Analysis of a Double-Stage Thermoelectric Refrigerator

Sudhanshu Sharma and Vijay Kumar Dwivedi

Abstract In the present work, a two-stage thermoelectric refrigerator that consists of 50 elements has been analyzed on the basis of the laws of thermodynamics. The elements are arranged as there are 49 elements on hotter side and only one element on colder side. A non-dimensional parameter 'x' has been considered that represents the ratio of number of elements on hot surface and that on cold surface. Refrigeration effect and coefficient of performance have been calculated to evaluate the operating performance of the double-stage thermoelectric refrigerator with $x = 49$. Refrigeration effect or rate of refrigeration and coefficient of performance both are the desirable performance measuring parameters of a refrigerating device. This work obtains the value of current input at which both the desirable parameters have the maximum value, therefore, the best performance of the device with this configuration.

Keywords Thermoelectric refrigerator · Laws of thermodynamics · Coefficient of performance · Rate of refrigeration

Nomenclature

COP	Coefficient of performance
TER	Thermoelectric refrigerator
A	Cross-sectional area of thermoelectric element
I	Input current of TER, A
T	Temperature
R	Electrical resistance of thermoelectric element, Ω
K	Thermal conductance of thermoelectric element, W/K
N	Number of thermoelectric elements

S. Sharma (✉) · V. K. Dwivedi
Department of Mechanical Engineering, Galgotias College of Engineering and Technology, Greater Noida, Uttar Pradesh 201306, India
e-mail: sudhanshu.shr@gmail.com

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Q	Cooling or heating capacity, W
x	Ratio of thermoelectric elements in hooter and colder stages
L	Length of thermoelectric element
W	Power input in TER
j	Ratio of current in hotter and colder stages

1 Introduction

Thermoelectric devices are pollution-free devices. A thermoelectric generator (TEG) can be used as heat engine (using Seebeck effect), and a thermoelectric refrigerator (TER) can be used as a refrigerator or heat pump (using Peltier effect). They possess encouraging potential in comparison with existing engines and refrigerators. As there is no moving component which increases durability, reduces maintenance cost, and improves span of system life. With change in direction of current, only the same device can be used as refrigerator as well as heat pump. The non-existence of refrigerant removes the hazards of leakages to the environment which are serious issues in vapor compression refrigeration systems. As there is no compressor used, it reduces the noise in the system. Nowadays, lots of new methods have been developed to make better-performing semiconductor materials; therefore, more and more applications and investigations to improve operating of thermoelectric devices are gaining importance. TERs are more appropriate in areas such as small electronic circuits, laser diodes, communication gadgets, medical, pharmaceutical, and defense operations where precise control of temperature is more important than other things. Thermoelectric refrigerators are better choice for the green environment as the ozone depleting working substances do not exist in the system. Due to these advantages, thermoelectric devices are being used in wide areas. The fundamental concepts of the thermoelectric technology and the bright likelihood of their applications have been studied by number of researchers and engineers [1]. As the gap between the energy consumption and finding energy sources in the world is widening day by day. The entire world is trying to face the challenges of discovering new energy sources to satisfy the spurt in energy consumption in day-to-day life along with degradation of the environment with pollution.

Thermoelectric devices are being viewed as alternatives systems which can utilize the waste heat for power generation. The low efficiency of nano-engineering thermoelectric device can be improved by decreasing the thermal conductivity of the materials, and they become a promising alternative for large-scale use because of their excellent performances [2]. A review of the research work to enhance the working of thermoelectric cooling systems was compiled. This work embraces the review of work related to enlist the new materials suitable for designing, making, and analyzing thermoelectric modules [3]. Thermoelectric refrigerators are found more appropriate for the applications where a regulation of temperature in limited space is required. A model was developed and studied to modify vapor content of air using TEC channels. It was observed in this particular work that the model speculated the variation in the temperature of air along the channel with minor error. Numerous experimental and simulation-based analysis of the thermoelectric devices have been done to evaluate their performance in various applications related to power generation and refrigeration or air-conditioning [4, 5].

A thermo-economic optimization based on exergy analysis of a vapor compression and vapor absorption refrigeration systems has been carried out. Thermo-economic analysis in these systems plays a very important role to obtain feasible life-cycle cost. Many researchers have established thermodynamic and thermo-economic objective functions based on second law and thermo-economic principles [6–9]. Comparison of single-stage and double-stage thermoelectric cooler has been done for specific designs [10]. Exergo-economic analysis has also been done for a multistage thermoelectric cooler [11, 12]. A three-dimensional numerical simulations' investigation has been done to optimize the design of a thermoelectric cooler using finite element method [13]. A study to evaluate the effect of input parameters on exergy flow in a thermoelectric system was done to establish the optimum value of input parameters [14].

