Chapter 3
Real Options

3.1 Introduction

A real options analysis values future flexibility in a real asset context. The flexibility relates to having a future choice between alternatives, or having the future ability to influence the directions of a project or venture, for example to contract, abandon or expand a project in order to improve envisaged outcomes. Real options have wide relevance throughout industry, including applications in resources, research and development, patents, contracts and adaptation to shifting climates and demographics. Examples in this book emphasize the diverse range of applications.

Historically, the published literature on real options adopted established financial options methods, such as the Black-Scholes equation (Black-Scholes) and binomial lattices and equivalent calculations using Monte Carlo simulation (as distinct from Monte Carlo simulation as an alternative to a second order moment analysis as mentioned in Chap. 2), and applied them analogously to real assets. This is understandable because financial options analysis existed before real options were formalized. Unfortunately, many assumptions applying to financial options do not translate well to real options.

Using the book's cash flow approach and Eq. (1.1), this chapter covers a complete collection of plain and compound real options—contract, abandon, choice, switch, delay/deferment, sequential, parallel, and rainbow options. A common treatment of all real options is given. This is demonstrated on examples. It is shown that there is no need to distinguish between the different real option types; each is reduced to its respective cash flows, and analysed using the approach outlined in Chaps. 1 and 2.

The literature on real options is very large, but its usage has been inhibited by its adoption of financial options assumptions and modelling, with an associated high level mathematical requirement.

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Real options share some of the characteristics of financial options, but also have some important differences. Chief among the differences are:

- Financial options rely on the movement in the price of a market underlying and this movement is largely not able to be influenced by the investor, whereas the cash flows in a real option can derive from and be dependent on influenceable sources including management of the real asset.
- Real options and real assets are not traded, are specific to an organization, are not proprietary in nature, and generally have no market comparables.
- Traditional financial options analysis is based on the price of a market underlying described by a time series with volatility. For most real options there is no equivalent market underlying (but rather, there is a real asset). Volatility may have no transferable meaning for non-traded assets.
- Financial options are embedded in contracts specifically defining exercise prices and expiry dates, and exercising rules are clearly defined, whereas real options commonly are embedded within management decisions with discretionary parameters.
- The exercising of financial options can bring about an instant return, whereas exercising and the benefits of exercising a real option may occur over a long period.

For these reasons, traditional methods of analysis, developed for financial options, are not directly applicable to real options and have been criticized. These differences with financial options, together with the analogy assumptions (real equivalents to underlyings, exercise price and volatility), lead to much of the criticism of adopting traditional financial option pricing tools for real options, even though generally the value of a real option is dependent on similar influences to those that determine the value of a financial option. But having said that, many people are comfortable using traditional financial options methods to analyse real options situations.

A number of writers have pointed out the deficiencies in using traditional financial option pricing methods for real options [7]. A main deficiency centres on the treatment of volatility of the asset price or value, with no real agreement as to how this should be done, and the assumption that it remains constant over time. Other deficiencies relate to deterministic exercise prices, the assumption of geometric Brownian motion or similar, a market place for the asset, lognormal assumptions, exercising instantaneously rather than over a period of time, known limits on the time of exercising rather than times that might be poorly defined, continuous time discounting, the zero-sum outcome, and exercising not affecting the asset value [7].

The chapter outlines a common treatment of all real options. This is demonstrated on examples. Because, among other things, all the cash flows in the following are random variables, financial options methods are not applicable. However, for the expand and contract example calculations, comparisons with Black-Scholes values are given, based on restricting the assumptions in the examples. Numerical comparisons in the examples are given as the difference in the calculated option values, normalized with respect to the exercise cost (or equivalent); the volatility-variance conversion given in Appendix 2.11.4 is adopted for these comparisons. Other numerical comparisons are given in Carmichael et al. [7] and Carmichael [5, 6].

3.2 Real Option Types

Real option types might be classified as:

- Single or plain (stand-alone) options (expand, contract, abandon, switch, delay/defer, rainbow); or
- Compound or combined (multiple plain) options (sequential, parallel, choice).

Compound options may have embedded independent and dependent options. An option is independent if its value can be calculated separately from what is happening with other options. An independent option may affect another option's cash flows or prevent the exercising of another, and may be a pre-requisite or co-requisite for another option. The value of a collection of independent exercisable options is the sum of the individual option values. Dependence between any of the options in this collection will give a total value different to this.

The cash flows and option values for each option type are discussed here. It is seen that there is no need for a classification distinguishing between plain option types, because the same thinking applies to all.

The notation follows that given in Chap. 2. The cash flow component k in period i, is denoted Y_{ik}, $i = 0, 1, 2, \ldots, n$; $k = 1, 2, \ldots, m$. These get discounted to the present worth, PW, following Appendix 2.11.2. A normal distribution is assumed for PW in calculating the option value, OV. Interest rate, r, is set at 10% for the examples.

3.3 Option to Expand

A typical expand scenario is where (future) additional capital investment leads to an enlarged operation, perhaps to exploit a new or growing market, and generating additional cash inflow. Examples include upgrading infrastructure, follow-on development of a manufacturing project and investment in research and development.

The relevant cash flows to use in the option calculation, for a greenfield-style expansion, are the cost of expansion (negative) together with any cash flows resulting from the expansion (usually both positive and negative). Where an expansion affects existing cash flows (brownfield), the relevant cash flows to be used in the analysis are the cost of expansion together with the difference between what would have existed (assuming no expansion) and what new cash flows result from the expansion (usually both positive and negative).

Example Consider a possible expansion at year 3. The expansion cost is estimated as: $E[Y_{32}] = -\$90k$ and $Var[Y_{32}] = (\$12k)^2$. This will generate both positive and negative cash flows. The positive cash flows are estimated as: $E[Y_{i1}] = \$20k$ and $Var[Y_{i1}] = (\$5k)^2$; the negative cash flows are estimated as: $E[Y_{i2}] = -\$4k$ and $Var[Y_{i2}] = (\$1k)^2$, i = 4, 5, …, 10 (Fig. 3.1). Y_{i1} and Y_{i2}, i = 4, 5, …, 10 are assumed independent, while the X_i, i = 3, 4, …, 10 are assumed well correlated.

Discounting, $E[PW] = -\$9.10k$, $Var[PW] = (\$9.64k)^2$. The upside of the present worth distribution is given in Fig. 3.2. From Eq. (1.1), OV = \$0.89k.

