Chapter 1
Certain Areas of Industrial and Applied Mathematics

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Abstract This chapter is based on Presidential address at the International Conference and 14th Biennial Conference of Indian Society of Industrial and Applied Mathematics, GNDU, Amritsar, Feb 2–4, 2018.

Keywords Walsh and Haar systems · 2-dimensional analogues of Banach space · Variational inequality · Wavelet theory · Fractals · Shearlets

1.1 Introduction

Here we introduce our contribution in certain areas of industrial and applied mathematics such as dyadic harmonic analysis, variational inequalities, wavelet theory, fractals, financial mathematics. People interested in these areas may go through books authored and edited by myself and colleagues and research papers written jointly by research scholars and colleagues. See selected research papers, books authored and edited volumes.

1.2 Areas Pursued By Our Research Group

i. Fourier and Dyadic Harmonic Analysis: I tried to popularize certain areas with which researchers in our country were not familiar. In the 60s, most of the researchers in northern India were engaged in Summability theory of sequences and special functions. I tried to introduce to Indian researchers, new areas such as classes of Fourier coefficients, approximation by Fourier series, and other orthonormal systems such as Walsh and Haar Systems. I pursued vigorously Walsh Fourier analysis and collaborated with Indian engineers (professors of electrical engineers IIT Kanpur) and Hungarian Mathematicians like F. Moricz, S. Fridli and F. Schipp. We learnt from Hungarian Mathematician's approximation by Walsh functions and Haar–Vilenkin

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system. I also benefited from Prof. William Wade, USA who is a well-known expert in this field. References of my joint work with Moricz and Fridli [14, 39] are given at the end. One can find fairly complete research work up to 1977 in my book of 1978 on Walsh functions, AMU Press. Our book to be published soon by Springer Nature named "Construction of Wavelets through Walsh Functions" provides very recent work by Farkov and research group of Manchanda, especially Meenakshi. These are the relevant Refs. [40, 56, 59, 72].

ii. Two-Dimensional Analogues of Banach Spaces: The concept of two-dimensional analogues of metric normed and Banach spaces were introduced and studied by Siegfried Gaehler of the German Academy of Science, Berlin around 1963. I introduced this topic to researchers in India, Iran, and Algeria. I also spent quite some time with Dr. Gaehler in Berlin. I wrote several research papers on non-Archimedean (ultra) 2-metric and 2-normed spaces quasi-normed spaces, 2-semi-inner product spaces, orthogonally in 2-normed spaces, ultra m-metric, and non-Archimedean m-normed spaces. Some typical results obtained by me and my collaborators will be given in the next section. However a list of papers on theme is given in the bibliography, see for example [20, 72].

iii. Non-Archimedean functional analysis: Besides the concept of compact operators and a fixed point in non-Archimedean functional analysis, we studied the concept of invariant means in non-Archimedean functional analysis which is published in Springer lecture notes in Math vol. 399, Springer, Berlin, 1974. Prof. Grande Kimpe of Belgium, a well-known expert of the field visited AMU, Constantine University, Algeria and also invited me to interact on research problems in non-Archimedean functional analysis. Several Ph.D. students of AMU worked under supervision of Prof. Siddiqi on different topics of this field.

iv. Variational Inequalities: Variational inequalities introduced and studied by celebrated mathematicians J. L. Lions, G. Stampacchia, and G. Fichera in the early 60s were not known in the Indian subcontinent till the 80s. I tried to popularize the subject in the Indian subcontinent and guided successfully 10 research scholars in this field. Some of them are well-known mathematicians of the present time. Four of these research scholars are full professors in the Aligarh Muslim University (AMU); eight of their research scholars are attending this conference. I studied applications of variational inequalities in areas like superconductivity, elasticity (rigid punch problem), and American option pricing references of our research papers in these areas which are given at the end, see for example [3–7, 11, 17, 18, 25, 47–49, 52, 61, 62, 69–71, 84–90].

v. Wavelet Theory: You will be surprised to know that wavelet theory was introduced to me by Helmut Neunzert while driving me from Kaiserslautern to Darmstadt in June 1990. He tried to introduce this subject to his Ph.D. students in the early 90s and other Indian researchers particularly, Prof. Manchanda. I joined KFUPM, Saudi Arabia in 1998 and devoted my full time to promote the study of wavelet theory and its applications. Some of my efforts yielded in publications of two special volumes of Arabian Journal of Science and Engineering, see references of edited volumes [58]. I published several research papers with my research collaborators and completed five research projects related to this theme. See Refs. [2, 31, 94] and edited volumes [35, 83, 91] and authored books [54, 64, 96] for details. The following are additional Refs. [12, 13, 15, 28–30, 38, 73, 80–82, 93].

