Comparative Analysis of Digital Image Watermarking Based on Lifting Wavelet Transform and Singular Value Decomposition

Sameena Naaz, Ehtesham Sana and Iffat Rehman Ansari

Abstract The various kinds of unauthorized use and modification of data has been constantly increasing day by day and very common due to the enormous growth of multimedia information on the Internet. Thus, authenticity of the data is one of the major issues in the defence data transmission as well as in secured transmission. To overcome this sort of problem, digital watermarking is supposed to be one of the legitimate solutions as it provides possible validation, safe, and secure transmission of secret data. Here, a robust and secure digital image watermarking method for the protection of copyright has been developed which involves LWT in combination with SVD. The latest approach to wavelet transform is LWT while the significant transform technique is SVD and the obtained experimental results are cross-validated by using inverse LWT and SVD. In order to create and insert a digital signature, the signature mechanism has been used after embedding the watermark and the ownership is authenticated before extracting watermarks in this research work. The robustness of the proposed method for several common image processing attacks like Mean, Median, Gaussian, Shear, Rotation, and Crop is checked by computing the two significant performance metrics, namely, PSNR and MSE. Here, the similarity is also checked by determining the NCC values. Finally, the watermarking scheme suggested in this work is compared with various existing research works which show that the algorithm used in the proposed scheme works fairly in terms of PSNR, MSE, and NCC.

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1 Introduction

The multimedia messages are being shared on a very large scale at present and are tremendously increasing day by day too. These messages may include the information available on the Internet in the form of images, audios, and videos as a result of which it is a problem of concern to secure the digital information. In this way, technocrats are trying endeavors to make innovations for the security of information. The technique that is employed to communicate interactive multimedia information secure over the network by giving proprietorship rights in Digital Watermarking and it is supposed to be one of the safe, secure, and progressed scheme as compared to other effective current strategies. The branch of applied science which basically reviews the digital images and changes in these images to enhance the quality or to extract data is referred to as Digital Image Watermarking [1, 2].

The secret information of the signal can be hidden within the cover signal itself by a technique known as Watermarking and hence the cover signal can be protected. The interactive media objects within which the watermark is inserted are typically known as the original image/cover image. When the watermark is inserted, the uniqueness of the original/cover image should not be changed and later on, it can be extracted as a proof of authenticity owner. In the process of watermarking, the watermark insertion procedure is generally known and therefore it is not necessary for the message to be secret. The essential requirements of a watermarking system are listed below:

- Imperceptibility
- Robustness
- Security
- Data Payload
- Computational Complexity
- Inevitability [3].

The watermarking techniques can be typically categorized as follows:

- Spatial domain
- Transform domain.

In spatial domain methods, the watermark is inserted by changing the intensity values of the image and these methods are less complex as well as easy to implement but they are typically fragile to image processing attacks. On the other hand, in frequency domain methods, the watermark is inserted in the frequency elements of the image and these methods are simple than the spatial domain methods. However, the transform domain methods are very popular as they are more resistant to noise and the commonly used techniques are LWT, DCT, DFT, DWT, and SVD. In order to perform multilevel decomposition, DWT is generally used that breaks down the signal/image into approximation and detail coefficients at each level and this kind of process is similar to low-pass and high-pass filtering [4].

In this research paper, digital image watermarking scheme based on LWT-SVD technique is presented to insert the watermark image into the cover/original image, thereby maintaining overall robustness property of the watermark, excellence of carrier media and security. The proposed scheme is also compared with the other existing strategies on the basis of PSNR, MSE, and NCC.

2 Literature Review

In order to hide data in digital color images, literature review survey was done on wavelet transformation and wavelet transformation in combination with SVD techniques.

The insertion of watermark into a cover image based on 2-level DWT and SVD has been proposed in [5] and the algorithm used in this work is very secure for guarding the multimedia objects. The qualitative as well as quantitative results confirm the robustness, imperceptibility, and security of the watermark.

A number of digital watermarking techniques for the protection of copyright based on SVD, DWT, and DCT have been proposed in [6–10]. In order to check the robustness of these methods, various attacks have been applied to the watermarked images. The PSNR and correlation coefficients were used as parameters for estimating the quality of the watermarked images. The simulation results obtained show that the watermarks are imperceptible in the domain.