(Black–Scholes cannot deal with this example directly. To compare with Black-Scholes, a deterministic expansion cost (strike or exercise price) is required—for this situation, \$90k at year 3. Consistent with established real options practice, the asset value at year 3 is obtained from the net cash flows after year 3 discounted to year 3. The asset value at year 0 is the asset value at year 3, discounted to year 0. Asset value variance is converted to volatility as indicated above. Black-Scholes gives an option value approximately 1.5% lower, as a proportion of the exercise price.)

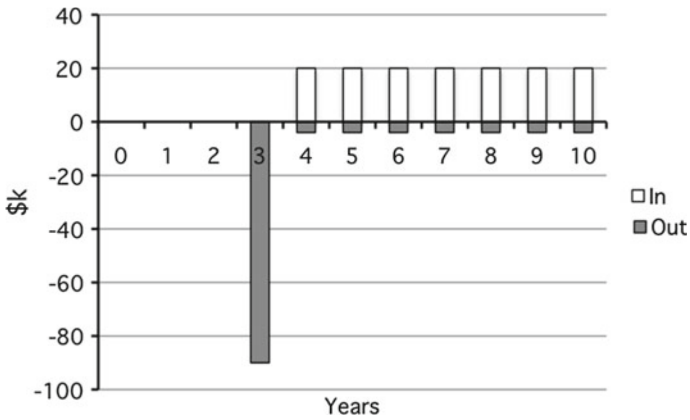

Fig. 3.1 Example—option to expand; expected values of cash flows

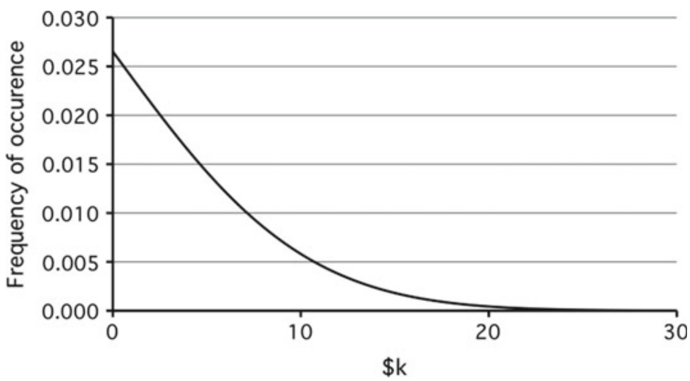

Fig. 3.2 Upside of PW distribution; expand example

3.4 Option to Contract

An example contract scenario involves an organization cutting back on operations, downsizing, or selling part of the organization (in the future), as a result of an anticipated or real downturn in demand. It could involve the income from the sale of part of an asset, or the cost involved with disposal of part of an asset. The option is exercised if the benefits from contracting outweigh those from continuing the status quo. The option to contract can protect an organization from losses in the future resulting from the effects of an unfavourable market. Contraction might allow resumption to former or greater production levels through expansion further into the future, when the market improves.

The relevant cash flows to use in the option calculation are the difference between what would have existed (assuming no contraction) and what new cash flows result from the contraction (usually both positive and negative), together with any sale income or disposal costs.

Example An option to contract, for example through a reduction in operations at a future time, may result in benefits foregone. Consider a possible sale of part of an operation at year 2. The sale price is estimated as: $E[Y_{21}] = \$71k$ and $Var[Y_{21}] = (\$5k)^2$. Without the sale, the anticipated net positive cash flows of the operation have expected values and variances, respectively, of \$27k and $(\$3k)^2$, giving: $E[Y_{i2}] = (–)\$27k$ and $Var[Y_{i2}] = (\$3k)^2$, i = 3, 4, …, 7. With the sale, the net positive cash flows are anticipated to reduce to: $E[Y_{i1}] = \$8k$ and $Var[Y_{i1}] = (\$1k)^2$, i = 3, 4, …, 7 (Fig. 3.3). Y_{i1} and Y_{i2}, i = 3, 4, …, 7, and the X_i, i = 2, 3, …, 7 are assumed well correlated.

The option to contract is dealt with in the same way as the option to expand. From the investor's viewpoint the cash flows might be thought of as being reversed.

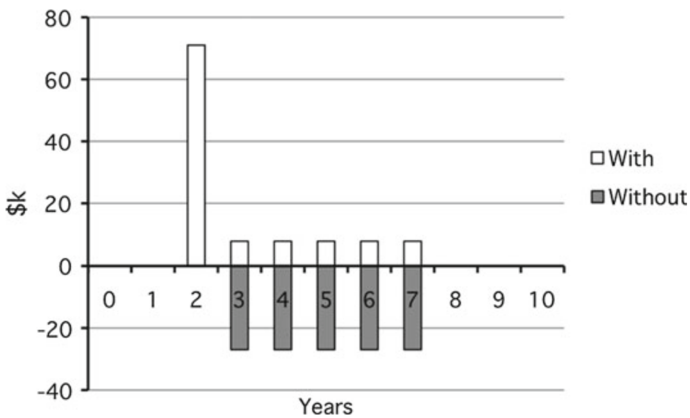

Fig. 3.3 Example—option to contract; expected values of cash flows

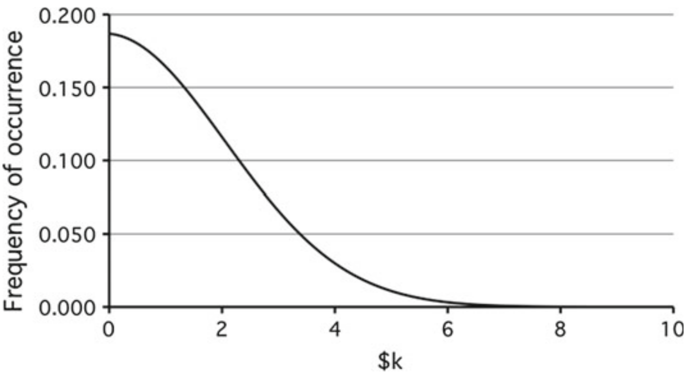

Fig. 3.4 Upside of PW distribution; contract example

Discounting, E[PW] = −\$0.85k, Var[PW] = (\$2.13k)^2. The upside of the present worth distribution is given in Fig. 3.4. From Eq. (1.1), OV = \$0.80k.

(Black–Scholes cannot deal with this example directly. To compare with Black-Scholes, a deterministic sale price (strike or exercise price) is required—for this situation, \$71k at year 2. Consistent with established real options practice, the asset value at year 2 is obtained from the net cash flows after year 2, discounted to year 2. The asset value at year 0 is the asset value at year 2, discounted to year 0. Asset value variance is converted to volatility as indicated above. Black-Scholes gives an option value approximately 1% lower, as a proportion of the exercise price.)

