A Comparative Deterministic and Probabilistic Stability Analysis of Rock-Fill Tailing Dam

Tanmoy Das and A. Hegde

Abstract This paper presents a comparative study between simple deterministic stability analysis and probabilistic analysis, considering the case of an existing rock-fill tailing dam of 51 m height located in Rajasthan, India. A detailed seismic stability analysis was carried out considering the pseudostatic approach. All the analyses were carried out in CAD-based 2-D limit equilibrium program SLIDE 2D. In order to integrate the soil heterogeneity, stochastic Monte Carlo Simulation (MCS) technique was used. A minimal random number generator developed by Park and Miller (Association for Computing Machinery 31(10):1193–1201, 1988) was used in the analysis. The factor of safety values were calculated using Spencer's method by considering circular failure surfaces. The cohesive strength (c), the angle of friction (φ) and the acceleration (α_h) due to earthquakes were considered as the random variable in the study. For the critical geometry of the slope, the observed factor of safety values in case of upstream slope (1.67) and downstream slope (1.15) were found to be higher than the values specified in the IS 7894 (Code of practice for the stability analysis of the earth dams. Indian Standard, New Delhi, 1975) (reaffirmed in 1997) and ANCOLD (Guidelines on tailings dam design, construction and operation. Australian National Committee on Large Dam, 1999). The seismic deformation analysis was also carried for the downstream slope using the Newmark displacement method. Permanent displacement of the slope was found within the tolerable limits. Further, the results revealed that the spatial variability of the soil significantly influences the factor of safety values. Hence, the present study recommends the probabilistic stability analysis over the deterministic stability analysis for the rock-fill tailing dams.

Keywords Tailing dam · Monte Carlo simulation · Random number generator · Newmark displacement method · Inherent spatial variability

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1 Introduction

Indian mining industries are contributing immensely to the economic growth of the country. About 3100 mines are spread all over India. Out of which, the major portion is used for extraction of nonmetal products and the rest of are metal and fuel products. Demand for minerals has increased tremendously from the past few decades due to infrastructure development and automotive production. As a result, the waste disposal problem has also arisen heavily in the country. Indian Bureau of mines [10] defines tailings as valueless mineral remaining at the "tail" end of the mineral extraction operation. For the retention of tailings slurry, the tailing dams are constructed using mill tailings, mine waste or rock. This tailing slurry has a high potential energy that causes dam break risk. Tailings dam failure leads to loss of life, economic losses and has social as well as environmental impact. According to statistics, the tailing disaster is one of the major disasters in the world after the earthquake, cholera, floods, and bomb blast [25]. Approximately, 147 cases of tailing dam failures were reported at present, out of these 26 cases from Europe and 57 cases from USA [18]. Hence, to ensure the safety of the tailing dam has become the topmost priority of the scientists and engineers. The traditional deterministic analysis methods may not ensure the complete safety of the tailing dams. Soil inherent spatial variability is a major factor that influences the overall stability of the dam. Unfortunately, the conventional methods do not consider the effect of spatial variability in the analysis. Apart from inherent randomness, several features like measurement errors, approximations, assumptions, model parameter uncertainties, absence of geological details make the analysis more uncertain. The reliability-based methods explicitly consider the uncertainties involved in stability analyses [5].

In general, the factor of safety is defined as the ratio between the shear strength of the soil and shear stress required for equilibrium. In deterministic approach, the factor of safety value greater than 1 represents the stable slope. Whereas, in probabilistic approach, mean value and variance of strength parameters are taken into consideration while addressing the uncertainty. In the reliability-based analysis, the factor of safety is expressed in terms of its mean value and variance [8]. In addition to a range of safety factor, probabilistic approach calculates the probability of failure (POF) and reliability index (β) for a particular slope.

One of the well-known robust and conceptually simple tools for analyzing the reliability of slope considering spatial variability is Monte Carlo Simulation (MCS) [6, 27, 28]. Monte Carlo Simulation (MCS) is a numerical process of repeatedly calculating a mathematical or empirical operator in which the variables within the operator are random or contain uncertainty with prescribed probability distribution [2]. The critical slip surface varies spatially and needs to be located for each random sample generated during MCS. A common approach is to locate the critical slip surface from the deterministic analysis and calculate the POF corresponding to that surface [22]. However, the critical surface with the minimum factor of safety may not be the surface with maximum probability of failure [7]. Under the probabilistic framework, POF of any slope is determined on the basis of the slip surface which is greater than the failure probability of any individual potential slip surface present on that particular slope. Many of the previous studies have focused on various slope failure modes caused by stratification. However, the inherent spatial variability of soil properties in soil layer is rarely considered in the analysis [13].

