



Force Tracking Control of Nonlinear Active Suspension System with Hydraulic Actuator Dynamic

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Abstract. This paper delivers findings on optimal control studies of two degree of freedom quarter car model. Nonlinear active suspension quarter car model is used which considering the strong nonlinearities of hydraulic actuator. The investigation on the benefit of using Sliding Mode Control as force tracking controller with the utilization of Particle Swarm Optimization is done in this paper. The controller is designed to improved trade-off performance between ride comfort and road handling ability. Comparison between proposed controller with PID control and conventional suspension system showed that performance of the proposed controller is significantly improved. Results illustrated via simulation runs using MATLAB.

Keywords: Active suspension system · Hydraulic actuator · Sliding mode control

1 Introduction

The suspension system provides a control towards the vehicles itself for having a good road handling and ride comfort in case facing an external disturbances and road irregularities while driving. Nowadays, most local automotive industry mostly implies conventional suspension system's design that usually having an issue with load carrying, passenger comfort and road handling [1]. It is difficult for traditional suspension system to achieve the trade-off between ride comfort and direction control of the vehicle. There are three main types of suspension that has been used/studied in vehicle systems, passive, semi-active and active suspension. Current automobile suspension system implies passive components also known as conventional suspension system that provide a non-controllable spring and damping coefficients with a fixed parameter. However, the trade-off between ride comfort, handling quality and load varying are difficult to achieve since the parameters are fixed. The difference for semi active suspension systems is the coefficient of dampers can be controlled [2]. In contrast to passive and semi-active, an active suspension system able to enhance energy externally by the use of force actuator to provide a closed loop response for the system rather than dissipates the energy by the use of springs in passive. Many researchers nowadays are

interested in developed suspension with active control automobile systems. Previous works of automobile system focusing on linear model of active suspension are adopted to propose various control strategy for different components to be controlled [1–7]. The proposed controller in the mentioned references have greatly improved the suspension performance however the dynamical effect of the system’s behaviors are being ignored. In fact, it is well known that the actuator behaves far from ideal in real life. The scopes for this paper is narrowed to hydraulic actuators for active suspension system model where the real implementation with its dynamic could easily be controlled to track a desired force with adequate techniques [8–13].

In this paper, the dynamic of an electro-hydraulic actuator is included in the design of non-linear active quarter car suspension system. The proposed control approach is designed based on sliding mode control algorithm to track the desired force trajectory generated by the skyhook damping dynamics. It has been proven that tuning the controlled parameters manually is a challenging task. Therefore, this paper proposed to utilized a particle swarm optimization (PSO) algorithm as tuning method in obtaining controllable gain and switching surface values that can minimise the effect of mismatched uncertainties.

2 Methodology

2.1 Non-linear Quarter Car Model

The car model studied in this paper comprises of one-fourth of the entire body with two degree of freedom, as illustrated in Fig. 1, The interconnection set-up using stiffness spring k_s , damper or shock absorber b_s , and a variable active force actuator F element which positioned between the sprung m_s and unsprung masses m_{us} .

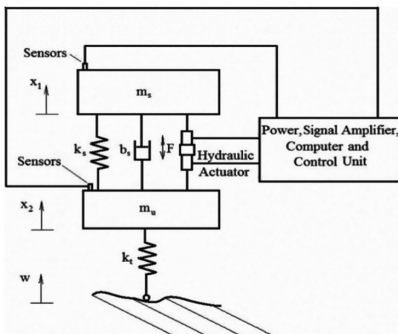


Fig. 1. Simplified quarter car of active suspension system [15]