2 Two-Stage Thermoelectric Refrigerator

A double-stage thermoelectric refrigerator (TER) is an arrangement of *n* and *p* type elements in which two single-stage TERs are connected thermally in series. That means the total heat absorbed from first stage will be transferred to second stage. There may be equal or different number of elements in the two stages. Figure 1 shows a double-stage TER. The energy balance equation for a two-stage TER can

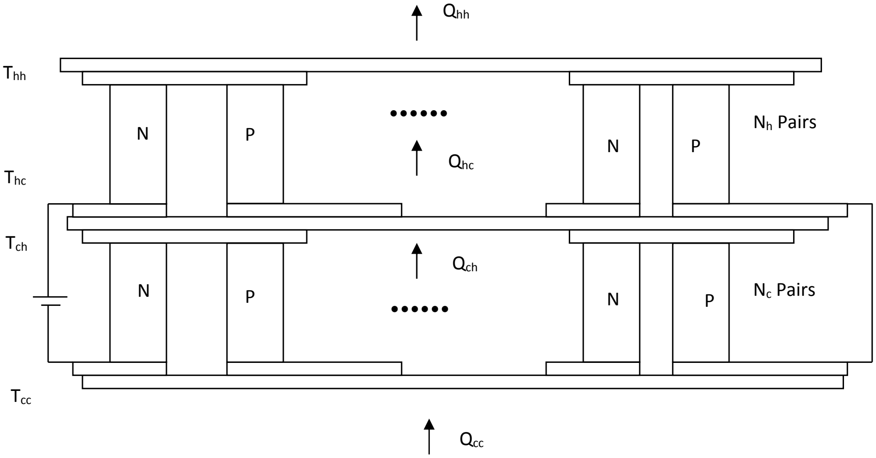

Fig. 1 Double-stage TER

be represented as:

$$Q_{\mathrm{cc}} = \left[\alpha I_{\mathrm{c}} T_{\mathrm{cc}} - \frac{I_{\mathrm{c}}^2 R}{2} - k(T_{\mathrm{ch}} - T_{\mathrm{cc}})\right] N_{\mathrm{c}} \tag{1}$$

$$Q_{\mathrm{ch}} = \left[\alpha I_{\mathrm{c}} T_{\mathrm{ch}} + \frac{I_{\mathrm{c}}^2 R}{2} - k(T_{\mathrm{ch}} - T_{\mathrm{cc}})\right] N_{\mathrm{c}} \tag{2}$$

$$Q_{\mathrm{hc}} = \left[\alpha I_{\mathrm{h}} T_{\mathrm{hc}} - \frac{I_{\mathrm{h}}^2 R}{2} - k(T_{\mathrm{hh}} - T_{\mathrm{hc}})\right] N_{\mathrm{h}} \tag{3}$$

$$Q_{\mathrm{hh}} = \left[\alpha I_{\mathrm{h}} T_{\mathrm{hh}} + \frac{I_{\mathrm{h}}^2 R}{2} - k(T_{\mathrm{hh}} - T_{\mathrm{hc}})\right] N_{\mathrm{h}} \tag{4}$$

where Q_{cc} is the heat absorbed at the cold side of the colder stage; Q_{ch} is the rate of heat rejected at the hot side of the colder stage; Q_{hc} is the heat absorbed at the colder side of the hotter stage; and Q_{hh} is the rate of rejected at the hotter side of the hotter stage. I_{c} is the current flow in colder side, and I_{h} is the current flow in hotter side of TER. In this analysis, the two stages are electrically is series, so the current I_{c} and I_{h} are equal. N_{h} and N_{c} are number of elements in colder and hotter sides of TER, respectively. For this analysis $N_{\mathrm{h}} + N_{\mathrm{c}} = 50$ has been considered. T_{cc} and T_{ch} represent the temperatures of the cold side and hot side of colder stage, and T_{hc} and T_{hh} represent the cold side and hot side temperature of hotter side. It can be assumed that there exists a junction temperature T_{m} which can be calculated by equating Q_{ch} and Q_{hc}. Since the two stages are thermally in series so the heat rejected from first stage will be transferred completely to second stage.