This paper presents a comparative study between simple deterministic stability analysis and probabilistic analysis. The cross section of the tailing dam reported by Sitharam and Hegde [21] was considered, which belongs to Rampura-Agucha mines in Rajasthan, India. In addition, the seismic deformation analysis was also carried out using the Newmark displacement method.

2 Methodology

All the analyses were carried out using the limit equilibrium method based SLIDE 7.0 software. It is a 2D slope stability analysis package with CAD-based graphical interface. SLIDE provides a numerous number of slope stability methods for analysis. Out of these, Spencer's method was chosen for both deterministic as well as probabilistic analysis. Spencer [24] method satisfies both force equilibrium as well as moment equilibrium equations and is applicable to failure surfaces of any shape. This method is also known for its accuracy in calculating the factor of safety [22]. The probabilistic analysis is carried out on the global minimum slip surface located by the regular (deterministic) slope stability analysis. The safety factor was re-computed for 1000 number of samples for the global minimum slip surface, by virtue of a different set of input variables that are randomly generated. MCS is a stochastic technique for using random or pseudo-random numbers to sample from a given input probability distribution. This method is generally used to model the probability of different outcomes in a process that cannot easily be predicted due to the intervention of random variables and uncertainty. MCS method is widely used in geotechnical engineering, especially for slope stability related problem where uncertainty in geotechnical properties plays a major role. Using MCS, the system probability of failure (POF) can be calculated using Eq. 1 as suggested by Li et al. [15] and Jiang et al. [13].

$$P_f = \frac{1}{N_t} \sum_{k=1}^{N_t} I\,(\mathrm{FS}_{\min} < 1) \tag{1}$$

where P_f = probability of failure; N_t = total number of samples generated; $\mathrm{FS}_{\min}$ = minimum factor of safety among the factor of safety values of a large but finite number of potential slip surfaces for a given set of random samples. For a given random sample, I ($\mathrm{FS}_{\min} < 1$) is considered as 1 when $\mathrm{FS}_{\min} < 1$ occurs, otherwise it is equal to zero. POF is nothing but the number of analyses with safety factor less than one, divided by the total number of samples. It is calculated for the most critical failure surface with a minimum factor of safety [19].

Table 1 Soil properties used in the analysis [22]

Material	Property	Distribution	Mean	Min	Max	Standard deviation
Mine muck	(a) Cohesion, c (kPa)	Normal	2	0	4	1
	(b) Friction angle, φ (°)	Normal	39	30	48	3
Mine tailings	(c) Cohesion, c (kPa)	Normal	1	0	2	1
	(d) Friction angle, φ (°)	Normal	35	26	44	3
–	Horizontal seismic coefficient, α_h	Exponential	0.06	0	0.12	–

MCS uses uniformly distributed random numbers with a large number of iterations. The cohesion (c), friction angle (φ), and horizontal seismic coefficient (α_h) were considered as random variables. For cohesion and the friction angle, normal distribution was assumed. The horizontal seismic coefficient was assumed to follow the exponential distribution. Usually, the normal distribution is assumed for the soil parameters [26]. If the occurrences of an event constitute a Poisson process, then the recurrence time would be described by the exponential distribution [2]. Also, if the waiting time before a given event occurs is unknown, it is often appropriate to consider the exponential distribution of the random variables. Hence, it is suitable to use exponential distribution in case of earthquake acceleration coefficient. Table 1 lists out the different random variable considered in the analysis and their properties as reported by Sitharam and Hegde [22].

To simulate a true random analysis, a new seed value was generated after each run, based on the current time on the user system. This means that the analysis results will be different each time after re-running the analysis [19]. A minimal random number generator developed by Park and Miller [17], which can produce an almost bottomless sequence of distinct random numbers (approximately 2 ^ 31) was used in the analysis. It is a type of linear congruential generator (LCG) that operates in the multiplicative group of integers modulo m.

