# Chapter 6
# Project Delivery—PPP Guarantees

## 6.1 Introduction

A public-private partnership (PPP, P3) may be classified as belonging to what are generally referred to as concessional delivery methods, which also incorporate privately-financed initiatives/projects (PFI, PFP), and different build-own-operate-transfer (BOOT) varieties [2]. PPP delivery is popular with the public sector because it enables infrastructure to be designed, constructed and operated using private funding. It can also be used by the private sector, for example coal-washing facilities on mine sites, though the majority of applications appear to be with the public sector. The relevant public sector authority (referred to below as 'the authority') uses the finance and skills of a private sector consortium (referred to below as 'the concessionaire') in this delivery. In return, the concessionaire is given time (a concession period) over which its investment can be recovered.

The following is written for public-private partnership (PPP, P3) toll road projects, but it applies generally to all PPP type projects. The uncertainty, risk and fairness in PPP agreements in toll road projects, specifically the financial aspects of such agreements, may be addressed in part by having options. Within the context of financial agreements and toll roads, the option for example may translate to adjusting future revenue in response to uncertain and shifting future road demand. The option right may come about in return for some direct or indirect cost (premium) to the option holder.

Depending on how a PPP agreement is structured, the concessionaire carries differing degrees of financial risk, primarily arising from road usage, patronage or demand uncertainties. While capital costs and ongoing operation costs are reasonably predictable, demand is not, and is influenced by the magnitude of the tolls being charged, travel times, vehicle operating cost, and the availability of alternative roads and transport. The viability analysis of the project from the concessionaire's viewpoint, among other things, looks at the financial risk carried, and attempts to reduce



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this risk through adjusting the agreement between the parties. The concessionaire may request subsidies or guarantees from the authority. The authority may, in turn, request reciprocal guarantees. An agreement involving guarantees, if properly structured, allows the risk to each party to be managed, and make the project more viable. If the risk being carried is considered unacceptable, either party might withdraw from the project.

The existing literature focuses on revenue-related guarantees introduced to deal with uncertainties in demand during the operational phase. These guarantees can take different forms. Third parties, such as insurance companies, may be involved, but the intent is the same whether third parties are involved or not, namely to assist the authority and the concessionaire with viability and risk management. Such guarantees may be valued using an options analysis. In general, the revenue is uncertain, being based on road usage, patronage or demand. This uncertainty needs to be captured in any analysis. Typically, the literature uses traditional financial market options techniques, and applies these by analogy. Each option is presented and analysed in stand-alone papers, and relies on the high level of mathematical skills of their authors. In contrast, this chapter presents a single unifying approach for analysing all PPP toll road options.

The chapter is structured as follows. The literature on PPP toll road options is first reviewed. The book's unifying approach is then presented. Each existing proposed PPP toll road option is presented and interpreted in terms of this book's approach. Discussion and conclusions follow. The chapter is written in terms of two parties to the PPP agreement, namely the authority and the concessionaire.

It is emphasized that only options dealing with financial aspects are dealt with. Options of a physical nature (although involving money), for example, in terms of increasing the number of lanes of a road or delaying the construction of a road are addressed in Chap. 4.

The chapter provides an original and unified approach to PPP toll road options. The chapter will be of interest to anyone involved in PPP toll road projects.

## 6.2 Background

PPP financial agreements between the authority and the concessionaire can involve a range of guarantees or adjustments under differing names or descriptors:

- Minimum revenue guarantee (MRG). The guarantee involves the authority paying the concessionaire if the actual toll revenue falls below a pre-agreed threshold. This puts a limit on the revenue downside for the concessionaire.
- Buyout. The authority holds the right to buy the concession back before the end of the concession period, at a predetermined exercise price, subject to certain conditions.
- Revenue-sharing. The authority holds the right to claim a percentage share of excess revenue when the revenue exceeds an agreed upper limit or threshold.

- Restrictive competition guarantee. This guarantee secures a road's revenue against loss caused by competing roads.
- Collar. A collar combines both lower and upper revenue thresholds to create a band. The concessionaire holds an option on low revenue. The authority holds an option on high revenue.
- Traffic floor and ceiling (TFC). Traffic floor and ceiling is based on pre-agreed lower (floor) and upper (ceiling) traffic levels. The concessionaire holds the traffic floor option, while the authority holds the traffic ceiling option. It is the same as a collar, but with two differences—an up-front cost or premium, and a guarantee covering only part of any traffic shortfall or traffic exceedance (referred to as a partial coverage guarantee).
- Toll adjustment mechanism (TAM). This is similar in intent to MRG (a guaranteed minimum revenue to the concessionaire), however TAM gives the concessionaire the right to adjust tolls to achieve a desired revenue.

Within existing publications, traffic or revenue is commonly assumed to follow a time series such as geometric Brownian motion with associated volatility measure. Monte Carlo simulation may be used to generate realizations, or the Black-Scholes equation might be used if applicable. By contrast, this book's approach does not have restrictive assumptions on time series or volatility, nor does it require exercise prices to be deterministic or at a single point in time; uncertainty is incorporated through variance estimates. There is also no need to distinguish between option types, for example a call (equivalent to a purchase) option or put (equivalent to a sale) option [2, 3], rather each case considers the cash flows from the viewpoint of whoever holds the option.

## 6.3 Cash Flows

Typically, for road PPPs, the cash flows involving an option can be thought of in terms of a cash outflow and a cash inflow at time $i = T$, the time of exercising the option (Fig. 6.1). $1 \leq T \leq n$, where n is the concession period. In Fig. 6.1, $Y_{T1}$ and $Y_{T2}$ refer to cash inflow and cash outflow respectively at time T, such that, with respect to Appendix 2.11.1, $X_T = Y_{T1} - Y_{T2}$. $Y_{T1}$ and $Y_{T2}$ may be deterministic or probabilistic, and are from the viewpoint of the option holder. The origins of these cash flows differ in each application, and are explained below. In some applications a cash flow may be revenue foregone, while in other applications, the two cash flows may represent the cash flows associated with exercising and not exercising an option.