Hence $Q_{\mathrm{ch}} = Q_{\mathrm{hc}}$

$$T_{\mathrm{m}} = \frac{k(T_{\mathrm{cc}} + xT_{\mathrm{hh}}) + \left(xj^2 + 1\right)\frac{1}{2} I_{\mathrm{c}}^2 R}{k(x+1) + I_{\mathrm{c}}\alpha(xj - 1)} \tag{5}$$

where $x = N_{\mathrm{h}}/N_{\mathrm{c}}$ and $j = I_{\mathrm{h}}/I_{\mathrm{c}}$.

The energy balance equations can be written as:

$$Q_{\mathrm{cc}} = \left[\alpha I_{\mathrm{c}} T_{\mathrm{cc}} - \frac{I_{\mathrm{c}}^2 R}{2} - k(T_{\mathrm{m}} - T_{\mathrm{cc}})\right] N_{\mathrm{c}} \tag{6}$$

$$Q_{\mathrm{ch}} = \left[\alpha I_{\mathrm{c}} T_{\mathrm{m}} + \frac{I_{\mathrm{c}}^2 R}{2} - k(T_{\mathrm{m}} - T_{\mathrm{cc}})\right] N_{\mathrm{c}} \tag{7}$$

$$Q_{\mathrm{hc}} = \left[\alpha I_{\mathrm{h}} T_{\mathrm{m}} - \frac{I_{\mathrm{h}}^2 R}{2} - k(T_{\mathrm{hh}} - T_{\mathrm{m}})\right] N_{\mathrm{h}} \tag{8}$$

$$Q_{\mathrm{hh}} = \left[\alpha I_{\mathrm{h}} T_{\mathrm{hh}} + \frac{I_{\mathrm{h}}^2 R}{2} - k(T_{\mathrm{hh}} - T_{\mathrm{m}})\right] N_{\mathrm{h}} \tag{9}$$

COP of double-stage TER:

$$(\mathrm{COP})_{\mathrm{act}} = \frac{Q_{\mathrm{cc}}}{W_{\mathrm{hh}} + W_{\mathrm{cc}}} = \frac{Q_{\mathrm{cc}}}{(Q_{\mathrm{hh}} - Q_{\mathrm{hc}}) + (Q_{\mathrm{ch}} - Q_{\mathrm{cc}})} = \frac{Q_{\mathrm{cc}}}{(Q_{\mathrm{hh}} - Q_{\mathrm{cc}})} \tag{10}$$

W_{hh} and W_{cc} are the work inputs in hot and cold sides, respectively. Since the two-stage TER is a combination of two single-stage TER's which are thermally in series, so the total work input would be equal to the sum of the work input to individual stages. But Q_{cc} would be the heat absorbed at the cold side plate only.

N_{h} and N_{c} are number of elements in hot and cold sides, respectively, and $N_{\mathrm{h}} + N_{\mathrm{c}} = 50$. There may be three possibilities to select N_{h} and N_{c} such that $N_{\mathrm{h}} > N_{\mathrm{c}}$, $N_{\mathrm{h}} < N_{\mathrm{c}}$, and $N_{\mathrm{h}} = N_{\mathrm{c}}$. A non-dimensional parameter 'x' has been used which is the ratio of number of elements in hot side and cold side in a double-stage TER.

The value of 'x' may vary from 0.0204 ($N_{\mathrm{h}} = 1$ and $N_{\mathrm{c}} = 49$) to 49 ($N_{\mathrm{h}} = 49$ and $N_{\mathrm{c}} = 1$). If the value of x is considered to be the maximum, i.e., 49, the performance of TER is as in Table 1.

Table 1 Variation in performance parameters of double-stage TER with I ($x = 49$)

I	T_{m}	Q_{cc}	Q_{ch}	Q_{hc}	Q_{hh}	COP
4	281.715	0.016	0.071	0.071	2.901	0.005
5	277.415	0.135	0.208	0.208	4.563	0.030
6	273.409	0.247	0.342	0.342	6.525	0.039
7	269.682	0.354	0.473	0.473	8.776	0.042
8	266.220	0.455	0.601	0.601	11.309	0.042
10	260.039	0.641	0.850	0.850	17.182	0.039
11	257.298	0.726	0.972	0.972	20.507	0.037
12	254.773	0.806	1.092	1.092	24.081	0.035
13	252.457	0.881	1.211	1.211	27.899	0.033
14	250.338	0.951	1.329	1.329	31.953	0.031
15	248.408	1.016	1.446	1.446	36.237	0.029
16	246.659	1.077	1.563	1.563	40.747	0.027
17	245.083	1.133	1.679	1.679	45.477	0.026
18	243.673	1.185	1.795	1.795	50.422	0.024
19	242.420	1.233	1.912	1.912	55.578	0.023
20	241.320	1.276	2.028	2.028	60.940	0.021
21	240.365	1.316	2.146	2.146	66.504	0.020
22	239.550	1.351	2.263	2.263	72.266	0.019
23	238.868	1.382	2.382	2.382	78.223	0.018
24	238.316	1.409	2.501	2.501	84.370	0.017