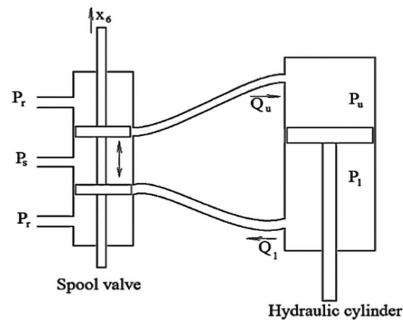


Fig. 2. Schematic diagram of electro-hydraulic actuator

A state variable of x_1 , x_2 , and w represents the vertical displacement of car body (chassis), vertical displacement of wheel and road input disturbances, respectively. By applying a Newton’s second law of motion, the governing equations that indicate nonlinear nature of the system model are derived as follows;

$$m_s \ddot{x}_1 = -F_s - F_b + F \quad (1)$$

$$m_u \ddot{x}_2 = F_s + F_b - F_w - F \quad (2)$$

Where F_s and F_b are spring and damping forces acting on the suspension respectively. The suspension components contain three different elements which are linear, symmetric and nonlinear as function of both suspension travel and velocity in each of applied forces [14]. Meanwhile, F_w is the force produced by road input disturbances and F is the generated force by actuator.

From Fig. 2, clearly showed the piston of the actuator is controlled by means of the voltage/current input to the electro-hydraulic servo valves in a three lane four-ways critical spool valve system [15, 16]. The hydraulic actuator force is produced through the high-pressure differences occur in the piston due to the movement of spool valve (P_L) multiplied with the cross-sectional area (A) of piston itself. The governing equations for the electro-hydraulic actuator can be structured into a simple form as modelled in [8, 14–16], where the derivatives of the load pressure is given by

$$\dot{P}_L = \alpha Q_L - \beta P_L - \alpha A (\dot{x}_1 - \dot{x}_2) \quad (3)$$

By upon substitution of α, β, γ , which expressed as,

$$\alpha = \frac{4\beta_e}{V_t}; \quad \beta = \alpha C_m; \quad \gamma = \alpha C_d w \sqrt{\frac{1}{\rho}}$$

where V_t is total volume of actuator, β_e is the effective bulb modulus, C_m is the total leakage coefficient of piston, C_d is the discharge coefficient, w is the spool valve area gradient, x_v is the servo-valve displacement, and ρ is the hydraulic fluid density. In the meantime, the resulting hydraulic flow rate, QL can be written as,

$$Q_L = \gamma x_v \sqrt{P_s - \text{sgn}(x_v) P_L} \quad (4)$$

Then, assumed that servo valve that controls a motion of spool valve, x_v as approximately a first-order linear system with a time constant, τ [9], as described in (5),

$$\dot{x}_v = \frac{1}{\tau} (u - x_v) \quad (5)$$

2.2 Suspension System with Hydraulic Actuator

The dynamics of the system with hydraulic actuator can be further re-arranged into a state space form. The state variables are defined as follows;

$$\dot{x}_1 = \dot{x}_1 - \dot{x}_2 = x_2 - x_4; \quad \dot{x}_2 = \ddot{x}_1; \quad \dot{x}_3 = \dot{x}_2 - \dot{w}; \quad \dot{x}_4 = \ddot{x}_2; \quad \dot{x}_5 = \dot{P}_L; \quad \dot{x}_6 = \dot{x}_v$$

The dynamics equation of motion for a nonlinear quarter car model can be obtained as;

$$\begin{aligned} \dot{x}_1 &= x_2 - x_4 \\ \dot{x}_2 &= -\frac{1}{m_s} \left[k_{sl}(x_1 - x_2) + k_{snl}(x_1 - x_2)^3 + b_{sl}(\dot{x}_1 - \dot{x}_2) - b_{ssym}(\dot{x}_1 - \dot{x}_2)^2 \right. \\ &\quad \left. + b_{snl} \sqrt{(\dot{x}_1 - \dot{x}_2)} \operatorname{sgn}(\dot{x}_1 - \dot{x}_2) + Ax_5 \right] \\ \dot{x}_3 &= \dot{x}_2 - \dot{w} \\ \dot{x}_4 &= \frac{1}{m_u} \left[k_{sl}(x_1 - x_2) + k_{snl}(x_1 - x_2)^3 + b_{sl}(\dot{x}_1 - \dot{x}_2) - b_{ssym}(\dot{x}_1 - \dot{x}_2)^2 \right. \\ &\quad \left. + b_{snl} \sqrt{(\dot{x}_1 - \dot{x}_2)} \operatorname{sgn}(\dot{x}_1 - \dot{x}_2) - K_f(x_2 - w) - Ax_5 \right] \\ \dot{x}_5 &= -\beta x_5 - \alpha A(\dot{x}_1 - \dot{x}_2) + \gamma x_6 \sqrt{P_s - \operatorname{sgn}(x_6)x_5} \\ \dot{x}_6 &= \frac{1}{\tau} (u - x_6) \end{aligned} \quad (6)$$

The values of parameters system applied for the simulation in this nonlinear quarter car model can be referred to the previous literature research in [17], as defined in Appendix.