Since the tailing dam was constructed using waste rocks, the possibility of pore water pressure generation is negligible. The upstream face of the dam was covered with impervious clay liner, thus the steady seepage situation also does not occur. Furthermore, in this case, the drawdown situation does not occur [21]. Hence, only the factor of safety against end of construction with and without earthquake condition was analyzed in this study. The minimum threshold values of factor of safety were selected as per IS 7894 [12] (reaffirmed in 1997) and ANCOLD [1].

The tailing dam is located in the earthquake zone-II as per seismic zonal divisions in India (IS 1893: 1984). The value of the horizontal seismic coefficient was considered as 0.06, as per the guidelines of IS: 1893 [11]. The minimum specified

Table 2 Minimum desired values of factor of safety for different loading conditions as per IS 7894 [12] (reaffirmed in 1997)

Case no.	Loading condition of the dam	Slope most likely to be critical	Minimum desired factor of safety
I.	Construction condition with or without partial pool	Upstream and downstream	1
II.	Reservoir partial pool	Upstream	1.3
III.	Sudden drawdown:		
	(a) Maximum head water to minimum with tail water at maximum	Upstream	1.3
	(b) Maximum tail water to minimum with reservoir full	Downstream	1.3
IV.	Steady seepage with reservoir full	Downstream	1.5
V.	Steady seepage with sustained rainfall	Downstream	1.3
VI.	Earthquake condition:		
	(a) Steady seepage	Downstream	1
	(b) Reservoir full	Upstream	1

values of factor of safety as per IS 7894 [12] (reaffirmed in 1997), also conforming to ANCOLD [1] are summarized in Table 2.

Furthermore, seismic displacement analysis was also performed on the downstream slope using the Newmark displacement method. The purpose of the Newmark [16] method is to estimate the slope deformation for those cases where the pseudo-static factor of safety is less than 1 [4]. The acceleration time history of the past earthquake of the particular region was taken into consideration for the displacement analysis. The obtained displacement value was checked with a maximum value of displacement, suggested by several researchers.

3 Results and Discussions

The results of both deterministic and probabilistic analysis are presented in this section. Sitharam and Hegde [21] reported the deterministic stability analysis of the 51 m high tailing dam of the Rampura-Agucha mine. In the present study, the similar dam section was considered. Initially, the deterministic analysis was performed similar to Sitharam and Hegde [21]. The deterministic analysis was also used for the validation. The cross-section and the material properties similar Sitharam and Hegde [21] were used in the deterministic analysis. Figures 1 and 2 shows the results of the