(continued)

Table 1 (continued)

I	T_m	Q_{cc}	Q_{ch}	Q_{hc}	Q_{hh}	COP
25	237.887	1.433	2.622	2.622	90.706	0.016
26	237.576	1.453	2.743	2.743	97.226	0.015
27	237.381	1.469	2.866	2.866	103.928	0.014
28	237.295	1.481	2.990	2.990	110.808	0.014
29	237.315	1.490	3.116	3.116	117.865	0.013
30	237.438	1.495	3.244	3.244	125.095	0.012
31	237.658	1.497	3.373	3.373	132.496	0.011
32	237.974	1.495	3.504	3.504	140.066	0.011
33	238.381	1.490	3.637	3.637	147.803	0.010
34	238.876	1.482	3.772	3.772	155.704	0.010
35	239.457	1.470	3.910	3.910	163.768	0.009
36	240.120	1.455	4.049	4.049	171.992	0.009
37	240.862	1.437	4.191	4.191	180.375	0.008
38	241.681	1.415	4.334	4.334	188.915	0.008
39	242.575	1.391	4.481	4.481	197.610	0.007
40	243.540	1.363	4.630	4.630	206.459	0.007
41	244.575	1.332	4.781	4.781	215.460	0.006
42	245.678	1.298	4.935	4.935	224.612	0.006
43	246.846	1.261	5.091	5.091	233.913	0.005
44	248.076	1.221	5.251	5.251	243.362	0.005
45	249.368	1.178	5.413	5.413	252.958	0.005
46	250.720	1.132	5.578	5.578	262.700	0.004
47	252.129	1.083	5.745	5.745	272.585	0.004
48	253.594	1.031	5.916	5.916	282.614	0.004
49	255.112	0.977	6.090	6.090	292.785	0.003
50	256.684	0.919	6.266	6.266	303.097	0.003
51	258.306	0.858	6.446	6.446	313.549	0.003
52	259.978	0.795	6.629	6.629	324.140	0.002
53	261.698	0.729	6.814	6.814	334.869	0.002
54	263.465	0.660	7.003	7.003	345.735	0.002
55	265.277	0.588	7.195	7.195	356.737	0.002
56	267.134	0.514	7.391	7.391	367.875	0.001
57	269.033	0.437	7.590	7.590	379.148	0.001
58	270.974	0.357	7.792	7.792	390.555	0.001
59	272.956	0.274	7.997	7.997	402.094	0.001
60	274.978	0.189	8.205	8.205	413.767	0.000

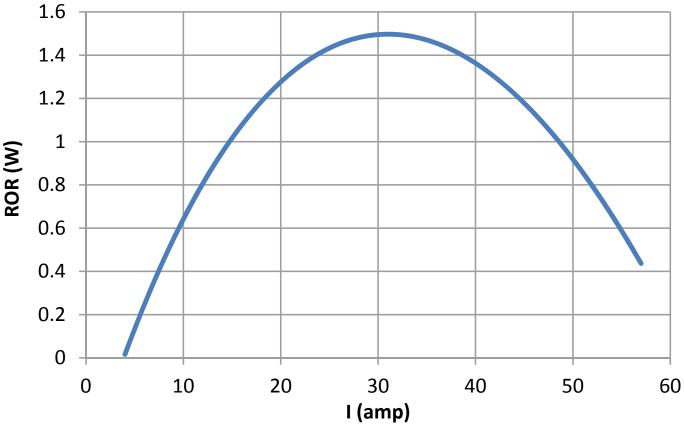

Fig. 2 ROR of double-stage TER with $x = 49$

Figure 2 shows the variation in ROR for a double-stage TER with $x = 49$ with current. The peak value of ROR is 1.49 at $I = 31$ Å.

Figure 3 shows the variation in COP for a double-stage TER with $x = 49$ with current. The peak value of COP is 0.042 at $I = 7$ Å.