Where cash flows connected to the option extend beyond T, that is, over time periods $i = T + 1, T + 2, \ldots, n$, then these are collectively discounted to time T, such that

$$E[Y_{T1}] = \sum_{i=T+1}^{n} \frac{E[Y_{i1}]}{(1+r)^{i-T}}$$

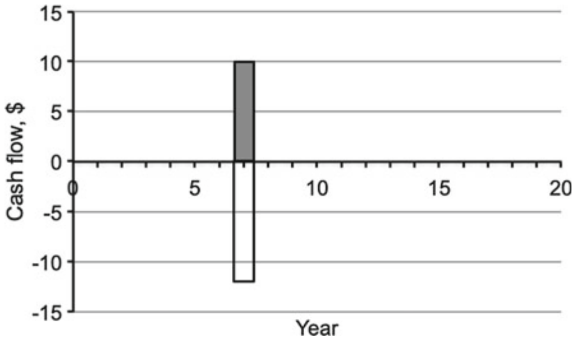


**Fig. 6.1** Example cash flows involved in an option at time T

$$\mathrm{Var}[Y_{T1}] = \sum_{i=T+1}^{n} \frac{\mathrm{Var}[Y_{i1}]}{(1+r)^{2(i-T)}} + 2 \sum_{i=T+1}^{n-1} \sum_{j=i+1}^{n} \frac{\mathrm{Cov}[Y_{i1}, Y_{j1}]}{(1+r)^{i+j-2T}}$$

Similar expressions apply for $Y_{T2}$. In this sense, $Y_{T1}$ and $Y_{T2}$ become an 'equivalent' cash inflow and an 'equivalent' cash outflow, respectively, in year T.

The content of Appendix 2.11.2 applies. The option value follows from Eq. (1.1).

## 6.4 Existing PPP Road Options

### *6.4.1 Outline*

Existing PPP toll road option cases are grouped according to the following descriptors: Minimum revenue guarantee (MRG); Buyout; Revenue-sharing; Restrictive competition guarantee; Collar; Traffic floor and ceiling (TFC); and Toll adjustment mechanism (TAM). In each case, this book's analysis is compared with the existing literature. To do this comparison, assumptions are made as compatible as possible with the existing literature, but necessarily not exactly the same, primarily because this book's approach uses variance instead of volatility, and there is no universal agreement as to what volatility should be used or to the conversion between volatility and variance [8]. Appendix 2.11.4 gives the result used to guide the conversion between variance and volatility. In the general case, the daily traffic, toll and any thresholds or equivalent could be anticipated to vary over time. The form of the analysis presented in this chapter remains the same, should any of these vary from year to year or be constant from year to year [4].

### 6.4.2 Specific Notation

The following gives the particular toll road notation used.

*Revenue related*

| | |
|---|---|
| $C_i$ | maximum revenue guarantee (threshold) in year i (pre-agreed); a revenue cap; related to $TC_i$ where a traffic ceiling is defined |
| f | revenue cap growth rate (constant cap growth per year) |
| $f_c$ | minimum threshold growth rate (percent/year, compounding yearly) |
| $F_i$ | minimum revenue guarantee (threshold) in year i (pre-agreed); a revenue floor; related to $TF_i$ where a traffic floor is defined |
| g | revenue growth rate (constant revenue increase per year) |
| $g_c$ | revenue growth rate (percent/year, compounding yearly) |
| $R_i$ | revenue in year i |
| $\alpha, \beta$ | percentages, or fractions $0 \leq \alpha, \beta \leq 1$ |

*Traffic related*

| | |
|---|---|
| $TC_i$ | traffic ceiling guarantee in year i, $TC_i = Tcr \times E[v_i]$ |
| Tcr | traffic ceiling ratio, used in establishing the traffic ceiling guarantee $TC_i$ |
| $TF_i$ | traffic floor guarantee in year i, $TF_i = Tfr \times E[v_i]$ |
| Tfr | traffic floor ratio, used in establishing the traffic floor guarantee $TF_i$ |
| $Toll_i$ | toll per vehicle in year i (possibly, pre-agreed) |
| $Tollcap_i$ | toll cap in year i (pre-agreed) |
| $v_i$ | traffic (vehicles/year) in year i |
| $\gamma$ | traffic growth rate (constant number of vehicles per year) |
| $\gamma_c$ | traffic growth rate (percent/year, compounding yearly) |
| $\phi_c$ | toll growth rate (percent/year, compounding yearly) |

*General*

| | |
|---|---|
| c, a | superscripts denoting concessionaire and authority, respectively |
| wi, wo | superscripts denoting with-invoking and without-invoking TAM, respectively |

Where the subscript i is omitted, then the variable takes a constant value for all i.

*Currencies.* USD US dollar, $; HKD Hong Kong dollar (HKD1.00 ≈ USD0.13); CNY Chinese yuan (CNY1.00 ≈ USD0.16); INR Indian rupee (INR1.00 ≈ USD0.016)

### 6.4.3 Minimum Revenue Guarantee

Minimum revenue guarantee (MRG) refers to a mechanism for limiting the revenue downside to the concessionaire, resulting from revenue uncertainty. The authority provides a guarantee of a minimum annual revenue (a threshold value), $F_i$, to the concessionaire. This can be viewed in terms of the concessionaire holding an option in each year of the concession period. The concessionaire exercises each option, and claims the revenue shortfall, when the actual annual revenue is lower than this defined minimum threshold. The total value of having the yearly options is the sum of the yearly option values.

In any year $i = T$ (Fig. 6.1), the option is only exercised if the revenue shortfall, $X_T > 0$, where: $Y_{T1}$ = the minimum guarantee (threshold) value ($F_i$ at $i = T$); and $Y_{T2}$ = the revenue ($R_i$ at $i = T$).

*Example.* Adapting Brandao and Saraiva [1], using a comparable traffic volume standard deviation equal to 30% of its expected value: concession period, $n = 25$ years; interest rate, $r = 15\%$ per annum; minimum guarantee level (constant), $E[F_i] =$ \$1.5B, $Var[F_i] = 0$; revenue (constant), $E[R_i] =$ \$1.9B, $Var[R_i] = (\$0.57B)^2$. Here, $F_i$ is 80% of $E[R_i]$. $F_i$ and $R_i$ are assumed to repeat for all $i = 1, 2, \ldots, n$. For $i = T$ (using Appendix 2.11.2), $E[X_T] = -\$0.4B$, and $Var[X_T] = (\$0.57B)^2$. From this, E[PW] and Var[PW] can be obtained and the option value for any year calculated (Fig. 6.2—solid curve).

*Comparison with the literature*. The sum of the yearly option values for 25 years is approximately \$0.52B. This is higher by approximately 0.4%, as a proportion of the threshold value, when compared with Brandao and Saraiva [1]. The yearly option values vary with the level of guarantee as shown in Fig. 6.2. The level of guarantee, that constitutes the minimum threshold in year i, is defined as a percentage of the expected annual revenue in year i, namely $E[R_i]$. The yearly option values increase as the level of guarantee increases, as anticipated, and this trend agrees with the observations of Brandao and Saraiva [1].