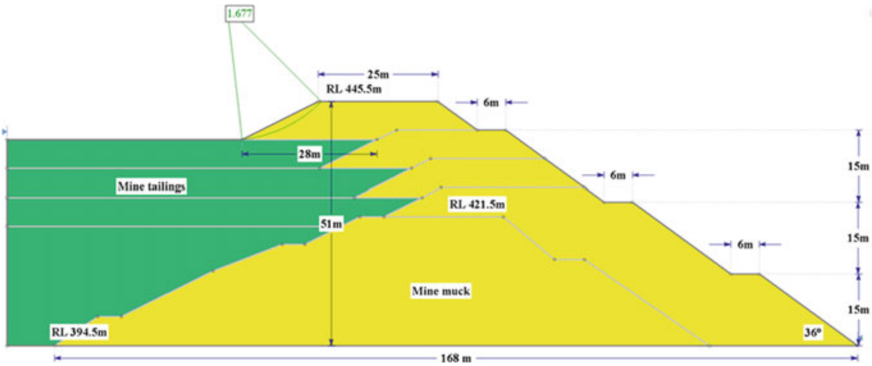


Fig. 1 Deterministic analysis results of upstream slope

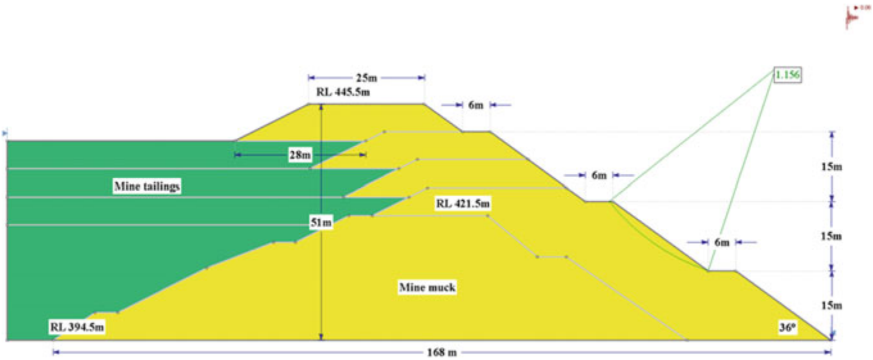


Fig. 2 Deterministic analysis results of downstream slope

deterministic analysis performed on the upstream and downstream slope respectively. The global minimum factor of safety of 1.67 and 1.15 was observed for upstream and downstream slopes respectively. The calculated FS values were higher than the threshold value suggested by IS 7894 [12] (reaffirmed in 1997). These values were found to be in good agreement with the values reported by Sitharam and Hegde [21]. Once the validation was completed, the same cross section was used for the probabilistic analysis.

The probabilistic analysis results for upstream and downstream slopes are shown in Figs. 3 and 4 respectively. Each analysis was re-run for 100 times to study the effect of re-running the analysis, using different random numbers generated from different seed values with respect to time. These results are summarized in Table 3. It was observed that the mean factor safety obtained from the probabilistic analysis is more than that of deterministic analysis. The upstream slope is fully safe with 0% POF. Whereas, the downstream slope had the POF value in the range of 5–9% with a reliability index of 1.3–1.6%. According to Sjoberg [23], the probability of

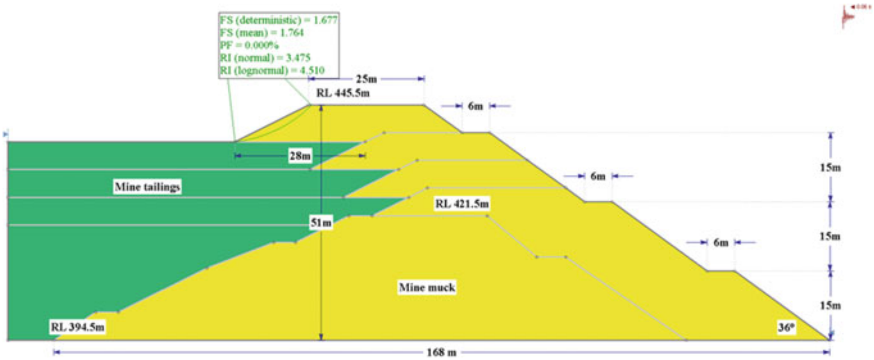


Fig. 3 Probabilistic analysis results of upstream slope

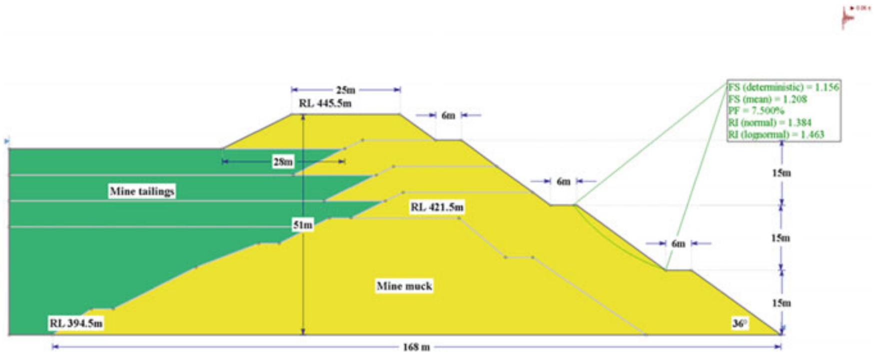


Fig. 4 Probabilistic analysis result of downstream slope

Table 3 Probabilistic analysis results for upstream and downstream slope

Slope type	Deterministic FOS	Mean FOS	POF (%)	Reliability index
Upstream slope	1.677	1.7–1.8	0	3.2–3.6
Downstream slope	1.156	1.19–1.22	5–9	1.3–1.6

failure less than 10% is considered as a safe condition. Hence the embankment was considered safe for height 51 m.

The histogram was plotted for factor safety for both upstream and downstream slope analysis as shown in Fig. 5a, b. The safety factor values less than 1 were highlighted. The POF can be graphically illustrated with the help of the histogram. Here, the POF is equal to the area of the histogram having the factor of safety less than 1, divided by the total area of the histogram.