Other robust watermarking methods based on DWT and SVD has also been suggested in [11–15]. The results achieved from these methods are very effective in terms of imperceptibility, security, capacity, and robustness and these techniques are found to be very robust against various types of attacks.

3 Preliminaries

The main objective of watermarking is the embedding of secret information that cannot be extracted easily. To do this, information should be embedded in particular parts of the image. The main terminologies employed in the proposed system are the cover image, LWT, and SVD and these are explained as follows.

3.1 Cover Image

It is the image in which the watermark will be inserted. There are various schemes available to protect this type of signal when it is transmitted over the network. Thus, the cover image is considered as the main carrier signal.

3.2 Lifting Wavelet Transform (LWT)

It is also known as second-generation wavelet transform or forward wavelet transform (FWT) and was proposed in [16]. Let I (m, n) be a 2-dimensional signal. First, the 1-dimensional wavelet transform is done in the vertical direction and after that in the horizontal direction on the given signal without any loss of generality. Each 1-dimensional wavelet transform is factored into one or more lifting stages. The basic principle and structure of lifting stage are mentioned in [17]. As far as the lifting stage is concerned, it comprises three steps and these are named as split, predict, and update.

1. **Split operation**: It is also known as lazy wavelet transform. Here, all the samples will be divided into two sets, namely, even polyphase sample set, i.e., Ie (m, n) as defined in Eq. (1) and the other is odd polyphase sample set, i.e., Io (m, n) as given in Eq. (2):

$$\mathrm{Ie}(m, n) = \mathrm{I}(2m, 2n) \tag{1}$$

$$\mathrm{Io}(m, n) = \mathrm{I}(2m, 2n + 1) \tag{2}$$

2. **Predict operation**: Predict operation is referred to as dual lifting. In this operation, the polyphase samples which are odd can be predicted from the neighboring polyphase samples which are even. The error is evaluated as the prediction of the odd samples from the even samples by using a prediction operator P and this error is actually the high-pass coefficient h (m, n) which is given by Eq. (3):

$$h(m, n) = \mathrm{Io}(m, n) - P[(\mathrm{Ie}(m, n)] \tag{3}$$

Now, the odd sample set can be recovered as given by Eq. (4):

$$\mathrm{Io}(m, n) = h(m, n) + P[\mathrm{Ie}(m, n)] \tag{4}$$

3. **Update operation**: Update operation is also known as primal lifting. In this particular operation, the even samples are updated with the updating value Uh (m, n) to generate the low-pass coefficient l (m, n) as defined in Eq. (5):

$$\mathrm{l(m, n) = Ie(m, n) + Uh(m, n)} \quad (5)$$

The predictions continuously come back from the vertical neighboring polyphase samples or even polyphase samples in the traditional lifting.

LWT is also known as second era quick wavelet transform could be a substitute methodology for discrete wavelet transform and in this particular transform, split and merge replaces the up- and downsampling in each of its levels. The computational complexities are decreased to half by the split and merge process in LWT. The loss of information is also less on comparing to DWT procedure because up and down sampling are not used in LWT-based procedure. In a parallel process, both even and odd polyphase segments of the signal are filtered with the help of their wavelet filter coefficients, thus produce the best result and an image reconstructed by lifting wavelet results in increased smoothness and decreased aliasing effects as compared to general wavelets [18]. Thus, by employing LWT flawlessness of embedded watermark within the image is increased, information loss is decreased, and increases the robustness of watermark. There are several advantages of LWT [19, 20] such as fewer memory prerequisites, low aliasing effects, distortion, reconstruction is good, and computational complexities are less. One of the demerits of lifting wavelet transform is that it will become difficult to take decisions on coefficient values as small values represent large coefficients. During this disintegration, filter coefficients will be regenerated into lifting coefficients represented as predict s (z), update t (z), and scaling (k) utilizing Euclidian procedure. The original image is divided into an odd set of samples and even set of samples and then approximate and detailed sub-bands are obtained by applying lifting coefficients to the sampled original image.

3.3 Inverse Lifting Wavelet Transform (ILWT)

As far as the inverse LWT is concerned, it is just exactly the reverse method of LWT/FWT and is determined by simply altering the sign. Hence, it can be easily computed. It consists of the following three main steps:

1. **Inverse Primal lifting operation**: This operation is also called as undo update. In this particular operation, original even samples can be reconstructed by just subtracting the update information.
2. **Inverse Dual lifting operation**: This is also known as undo predict. When prediction information is simply added to the loss of information, original odd samples can be recovered.
3. **Merge operation**: In this operation, the original signal can be obtained by merging together the recovered odd and even samples. In order to perform merging of the signal, even samples are interpolated by inserting zeros in between the samples and after that the odd samples are placed in place of zeros.