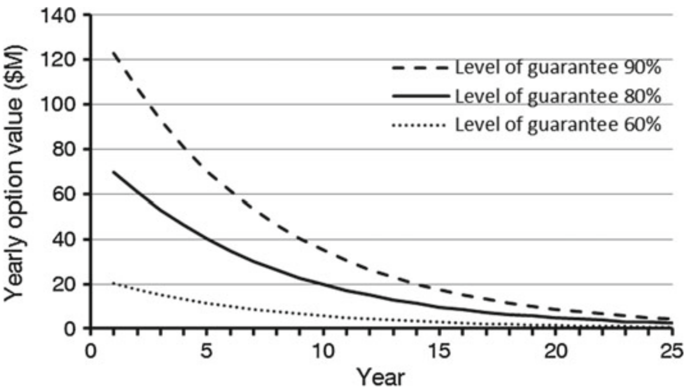


**Fig. 6.2** Minimum revenue guarantee example; yearly option value versus time for different levels of guarantee

### 6.4.4 Buyout

A buyout option gives the authority the right to buy the concession back before the end of the concession period, at a predetermined buyout cost (exercise price), subject to certain conditions. The buyout option may be exercised, at time T, only if the updated value of the revenue remaining till the end of the concession period (that is, over the period from i = T to i = n) exceeds a pre-agreed level (equivalently, a buyout cost). The updated value is the (updated) present worth (discounted to T) of the remaining revenue cash flows. Expressed differently, this means taking all the cash flows from i = T to i = n and discounting them to time T to give a collective discounted value—the updated value. The exercising may be defined to occur at a pre-specified year within the concession period, or in any year of the concession period.

In any year i = T (Fig. 6.1), the option is exercised if $X_T > 0$, where: $Y_{T1}$ = the revenue ($R_i$) for years i = T, T + 1, T + 2, …, n discounted to year T; and $Y_{T2}$ = the buyout cost ($F_i$ at i = T).

*Example.* Consider the buyout option example of Power et al. [10], using a comparable traffic volume standard deviation equal to 25% of its expected value: traffic (million vehicles) in year 1, $E[v_1] = 36.085$; $Var[v_1] = 9.021^2$; toll per vehicle (constant), Toll = \$10; traffic growth (million vehicles per year), $E[\gamma] = 5$; $Var[\gamma] = 1^2$; buyout cost at year 6 (buyout multiplier of 1.5), $F_6$ = \$8,600M; interest rate, r = 11.6% per annum; concession period, n = 25 years. The buyout cost (here a constant) equals the product of a buyout multiplier (here, 1.5) and the present worth (at year 0) of revenue over the 25-year concession period (here, \$5,700M). The revenue in year i follows from Results (I) and (II) of Appendix 6.8, with $E[R_1]$ = \$361M and $Var[R_1] = (\$90M)^2$.

Consider exercising the buyout option in year 6, interpreted from the viewpoint of the option holder (the authority). The buyout cost (a known amount), $E[Y_{62}] = F_6$ = \$8,600M, and $Var[Y_{62}] = 0$ based on Power et al. [10]. In calculating $Y_{61}$ according to Appendix 6.8, the $R_i$, i = 6, 7, 8, …, 25, could be assumed to be strongly correlated. Discounting, E[PW] = –\$399M; $Var[PW] = (\$541M)^2$. From Eq. (1.1), OV = \$72.5M.

Figure 6.3 (solid curve) shows how the option value varies with year of exercising, assuming that the buyout cost remains the same at \$8,600M. The current set of values might be considered favourable to the authority over the concessionaire in terms of early buy out, and would need negotiation, and in particular negotiation perhaps on a variable buyout cost. Figure 6.3 shows the influence of altering the value of the buyout multiplier through using example buyout multipliers of 1.5 (as in the numerical example above) and a slightly larger 1.6. The buyout cost equals the product of the buyout multiplier and the present worth (at year 0) of revenue over the 25-year concession period.

With a buyout cost, $F_i$, defined as a multiplier of the present worth of the revenue remaining till the end of the concession period (buyout cost reducing with time),


**Fig. 6.3** Buyout example (constant buyout cost); option value versus time for different levels of buyout

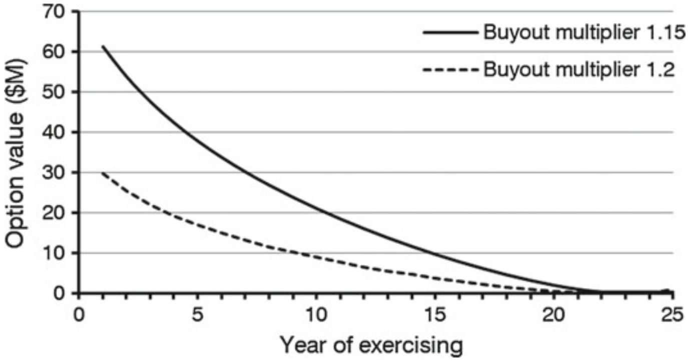


**Fig. 6.4** Buyout example (reducing buyout cost); option value versus time for different levels of buyout

Fig. 6.4 shows how the option value varies with year of exercising and buyout multiplier magnitude. This result is similar to revenue-sharing. Whereas revenue-sharing allows the authority to collect excess revenue if revenue is higher than an upper threshold, buyout gives the authority the right to buy back the project, and collect the full revenue for the remainder of the concession period.

*Comparison with the literature.* This book's approach gives an option value (based on exercising in year 6, with a fixed buyout cost) higher by approximately 0.5% as a proportion of the buyout cost, when compared with Power et al. [10].

Figure 6.3 shows that the optimal value of the buyout option occurs near the middle of the concession period, where the plot peaks. This reflects the increasing yearly revenue with time, countered by the decreasing present worth of future revenue with time. This optimality finding agrees with the conclusion of Power et al. [10]. The option value varies with year of exercising and decreases with increasing level of buyout cost for both constant and varied buyout cost. These trends agree with the observations of Power et al. [10].

### *6.4.5 Revenue-Sharing*

A revenue-sharing option gives the authority (as option holder) the right to claim a percentage share, β, of the revenue that exceeds an upper limit (maximum revenue cap) in any year. The cap is adjusted upwards by a constant amount each year. The concessionaire retains a (1 − β) share of this excess revenue. The option is exercised in any year when the revenue exceeds the pre-agreed limit. The total value of having the yearly options is the sum of the yearly option values.

In any year $i = T$ (Fig. 6.1), the option is exercised if the excess revenue above the cap, $X_T > 0$, where: $Y_{T1}$ = the revenue ($R_i$ at $i = T$); and $Y_{T2}$ = the maximum revenue cap ($C_i$ at $i = T$). The authority receives β percent of the revenue excess, that is, $\beta X_T$. The option value is calculated based on the present worth derived from an expected value $\beta E[X_T]$, and a variance $\beta^2 Var[X_T]$. (This follows from the results in Appendix 6.8.)

