Chapter 12
Real Estate Options

12.1 Introduction

Economic, political and other external factors influence the value of real estate or property and the uncertainty in this value. One way of dealing with this uncertainty and associated risk is hedging through options. An option grants the option holder the right, but not the obligation, to typically buy or sell something in the future. Black-Scholes [3], binomial lattices and equivalent Monte Carlo simulation are commonly used pricing tools to value these options.

However, in atypical markets, where property price fluctuations and price uncertainty are anticipated to not follow any usual patterns, such pricing tools may not apply [6]. This chapter shows how options in such atypical markets or atypical situations can be valued according to the book's approach, which also applies in regular markets. The approach is straightforward to implement, intuitive to understand and requires low mathematical sophistication. Future cash flow estimates are informed by experience, knowledge and education related to the market, together with any available analysis (Chap. 2). Although no time series or trend lines describing the time evolution of property prices are used, if available they can act as information supplemental to the experience, knowledge and education of the person doing the calculations.

The approach is demonstrated on a range of different types of real estate options, and compared with Black-Scholes. The range includes property (purchases, sales, expansion, delay and abandonment), stock exchange indices, mortgages (prepayment, default/close, change), and leases.

The approach is one of interpreting an option as giving rise to equivalent future cash flows, discounting these to the present day using probabilistic present worth, and interpreting the resulting present worth distribution. Cash flows not associated with exercising are not considered, but are nevertheless important from any overall investment viability viewpoint. The cash flows are interpreted from the option

D. G. Carmichael, *Future-proofing—Valuing Adaptability, Flexibility, Convertibility and Options*, Management in the Built Environment,
https://doi.org/10.1007/978-981-15-0723-6_12

holder's viewpoint as in usual property investments. Thus there is no need to distinguish between expand or contract style options. For typical options, the results agree with Black-Scholes, but the approach is less restrictive on assumptions than Black-Scholes. The structure behind the approach is consistent with Black-Scholes. However, any information available to the person doing the calculations, qualitative or quantitative, can contribute to making informed estimates or forecasts of future cash flows. The approach is also generally applicable to all real estate options, irrespective of the underlying—real property, indices, rentals or other.

The outline of this chapter is as follows. Firstly, some background on real estate options and the book's approach is given. A range of real estate options is then shown how they can be interpreted in terms of the book's approach. A conclusion follows. All options presented in this chapter are valued from the viewpoint of the option holder. The chapter does not look at the effects on other parties, surrounding circumstances, people behaviour, or industry issues. Taxation and duties applicable on property transactions are country dependent, and mention of these is omitted from the following, but would be included in an overall viability calculation of any transaction. The chapter will be of interest to anyone involved with options in real estate.

12.2 Background

The body of knowledge on financial options is typically borrowed when dealing with real estate options. In real estate, the underlying may be property (including land) belonging to one of the parties to the option, may be some index, or may relate to mortgages and leases.

Options protect against losses beyond the premium paid, but are rewarded for gains. Options are used for hedging, speculation and conducting arbitrage transactions. Greater uncertainty in the underlying price gives greater value to the option. Options in real estate may be used, for example, to hedge against fluctuating prices, at property buying time to assist gaining finance, or in anticipation of land rezoning. For a property purchaser, an option can put a property on hold up until the expiry date of that option, thereby providing time to secure appropriate finance, insurance and permits before committing to the purchase.

With property as the underlying, the options tend to be traded directly between the parties, but perhaps overseen by agents. The option contracts are non-standard and are tailored specifically to the parties' needs, for example in terms of time to maturity; not being standardised leads to an absence of contract writing guidelines, regulations and details. American or European option styles can be used. Factors affecting the value of the underlying, and hence the value of the option premium include the property's size, location, age and condition, as well as property demand and price forecasts. A characteristic of property is that it is relatively illiquid when compared to stocks and bonds, and characteristics of property options are that they are private transactions, infrequently traded, with higher fees, information asymmetry and many local markets. This leads to decreased numbers of arbitrageurs in real estate option trading [6]. Options contracts can be sold on a secondary market to another interested buyer, if available, should the original option holder not wish to proceed with the purchase or not be able to get the necessary finance, insurance or permits. The sale terms on the secondary market are negotiable, dependent on the amount of time remaining until expiry of the option, demand for the property and how prices may have shifted since the original option contract was initiated.