2.3 Controller Design

a. Feedback Linearization

This process basically involves dynamic inversion by means to invert the dynamic of an original systems. In general, a linearization for Input-Output such as SISO class can be defined in the form below,

$$\dot{x} = f(x) + g(x) \quad (7)$$

$$y = x(t) \quad (8)$$

which the detail works can be seen in [18]. The linearization of the system is done where the relative degree of the system become one that same goes to the order of the system, and the control input, u will be produced by taking one time differentiation on the output [8]. The linearizing feedback control can illustrate as,

$$u = \frac{1}{g(x)}(-f(x) + v) \quad (9)$$

Therefore, \dot{x} in Eq. (7) now is,

$$\dot{x} = v \quad (10)$$

where v can be designed with any method that gives to a good tracking performance for the actuator output forces to be supplied to the system.

b. Sliding Mode Control

In the early 50's, sliding mode control (SMC) with variable structure control (VSC) was proposed by [19, 20] in the Soviet Union, Russia. Until now, sliding control approaches are applied in a wide variety of engineering system by considering an actuator dynamic as in related previous works [21–24]. Here, the design of sliding control can be separated in two stages. First to determine the necessary spool valve position in order to generate the force desired by actuator. Second, to generate the control input, u to the servo valve in order to obtain the desired spool valve position. In addition, the particle swarm optimisation (PSO) algorithm also will be used to provide the best values of proportional gain k in a way to meet a necessary sliding condition, as given in [9].

To begin with, sliding surfaces is defined as an error between the state variable and the desired value needs for the system. Based on the linearizing feedback control in Eq. (9), a special case of this controller yields as below,

$$v = -k_i s_i + \dot{x}_{desired} \quad (11)$$

The pressure generated in the dynamic system by electro-hydraulic can classify in the linearized form as in Eq. (7),

$$\begin{aligned} \dot{x}_5 &= -\beta x_5 - \alpha A (\dot{x}_1 - \dot{x}_2) + \gamma x_6 \sqrt{P_s - \text{sgn}(x_6)x_5} \quad \text{with} \\ f(x) &= -\beta x_5 - \alpha A (\dot{x}_1 - \dot{x}_2) \quad \text{and} \quad g(x) = \gamma x_6 \sqrt{P_s - \text{sgn}(x_6)x_5} \end{aligned}$$

Thus, the necessary spool valve position that will guarantee the actual output force by actuator to approach the desired force can be given as,

$$x_{6desired} = \frac{1}{\gamma \sqrt{P_s - \text{sgn}(x_6)x_5}} \left(\beta x_5 + \alpha A (\dot{x}_1 - \dot{x}_2) - k_i s_1 + \dot{x}_{5desired} \right) \quad (12)$$

The desired actuator force which generated by the skyhook damping dynamics [9], can be obtained as,

$$x_{5desired} = \frac{-Cx_2}{A} \tag{13}$$

Next, the second stage of sliding control is to induced the control input, u to the servo valve in a way to obtain the needed spool valve position (X_{6d}). After satisfying the same sliding condition through a proper chosen the gain k_2 by PSO algorithm, the control input of the second sliding surfaces can be obtained as follows,

$$u = x_6 + \tau \dot{x}_6 = x_6 + \tau (x_{6d} \dot{-} k_2 s_2) = (1 - \tau k_2)x_6 + \tau k_2 x_{6d} + \tau \dot{x}_{6d} \tag{14}$$

3 Simulation Results

The simulation design of the nonlinear active suspension systems is carried out using MATLAB and Simulink. For the assessment of the controller response, a double bump road input profile was implemented in this system as a road disturbance, referred from previous research [5]. The characteristic equation of the road input is described as

$$r(t) = \begin{cases} \frac{a}{2}(-\cos 8\pi t) & \text{if } 0.50 \leq t \leq 0.75 \text{ and } 3.00 \leq t \leq 3.25 \\ 0 & \text{Otherwise} \end{cases} \tag{15}$$

where a is the amplitude of bump input. In detailed, amplitude of first and second bump were equally set to 0.11 m and 0.5 m respectively as illustrated in Fig. 3.