3.5 Option to Abandon

The option to abandon might be regarded as a particular case of contraction (or contraction might be regarded as partial abandonment). At abandonment, there may be an additional one-off cash flow—the salvage cost (negative, cash outflow) or residual (positive, cash inflow) value. The option is exercised if the savings made from abandonment outweigh future revenues that would have existed without abandonment. An example of abandonment is the early termination of a research and development project due to anticipated failure should the research continue. Abandonment might occur at any stage of any project.

Example The option to abandon is analysed as a particular case of contraction, or contraction might be regarded as partial abandonment. The cash flows are similar in nature for contraction and abandonment. For example, consider the case analysed in Sect. 3.4. If this was abandonment rather than contraction, then $E[Y_{i2}]$ and $Var[Y_{i2}]$, i = 3, 4, …, 7, are all zero (Fig. 3.5).

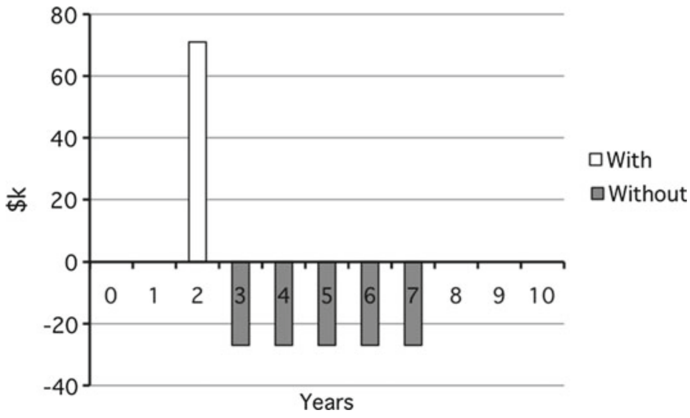

Fig. 3.5 Example—option to abandon; expected values of cash flows

Sell-back agreement. Consider acquiring an asset accompanied by a sell-back option after one year. Two possible option situations or cases relate to: (1) the sell-back price is pre-agreed (certain, or deterministic); and (2) the sell-back price is uncertain and depends on the market. Case 1 closely mirrors a traditional financial options framework, and might be dealt with analogously using Black-Scholes. However Case 2 does not fit a traditional financial options framework.

To demonstrate numerically, for Case 1, let the sell-back price be $E[Y_{11}] = \$1M$, while for Case 2, assume that this is the (market-based) estimated sell-back price expected value with a (market-based) estimated sell-back price variance of $Var[Y_{11}] = (\$0.1M)^2$. Let the estimated expected value and variance of the asset value at the end of year 1 be $E[Y_{12}] = (-)\$1.1M$, and $Var[Y_{12}] = (\$0.05M)^2$ respectively. The cash flow diagram, for both cases, from the viewpoint of the holder of the sell-back option is shown in Fig. 3.6.

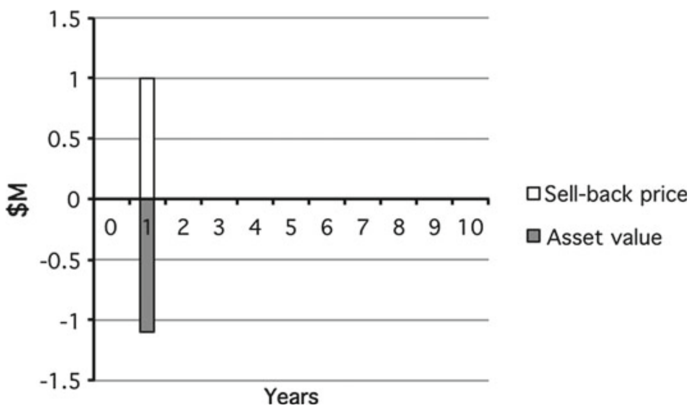

Fig. 3.6 Cash flows—sell-back example

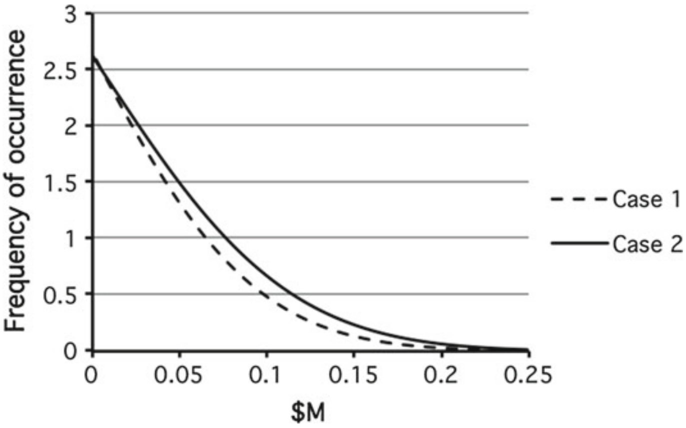

Fig. 3.7 Upsides of PW distributions—sell-back example

Discounting Y_{11} and Y_{12} gives E[PW] = –$0.091M for both cases, and Var[PW] of $(\$0.091M)^2$ and $(\$0.102M)^2$ respectively for Case 1 and Case 2. This gives an option value, OV, of $0.0076M and $0.0103M respectively for Case 1 and Case 2.

(For Case 1, Black-Scholes gives an option value approximately 2.5% different, as a proportion of the sell-back price. Generally, it has been found that agreement with Black-Scholes is better with call-style options rather than put-style options, as in this example. This is believed to be due mainly to the differing distribution assumptions between Black-Scholes and this book's approach (the values given here are based on a normal distribution for PW). Assuming a different PW distribution, in conjunction with the book's approach, could give closer agreement with Black-Scholes.)