The factor of safety values were found to be fitted nicely with gamma distribution as compared to other distributions. The gamma distribution is a two-parameter family of continuous probability distribution and exponential distribution. In this study, two

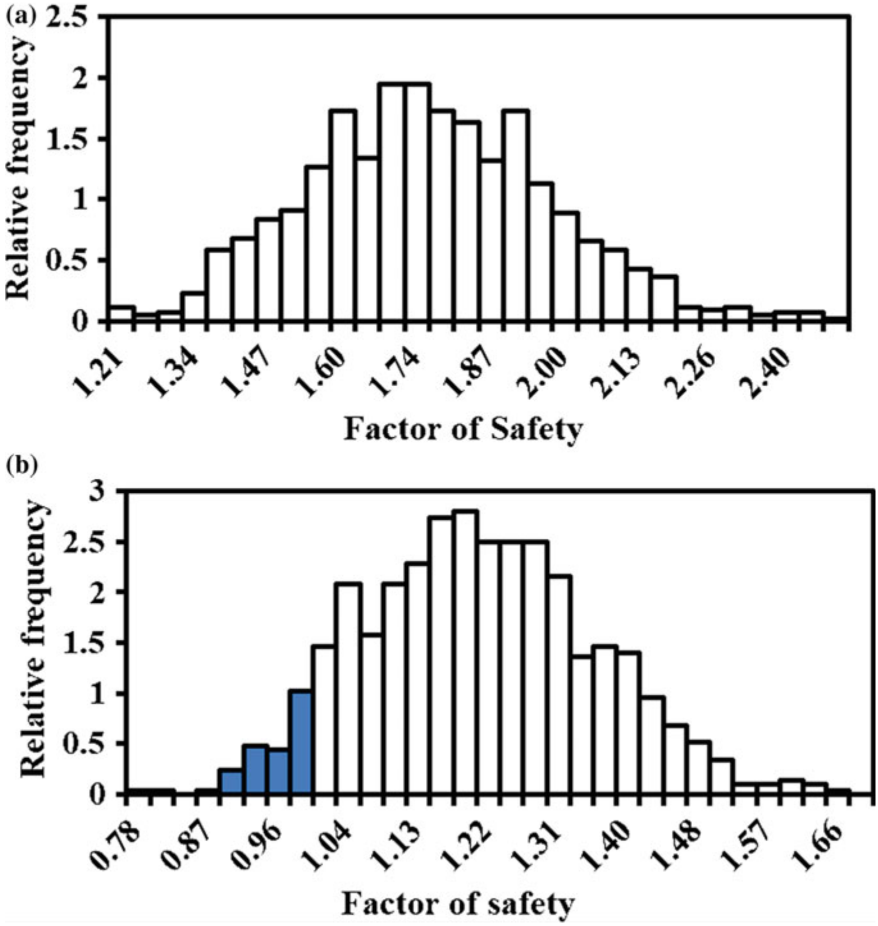


Fig. 5 **a**, **b** Histogram plot of factor of safety values: **a** upstream slope; **b** downstream slope

different distributions in the form of normal and exponential were considered in the case of input parameters. However, the outputs are following gamma distribution.

To analyze the dependency of factor of safety on the soil parameters, scatter plots has been plotted for the downstream slope between the factor of safety and two major soil properties namely, cohesion and friction angle as shown in Fig. 6a, b. The plot between the factor of safety and friction angle shows a strong correlation, as all the points on the plot are converging towards the regression line. The correlation coefficient was found to be 0.8; which is close to 1. On the other hand, less dependency of safety factor on the cohesion was observed with a correlation coefficient of magnitude 0.3. It indicates that the friction angle has a strong influence on the factor of safety values and probability of failure too.