vi. Fractals: I was motivated by a distinguished electrical engineering professor (Prof. M. N. Faruqi, Former Deputy Director, IIT, Kharagpur and Vice Chancellor, AMU) in 1993 to take up the study of fractals and their applications in image processing. I guided research in this area and one of my Ph.D. students (Mr. Aiman Mukheimar) worked in this area got a Ph.D. degree and is now HoD of Applied Sciences in the Prince Sultan University, Riyadh, Saudi Arabia. I have published a good number of papers given in the bibliography, for example, application of fractal methods in metrological studies some leading scientists have cited this work [42, 43]. see also Refs. [32, 58].

vii. Industrial and Financial Mathematics: Writings of Prof. H. Neunzert inspired me to take up the study of Industrial and applied mathematics. I made serious efforts to initiate teaching and research on industrial mathematics in India. We established the Indian Society of Industrial and Applied Mathematics (ISIAM) with the help of senior academicians of our country such as Prof. J. N Kapur, Prof. H. P. Dixit, Prof. U. P. Singh, Prof. D. Sinha, Prof. V. P. Saxena, Prof. G. C. Sharma, Prof. N. K. Gupta, Prof. Bhola Ishwar, Prof. Karmeshu, Prof. O. P. Bhutani, and Prof. Rudraiah. We made serious efforts for organizing Biennial Conferences of the society and International Conference since 1992. The proceedings of these conferences are published by reputed publishers. The society is publishing its journal named Indian Journal of Industrial and Applied Mathematics. Researchers from all over the world are associated with this journal. Famous publisher Springer, now Springer Nature has started publishing a series called Industrial and Applied Mathematics Series jointly with the society. Some useful references are [9, 10, 19, 33, 34, 37, 55, 57, 60, 66, 95].

viii. Oil Exploration and Wavelets: I worked as a consultant in research projects of the largest oil companies in the world named ARMACO. I also completed research projects on applications of wavelets to meteorological data of the kingdom of Saudi Arabia. See [42, 43] and edited volumes [53, 58].

ix. Wavelets Inverse Problems and Medical Signals: I have worked on the following themes with my research scholars in Sharda University, Greater Noida, for example, Ruchira Aneja, Nagma Irfan, Noor-E- Zahra have already got their Ph.D. degree in 2016. Other researchers with whom I have collaborated are Vivek Singh Bhadouria, Shafali Pande, Amita Garg, Padmesh Tripathi, and Nitender Kumar Shukla. Themes pursued are scalar tomography, vector value tomography, seismic tomography ANFIS, SVM and neuro-fuzzy methods, MRI, inverse problems related to heat equation, etc., see Refs. [35, 96] and research papers [21–24, 26, 27, 44–46, 75, 76, 92, 97].

1.3 Ruchira Aneja—Variants of Wavelet in Medical Imaging

This section is based on [46]: Ruchira Aneja and A. H. Siddiqi, **A Hybrid Shearlet-based compression coefficients and ROI Detection, Journal of Medical Imaging and Health Informatics, USA**.

Need For Geometric Transformations

The need to understand geometric structures arises since it is essential to efficiently analyze and process the data. Data are highly correlated and it is essential to extract the relevant information. This relevant information can be extracted and can be grouped into a certain class if we have an understanding of its dominant features, which are associated with their geometric properties. For instance, edges in natural images. One major goal of applied harmonic analysis is constructing classes of analyzing elements that capture the most relevant information in a certain data class.

Properties of shearlets Shearlets are well localized; they exhibit high directional sensitivity and satisfy parabolic scaling; they are spatially localized and optimally sparse.