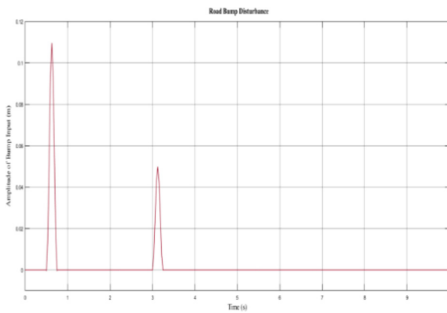


Fig. 3. Double bump road input

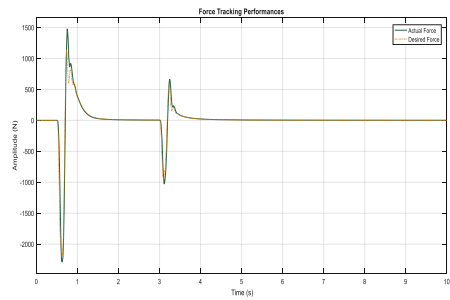


Fig. 4. Performance of force tracking of actuator

The implementation of PSO fed to the algorithm in sliding control includes the number of particles in each population that is 30, the maximum number of iterations is 15, and the dimension of problems is 2. Integral Absolute Error (IAE) was utilised as an objective function that used to calculate the minimum error produced in searching the best values. The car body acceleration (sprung mass) was considered as the fitness function, described as

$$J = \int_0^T |\ddot{X}_s| d(t) \tag{16}$$

In order to meet the satisfying sliding trajectory, the optimal values of proportional gain, k_1 and k_2 obtained from particle swarm activity at the end of searching process up to its maximum iterations are 1 and 12.9930 respectively.

In the simulation, Proportional Integral Derivative (PID) control is utilised as comparative method to evaluate the performances of proposed controller that has been presented in this paper for nonlinear active suspension system in terms of vertical car body acceleration, suspension travel, and wheel deflection. The transient response for both active suspension controls and passive system are determined in time domain analysis under double bump input road profile. It is important to check is the controllability of the force tracking controller [16]. The force tracking error of the hydraulic actuator model is measured using sliding control for the particular road profile as function of target force is shown in Fig. 4 which shows that the output force generated by electro-hydraulic actuator able to track the desired force well.

The results of comparative performances of active suspension systems over passive regarding the root mean square (RMS) values are given in Table 1. From this, the overall result is significantly improved as compared to PID control technique. In addition, the proposed method has simpler controller design where it only requires one control loop to achieve the trade-off performances despite of using two control loops in the PID controller design.

Table 1. RMS values of Performances of Suspension System

Performances	RMS value of suspension system			% of reduction	
	Active		Passive	Sliding mode/PID	Sliding mode/passive
	Sliding mode control	PID control			
Car body acceleration (m/s ²)	0.0032	0.0049	0.0063	34.7%	49.5%
Suspension travel (m)	0.0113	0.0145	0.0147	22.1%	23.2%
Wheel deflection (m)	0.0018	0.0021	0.0027	14.3%	33.3%

Figure 5 described a vertical car body acceleration of active and passive suspension system. It clearly shown that the vibration of the vertical acceleration can be effectively track reference system as well as suppressed by the proposed sliding mode control. The RMS value of car body acceleration are strictly related to the ride passenger comfort by means to specify the amount of acceleration transferred to the car body. It shows as much as 34.7% of percentage reduction in sliding control compared to PID control for active suspension which means the proposed method was successfully minimised in terms of less noticeable effect on vibration felt by passenger that guarantee the stability of vertical motion of car body.