Instead of potentially taking ownership of property, stock exchanges have real estate indices enabling investors to speculate using options. This facilitates an affordable way of investing in the property market; option contracts can be at relatively low cost with low trading and brokerage costs, and there are reduced borrowing requirements and expenses compared to taking ownership of the property with all its associated costs.

Within mortgages a number of options exist; these include options related to prepayment, default/close and change. Within leases, also a number of options exist; these include subleasing, lease cancellation, lease renewals, property purchase and revenue sharing.

12.3 Option Valuation

Black-Scholes, binomial lattices and Monte Carlo simulation equivalents, and their variations, are commonly accepted ways of valuing options. These require assumptions on the movement (such as Geometric Brownian motion) in the underlying price, assumptions on the distribution of prices (for example log normally distributed), and other assumptions such as a constant risk-free interest rate. Adjustments are made to suit particular uses. Such methods are not followed here in favour of the book's approach.

A typical call option involves paying a premium now, and in return having the option to purchase the property for an amount K (exercise price) at some time T in the future. Let the estimate of the property value (or underlying price) at time T be S_T. From the option holder's viewpoint, K can be regarded as a negative cash flow, and S_T a positive cash flow. The option is exercised if it is worthwhile (in the money), that is $S_T > K$. $S_T - K$ can be discounted to give a present worth, and the option value follows from Eq. (1.1).

Equation (1.1) provides a single way of valuing all options, whether call-style or put-style, whether related to property, indices, mortgages or leases. Later, the cash flows S_T and K are generalised to be cash flow equivalents, interpreted from the option holder's viewpoint. Favourable cash flows resulting from exercising the option are regarded as a cash inflow, while unfavourable cash flows are regarded as a cash outflow. Strict accounting conventions need not be used. Monte Carlo simulation or a second order moment analysis can be used to discount the cash flows (Chap. 2). This book uses the latter for convenience. It requires no assumptions to be made on the probability distributions of the cash flows, but rather works in terms of the moments, E[] and Var[], of the cash flows. Possible ways of estimating the expected values and variances of underlying prices or values are discussed in Chap. 2 and the next section.

Agreement between the book's approach and Black-Scholes is good. Section 12.5 gives a comparison example. Higher uncertainty in an underlying price or value leads to greater spread in the present worth distribution, and hence higher option values. Put-call parity can be shown to hold [4]. Exact agreement with Black-Scholes could not be anticipated because Black-Scholes and the book's approach are based on different assumptions, most notably (with the book's approach): volatility is replaced with variance and variance need not be constant over time; there are no required time series or probability distribution assumptions; the exercise price need not be deterministic or at one point in time; interest rates can vary over time and be different for the exercise price and the underlying price or value; and discounting can be continuous time or discrete time.

12.4 Estimates of Future Underlying Prices or Values

Estimating practices vary between people, and invariably involve combining knowledge of: the market; historical trends; industry data; and national and international economies and events. This is coupled with experience, education and 'gut feel' (a combination of logic and emotion). Any fitted time series models or trend functions available can be incorporated and to whatever extent desired, but will need modification in atypical markets, perhaps through qualitative manipulation, where the past is not repeating. There is no one single agreed method for estimating. What is required is a best estimate based on available information. Estimating is not dealt with in more detail in Chap. 2 All underlying prices and values—property prices, indices, rental costs etc.—contain uncertainty.

In applying Eq. (1.1), persons doing the calculations are free to adopt whatever method they like in order to estimate future underlying prices or values. In the following examples, estimates for the underlying prices or values at time T, S_T, are made by first estimating optimistic (a) most likely (b) and pessimistic (c) values, as in Chap. 2. But the book's approach can use estimates obtained by any method, including where uncertainty increases with time. The exercise price K is deterministic and is established from the options contract. Correlations between variables similarly can be estimated in any way considered suitable, but the results will be bound by assumptions of complete correlation and independence.