Shearlet system is a special case of Composite Wavelet systems which provide optimally sparse representation for a large class of bivariate functions. A Composite Wavelet system in dimension $n = 2$ is

$$\Psi_{j,k} = |detA|^{i/2}\Psi(B^j A^i - k) : i, j \in Z, K \in Z^2$$
$$\text{where } A = \begin{pmatrix} 2 & 0 \\ 0 & \sqrt{2} \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$G = (M, t) : M \in D_{a,t} \in R^2$$

where for each $0 < \alpha < 1$, $D_\alpha < L_2(R)$ is a set of matrices

$$D_\alpha - \{M - M_{\alpha s} - \begin{pmatrix} a & -a^\alpha s \\ 0 & a^\alpha \end{pmatrix}\}\text{where } a > 0, s \in R.$$

Continuous Shearlet Transform For $\Psi \in L^2(R^2)$, the continuous shearlet system $SH(\Psi)$ is define by

$$SH(\Psi) = \Psi_{a,s,t} = T_t D_{A_\alpha} D_{S_s}\Psi : a > 0, s \in R, t \in R^2.$$

$$A_a = \begin{pmatrix} a & 0 \\ 0 & a^{1/2} \end{pmatrix}.$$

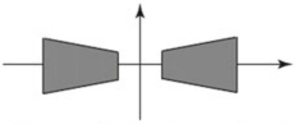
Shearing operator is D_{S_s}, $s \in R$, where the shearing matrix S_s is given by parabolic scaling

$$S_s = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}.$$

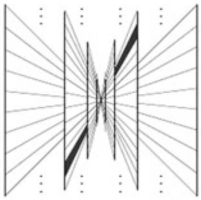
T_t is the translation operator on $L^2(R^d)$, defined by

$$T_t \Psi(x) = \Psi(x - t), \text{ for } t \in R^d.$$

Discrete Shearlet Transform Discrete versions of shearlet systems can be constructed by appropriate sampling of the continuous parameter set S or Scone. Various approaches have been suggested, aiming for discrete shearlet systems which preferably form an orthonormal basis or a tight frame for $L^2(R^2)$ A (regular) discrete shearlet system associated with Ψ, denoted by $SH(\Psi)$, which is defined by

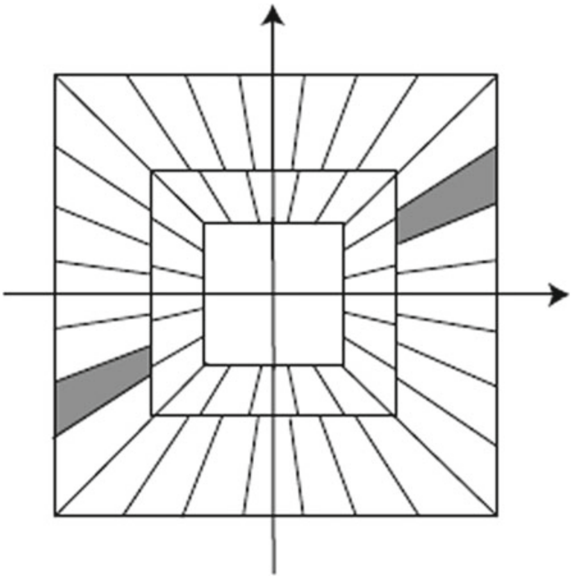


(a) Support of the Fourier transform of a classical shearlet.



(b) Fourier domain support of several elements of the shearlet system, for different values of a and s.

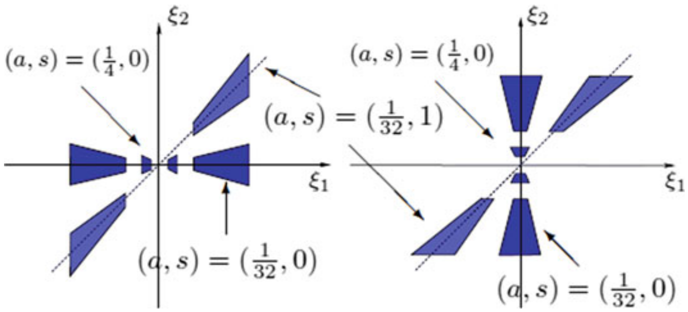
$$SH(\Psi) = \Psi_{j,k,m} = 2^{\frac{3j}{4}} \Psi(S_k A_{2^j} - m) : j, k \in Z, m \in Z^2.$$



Shearlet systems are derived from a single or finite set of generators. It also ensures a unified treatment of the continuum and digital world due to the fact that the shear matrix preserves the integer lattice.