3.4 Singular Value Decomposition

It is one of the numerical examination strategy. It has been found long back and previously used to serve numerous applications. The SVD plan was presented in the 1870s by Beltrami and Jordan, utilized for real square matrices and afterward for complex matrices in 1902 by Autonne's. The researchers Eckart and Young enhanced the SVD system to incorporate rectangular matrices in 1939. At present, the SVD plan is successfully used in image watermarking, image compression, image hiding, and reduction of noise [21]. An SVD-based image watermarking scheme in the spatial domain is found to be very robust and is discussed in [22]. As far as SVD scheme is concerned, it is actually derived from linear algebra theorem. In this, a rectangular matrix I can be decomposed into three matrices in which the two matrices are singular matrices and third matrix is the diagonal matrix. Let I be a 2-D matrix having M*N dimensions. When SVD is applied to I, it gets disintegrated as defined by Eq. (6):

$$\mathrm{I} = \mathrm{U} * \mathrm{S} * \mathrm{V}^{\mathrm{T}} \tag{6}$$

$$\mathrm{I} = \sum_{i=1}^{r} u_i s_i v_i^T$$

$$\mathrm{U} = [\mathrm{u}_1, \mathrm{u}_2 \ldots \mathrm{u}_{\mathrm{N}}]$$

where symbols U and V are singular orthogonal matrices and I denotes diagonal matrix. They actually determine the detailed geometry of the original image. Let r ($r \leq n$) be the rank of the matrix I. After that the diagonal values of the matrix S are placed in the descending order and now, the matrix I can be defined by Eq. (7):

$$\sigma 1 \geq \sigma 2 \geq \sigma 3 \geq \cdots \sigma r \geq \sigma r + 1 \geq \cdots \sigma n = 0$$

$$\mathrm{I} = \sum_{k=1}^{r} \sigma_k u_k v_k^T \tag{7}$$

where $\mathrm{u_k}$ and $\mathrm{v_k^T}$ are the kth eigenvector of U and V and $\mathrm{s_k}$ denotes the kth singular value, respectively. The quality of the cover image in which the watermark is to be inserted can be very well maintained by using SVD. So rather than inserting a watermark into 2-D cover, it is much better to insert the watermark into one of the decomposed matrices as given by Eq. (7).

4 Proposed System

The watermarking scheme proposed in this research work is using an LWT method in combination with SVD. Our objective is to provide improvement and enhancement in the robustness of the watermark that also results in its improved imperceptibility. Here, the image is decomposed first into four frequency bands, namely, LL, LH, HL, and HH bands, respectively. LL depicts the band of low-frequency providing the approximate details, LH and HL both are the bands of middle frequency providing the vertical details and horizontal details, respectively, while the HH band depicts the band of high frequency providing the diagonal details of an image. In the proposed scheme, HH band has been used for inserting the watermark as it consists of the finer details of the image. The basic model of digital image watermarking consists of two main sections, namely:

- Watermark embedding
- Watermark extraction.

The method of inserting the watermark into the original/cover image is referred to as watermark embedding. The watermarked image is the output of watermark embedding process. This procedure is done on sender's side while the watermark extraction is the method of watermark detection from the watermarked image. The scheme proposed here comprises two main algorithms, namely, watermark embedding algorithm and watermark extraction algorithm as shown in Fig. 1a, b.

5 Performance Evaluation Metrics

In this research paper, the following quality measures have been used to estimate the performance of the watermarked images:

5.1 Peak Signal-to-Noise Ratio (PSNR)

Here, the imperceptibility between the original host image and the watermarked image can be measured by PSNR. As far as the watermark is concerned, it should not be noticeable to the user as well as it should not corrupt the quality of the original image. For a gray-scale image, PSNR is defined as

$$PSNR = 10\log_{10}\left[\frac{255^2}{MSE}\right] \tag{8}$$

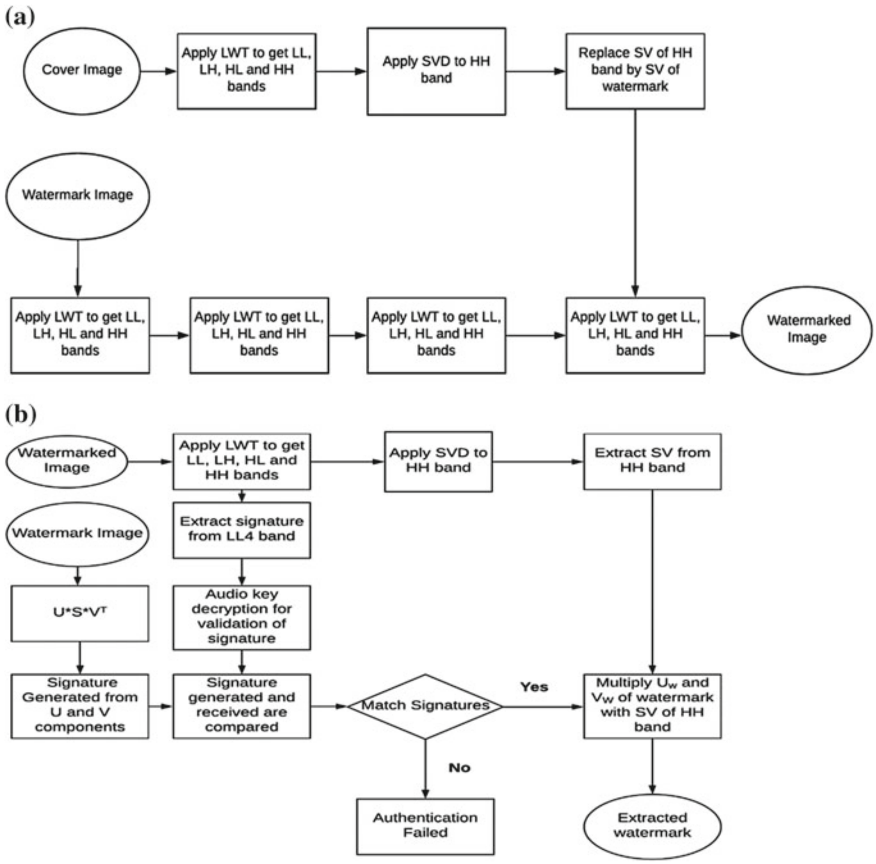

Fig. 1 **a**. Watermark embedding procedure. **b.** Watermark extraction procedure

5.2 Mean Square Error (MSE)

Another performance metric used in this work is MSE that can be defined as the mean square error between the cover image and the watermarked image. The MSE can be defined as

$$MSE = \frac{1}{M \times N} \sum_{i} \sum_{j} [I(i, j) - I_w(i, j)]^2 \tag{9}$$

where I (i, j) indicates the original host image consisting of M × N pixels and I_W (i, j) denotes the watermarked image.

5.3 Normalized Cross-Correlation (NCC)

To check the similarity between the two images having same size, the correlation coefficient (CC) is used. Its value varies between 0 and 1 but the ideal value for CC is supposed to be 1, i.e., unity. Here, the similarity between the inserted and the extracted watermark can be measured by the correlation coefficient which is defined as follows:

$$\rho(W, W^*) = \frac{\sum_{i=1}^{N}\sum_{j=1}^{N}\left(w_{ij} - \mu_w\right)\left(w_{ij}^* - \mu_w^*\right)}{\sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N}\left(w_{ij} - \mu_w\right)^2}\sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N}\left(w_{ij}^* - \mu_w^*\right)^2}} \quad (10)$$

where w denotes the watermark inserted, w^* denotes the watermark extracted, μ_w indicates the mean value of the inserted watermark, and μ_w^* indicates the mean value of the extracted watermark, respectively.

6 Experimental Results

In this research work, LWT–SVD scheme is proposed and the simulation of this scheme has been implemented in MATLAB. The digital color image “Lena” of dimension 512 × 512 is chosen as the original/cover image while the digital color image “Cherry” of dimension 512 × 512 is used as the watermark image to perform the experiment. Here, the 1-level and the 4-level LWT decomposition of the cover/original image is also done as SVD which is applied to only HH band and the digital signature is embedded into LL4 band.