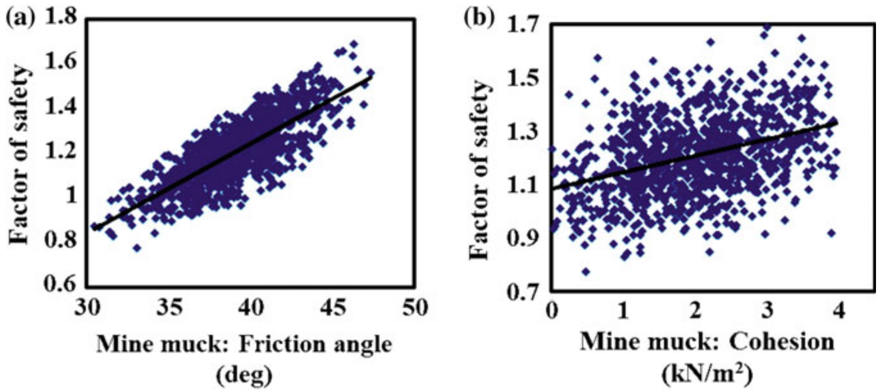
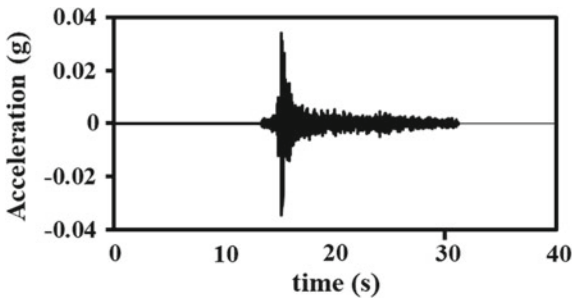


Fig. 6 **a**, **b** Scatter plots: **a** factor safety versus friction angle; **b** factor safety versus cohesion

According to ANCOLD [1], if the factor of safety is lower than 1.3 or 1.5, deformations should be estimated using the simplified deformation analysis. In that context, Newmark displacement analysis was performed for downstream slope. Generally, this method is used to calculate the post-earthquake displacement of a rigid sliding mass, first suggested by Newmark [16]. The analysis assumes one directional displacement of the sliding rigid soil block on a plane surface [5]. The acceleration time history data of the 2006 Alwar earthquake (shown in Fig. 7) was used in the analysis. Alwar is located in the northeastern part of Rajasthan.

The analysis was performed on the downstream slope of the tailing dam. Figure 8 shows the results of the displacement analysis. The obtained displacement was found to be within the tolerable limit suggested by various researchers [3, 9, 14, 20]. Hence, the dynamic stability of the tailing dam was fully satisfied.

Fig. 7 Time history curve of Alwar Earthquake (2006)



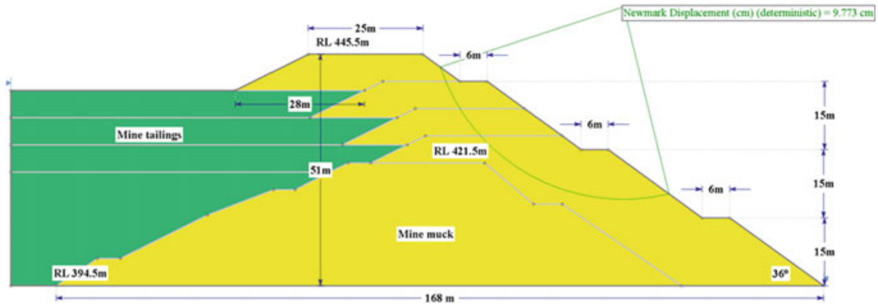


Fig. 8 Seismic displacement analysis of downstream slope using Newmark method

4 Conclusions

The comprehensive slope stability analyses of a rock-fill tailing dam were performed in the present study. The results of the deterministic and probabilistic analysis were compared. In the probabilistic analysis, soil properties like cohesion, friction angle and horizontal seismic coefficient were considered as random variables. The factor of safety values of upstream and downstream slopes were found to be higher than the specified values recommended by IS 7894 [12] (reaffirmed in 1997) and ANCOLD [1]. The mean factor safety obtained from the probabilistic analysis was found to be higher than that of deterministic analysis. Friction angle found to be the predominated factor that highly influences the factor of safety. Further, the probability of failure value was less than 10% for both upstream and downstream slopes. The seismic displacement analysis suggested that slope displacements were within the tolerable limits. In overall, the existing tailing dam was found to be stable against all possible modes of failure. As Duncan et al. [5] stated, knowing the values of both factors of safety and probability of failure is more useful than knowing either one alone. Considering the safety of downstream public, it is always recommended to perform the probabilistic stability analysis along with the deterministic analysis.

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