12.5 Applications

12.5.1 Outline

A range of example option valuations is given using the book's approach. The approach applies to both real property, indices, mortgages and leases without modification.

Black-Scholes, with alteration, and similar approaches, rely on future fluctuations in the underlying prices to be the same as the past (history repeating). Such approaches are well established and are not repeated here. Where the market is atypical, Black-Scholes and similar approaches would generally not work [6]. This book's approach however can deal with any market irregularities or regularities, and also incorporate uncertainty in all the variables.

Present worth, PW, when calculated through a second order moment analysis, requires estimates of the moments $E[S_T]$ and $Var[S_T]$. S_T and K are discounted to time 0, to give PW (Chap. 10 and Appendix 2.11.2),

$$E[PW] = \frac{E[S_T] - K}{(1 + r)^T}$$

$$Var[PW] = \frac{Var[S_T]}{(1 + r)^{2T}}$$

Knowing E[PW] and Var[PW] allows any two-parameter probability distribution for PW to be fitted. The distribution choice is discretionary. The option value, OV, then follows from Eq. (1.1). The process steps become (Chap. 2):

- Estimate $E[S_T]$ and $Var[S_T]$;
- Discount these together with K to give E[PW] and Var[PW];
- Fit any reasonable probability distribution to PW; and
- Calculate Φ and M, and hence OV.

The call option looks at one tail of the PW distribution. The put option looks at the other tail of the PW distribution. Whichever is being looked at, cash flows are interpreted from the option holder's viewpoint, namely a cash inflow being positively viewed and a cash outflow being negatively viewed, irrespective of any accounting conventions.

The analysis is done for any time of exercising, T. To establish the optimum time to exercise, repeated similar analyses are done for different times, the resulting option values compared, and the best selected by enumeration.

Normal distributions for present worth are used in the following examples, but any distribution thought appropriate can be used. An interest rate of 8% per annum (p.a.) is adopted.

The choice of interest rate and probability distribution for present worth is up to the person doing the calculations. The choice is based on whatever is thought to be the most suitable. With interest rate choice, it is noted that uncertainty in the property/index price is already incorporated, and so a choice approaching a ‘risk free’ rate might be preferred.

Some examples follow.

12.5.2 Property Purchase

Consider purchasing a property for redevelopment. The property is currently valued around $1M but the buyer needs time to raise finance and get necessary development permits. The property value is anticipated to increase. To hedge against a potential price rise, the investor purchases a call option on the property at an exercise price of $1M and an expiry date of one year. Optimistic, most likely and pessimistic estimates for the property in one year are $1.05M, $0.99M and $0.95M respectively. That is, $E[S_T] = \$0.993333M$ and $Var[S_T] = (\$0.016667M)^2$. This gives $\Phi = 0.34$, $M = \$10,310$ and $OV = \$3,550$. The premium that the purchaser should be looking to pay will be something close to this OV.

In some option contracts, the final purchase price of the property might be reduced by the premium already paid by the option buyer, if the option is exercised. This does not affect the option value calculation, but does affect overall purchasing viability.

The above example involves usual market forces on property prices. A second example that uses the same calculations involves rezoning. Perhaps, rezoning of rural land might be imminent, and maybe dependent on a different political party in power. A developer may pay the land owner an option premium (call option) such that the developer has the right to purchase the land at some defined future time for a defined exercise price. Optimistic estimates (a) for S_T are now not based on historical trends, but rather require experience in similar previous situations, along with a sense of what the demand might be for a product that currently does not exist. Pessimistic estimates (c) will be based on the existing political party staying in power. Most likely estimates (b) will be based on the likelihood of a different political party. For example, with a probability p of the political party staying the same, $b = pa + (1 - p)c$.

The land owner receives the premium now. The developer’s gain with a new political party is not limited. The developer’s loss with the same political party is capped at the premium.