6.1 Test for Imperceptibility

Imperceptibility is the measure of transparency. By using performance metrics such as PSNR and MSE, imperceptibility can be measured and it is good when the watermarked image almost resembles/looks like the original/cover image. Hence, the watermark embedding process hardly affects the cover image. The cover image and the watermarked image are shown in Fig. 2a, b, respectively. The watermark logo/image or simply the watermark is shown in Fig. 2c. The values of PSNR and MSE for the watermarked image before and after the attack with respect to the original cover image are given in Table 1. Figure 3a shows the PSNR values for original/cover and watermarked image while Fig. 3b shows the MSE values for the original/cover and watermarked image.

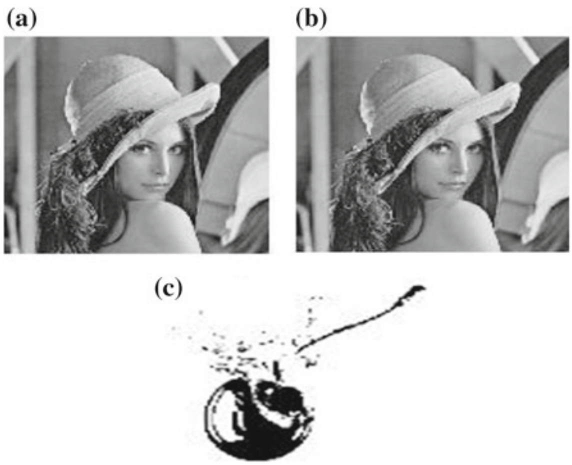

Fig. 2 **a**. Cover image **b**. Watermarked image signed with secret key **c**.Watermark logo

Table 1 Values of PSNR and MSE for cover and watermarked image

Attacks	Performance metrics	
	PSNR	MSE
No attack	70.31	0.0059
Mean	28.28	96.42
Median	35.21	19.59
Gaussian	39.28	7.675
Rotation	10.43	5.8851e + 03
Shear	9.07	8.0404e + 03
Crop	13.90	2.6480e + 03

6.2 Test for Robustness

The robustness of the method proposed in this work has been checked by performing various geometric attacks on the watermarked image and then extracting the watermark. The robustness of the image was checked by comparing the similarity of the extracted watermark with the original watermark. The robustness of the watermarking scheme against various types of attacks can be checked by using values of NCC. The robustness is tested under six types of attacks, namely, mean attack, median attack, Gaussian attack, rotation attack, shear attack, and crop attack, respectively. Figure 4 shows the extracted watermark before the attack. Figure 5 shows the extracted watermark after applying different types of attacks. In this research work, the values of the correlation coefficient between extracted watermark and the original watermark for original/cover image "Lena" before and after the attack is given in

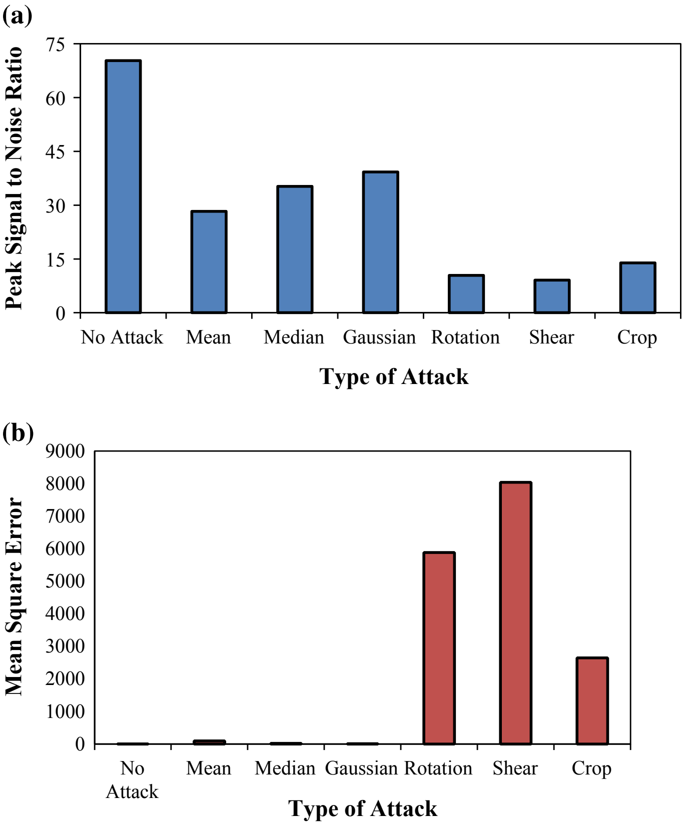

Fig. 3 **a**. PSNR values for cover and watermarked image. **b.** MSE values for cover and watermarked image