Figure 3.7 plots the PW upsides, showing the difference between the two cases. The option value for Case 2 is higher because of the increased uncertainty, which is reflected in the higher variance of PW

3.6 Choice in Option Types

At a future point in time, there may be, say, the possibility either to expand, to contract, to abandon or to continue operations unchanged. For example, dependent on ore prices, yield and reserves, a mining company might expand or contract operations, abandon the mine, or continue operations unchanged. In this scenario, only the most attractive option is exercised. That is, each option is valued separately and the option yielding the highest value, at the future time, is exercised if it is better than continuing operations unchanged. Each option is mutually exclusive because it is not possible to exercise more than one option at a given time; for example, an organization cannot simultaneously expand and contract a project. Also, exercising one of the possible options alters the subsequent investment cash flows, and creates

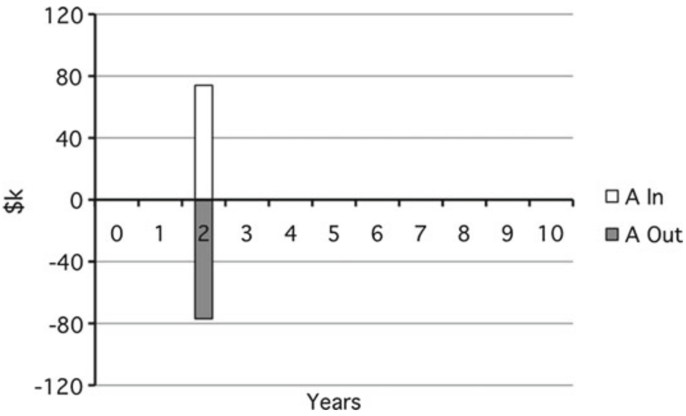

Fig. 3.8 Example—expand option A at year 2; expected values of cash flows

the need to recalculate the values of the other options based on the new cash flows. In principle, the value of having a choice in options should be higher than or equal to the value of any of the component options, because of the enhanced flexibility.

Care needs to be exercised when comparing options that are capable of being exercised at different times, or have cash flows of different orders of magnitude, much like the situations in conventional investment preference calculations using deterministic present worth.

Example Consider the possibility of selecting between two expand options, capable of being exercised from year 2 onwards. Only one option would be exercised, not both. For the first option (A), the expansion cost discounted to year 2 is: $E[Y_{22}] = -\$77k$, $Var[Y_{22}] = (\$5k)^2$, while the return discounted to year 2 is: $E[Y_{21}] = \$74k$, $Var[Y_{21}] = (\$7k)^2$ (Fig. 3.8). For exercising in later years, the standard deviation of the return is assumed to grow by 12.5% per year, with other values staying constant. For the second option (B), the expansion cost discounted to year 2 is: $E[Y_{22}] = -\$75k$, $Var[Y_{22}] = (\$5k)^2$, while the return discounted to year 2 is: $E[Y_{21}] = \$74k$, $Var[Y_{21}] = (\$12k)^2$. For exercising in later years, the expansion cost is assumed to grow by 1% per year, with other values staying constant. Within each option, the expansion cost and return are assumed to be well correlated.

Discounting to give E[PW] and Var[PW], and using Eq. (1.1) for OV, Fig. 3.9 shows option value with time, indicating a crossover of preference of one option over the other. The value of having a choice follows the upper envelope represented by the preferred option at any time. If either option A or B is exercised, this alters any subsequent analysis.

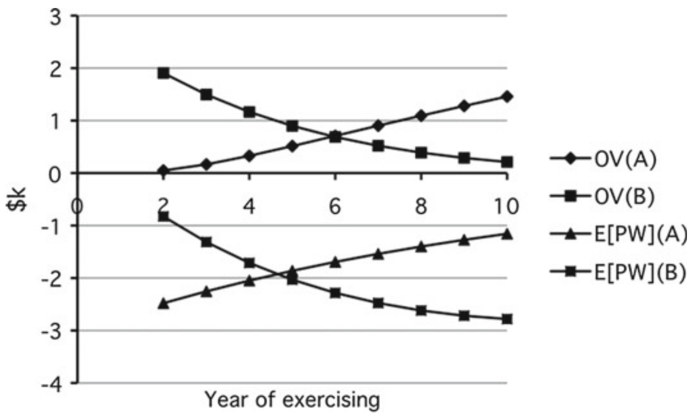

Fig. 3.9 Option value and expected present worth versus time; choice between two options (A and B)

3.7 Switch in Practices, Change in Form

Organizations may decide to change or switch the way they do things, for example contractually, or through changed method or changed resource usage, based on an option to switch or change. There may be a cost associated with this switching. The relevant cash flows to use in the option calculation are the difference between what cash flows would have existed (assuming no switching) and what new cash flows result from the switch (usually both positive and negative).

Convertible contracts. (Note, this is a reference to contracts as legally enforceable agreements, as distinct from the option usage of the term ‘contract’ above.) Consider a switch between contract payment types (here, cost reimbursable to lump sum) within a project, at year 1 of the project. The cost of doing this is based on the lump sum estimate, while the gain (cost foregone) is based on the cost reimbursable estimate. For the work remaining after 1 year, the cost reimbursable estimate is: $E[Y_{11}] = \$680k$ and $Var[Y_{11}] = (\$56.7k)^2$, where Y_{11} refers to cost reimbursable values; while the lump sum estimate expected value and variance are, respectively, \$650.0k and $(\$32.5k)^2$, giving: $E[Y_{12}] = (–)\$650.0k$ and $Var[Y_{12}] = (\$32.5k)^2$, where Y_{12} refers to lump sum values (Fig. 3.10). Since values for Y_{11} and Y_{12} are based on similar estimating principles, it is reasonable to assume that Y_{11} and Y_{12} are close to being perfectly correlated.

Present worth is based on discounting the difference between Y_{11} and Y_{12}, giving $E[PW] = \$27.3k$ and $Var[PW] = (\$81.1k)^2$. Equation (1.1) gives $OV = \$45.1k$ (approximately, 7% of the contract sum). This is the value of having convertibility within the contract. With a prescribed definite switch in payment types within the contract (compared with the option to switch as just calculated), this leads to a lower value (namely $E[PW] = \$27.3k$ or 4% of the contract sum).

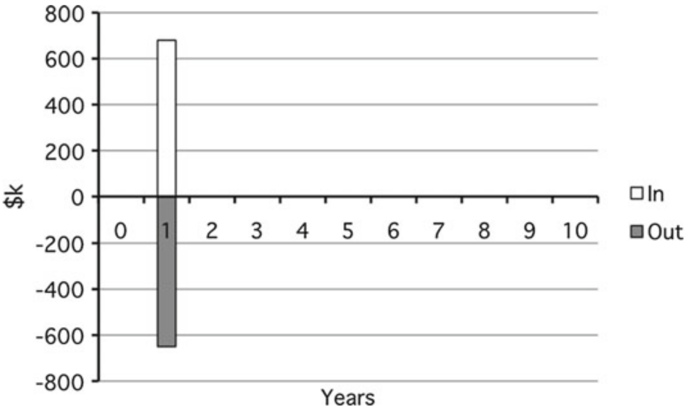

Fig. 3.10 Example—convertible contract; expected values of cash flows

Adaptable infrastructure. Adaptability related to infrastructure may be thought of in terms of: (i) Designed-in adaptability versus non designed-in adaptability; and (ii) Preset adaptation versus optional adaptation. With designed-in adaptability, the asset is deliberately designed, from the beginning, with the view that adaptation could (but not necessarily) take place in the future. With preset adaptation, explicit dates for adaptation may be set, and/or explicit adaptation may be prescribed.