The suspension travel for both active and passive suspension system are also analysed as shown in Fig. 6, clearly demonstrates that the suspension travel will travel between ± 8 cm range. The performance of active suspension with sliding mode control has improved the rattle-space dynamics from passive suspension system by 23.2% and PID control by 22.1% of percentage reduction, which extent improved the passenger ride comfort.

For road handling ability, considering tire or wheel deflection that determines how well the car wheel make a contact with road surfaces in which in this case the output signals produced closed to zero. Figure 7 prove that sliding mode control has better contact between the tire to road surface with the improvement of 14.3% and 33.3% reduction over the PID control and passive system, respectively as shown in Table 1. The reduction in vibration of wheel deflection in proposed active suspension had ensured a good road handling ability.

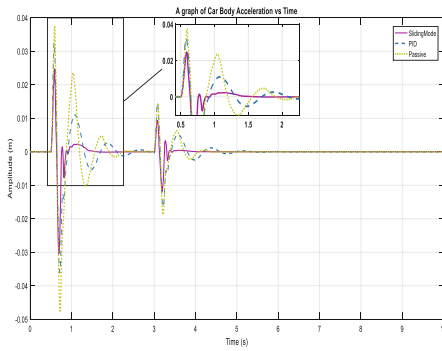


Fig. 5. Car body Acceleration

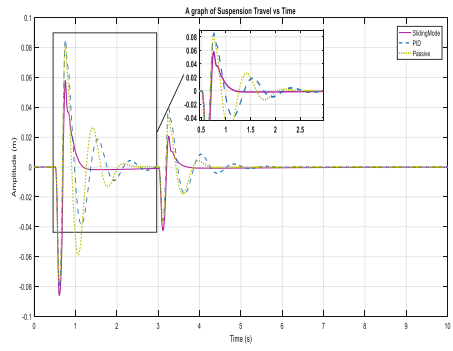


Fig. 6. Suspension Travel

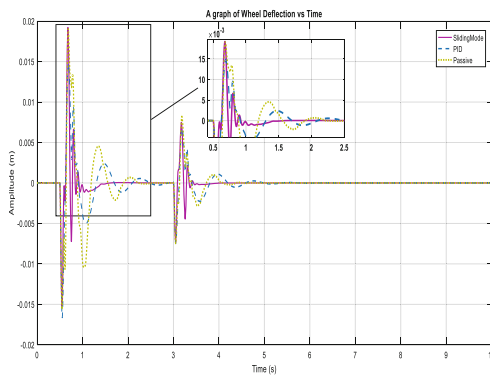


Fig. 7. Wheel Deflection

4 Conclusion

In this work, the approach control technique for non-linear active suspension system with electro-hydraulic actuator dynamic consideration using sliding control through an optimally controlled Sky-hook as reference model was presented. PSO algorithm is adopted to serve an optimal value of switching gain in sliding control for a smooth version of tracking performance. The control system focused on tracking the desired force of nonlinear system which the major nonlinearity was caused by an actuator dynamic and some with nonlinear spring stiffness and damping coefficient in the quarter car model itself. This adaptation to obtain the real effects and results of model used in practical manners. In overall, the performances of the proposed sliding mode control method for active suspension system in terms of vertical car body acceleration, suspension travel and wheel deflection offering a good improvement in ride passenger comfort with a minimum rattle-space and road handling ability as compared to active suspension with PID control as well as passive suspension system with travel under similar road input disturbances.

Appendix

See Table 2.

Table 2. System parameters in nonlinear Quarter Car model [17]

Parameter	Symbol	Value
Sprung mass	m_s	290 kg
Unsprung mass	m_u	40 kg
Linear and non-linear damping coefficient	b_{sl}, b_{snt}	980 Ns/m, 400 Ns/m
Damping symmetry	b_{ssym}	400 Ns/m
Linear and non-linear constant spring stiffness	k_{sl}, k_{snt}	2.35×10^4 N/m, 2.35×10^6 N/m
Constant tire spring stiffness	k_t	1900000 N/m
Actuator parameters	α, β, γ	4.515×10^{13} , 1, 1.545×10^9
Area of piston	A	3.35×10^{-4} m ²
Pressure supply to piston	P_s	10 342 500 Pa
Time constant	τ	0.0333 s

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