*Example.* Consider the example of Song et al. [12]: revenue in year 1, $E[R_1]$ = CNY138M; $Var[R_1]$ = (CNY48M)$^2$; revenue growth per year, $E[g]$ = CNY20M; $Var[g]$ = (CNY8M)$^2$; revenue cap in year 1, $E[C_1]$ = CNY180M; $Var[C_1]$ = (CNY42M)$^2$; revenue cap growth per year, $E[f]$ = CNY27M; $Var[f]$ = (CNY9M)$^2$; interest rate, $r = 15\%$ per annum; concession period, $n = 25$ years. In Song et al. [12], a single volatility has been assumed; with revenue, revenue growth, revenue cap and revenue cap growth all random variables, comparable standard deviations of 30%, 40%, 25% and 30%, respectively, of their expected values have been assumed here. Results (III) and (IV) of Appendix 6.8 apply. The revenue,$R_i$, and the revenue cap, $C_i$, $i = 1, 2, \ldots, 25$, could be anticipated to be independent, and this is the case assumed in the calculations here, but the calculations are essentially the same should that not be the case.

Figure 6.5 (solid curve) shows the option value in any year, calculated for β = 80%. The change in the decline rate of the option value, most noticeably around year 10, occurs because E[PW] and Var[PW] decline at different rates over time. The trend, however, remains downward.

*Comparison with the literature.* While exact numerical comparison is not possible because of the method used in Song et al. [12], the trend in Fig. 6.5, showing how the yearly option value varies with percentage sharing β, is consistent with the trend given in Song et al. [12]. The yearly option value varies with the level of the revenue cap, $E[C_1]$, as shown in Fig. 6.6. This trend is also consistent with that given in Song et al. [12].

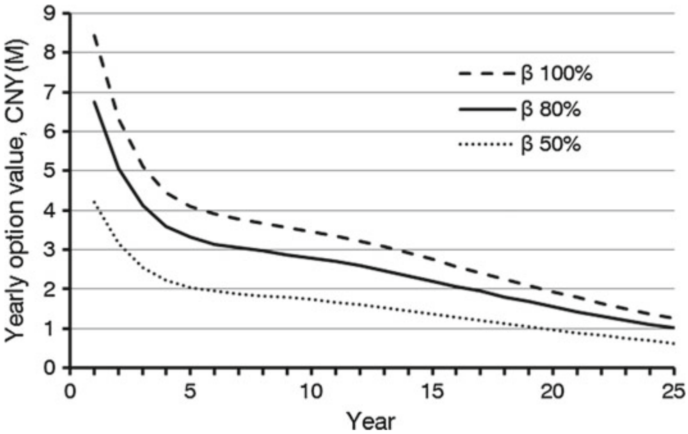


**Fig. 6.5** Revenue-sharing example; yearly option value versus time for different β

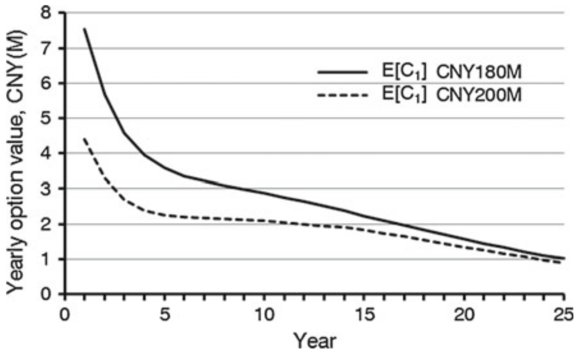


**Fig. 6.6** Revenue-sharing example; yearly option value versus time for different revenue caps; β = 80%

## *6.4.6 Restrictive Competition Guarantee*

Traffic on one road can be influenced by the presence of alternative roads. Alternative roads compete with each other for vehicles and, where roads are tolled, lead to lower revenue. In order to address revenue loss caused by competing roads, a restrictive competition guarantee (non-compete clause) could be used. Here, the authority promises either to not approve any competing road during the concession period, or to provide reimbursement to the concessionaire. This guarantee secures the said road's revenue against loss caused by competing roads.

If the authority approves and/or builds a competing toll road in year i, the concessionaire (option holder) can exercise the option to claim reimbursement from the authority. The authority then compensates the concessionaire a percentage, α, of the revenue shortfall when the revenue is lower than a pre-agreed amount (revenue threshold) in any year. The revenue threshold is adjusted each year. The value of a restrictive competition guarantee is the sum of the option values in each year of the concession period.

In any year i = T (Fig. 6.1), the option is exercised if the revenue shortfall, $X_T > 0$, where: $Y_{T1}$ = the minimum revenue threshold ($F_i$ at i = T); and $Y_{T2}$ = the revenue ($R_i$ at i = T). The concessionaire receives α percent of the revenue shortfall, that is, $\alpha X_T$. The option value is calculated based on the present worth derived from an expected value $\alpha E[X_T]$, and a variance $\alpha^2 Var[X_T]$. (This follows from the results in Appendix 6.8.)

*Example.* Consider an example adapted from Liu et al. [9], using a comparable traffic volume standard deviation equal to 40% of its expected value: revenue in year 1, $E[R_1]$ = CNY207M, $Var[R_1] = (CNY76M)^2$; revenue growth per year (rate), $g_c$ = 7.5%; minimum threshold at year 1, $E[F_1]$ = CNY140M, $Var[F_1] = (CNY20M)^2$; threshold growth per year (rate), $f_c$ = 8%; interest rate, r = 5% per annum; concession period, n = 20 years; reimbursement (percentage of revenue shortfall), α = 70%. Results (V) and (VI) of Appendix 6.8 apply. $F_i$ and $R_i$, i = 1, 2, …, 20, are assumed independent, but need not be.

Figure 6.7 (solid curve) shows how the option value varies with time. The upward trend shown is because, in the example calculations, the project revenue increases at a lower rate than the minimum threshold. Accordingly, the option has a higher likelihood of being exercised later in the concession period. Altering the interest rate leads to different option value trends (Fig. 6.7). Higher interest rate values lead to lower present worths, and in turn to lower option values. Figure 6.8 shows how the option value varies with α, the reimbursement percentage of revenue shortfall. Lower α values lead to lower option values, as anticipated. The influence of altering minimum threshold values is shown in Fig. 6.9. The yearly option values increase as the level of the revenue threshold increases.