Consider, for example, a seawall in the light of possible future climate shift and progressive sea level rising, both of which have uncertainties arising from multiple sources—environmental, human impact, social, political and so on. Climate shift projections, and resulting sea level rises, are informed approximations because of these uncertainties. Adaptation implies raising the seawall over time, in order to afford continual protection of adjacent inhabited regions.

For possible adaptation at a time T, based on sea level forecasts (Fig. 3.11):

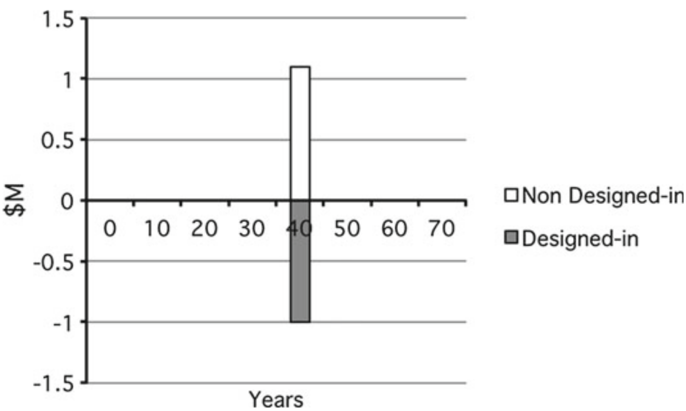

Fig. 3.11 Example—adaptable infrastructure; expected values of cash flows

(i) Designed-in adaptability versus non designed-in adaptability.
Let the cost of adaptation for the non designed-in version at T be Y_{T1}, and let the cost of adaptation for the designed-in version be Y_{T2}. Both Y_{T1} and Y_{T2} are random variables. This includes the situation where Y_{T1} corresponds to a completely new seawall being built.
(ii) Preset adaptation versus optional adaptation.
Let the cost of the adaptation (preset) at T be Y_{T1}. And let the cost of the adaptation (optional) be Y_{T2}. Both are random variables, but with Y_{T1} having a low variance.

For both (i) and (ii), the cost difference is examined; let $X_T = Y_{T1} - Y_{T2}$ (Fig. 3.11). The expected value and variance of X_T and PW follow. OV is obtained from Eq. (1.1). For viability in Case (i), OV is compared with any additional initial cost involved with designing-in adaptability features.

For Case (i), Y_{T1} and Y_{T2} will have similar variances and will be close to being perfectly correlated. This will lead to Var[X_T] being small, Var[PW] being small, and the majority of the present worth distribution occurring over positive PW values. Whether the designed-in version is viable will thus rest on E[PW] having to be greater than the additional initial cost involved in building-in adaptability. Even though future uncertainty is involved, it should be sufficient to only consider deterministic costs in a comparison of designed-in adaptability versus non designed-in adaptability.

For Case (ii), Y_{T1} and Y_{T2} could be anticipated to have similar expected values, that is Φ will be approximately 0.5, and hence optional adaptation will always be better than preset adaptation.

3.8 Delay, Deferment

The exercising of an option might be delayed or deferred until circumstances differ. The delay might be in anticipation of higher future market prices or advances in technology, or while waiting for regulatory approval. Delaying the exercising of an option, because of the resultant effect on cash flows, may lower or increase the option value, dependent on the circumstances.

The analysis of options involving delays is no different to the earlier plain or single options. All that needs to be done is to acknowledge the impact of the delay on the cash flows used in the analysis. This is so whether the project life, and hence the extent over time of the ensuing cash flows, is finite or very long. Delaying the exercising of an option may result in cash flows being lost, delayed or advanced, modified (increased or decreased), remaining as they were or a combination of these, dependent on the circumstances. But still the form of the analysis remains the same. Any additional direct cost (or gain) associated with a delay becomes yet another cash flow.

In establishing when is the best time to exercise an option, different available times can be considered in the analysis and option values compared via enumeration.

Examples A delay or deferment might result in cash flows being lost, delayed or advanced, modified, remaining as they were or a combination of these.

Consider an illustration. An expansion at year 2 costs: $E[Y_{22}] = -\$100k$ and $Var[Y_{22}] = (\$5k)^2$. This generates income each year following the expansion of: $E[Y_{i1}] = \$25k$ and $Var[Y_{i1}] = (\$6k)^2$, $i = 3, 4, \ldots, 7$ (Fig. 3.12). All cash flows are assumed strongly correlated. Discounting gives E[PW] and Var[PW], and Eq. (1.1) gives OV.

Three scenarios, with examples, can be considered:

(i) Cash flows lost. In delaying the expansion, the cash flow Y_{i1} in any year is consequently lost. For example, if the expansion is delayed 1 year, then Y_{31} is lost (Fig. 3.13).

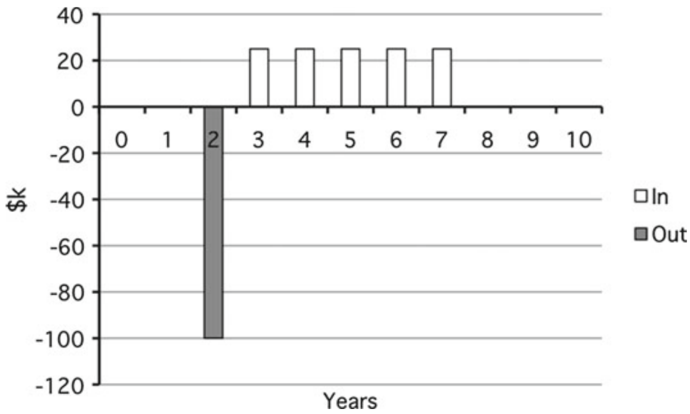

Fig. 3.12 Example—original expand option prior to delay; expected values of cash flows

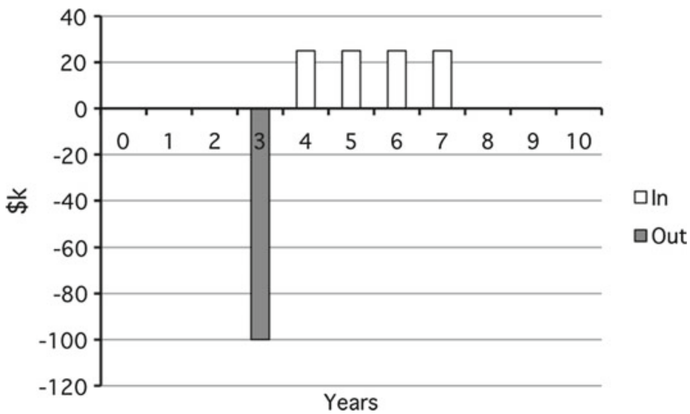

Fig. 3.13 Example—expand option, delay 1 year, cash flows lost; expected values of cash flows

(ii) Cash flows delayed or advanced. In delaying the expansion, the cash flows Y_{i1} in the following years are delayed. For example, if the expansion is delayed 1 year, then Y_{i1} now applies over i = 4, 5, …, 8 (Fig. 3.14).