12.5.3 Property Sale

Options can also be used to hedge in the sale of property in the future. A property owner wishes to sell a property in one year. The property value may decrease. The owner purchases a put option on the property for an exercise price of $1M and an expiry date of one year. Optimistic, most likely and pessimistic estimates for the property in one year are \$0.97M, \$1.01M and \$1.03M respectively. That is, $E[S_T] =$ \$0.993333M and $Var[S_T] = (\$0.01M)^2$. This gives $\Phi = 0.25$, M = \$5,540 and OV = \$1,400. The premium that the seller should be looking to pay, in order to lock in the \$1M sale price, will be something close to this OV. For example, for a premium of \$2000, a property value in one year greater than \$1.002M (option not exercised; property sold on the market) or less than \$0.998M (option exercised; property sold for \$1M) represents a gain to the seller. Between \$0.998M and \$1.002M sale price, the option holder suffers a loss, but the loss is capped at the breakeven values of \$1.002M and \$0.998M.

12.5.4 Property Expansion, Delay, Abandonment

Oppenheimer [6], among others, notes a number of real options existing with buildings and land—expansion, delayed development (wait), and abandonment. In a given situation, more than one of these options may exist, but on exercising one option, the other options are no longer present. The analysis of each of these options involves, as before, identifying the cash inflows and cash outflows resulting from exercising the option.

Expansion might be contemplated in order to exploit a new or growing market. Examples include upgrading buildings or follow-on housing project development. The cash outflows relate to the cost of expansion (including any possible disruption costs), and the cash inflows relate to the ensuing income generated by the expansion. All cash flows can contain uncertainty, and hence the assumption of a deterministic exercise price in Black-Scholes does not hold.

Abandonment and contraction are treated similarly. Contraction may involve selling real estate in anticipation of a downturn in demand. The relevant cash flows are the difference between what would have existed (assuming no contraction) and what new cash flows result from the contraction (usually both positive and negative), together with any sale income or disposal costs. If abandonment, there is a need to also include a one off demolition/disposal cost or residual value. Refer Chap. 3.

Delay or deferment of an option might occur in anticipation of some future event occurring or future market shifting, or future approval. The analysis of delays is no different to any of the previous options except that a delay may affect (increase or decrease) the magnitudes of the relevant cash flows. The presence of a delay may have its own cost. The option value may improve or decrease with a delay depending on the particular situation. The optimum delay time can be established by doing analyses for different exercising times and selecting the best result from these. Refer Chap. 3.

12.5.5 Stock Exchange Indices

Real estate performance is tracked with indices on stock exchanges. Options with these indices as underlyings are an alternative way to invest in real estate, and require relatively low capital compared with the outright purchase of property or stock. The exercise style is European, that is exercising upon maturity. Maturity may be quarterly and up to four quarters or a year total [1, 2]. In a rising market, an index call option may be bought, while in a falling market an index put option might be bought.

An example of the use of a real estate index option for hedging is that of an individual who wants to buy a house in one year's time but is anticipating that house prices will rise at a faster rate than the individual's income and savings. To counter this, the individual buys a call option on a real estate index, which has a high correlation with house prices. Another example is that of a homeowner who wants to sell in one year and would like to hedge against a decline in house prices. To counter this, the homeowner buys a put option on a real estate index. The hedging reduces exposure brought about by adverse price movements in associated assets.

Consider an example of an investor who buys a call option on a REIT (real estate investment trust) index. The exercise price is 1025 with a one-year maturity. Optimistic, most likely and pessimistic estimates are 1050, 1020 and 1000 respectively (at one year). That is, $E[S_T] = 1021.7$ and $Var[S_T] = (8.3)^2$. This gives $\Phi = 0.34$, $M = 5.13$ and $OV = 1.8$. The premium that the investor should be looking to pay would be considered relative to this OV—a premium lower than this OV would be desired by the investor. The discussion on gains and losses follows the earlier call option treatment; the loss is limited to the premium paid. Using the Appendix 2.11.4 expression for a volatility–variance relationship, Black-Scholes gives an option value of 3.0, which is about 0.1% different as a proportion of the exercise price. Assuming a present worth distribution other than a normal distribution, or assuming another volatility–variance relationship, could make this comparison closer.