*Comparison with the literature.* This book's approach gives a summed yearly option value higher by approximately 0.2% as a proportion of the revenue threshold value,

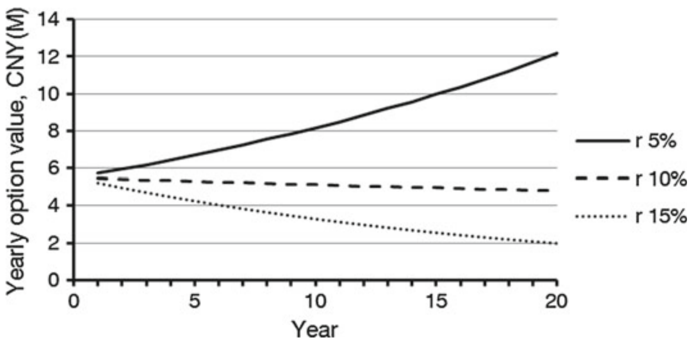


**Fig. 6.7** Restrictive competition guarantee example; yearly option value trends with different interest rates

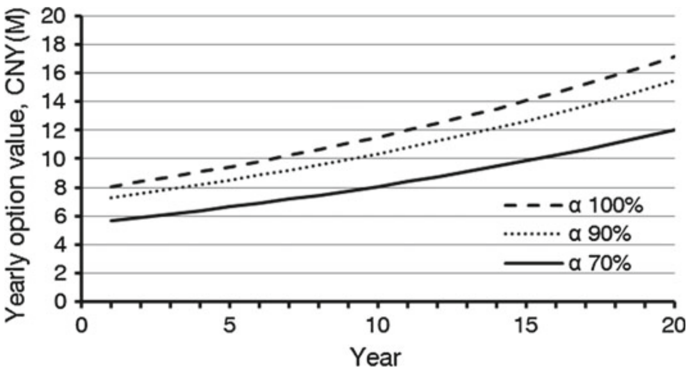


**Fig. 6.8** Restrictive competition guarantee example; influence of α

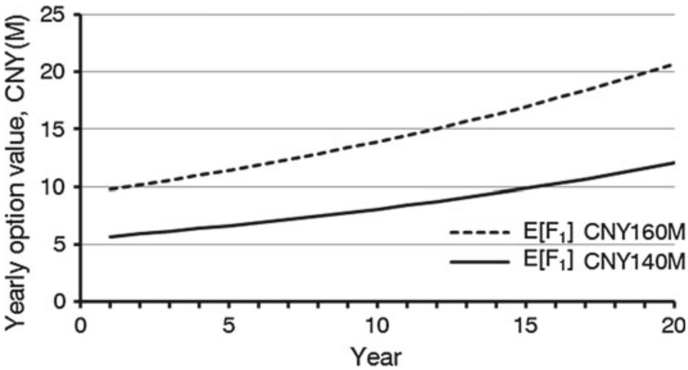


**Fig. 6.9** Restrictive competition guarantee example; influence of altering the revenue threshold

when compared with Liu et al. [9]. The summed yearly option value is obtained by using Eq. (1.1) for each year and then adding these values over all years of the concession period. The trend shown in Fig. 6.9 agrees with the observations of Liu et al. [9].

### *6.4.7 Collar*

A collar combines both lower and upper revenue thresholds to create a band. In any year, i, there are two possible options, and depending on the actual revenue, one or neither of these is exercised. The concessionaire holds an option on low revenue. If the actual revenue falls beneath a minimum revenue guarantee or revenue floor, $F_i$, the concessionaire has a right to claim the revenue shortfall. The authority holds an option on high revenue. If the actual revenue is higher than a maximum revenue guarantee or revenue cap, $C_i$, the authority has a right to collect the excess revenue. Revenue occurring within the thresholds' envelope or band is unaffected. Alternative to floors and caps in revenue, the situation may be expressed in terms of floors and caps in traffic.

The floor, $F_i$, and cap, $C_i$, can be determined in either of two ways, in terms of: zero-cost to both parties; or partial cost to one party, but not to the other party.

No cost to each party can be obtained by setting, in each year, the premium of the concessionaire's option equal to the premium of the authority's option. This is referred to as a zero-cost collar. The two option values are used as proxies for the two premiums. $F_i$ and $C_i$ are adjusted such that the two option values applying to the concessionaire and the authority are the same value. This might be done by first choosing $F_i$, calculating the associated option value, and then using this option value in a reverse calculation to give $C_i$.

For the partial cost collar, $F_i$ and $C_i$ are negotiated between the two parties. $F_i$ and $C_i$ can be adjusted to produce a narrower or wider band, with consequent different premiums and option values. Higher $F_i$ and lower $C_i$ separately lead to increased option values. The difference in the premiums represents a cost to one party. However, the parties may agree that there is no up-front cost to either party. In such cases, $F_i$ and $C_i$ might be adjusted according to what might be perceived as a 'fair' allocation of uncertainty to each party.

Using this book's cash flow approach, lower and upper revenue threshold values can be set asymmetrically in each year, and different for all years, without requiring any additional work, assumptions or considerations.

Introduce the superscript notation of c and a for concessionaire and authority, respectively. In any year $i = T$ (Fig. 6.1), the concessionaire's option is exercised if the revenue shortfall, $X_T^c > 0$, where: $Y_{T1}^c$ = the minimum guarantee (threshold) value ($F_i$ at $i = T$); and $Y_{T2}^c$ = the revenue ($R_i$ at $i = T$). This is the same as the minimum revenue guarantee above. In any year $i = T$ (Fig. 6.1), the authority's option is exercised if the revenue excess, $X_T^a > 0$, where: $Y_{T1}^a$ = the revenue ($R_i$ at $i = T$); and $Y_{T2}^a$ = the maximum revenue cap ($C_i$ at $i = T$). This is the same as revenue-sharing above where $\beta$ equals 100%.

*Example.* Consider the zero-cost collar example of Shan et al. [11], using a comparable traffic volume standard deviation equal to 25% of its expected value: traffic in year 1 (vehicles/day × 365 days), $E[v_1] = 25$ k; $Var[v_1] = (6.25\ k)^2$; traffic growth per year (rate), $\gamma_c$ (years 2–5) 6%, (years 6–10) 3.5%, (years 11–35) 2%; toll per vehicle in year 1, $Toll_1 = \$1.30$; toll growth per year (rate), $\phi_c$ (years 2–5) 5%, (years 6–10) 3%; (years 11–35) 2%; minimum revenue guarantee, year i, $F_i = 78.6\%$ of $E[R_i]$; interest rate, $r = 7.5\%$ per annum; concession period, $n = 35$ years. $F_i$, here, is deterministic and set as a percentage of the expected revenue (adapted from [6, 7, 11]. (For the minimum revenue guarantee example above, $F_i$ is taken as constant for all i.)

*Concessionaire's option.* In year 1, for example, $E[Y^c_{12}] = -\$11.86M$, $Var[Y^c_{12}] = (\$2.97M)^2$, $E[Y^c_{11}] = \$9.32M$. Then, $E[X^c_1] = -\$2.54M$ and $Var[X^c_1] = (\$2.97M)^2$ using the results in Appendix 2.11.2. Discounting, $E[PW] = -\$2.36M$, $Var[PW] = (\$2.76M)^2$. From Eq. (1.1), the concessionaire's option value is \$0.296M. The calculation is repeated for other years. Figure 6.10 (solid curve) shows the concessionaire's option value in any year.