(iii) Cash flows modified. In delaying the expansion, the expected values and variances of all the cash flows in the following years increase: for 1 year delayed, 5%; for 2 years delayed, 10%; and so on (Fig. 3.15).

Figure 3.16 shows the example option values versus years delayed for the three above-given scenarios.

For delay analysis, Fig. 3.16 applies for a particular set of cash flows and cash flow assumptions, and is not intended as general behaviour associated with delay.

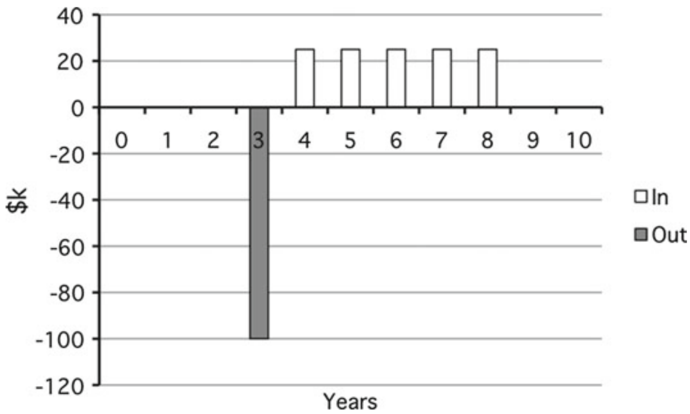

Fig. 3.14 Example—expand option, delay 1 year, cash flows delayed; expected values of cash flows

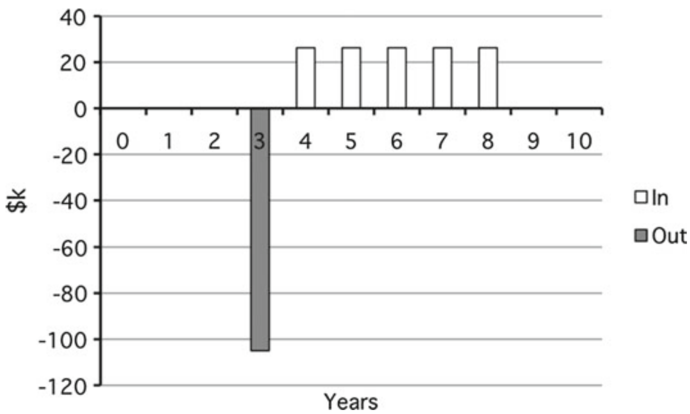

Fig. 3.15 Example—expand option, delay 1 year, cash flows increased; expected values of cash flows

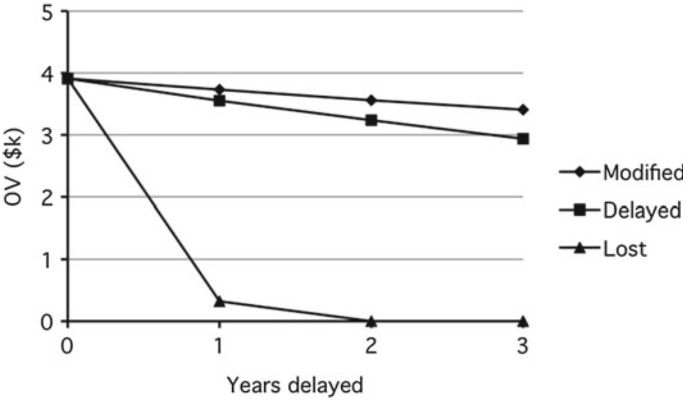

Fig. 3.16 Example delay analyses; option value versus days delayed

Generally, with traditional financial options, delay in exercising is rewarded, while with real options, delay may be penalized or rewarded.

3.9 Sequential Options

Sequential options are a chain of options. For example, in a project comprised of stages, there may exist an option at each stage, and this determines what subsequently happens with the project. The options in a chain might be any of the single options, in different sequences, and hence there is no one standard sequential options case. In the sequence of options, naturally a preceding (independent) option is exercised before a succeeding (dependent) option.

Where the exercising of each option generates its own cash flows, the value of the following option is included in the valuation of the option at hand. This is in line with Bellman's principle of optimality. Such sequential options are analysed backward in time, but each option analysis is no different to the earlier single options.

Examples Two typical sequential option cases are presented here, related to whether: (i) exercising the component options generates their own cash flows, or (ii) all component options need to be exercised in order to generate a cash flow. The example treatments below apply generally to any number of multiple component options.

(i) Each exercise generates cash flows

Consider sequential options occurring at $t = t1, t2, \ldots$ where $t1 < t2 < \ldots$ The following (dependent) option can only exist if the preceding (independent) option has been previously exercised. Exercising the first option at t1 generates positive and negative cash flows reducible or equivalent to $Y_{t1,1}$ and $Y_{t1,2}$ respectively, exercising the second option at t2 generates positive and negative cash flows reducible or equivalent to $Y_{t2,1}$ and $Y_{t2,2}$ respectively, and so on up to the last option at time tq.

Bellman's principle of optimality stated in the 1950s is relevant here [2]; see [3], p. 139 and [4]: *An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.*

Valuing the options in reverse order, the option value for the last option follows using $Y_{tq,1}$ and $Y_{tq,2}$ discounted to time tq–1, and is denoted OV_{tq}. The second last option is then valued based on cash flows $Y_{tq-1,1}$, $Y_{tq-1,2}$ and OV_{tq}, all discounted to time tq–2. This gives OV_{tq-1}. The procedure for the second last, third last etc. options is repeated in a backwards time sense, ending with the first option calculation, giving OV_{t1}, which is the option value for the sequential option.