12.5.6 Mortgages—Prepayment, Default/Close, Change

A number of authors have written on options that exist within mortgages. Some example options that exist within mortgages and mentioned by these authors relate to prepayment, default/close and change. These authors also discuss the circumstances when such options might be exercised and associated ramifications. The value of these options does not appear to be something that is explicitly calculated by the borrower, however the method given here can be used. At any particular time, T, the borrower looks at its effective cash flow position. The effective cash flow position at T is the present worth of all equivalent cash flows in and cash flows out, at and beyond T, discounted to time T. The net equivalent cash flow at T is then discounted to 0 to give PW, and the option value is calculated as above.

The options to the borrower for prepayment, default/close and change cannot exist at the one time. Exercising one option prevents the others from being exercised at the same time. Each option will have a different worth to the borrower, and it seems reasonable that the borrower would select the option which is best for it. Sensibly, an option is only exercised when it is 'in the money', but borrowers may have non-mortgage reasons for exercising.

All analysis is from the borrower's perspective—the borrower is the holder of the option. The lender's perspective can be informed by looking at this. Most publications tend to be from the lender's perspective in trying to establish the value of a mortgage in the presence of these options.

For dealing with options within mortgages over N years, the equations in Appendices 2.11.1, 2.11.2 and 2.11.3 can be used, and are a generalisation of the earlier Sect. 12.5.1 equations. For the option, equivalent cash inflows and cash outflows, denoted Y_{1i} and Y_{2i} respectively, will exist at and beyond T, namely for years i = T, T + 1, …, n. Y_{1i} (in place of S_T) and Y_{2i} (in place of K) can be random variables. In any year i, the net cash flow is $X_i = Y_{1i} - Y_{2i}$. That is, $E[X_i] = E[Y_{1i}] - [Y_{2i}]$ and $Var[X_i] = Var[Y_{1i}] + Var[Y_{2i}] - 2\rho_{12}\sqrt{Var[Y_{1i}]}\sqrt{Var[Y_{2i}]}$, where ρ_{12} is the correlation between Y_{1i} and Y_{2i}. The present worth of X_i, $PW_i = \frac{X_i}{(1+r_i)^i}$. Allowing for the interest rate r_i to also be a random variable and to vary with year i, this leads to the results in Appendix 2.11.3.

Appendix 2.11.3 is a generalisation of Appendix 2.11.2. The use of Eq. (1.1) follows. This is the option value when viewed at time i = 0. As time progresses, updated option values can be calculated by discounting the cash flows to the current time i. It is noted that this book's method allows for non-deterministic exercise price and non-deterministic interest rate, unlike traditional financial options methods.

Prepayment

Prepayment refers to the ability of a borrower to prepay any remaining loan balance during the loan period, and can be interpreted as a call option [6]. The borrower in return takes ownership of the property. Prepayment may be done in conjunction with external or internal refinancing. In a market where the differential—property value minus mortgage value—increases with time, the likelihood of exercising the option would increase. Holding the option may be of no consequence to the borrower however, because the borrower may not have access to the prepayment amount, and this may be the case for many borrowers. A lender would not be aware of this, and hence would not know at any time whether the borrower was likely to exercise the option.

Cash inflow—property value, Y_{1T}; $Y_{1i} = 0$, i = T + 1, T + 2, …, n

Cash outflow—mortgage value and transaction cost, Y_{2T}; $Y_{2i} = 0$, i = T + 1, T + 2, …, n

Default/close

Borrowers may close or default on their loan. This can be interpreted as an option. The lender sells the property in order to recover outstanding debt. The sale of the property is on the lender's terms and true market value may not be obtained. In a market where the differential—property value minus mortgage value—decreases with time or goes negative, the likelihood of exercising the option would increase, in order to reduce the borrower's losses. A lender would ordinarily be aware of the difference between mortgage value and property value, and current levels of interest rates, but non-mortgage information would not be known to the lender and hence whether the borrower is likely to exercise the option. For the property value greater than the mortgage value:

Net cash flow at T, $Y_{1T} - Y_{2T}$: Property value minus mortgage value minus transaction cost; $Y_{1i}, Y_{2i} = 0, i = T + 1, T + 2, \ldots, n$; there may also be a reputation (credit rating) cost extending beyond T.

