 200 | 1000 | 300 | 4 |

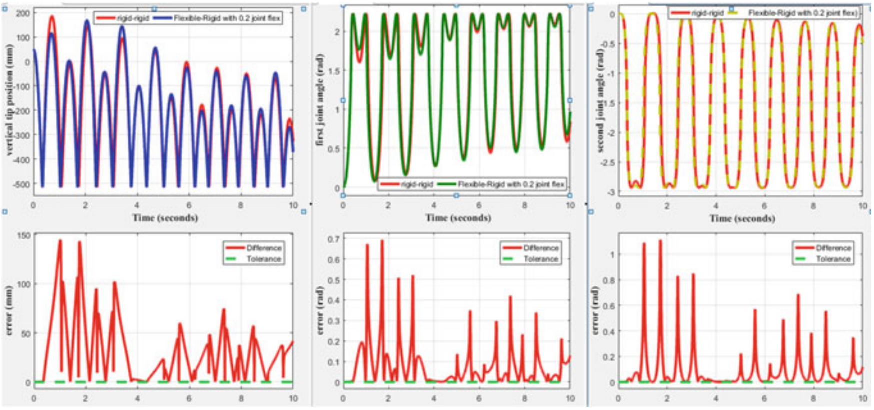

**Fig. 11** The error in tip position, joint 1 & joint 2 angles with PID controller

## 4.3 The Effect of the Controllers on the Input and Output Response

Figure 12 provides the effect of PID and PI control strategies to control and reduce the tip position's error and revolute joints 1 & 2 errors no control strategy is adopted. When PI control is adopted the maximum positional error reduced by 59.2 and 74.8% reduction using PID control. The angular error reduced by 49.2% at joint 1 and 45.7% at joint 2 using PI control. When PID control is used, the error at joint 1 reduced by 64.2 and 60.6% at joint 2. It is obvious from Fig. 10 that the PID controller performs

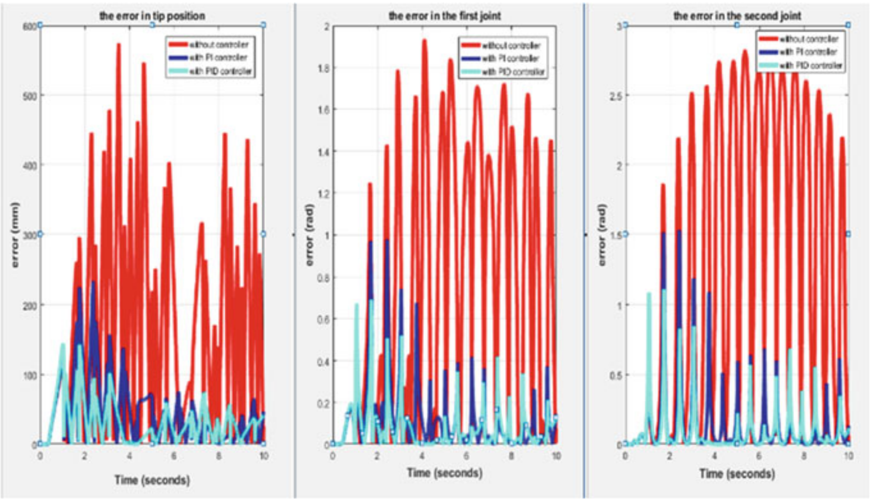

**Fig. 12** Effects of PI & PID control strategies

better than PI control for the given conditions and therefore it should be chosen to reduce error and follow the desired path.

## 5 Results and Discussions

When no control strategy was adopted to control link flexibility, the tip position's maximum error was equal to 561 mm and with average RMS error of 158.6 mm. The joints 1 & 2 maximum positional error is 1.88 radians and 2.79 radians respectively. Figure 4 provides these results and it can be deduced that these errors are very large and the flexibility has substantial effect on the tip position's accuracy. By including the joints clearances (0.2 mm clearance at each joint) along with link flexibility we can observe that the tip position's error increased to 574.28 and with average RMS error of 171.2 mm. The angular position error at joint 1 & 2 increased to 1.93 radians and 2.82 radians respectively. Figure 7 provides these results and it can be said, that the joint clearances will affect the tip position increasing the error.

To reduce these errors, two control strategies were used to determine tip position error and see that end-effector follows the specific vertical path.

## 6 Conclusions

A two-link rigid and flexible manipulator has been successfully co-simulated in Adams software and MATLAB. From the model created in ADAMS it is found that link flexibility significantly affects the system behavior. The tip position, first joint angle and second joint angle values are compared by using MATLAB and two types of controllers were (i.e., PID and PI) applied. From the above study, it can be concluded that when joint flexibility (i.e., 0.2 mm clearance) is added to already flexible links the end-effector deflection increased by 2.31%. Whereas when PI control is adopted the maximum positional error reduced by 59 and 74.8% reduction using PID control. The angular error reduced by 49% at joint 1 and 46% at joint 2 using PI control. When PID control is used, the error at joint 1 reduced by 65 and 61% at joint 2. It can be seen from Fig. 12 that PID controller performs better than PI control for the given conditions and therefore it should be chosen to reduce error and follow the desired path.

**Acknowledgements** The authors would like to thank University Grants Commission, South Eastern Region (SERO), Hyderabad for sponsoring this project, as this work is being carried out from UGC Sanctioned Minor Research Project No. F-MRP-6758/16 (SERO/UGC) dated June 30, 2017.

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