R&D example. Consider a three-sequential option, manufacturing R&D example. The data follow. Product introduction stage: $E[Y_{t1,1}] = -250M$, $Var[Y_{t1,1}] = (\$62M)^2$, $E[Y_{t1,2}] = \$182M$, $Var[Y_{t1,2}] = (\$45M)^2$; First expansion stage: $E[Y_{t2,1}] = -\$250M$, $Var[Y_{t2,1}] = (\$62M)^2$, $E[Y_{t2,2}] = \$245M$, $Var[Y_{t2,2}] = (\$61M)^2$; Second expansion stage: $E[Y_{t3,1}] = -275M$, $Var[Y_{t3,1}] = (\$68M)^2$, $E[Y_{t3,2}] = \$322M$, $Var[Y_{t3,2}] = (\$80M)^2$ (Fig. 3.17). The cash flows at $t1 = 1$, $t2 = 2$, and $t3 = 4$ years are assumed independent.

Each of these is an expand option, and calculated as discussed above. OV_{t3} is calculated first and equals \$56.4M. This is included in the calculation of OV_{t2}, which equals \$59.0M. This is included in the calculation of OV_{t1}, which equals \$31.6M, the option value for the total sequential option.

CDM projects. Clean Development Mechanism (CDM) projects can choose to have a 7-year crediting period that can be renewed twice, giving a total of 21 years [1, 8]. The choice of an initial 7-year crediting period, and the subsequent renewal or non-renewal for a following 7 or 14 years, can be seen as a sequential option, that is an option to discontinue or continue as a CDM project at the end of year 7, and later possibly at the end of year 14. Option calculations only consider cash flows that are

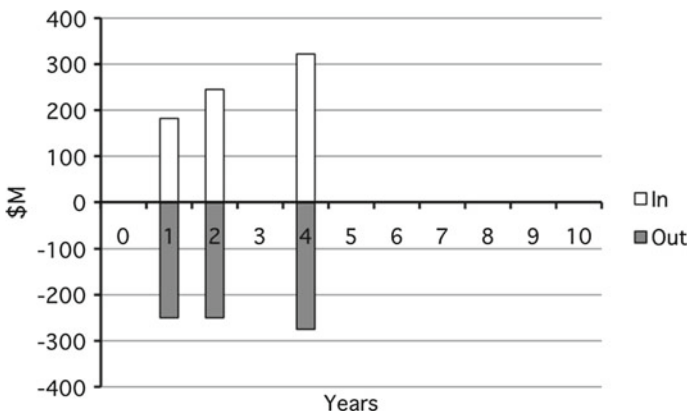

Fig. 3.17 Example—sequential options, each component with cash flows; expected values of cash flows

CER- or carbon-based (inflows and outflows) and not all project cash flows. (CERs are carbon emission reduction units, or carbon credits) The analysis of the option occurring at the end of year 7 (assuming renewal only for one more 7-year period) is calculated in terms of the anticipated carbon-based costs (CDM operational costs) and anticipated carbon-based revenues (based on forecast prices and project output of CERs) for years 8 to 14. All these costs and revenue contain uncertainty. This option value then needs to be compared with its 'premium', namely the (additional) cost of preparing updated documentation at the end of year 7. The option value would need to exceed this additional administration cost in order to proceed as a CDM project. That is, there could be value in having flexibility to either discontinue or continue a project at the end of year 7, dependent on anticipated costs and anticipated revenue at year 8 and over the following years up to year 14.

Similarly, if the choice at the end of year 7 is that of continuing CDM registration, there exists an option at the end of year 14.

The total option value of having options at the end of both years 14 and 7 is evaluated by working backwards in time. The option value at the end of year 14 is calculated based on the cash flows in years 15 to 21. This option value, together the additional administration cost at year 15 and the cash flows in years 8 to 14, give the option value at the end of year 7. This option value, together the additional administration cost at year 8 and the cash flows in years 1 to 7, give the option value at year 0. This is the total option value and is compared with the up-front CDM cost, in order to establish CDM viability.

(ii) All options needing exercising to generate cash flows

Consider sequential options occurring at $t = t1, t2, \ldots$ where $t1 < t2 < \ldots$ A following (dependent) option can only exist if the preceding (independent) option has been previously exercised. Exercising all options except the last generates no direct cash inflows, but may have cash outflows reducible or equivalent to $Y_{t1,2}$, $Y_{t2,2}$, …, while exercising the last option at tq has a cash outflow reducible or equivalent to $Y_{tq,2}$, and cash inflow reducible or equivalent to $Y_{tq,1}$. The option value might be thought of as evolving in time, however, intermediate option values are not realizable, since it is only on exercising the last option that a return is gained.

Consider a manufacturing plant example. Three project stages are involved: a land acquisition and permitting stage, a design stage and a construction stage, starting at $t1 = 1$, $t2 = 3$, and $t3 = 5$ years respectively. Cash outflows associated with the options at these points in time are: $E[Y_{t1,2}] = -\$30M$, $Var[Y_{t1,2}] = (\$3M)^2$; $E[Y_{t2,2}] = -\$90M$, $Var[Y_{t2,2}] = (\$11M)^2$; and $E[Y_{t3,2}] = -\$210M$, $Var[Y_{t3,2}] = (\$26M)^2$, respectively. The cash inflow occurs at the end of the construction stage (7 years), giving a discounted cash inflow at t3 of: $E[Y_{t3,1}] = \$402M$, $Var[Y_{t3,1}] = (\$100M)^2$ (Fig. 3.18). All cash flows are assumed well correlated. The option value for the total investment is calculated by discounting all cash flows to the present, with Eq. (1.1) giving OV = \$29M.

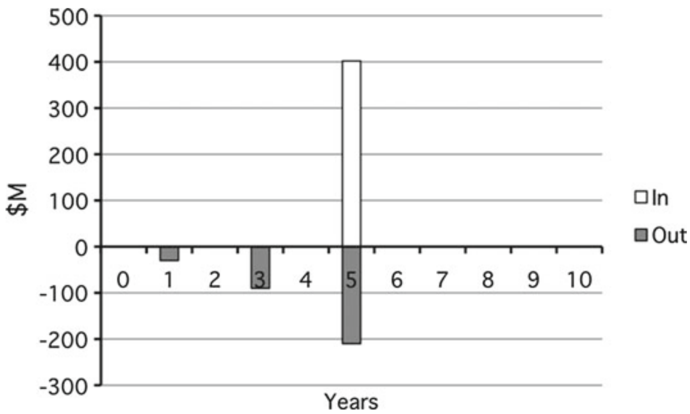

Fig. 3.18 Example—sequential options, all components needing exercising; expected values of cash flows

3.10 Parallel Options

Parallel options are characterized by a dependent option and an independent option, as in sequential options, but now applying over the same time frame, rather than in sequential time frames. The component options may be any of the single options. The independent option is exercised first. The analysis mirrors that of sequential options.