*Authority's option.* The authority's option value, in any year, would ordinarily be calculated based on the expected value, E[PW], and variance, Var[PW], of the present worth of the cash flows associated with the authority's option. However here, the authority's option value is set equal to the concessionaire's option value (just calculated). See Fig. 6.10 for how these option values vary over time. If $R_i$ is assumed to follow a symmetrical probability distribution, then setting $F_i$ and $C_i$ equidistant from $E[R_i]$ will lead to option values for the authority and the concessionaire being the same. Where $R_i$ does not follow a symmetrical probability distribution, then this $C_i$ value will need adjusting.

*Comparison with the literature.* Comparing available results at years 1, 2, 5 and 10, this book's approach gives option values higher by approximately 1% as a proportion of the concessionaire's minimum revenue guarantee in the corresponding years, when compared with Shan et al. [11]. The yearly option values (same for the concessionaire and the authority) vary with the level of guarantee as shown in Fig. 6.10. The base case referred to in Fig. 6.10 corresponds with the above example values. The alternative case referred to in Fig. 6.10 has the following different values: the expected daily traffic in year 1 is reduced to 23 k vehicles; the traffic growth rate (years 1–5) is reduced to 5%; and the minimum revenue guarantee is increased to 88.3% of the revenue, with the maximum revenue guarantee set equidistant at 111.7%. Moving the guarantee levels closer to $E[R_i]$ increases the option values; while a direct numerical comparison is not possible, this trend is consistent with intuition and the trend given in Shan et al. [11].

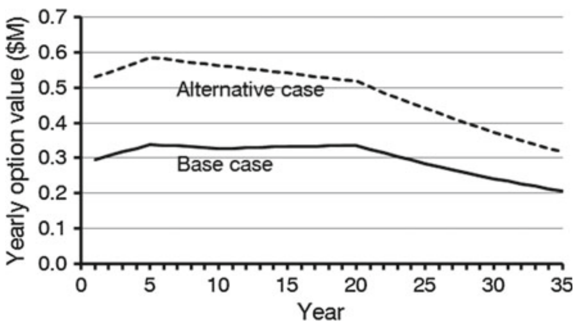


**Fig. 6.10** Collar example; yearly option values—concessionaire's (or authority's) options—with different input data. [Alternative case—reduced traffic, reduced traffic growth rate, raised minimum revenue guarantee percentage]

### 6.4.8 *Traffic Floor and Ceiling*

Traffic floor and ceiling (TFC) is based on pre-agreed lower (floor) and upper (ceiling) traffic levels or thresholds. These two traffic thresholds (floor and ceiling) can be converted to revenue thresholds by multiplying each traffic threshold by the toll ($F_i = TF_i \times Toll_i, C_i = TC_i \times Toll_i$). The concessionaire holds the traffic floor option, while the authority holds the traffic ceiling option, as with the collar. Percentages $\alpha$ and $\beta$ are nominated ($0 \leq \alpha, \beta \leq 1$), such that if the traffic in any year is lower than the pre-agreed floor level, the floor option is exercised and the concessionaire (option holder) claims a percentage $\alpha$ of the revenue shortfall, while if the traffic is higher than the maximum pre-agreed ceiling level, the ceiling option is exercised and the authority (option holder) receives a percentage $\beta$ of the revenue excess.

In each year, the TFC involves two options based on lower and upper traffic levels or thresholds (equivalently, revenue levels or thresholds). It is the same as a collar [11], but with two differences—an up-front cost or premium, and a guarantee covering only part of any traffic shortfall or traffic exceedance (a partial coverage guarantee):

- The zero-cost collar has no premium requirement from either party, while the TFC requires premium payments from both the concessionaire and the authority. Premiums, while affecting project viability, do not enter the option value calculations, and hence do not alter the above statements on collar options.
- The collar provides a full coverage guarantee (equivalently, ratios $\alpha, \beta = 1$) above the upper threshold, and below the lower threshold, whereas TFC offers partial revenue protection ($0 \leq \alpha, \beta \leq 1$).

Negotiation between the parties is needed on the traffic floor and ceiling values, and the lower and upper percentages $\alpha$ and $\beta$.

Using this book's cash flow approach, traffic floor and ceiling values can be set asymmetrically in each year, and different for all years, without requiring any additional work, assumptions or considerations.

In any year $i = T$ (Fig. 6.1), the concessionaire's option is exercised if the revenue shortfall, $X_T^c > 0$, where: $Y_{T1}^c$ = the minimum guarantee (threshold) value ($F_i$ at $i = T$); and $Y_{T2}^c$ = the revenue ($R_i$ at $i = T$). The concessionaire receives $\alpha$ percent of the revenue shortfall, that is, $\alpha X_T^c$. The option value is calculated based on the present worth derived from an expected value $\alpha E[X_T^c]$, and a variance $\alpha^2 Var[X_T^c]$. (This follows from the results in Appendix 6.8.) This is the same as the collar above, but with $\alpha$ introduced. In any year $i = T$ (Fig. 6.1), the authority's option is exercised if the excess revenue above the cap, $X_T^a > 0$, where: $Y_{T1}^a$ = the revenue ($R_i$ at $i = T$); and $Y_{T2}^a$ = the maximum revenue cap ($C_i$ at $i = T$). The authority receives $\beta$

percent of the revenue excess, that is, $\beta X_T^a$. The option value is calculated based on the present worth derived from an expected value $\beta E[X_T^a]$, and a variance $\beta^2 Var[X_T^a]$. (This follows from the results in Appendix 6.8.) This is the same as the collar above, but with β introduced.

*Example.* Consider the example of Iyer and Sagheer [6], using a traffic volume standard deviation equal to 25% of its expected value: traffic in year 1 (vehicles/day × 365 days), $E[v_i] = 20{,}654$, $Var[v_i] = (4957)^2$; traffic growth per year (rate), $\gamma_c = 6\%$; toll per vehicle, Toll = INR28.50; traffic floor ratio, Tfr = 80%; traffic ceiling ratio, Tcr = 130%; traffic floor at year i, $TF_i = Tfr \times E[v_i]$; traffic ceiling at year i, $TC_i = Tcr \times E[v_i]$; lower coverage ratio, $\alpha = 50\%$ of $X_i^c$; upper coverage ratio, $\beta = 50\%$ of $X_i^a$; interest rate, r = 12% per annum; concession period, n = 20 years. Result VII of Appendix 6.8 applies. For example in year 8:

*Concessionaire's option.* $E[Y|_{82}^c] = -INR323.1M$, $Var[Y|_{82}^c] = (INR78.2M)^2$, $E[Y|_{81}^c] = 80\%$ of $E[Y|_{82}^c] = INR258M$.