The results represented in Table 2 show that COP is maximum with value 0.042 at current input as 7 Å with ROR as 0.354. The comparison of this result with the

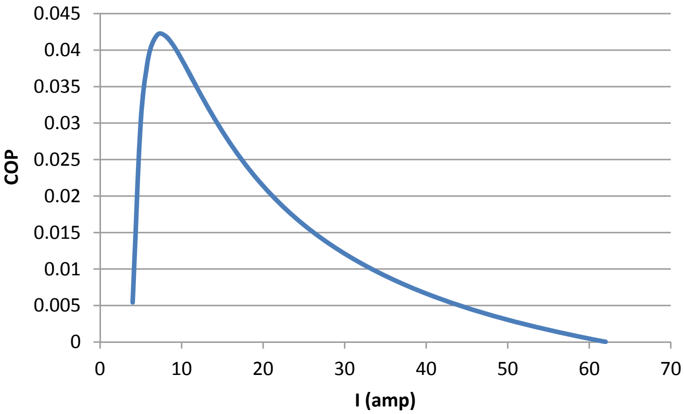

Fig. 3 COP of double-stage TER with $x = 49$

Table 2 Double-stage TER with $x = 49$ for COP and ROR

I (Å)	COP	ROR
07	0.042	0.354
31	0.011	1.49

results at 31 Å shows that the maximum ROR is obtained at 31 Å, i.e., 1.49 but with a loss in COP. There is a significant improvement in ROR at 31 Å. So these results show that the same device should be used at these different values of current to obtain the best performance according to the application and desired effect.

3 Conclusion

A two-stage thermoelectric refrigerator, with 49 elements on hotter side and only one element on colder side, has been analyzed. Rate of refrigeration and coefficient of performance have been calculated to measure the performance. As rate of refrigeration and coefficient of performance both are the desirable performance measuring parameters of a refrigerating device. This work shows the values of current input at which both the desirable parameters, rate of refrigeration and coefficient of performance have the maximum values, respectively. The results may be useful for a user to obtain the best performance according to the application and desired effect. Thermoelectric refrigerators are more useful in the applications where meticulous control of the temperature is needed. So this work may be extended to design a thermoelectric refrigerator for such applications.

References

1. Riffat SB, Ma X (2003) Thermoelectrics: a review of present and potential applications. Appl Therm Eng 23(8):913–935
2. Martín-González M, Caballero-Calero O, Díaz-Chao P (2013) Nanoengineering thermoelectrics for 21st century: energy harvesting and other trends in the field. Renew Sustain Energy Rev 24:288–305
3. Riffat SB, Ma X (2004) Improving the coefficient of performance of thermoelectric cooling systems: a review. Int J Energy Res 28(9):753–768
4. Jradi M, Ghaddar N, Ghali K (2012) Experimental and theoretical study of an integrated thermoelectric-photovoltaic system for air dehumidification and fresh water production. Int J Energy Res 36(9):963–974
5. Belanger S, Gosselin L (2012) Multi-objective genetic algorithm optimization of thermoelectric heat exchanger for waste heat recovery. Int J Energy Res 36(5):632–642
6. Lertsatitthanakorn C, Srisuwan W, Atthajariyakul S (2008) Experimantal performance of a thermoelectric ceiling cooling panel. Int J Energy Res 32(10):950–957
7. Kızılkan O, Şencan A, Kalogirou SA (2007) Thermoeconomic optimization of a LiBr absorption refrigeration system. Chem Eng Process 46(12):1376–1384
8. Ahmed JU, Saidur R, Masjuki HH (2011) A review on exergy analysis of vapor compression refrigeration system. Renew Sustain Energy Rev 15(3):1593–1600
9. Rezayan O, Behbahaninia A (2010) Thermoeconomic optimization and exergy analysis of CO_2/NH_3 cascade refrigeration system. Energy 36(2):888–895
10. Sharma S, Dwivedi VK, Pandit SN (2014) Exergy analysis of single-stage and multi stage thermoelectric coolers. Int J Energy Res 38:213–222
11. Sharma S, Dwivedi V, Pandit SN (2014) Thermoeconomic analysis of multi-stage thermoelectric coolers. Int J Therm Environ Eng 8(2):77–82

12. Sharma S, Dwivedi VK, Pandit SN (2016) Exergoeconomic analysis of two-stage thermoelectric cooler with genetic algorithm. Int J Adv Res Innov 4(1):24–28
13. Gong T, Gao L, Wu Y, Zhang L, Yin S, Li J, Ming T (2019) Numerical simulation on a compact thermoelectric cooler for the optimized design. Appl Therm Eng 146:815–825
14. Dizaji HS, Jafarmadar S, Khalilarya S, Pourhedayat S (2019) A comprehensive exergy analysis of a prototype Peltier air-cooler; experimental investigation. Renew Energy 131:308–317