Parallel options have an independent option and a dependent option as for sequential options, but these component options exist over the same time frame. In principle, the valuation for parallel options is no different to that for sequential options. The same two cases examined in sequential options above, could apply to parallel options. For the two-component parallel option case, in line with the terminology used for sequential options, let the time frame for exercising be t1. The independent option is evaluated over t1, while the dependent option is evaluated over a time frame of $t2 = t1 + \Delta t$, where Δt is a small increment in time (Fig. 3.19). That is, by imagining that the independent option is exercised at a very small time interval different from the dependent option, then the same thinking as sequential options applies, and the option analysis, again, is no different to the earlier expand and contract options. In effect, for calculation purposes, $t2 = t1$.

With compound options, because the difference between the estimates of the two forms of analysis can be both positive and negative, depending on the values taken by the variables, it is possible that the component option value estimates may balance each other out, rather than enlarge the difference between the estimates of the two forms of analysis. However, no general conclusion on this is possible.

With compound options, the existing literature works on traditional financial option-style examples using volatility and present day asset values, and hence there is little that the book's approach can directly compare to. The values of individual options, within the compound options, are consistent with Black-Scholes, and the

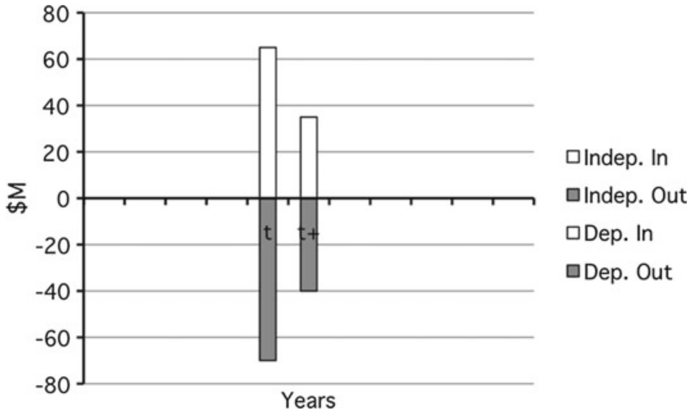

Fig. 3.19 Example—parallel options; expected values of cash flows

compound option approach is consistent with the existing literature, and so it is anticipated that the option values calculated will be satisfactory. Where comparisons are able to be made between the two forms of analysis, the trends in compound options, with respect to Φ, time of exercising, interest rate and uncertainty are similar to those trends identified above for plain options.

3.11 Rainbow Option

A rainbow option refers to there being more than one type of uncertainty present. For example in mining, the uncertainty in the cash flows can come from the price of the ore, ore yield and the mine output. While this prevents or complicates the calculation if a traditional financial options method is used (for example, a binomial tree goes to a quadrinomial tree where there are two sources of uncertainty), the calculations remain essentially the same if the analysis is using cash flows. The calculations are increased only slightly. All uncertainty sources are assembled, acknowledging any correlation present, into the estimates used for cash flows. Having incorporated all the uncertainty sources into the cash flows, the analysis proceeds as for any of the other options.

For example, consider a cash flow Y made up of price, Z_a, and output, Z_b, both random variables, in the form,

$$Y = Z_a Z_b$$

Then,

$$E[Y] = Cov[Z_a, Z_b] + E[Z_a]E[Z_b]$$

and for Z_a and Z_b independent,

$$Var[Y] = E^2[Z_a]Var[Z_b] + E^2[Z_b]Var[Z_a] + Var[Z_a]Var[Z_b]$$

Other expressions are available for different assumptions on the correlation of the variables containing uncertainty, as well as for more than two sources of uncertainty [5].

3.12 Closure

It is believed that the book's cash flow approach will make real options analysis more commonplace and more accessible to a greater numbers of people. In this form, it can be given as an add-on to conventional discounted cash flow analysis taught to undergraduates; no new thinking is required.

The cash flow approach to real option estimates is very straightforward, requiring minimal mathematical background. The approach does not distinguish between the type of option, for example whether it is an expand-style option, contract-style option, involves delays or contains multiple sources of uncertainty (correlated or not); the approach is the same for all options. It uses variance instead of volatility, a term which does not have meaning in most real options, and avoids trying to establish an analogous financial option in order to force fit established financial options methods. Any set of cash inflows and cash outflows, whether correlated or not, can be accommodated through the one approach. The present worth can take both positive and negative values; there are no inherent lognormal assumptions, rather the distribution for PW can be selected at the discretion of the person doing the calculations. And if a second order moment approach is adopted, there are no distribution assumptions necessary on the cash flows. Interest rates and rate mixes can also be chosen at the discretion of the person doing the calculations.

Heretofore, real options have been valued by adopting analogies with traditional financial options, and the number of technical publications using such methods is very large. Something new and different, in general across all human endeavours, may not be well accepted by people. Coupled with the book's cash flow approach being very simple, it is envisaged that there will be initial public resistance to adopting or public unwillingness to move to a cash flow approach.

For plain options, the relationship between the option value and the analysis input variables of Φ, time to exercising, uncertainty, and interest rate is similar between the approach and that of Black-Scholes. The book's approach gives essentially the same option values as Black-Scholes, as also observed in Carmichael et al. [7]. The larger differences with Black-Scholes occur with higher values of: Φ, time to exercising, interest rate and uncertainty. However, anticipated real option applications will have lower values of Φ, time to exercising, interest rate and uncertainty than those at which the book's approach starts to diverge from Black-Scholes. For compound options, where comparisons are able to be made, similar trends are observed.

Although the chapter compared option values from the book's approach with Black-Scholes, for a restricted number of cases, it is remarked that there is no 'correct' answer to compare the real option value with. It is only possible to qualitatively compare with methods such as Black-Scholes, because different situations are being looked at in real options compared to traditional financial options. However, the restricted comparisons show that the book's approach produces reasonable values.

The book's cash flow view of real options provides an intuitive user-friendly approach, requiring minimal mathematical background, devised to specifically value real options and aligned with usual investment calculations, without the need for financial options analogies, constraints, terminology and variables such as volatility. Accordingly, the cash flow view should be acceptable to most.

3.13 Extensions

The chapter provides the skeleton for a cash flow approach to valuing real options. Future developments could examine possible refinements, for example in the choice of the present worth distribution and assumed interest rates, and in examining special-case compound options.

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