Change

Changing the nature of a mortgage may be possible at some time in the future. For example, it may be possible to change between loans with different interest types—fixed rate (constant over a defined period or term) and variable (floating, adjustable) rate. This works like refinancing. The interest rate in variable rate loans generally fluctuates with the cash rate, which is set by a central bank, and fluctuates over time. When the cash rate is adjusted, commercial lenders make a decision on whether to pass on the same adjustment to their borrowers fully, partially or not at all. Commercial lenders may also adjust their rates independently of any cash rate movement. Borrowers and their loan repayments are thus exposed to the uncertainty of interest rate movements. Variable rate loan repayments increase with a rise in interest rates and decrease with a fall in interest rates. By comparison, fixed interest rate loans allow borrowers to lock in a definite interest rate for a given period or term; borrowers know exactly what their loan repayments are over the life of the loan. Fixing the interest rate protects the borrower from cash rate rises but not against rate falls, while such loans tend to be less flexible than variable rate loans, with restrictions on extra repayments and access to these, and also perhaps incurring greater costs for early termination. Having the ability to convert or switch between rate types can be viewed as an option.

Net cash flow at T, $Y_{1T} - Y_{2T}$: present worth (discounted to time T) of remaining (years > T) mortgage payments required before converting, minus the present worth (discounted to time T) of remaining (years > T) mortgage payments required after converting, minus transaction cost.

Repayments greater than required by the lender nay also be allowed. This will have the effect of reducing the principal and future interest amounts. Although not conventionally seen as an option, its value can be obtained similarly to the case just discussed.

12.5.7 Leases

Options can exist within leases. For example, Oppenheimer [6] mentions subleasing as a call option held by the lessor, and cancellation as a put option held by the lessee.

Others mention lease renewals as call options held by the lessor with renewal terms indexed to, for example, inflation or sales; the option to purchase at the end of a lease period is a call option held by the lessor; the case where the lessee receives a share of sales above a threshold is a call option held by the lessee.

The evaluation of all these options follows the same path as previously treated options. Estimates, including uncertainty, are made for equivalent cash inflows and cash outflows at time T with respect to the person holding the option; cash flows related to the option exercising but extending beyond time T are discounted to time T. Collectively, all the cash flows are discounted to time 0 and the option value calculated from Eq. (1.1).

12.6 Conclusion

Where the future movements in underlying prices or values or proxies are anticipated to be the same as the past, Black-Scholes, binomial lattices and equivalent Monte Carlo simulation, albeit with modification, can be used to value options. There is a large literature on this. Assumptions may be needed to extend such financial options tools to real options. Where the market is atypical, such methods may not be appropriate. This book's approach, however, has relaxed assumptions and can deal, for example, with any market irregularities or regularities as well as incorporate uncertainty in all the analysis variables.

The book's approach gives option values close to that of Black-Scholes. Trends in behaviour with rspect to analysis variables are also the same.

Property and index prices are uncertain, and real estate options provide one way of dealing with the associated risk. In atypical markets, including those influenced by political, economic and external factors, traditional option pricing tools may not apply. This book gives a practical method, suitable in such atypical markets as well as regular markets, by which real estate options could be valued. The approach is based on recognizing cash flows or cash flow equivalents that result from exercising an option, and the using a probabilistic present worth analysis. The approach's strengths are that it is intuitive to understand, straightforward to implement and requires low mathematical sophistication. Estimates of the cash flows or cash flow equivalents, including their uncertainty, can be made by any means available, and can incorporate experience, knowledge of the market, knowledge of anticipated shifts in economies or external factors, available time series or trends, or any other information known. There is no prescribed underlying price time series to follow, or method by which estimates should be obtained. The estimating method and incorporated knowledge is discretionary; all that is wanted is the best estimates possible.

It is anticipated, as with Black-Scholes, that users of the book's approach will adjust the approach over time to suit their experiences and particular requirements.

The chapter explored different real estate options including those related to property, indices, mortgages and leases, and demonstrated how these might be valued in atypical markets.

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