Fig. 4 Extracted watermark without attack

Fig. 5 Extracted watermark images after applying various attacks

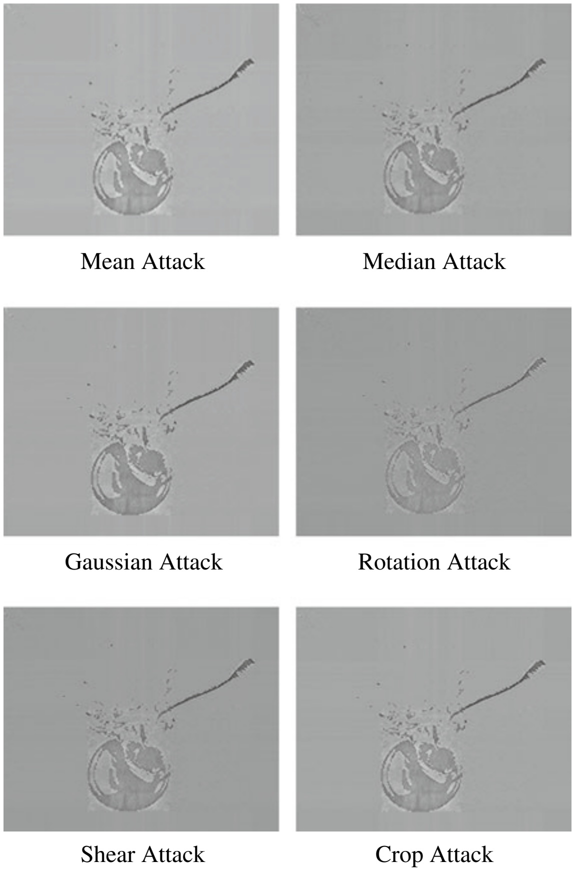

Table 2. The graph showing the values of the NCC for the extracted watermark is given in Fig. 6

Table 2 Values of NCC for extracted watermark image

Attacks	Performance metric
	Normalized cross-correlation
No attack	1.00
Mean	0.7687
Median	0.9592
Gaussian	0.9354
Shear	0.8154
Crop	0.8289

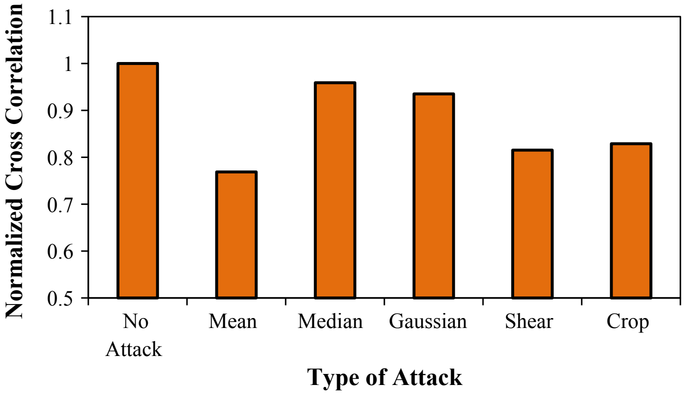

Fig. 6 Normalized cross-correlation values for extracted watermarks

6.3 Comparison of Proposed Method with Kaur et al. [5] and Radouane et al. [6] Based on Peak Signal-to-Noise Ratio

The results of the proposed scheme are compared with the results of [5, 6]. One of the schemes to embed a watermark into a cover image has been proposed in [5] which works on the combination of 2-level DWT and SVD. The other scheme has been proposed in [6] to insert the watermark in different coefficients of DWT (LH, HL, HH) and SVD technique was used. Other than this, in this particular scheme, an optimal block based on maximum entropy has been utilized for inserting the watermark in the original/cover image. Table 3 gives a comparison of the proposed work with that of [5, 6] in terms of imperceptibility which can be measured by one of the significant performance metrics, i.e., PSNR. The proposed algorithm clearly outperforms the two algorithms based upon the quality of watermarked image. The results are plotted in the form of bar chart as shown in Fig. 7.