*Authority's option.* $E[Y_{81}^a] = INR323.1M$, $Var[Y_{81}^a] = (INR78.2M)^2$, $E[Y_{82}^a] = 130\%$ of $E[Y_{81}^a] = -INR420M$.

The option values in each year are shown in Fig. 6.11.

*Comparison with the literature.* This book's approach gives concessionaire's and authority's summed yearly option values lower by approximately 1% and 0.7%, respectively, as a proportion of the sum of revenue threshold values for 20 years, when compared with Iyer and Sagheer [6]. Figures 6.12 and 6.13 show how the concessionaire's and authority's option values vary with Tfr and Tcr, respectively. The yearly option values increase as a result of a higher minimum floor guarantee and a lower maximum ceiling guarantee, and this trend is consistent with the results of Iyer and Sagheer [6].

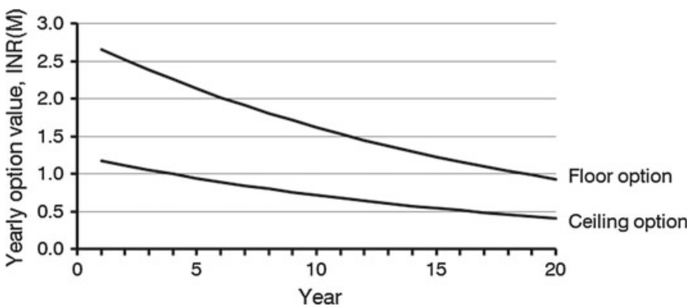


**Fig. 6.11** Traffic floor and ceiling example; yearly floor and ceiling option values, Tfr = 80%, Tcr = 130%

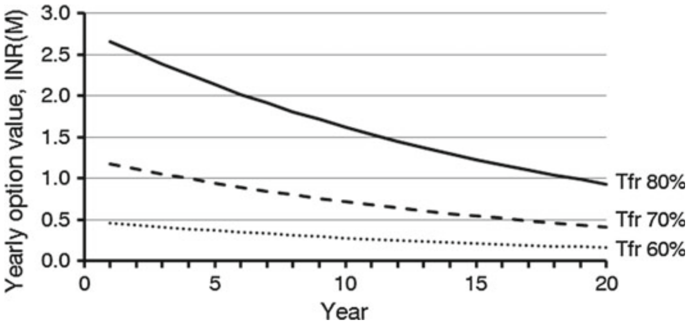


**Fig. 6.12** Traffic floor and ceiling example; yearly floor option value; influence of Tfr

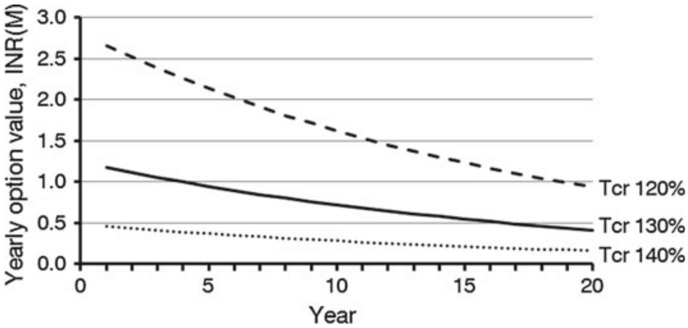


**Fig. 6.13** Traffic floor and ceiling example; yearly ceiling option value; influence of Tcr

### *6.4.9 Toll Adjustment Mechanism*

A toll-adjustment mechanism (TAM) is similar in intent to MRG, in that it provides protection for the concessionaire from revenue loss. Whereas MRG gives the concessionaire a guaranteed minimum revenue (the threshold), TAM gives the concessionaire the right to adjust tolls to achieve a revenue level negotiated between the concessionaire and the authority.

Tolls increase annually at a fixed growth rate. However, tolls can be raised further (up to a pre-agreed toll cap) by the concessionaire if the TAM is invoked, that is if the option is exercised. The TAM, if invoked at time $i = T$ when the actual revenue (to the concessionaire) in year T is lower than a pre-agreed level, gives the concessionaire the right to adjust (raise) the toll for the remaining periods $T + 1, T + 2, \ldots, n$. In any year, the toll cap represents the maximum toll that the concessionaire may charge, whereas the pre-agreed minimum revenue level defines the TAM exercise trigger. Both the toll cap and the pre-agreed minimum revenue level are included in the PPP agreement, and are established by negotiation between the concessionaire and the authority. The exercising can only be done once within the concession period—either (depending on the PPP agreement): at a pre-defined year; or in any year. Result VIII of Appendix 6.8 applies. It could be assumed that the traffic would decrease as the toll increases. Accordingly, the traffic with TAM adjustment (raises the toll) might be estimated lower than the traffic without TAM.

In any year $i = T$ (Fig. 6.1), the option is exercised, and the tolls raised for the remainder of the concession period, if the actual revenue in year T, $R_T$, is less than the agreed minimum revenue threshold. Then: $Y_{T1}$ = the revenue ($R_i^{wi}$) for years $i = T + 1, T + 2, \ldots, n$ discounted to year T; $Y_{T2}$ = the revenue ($R_i^{wo}$) for years $i = T + 1, T + 2, \ldots, n$ discounted to year T; and $X_T = Y_{T1} - Y_{T2}$ = revenue difference (with and without).

*Example.* Consider the TAM example of Chen et al. [5], scenario 5, using a comparable traffic volume standard deviation equal to 20% of its expected value. *Without adjusted tolls:* traffic in year 1 (million vehicles), $E[v_1^{wo}] = 73$; $Var[v_1^{wo}] = 14.6^2$; traffic growth per year (rate), $\gamma_c^{wo} = 2\%$ of $v_1^{wo}$; toll per vehicle in year 1, $Toll_1^{wo}$ = HKD18; toll growth per year (rate), $\phi_c = 6\%$ of $Toll_1^{wo}$. *With adjusted tolls (up to toll cap—an upper limit):* traffic in year 1 (million vehicles), $E[v_1^{wi}] = 36.5$; $Var[v_1^{wi}] = 7.3^2$; traffic growth per year (rate), $\gamma_c^{wi} = 2\%$ of $v_1^{wi}$; toll cap per vehicle in year i, $Tollcap_i^{wi}$, (all HKD) 20 (years 2, 3), 25 (4, 5), 30 (6, 7), 35 (8, 8, 10, 11), 45 (12), 55 (13), 65 (14), 75 (15), 85 (16–21), 100 (22–30); interest rate, r = 12% per annum; concession period, n = 30 years. The superscripts wi and wo denote with-invoking and without-invoking TAM, respectively. Result IX of Appendix 6.8 applies.