Shearlet and Curvelet in Image Processing

Digital signal and image processing is the most important technique to analyze, manipulate, and process real-world data and images. These types of signals and images may be time series, collection of numbers or measured values. These include audio signals, video images, real-world data like seismic data, rainfall data, biomedical data, and images. Edges are prominent features in images and their analysis and detection are an essential goal in computer vision and image processing. Indeed, identifying and localizing edges are a low-level task in a variety of applications such as 3D reconstruction, shape recognition, image compression, enhancement, and restoration.



Shearlet and curvelet are a novel directional multiscale mathematical framework which is especially adapted for identification and analysis of distributed discontinuities such as edges occurring in natural images. Multiscale methods based on wavelets have been successfully applied to the analysis and detection of edges. Despite their success, wavelets are however known to have a limited capability in dealing with directional information. The shearlet and curvelet approach is particularly designed to deal with directional and anisotropic features typically present in natural images, and has the ability to accurately and efficiently capture the geometric information on edges. As a result, the shearlet framework provides highly competitive algorithms, for detecting both the location and orientation of edges, and for extracting and classifying basic edge features such as corners and junctions. The shearlet framework provides a unique combination of mathematical rigidness and computational efficiency when addressing edges, optimal efficiency in dealing with edges, and computational efficiency.

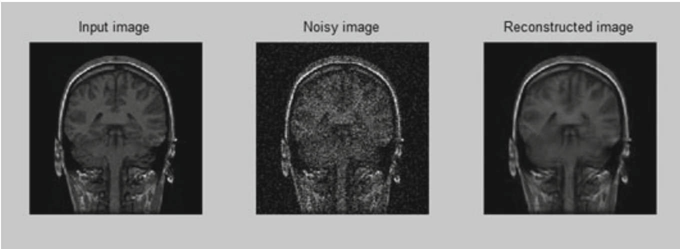
Shearlet in MRI of Brain

Image denoising is a process of recovering the original image from the image corrupted with various types of noise such as Gaussian, speckle, salt and pepper, impulse, etc. Shearlets can be used effectively for image denoising by using various shrinkage rules. The main steps of image denoising are

Table 1.1 Results cont...

MRI brain image	Gaussian noise		
Sigma	Noisy PSNR	PSNR	MSE
10	28.11	34.82	21.45
15	24.61	32.98	32.71
20	22.10	31.72	43.75
25	20.21	30.69	55.51

1. Compute shearlet transform of the noisy image.
2. Apply hard/ soft threshold to the obtained shearlet coefficients.
3. The thresholded shearlet coefficients are subjected to reconstruction to recover the original image (Table 1.1).



1.4 Wavelet Cross-Correlation

Two signals are said to be correlated if they are linearly associated, i.e., if their wavelet spectrum at a certain scale or wavelength is linearly associated.

Broadly speaking, graphs of a vs E(a) for two signals are similar (increase or decrease together).

Wavelet Spectrum Wavelet spectrum E(a) is defined as

$$E(a) = \frac{1}{C_g} \int_{-\infty}^{\infty} W(a, b)^2 db.$$

Wavelet spectrum defines the energy of wavelet coefficients for scale $'a'$.

Peaks in $E(a)$ highlight the dominant energetic scales within the signal.

For details see edited volume [53]. See also authored books [1, 8, 41, 50, 51, 54, 63, 64, 77, 79, 96] and edited volumes [16, 35, 36, 53, 65, 67, 74, 78, 78, 83, 91] for more recent developments in areas mentioned above.

1.5 Fractal Dimension and Predictability

$$D = 2 - H.$$

If the fractal dimension D for the time series is 1.5, there is no correlation between amplitude changes corresponding to two successive time intervals. Therefore, no trend in amplitude can be discerned from the time series and hence the process is unpredictable. However, as the fractal dimension decreases, to 1, the process becomes more and more predictable as it exhibits "persistence".

Predictably indices (denoted by PI_T, PI_P, and PI_R respectively) for temperature, pressure, and precipitation are defined as follows.

$$PI_T = 2|D_T - 1.5|;\; PI_P = 2|D_P - 1.5|,\; PI_R = 2|D_R - 1.5|.$$

Concepts of fractal and multifractal and their relevance to real-world systems were introduced by Benoit B. Mandeldort, for updated references and interesting introduction of the theme we refer to Benoit B. Mandelbort and Richard L. Hudson, 2004.

In many real-world systems, represented by time series, understanding of the pattern of singularities that is a graph of points at which time-series changes abruptly is quite a challenging task. The time series of rainfall data are usually fractal or multifractal. For details see Refs. [42, 43] .

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