Table 3 Comparison of proposed method with Kaur et al. and Radouane et al. based on peak signal-to-noise ratio

Attacks	Proposed method	Kaur et al.	Radouane et al.
No attack	70.39	68.79	60.052
Median	35.21	–	25.173
Gaussian	39.28	32.111	18.973
Crop	13.90	11.38	12.599

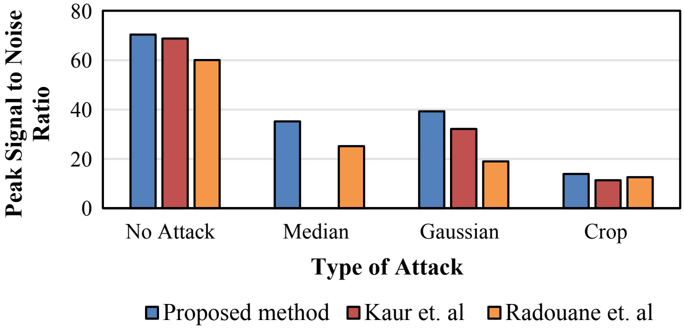

Fig. 7 Comparison of proposed method with Kaur et al. and Radouane et al. based on peak signal-to noise ratio

6.4 Comparison of Proposed Method with Kaur et al. [5], Loukhaoukha and Chouinard [21] and Zhang et al. [22] Based on Normalized Cross-Correlation

One of the hybrid robust watermarking methods based on LWT and SVD has been proposed in [21] in which the original/cover image was decomposed into a 2-level lifting LWT and then the ILWT of a selected SB was calculated. The singular values are also modified for the watermark embedding process. Another watermarking method based on an SVD was proposed in [22] in which the watermark was embedded in the largest singular value of the image blocks. The results presented in Table 4 show that our proposed scheme outperforms the other three methods based upon the normalized cross-correlation. The same results have been also depicted in Fig. 8.

Table 4 Comparison of proposed method with Kaur et al., Loukhaoukha and Chouinard and Zhang et al. based on normalized cross-correlation

Attacks	Proposed method	Kaur et al.	Loukhaoukha and Chouinard	Zhang et al.
No attack	1.0000	0.9707	1.0000	1.0000
Median	0.9592	–	0.4700	0.9386
Gaussian	0.9354	0.9754	0.8500	0.9693
Crop	0.8289	0.3254	0.7200	0.7828

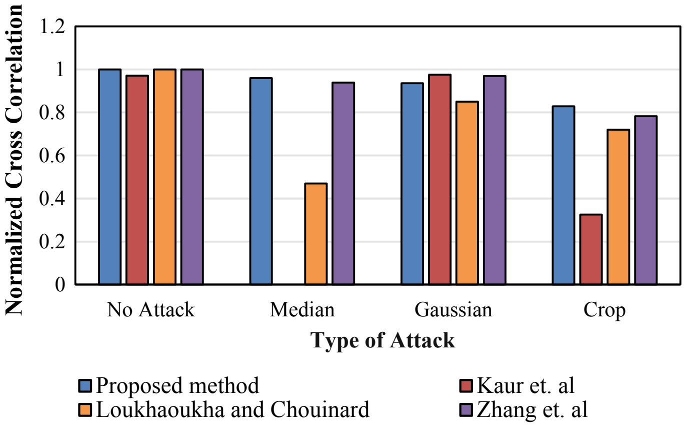

Fig. 8 Comparison of proposed method based on normalized cross-correlation

7 Conclusion

This research paper discusses an improved, robust, and secure image watermarking scheme based on LWT and SVD. The proposed scheme is implemented using MATLAB. Here, the steps used for image processing include embedding of the watermark (i.e., encoding), attacking the watermarked image, extracting the watermark (i.e., decoding), and evaluation of the results achieved. The scheme proposed here utilizes the properties of LWT as well as SVD transform to obtain the requirements of the watermarking, i.e., imperceptibility, robustness under various attacks and security. The main goal of the research is to provide improvement in the efficiency of digital image watermarking under various types of attack. To analyze the performance of the watermarked images in our work, various performance metrics have been computed, namely, NCC, PSNR, and MSE. This watermarking scheme gives NCC value 1 for no attacks with good PSNR and MSE values. The parameter values achieved by this scheme are good for most of the common image processing attacks and as far as the security is concerned, it is also improved and guaranteed by using a digital signature authentication mechanism.

The proposed method is compared with four different research works given in the literature and it has been found that our algorithm outperforms all the others in terms of PSNR, MSE, and NCC.

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