Assume that the TAM is exercised in year 10, and the toll cap applies in the following years. Then E[PW] = −HKD3,011M, and Var[PW] = $(HKD3022M)^2$. From Eq. (1.1), OV = HKD253.4M. For other years of exercising, Fig. 6.14 shows the corresponding option values. The plot's shape is influenced by the toll cap altering over time.

*Comparison with the literature.* The maximum yearly option value (year 11) is approximately 2% greater than that given in Chen et al. [5], as a proportion of the corresponding and equivalent minimum revenue threshold. Adapting two more

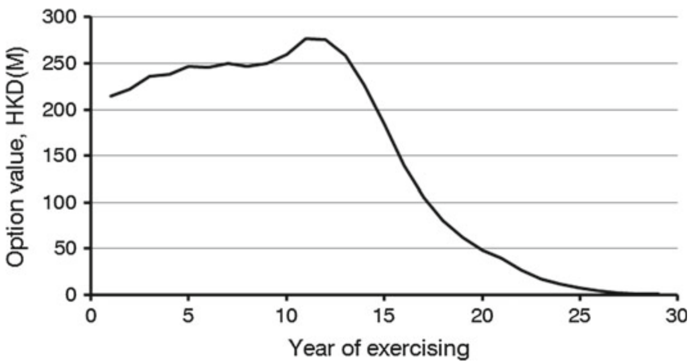


**Fig. 6.14** TAM example; option value versus year of exercising

assumed scenarios (scenarios 6 and 8) of Chen et al. [5], the yearly option values increase as the level of traffic volume and toll caps increase, and this trend agrees with the observations in Chen et al. [5].

## 6.5 Discussion

For all the different option cases analysed above, when compared with existing literature relevant to each case, this book's approach gives option values essentially the same. Supplementary sensitivity-style analyses, conducted by altering the values of the case variables, showed option value trends to be the same with the existing literature. The differences between this chapter's option values and those of existing publications, as a proportion of similar exercise costs, is less than a few percent. Exact agreement would not be anticipated, because of different assumptions applying between those in this book's approach (using variance) and those of existing publications (using volatility). Nevertheless, in this chapter's analysis, assumptions as compatible as possible with the existing literature are made.

A main difference between this book's approach and that of the existing literature is that this book's approach was developed especially for real options and uses conventional discounted cash flow thinking familiar to many. This is compared with the existing literature, which uses methods from the financial markets literature and draws analogies between underlying market variables and infrastructure variables. The book's approach accommodates uncertainty through the use of variances rather than volatility as in the existing literature. Volatility choice is based on analogies with the financial markets methods, and the term may not have direct transference to infrastructure.

The probability distribution for present worth can be assumed to be any appropriate distribution, though a normal distribution was used for convenience in the above calculations. Users of the book's approach are free to adopt an asymmetric probability distribution if they believe that reflects present worth more appropriately. The book's approach can deal with cash inflows and outflows with different levels of correlation, and different over time (that is, varying over the concession period), deterministic cash flows and interest rate variability and uncertainty.

The option valuation is done from the option holder's point of view, whether this is the concessionaire or the authority. Cash flows are established from the option holder's viewpoint. Cash flows, typically, are in terms of those that would exist without exercising the option, and those resulting from exercising the option. The cash flows, depending on the guarantee or agreement, may only be for the year in which the option applies, or over the years extending from the year of exercising the option to the end of the concession period. The approach is the same irrespective of the PPP guarantee or agreement, for example whether a minimum guarantee or a maximum guarantee. There is also no need to distinguish option type, as occurs in the financial markets literature.

## 6.6 Closure

Guarantees within PPP road project agreements are used by both the authority and the concessionaire to assist in improving project viability and to deal with risk and fairness. The guarantees are one way of addressing the uncertainty in usage or demand that is experienced by roads. Such guarantees can be analysed as options. Heretofore, every different PPP guarantee was presented in terms of its own one-off options analysis, relying on the high level of mathematical skills of the presenters.

With this as background, the chapter demonstrated the following:

- All guarantees (minimum revenue guarantee, buyout, revenue-sharing, restrictive competition guarantee, collar, traffic floor and ceiling guarantee, and toll adjustment mechanism) and their options analyses can be treated in the same way.
- The book's probabilistic present worth cash flow approach provides an original unified approach to PPP toll road options.
- The approach makes no unrealistic assumptions, the level of mathematics necessary is minimal, with knowledge of financial market options analysis techniques not being required.
- Both the concessionaire and the authority are readily able to evaluate, using the book's approach, the impact of any guarantees, providing a basis for the parties to negotiate their PPP agreement.
- The approach permits options that can be exercised yearly or discretely throughout a project's concession period, and these can be treated in a common way.
- The book's approach provides a way forward for analysing all PPP toll road options.

## 6.7 Extensions

The book's approach is extendable to all infrastructure types, not just roads, and to guarantees which are different to those covered in this chapter, and which may be proposed in the future. Heretofore, the possibilities have been limited because of the restrictive mathematics and assumptions of financial markets methods and establishing analogies with infrastructure.

## 6.8 Appendix: Some Common Results

For a relationship between any general variables, Z and $X_s$, and general constants, $a_s$, of the form, $Z = \sum_{s=1}^{m} a_s X_s$, then,

$$E[Z] = \sum_{s=1}^{m} a_s E[X_s] \qquad Var[Z] = \sum_{s=1}^{m} a_s^2 Var[X_s] + 2\sum_{s=1}^{m-1}\sum_{t=s+1}^{m} a_s a_t Cov[X_s, X_t]$$

These results are used in the chapter for expressions as follows:

| | |
|---|---|
| I | $R_i = Toll_i \times v_i$ |
| II | $R_{i+1} = R_1 + (i-1)Toll_i\gamma$ |
| III | $R_{i+1} = R_i + (i-1)g$ |
| IV | $C_{i+1} = C_i + (i-1)f$ |
| V | $R_i = (1+g_c)^{i-1}R_1$ |
| VI | $F_i = (1+f_c)^{i-1}F_1$ |
| VII | $Y_{i2} = (1+\gamma_c)^{i-1}Y_{12} \quad Y_{i1} = (1+\gamma_c)^{i-1}Y_{11}$ |
| VIII | $R_i^{wo} = Toll_i v_i^{wo} \quad R_i^{wi} = Tollcap_i v_i^{wi}$ |
| IX | $R_i^{wo} = Toll_i^{wo} v_i^{wo} = (1+\phi_c)^{i-1}(1+\gamma_c^{wo})^{i-1}Toll_1^{wo}v_1^{wo}$<br>$R_i^{wi} = Toll_i^{wi} v_i^{wi} = (1+\gamma_c^{wi})^{i-1}Tollcap_1^{wi}v_1^{wi}$ |

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