Risk Analysis of Lateral Collision of Military and Civil Aviation Aircraft Based on Event Model

Guhao Zhao[1,2(✉)], Shaojie Mao[1], Yi Mao[1], Xiaoqiang Zhang[2], and Yarong Wu[2]

[1] State Key Laboratory of Air Traffic Management System and Technology, Nanjing 210007, Jiangsu, China
zghlupin@163.com

[2] Air Traffic Control and Navigation College, Air Force Engineering University, Xi'an 710077, Shaanxi, China

Abstract. Collision risk analysis is an important part of airspace safety assessment. Based on Event model, the lateral position deviation probability of military aircraft is calculated. Combining with the probability model of lateral position deviation of civil aviation aircraft, the frequency of military aircraft collision box passing through separation sheet in high slope circling training is calculated. A collision risk assessment model between training flying aircraft in military training airspace and aircrafts flying in civil aviation route is constructed. Through the simulation calculation of the lateral collision risk of military and civil aviation, the size, layout, and use suggestions of military high slope circling training airspace are obtained, which can provide reference for the airspace safety assessment.

Keywords: Airspace safety assessment · Collision risk · Military training airspace · Air route

1 Introduction

In recent years, with the rapid development of civil aviation industry and the rapid growth of air traffic flow, at the same time, the renewal of Air Force Weapons and equipment is accelerating, and the demand for airspace for military flight training is also increasing. This makes the contradiction between military aviation and civil aviation in the demand for space resources increasingly prominent. On the premise of meeting the requirement of safety target grade, according to the requirement of military flight training subjects, it is an effective way to solve the contradiction between military aviation and civil aviation to delimit the airspace scope suitable for training needs and improve the utilization ratio of airspace.

In the 1960s, Reich first proposed the REICH collision risk model [1], which is used to analyze the safety of air routes. In 2003, Peter Brooker proposed Event model

Project supported by the National Natural Science Foundation of China (Grant Nos. 61601497).

Q. Liang et al. (Eds.): *Artificial Intelligence in China*, LNEE 572, pp. 76–86, 2020.
https://doi.org/10.1007/978-981-15-0187-6_9

[2] to assess the risk of lateral collision between civil aviation aircraft. With the advent of various collision risk models, many scholars at home and abroad have made in-depth research on them and put forward various improved methods [3]. However, the above studies only focus on the collision probability between civilian aircraft flying in the air route, and there is little literature on the collision probability between military aviation and civil aviation. In this paper, the Event model is used to model and calculate the collision risk between the civilian aviation aircraft in the air route and military aircraft in the adjacent training airspace.

2 Event Model

According to the original Event model, a separation sheet with no thickness is defined centering on aircraft "B." A collision box is defined centering on aircraft "A." In the process of passing through the separation sheet, the collision box leaves a projection on the separation sheet, which is defined as an extended collision box. According to the probability theory, the collision probability of two aircraft is equal to the product of the probability of the collision box passing through separation sheet and the probability that aircraft "B" is located in the extended collision box, as shown in Fig. 1.

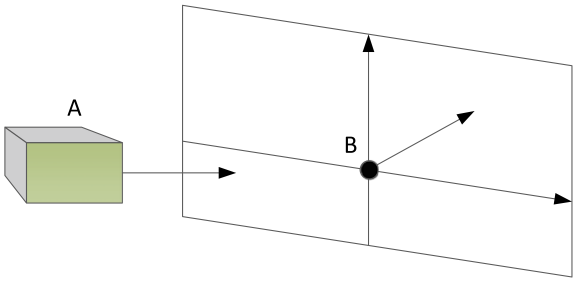

Fig. 1. Collision box passing through separation sheet

The collision probability is shown in Formula 1.

$$N_{\mathrm{ay}} = \mathrm{GERh}\frac{E(S)}{2L}\left(2\lambda_x + \frac{U2\lambda_{\mathrm{y}}}{V}\right)P_{\mathrm{z}}(0)\left(1 + \frac{W2\lambda_{\mathrm{y}}}{V2\lambda_{\mathrm{z}}}\right) \tag{1}$$

where GERh is the probability of lateral overlap of two aircrafts per hour, and L is the longitudinal separation standard. $E(S)$ is the logarithm of the aircraft flying in the same direction within a distance of $2L$. λ_x, λ_y, λ_z are the length, width, and height of the collision box, respectively. When aircraft "A" passing through the separation sheet, U, V, and W are the relative speeds of the two aircraft in the longitudinal, lateral, and vertical directions, respectively. $P_z(0)$ is the probability of vertical overlap between two aircraft at the same flight level.

In Eq. 1, the parameter GERh is statistical data, and however, such a statistical database is missing between military and civil aircraft. And in the air route, the aircraft heading at the same flight level is the same, and the heading of the military training flight is not fixed. For the above two reasons, the Event model cannot be directly applied to the assessment of the collision risk of military and civil aviation. Therefore, the Event model needs to be modified for the characteristics of military and civil aviation flights.

3 Collision Probability Model of Military and Civil Aviation

3.1 Hypothetical Conditions of the Model

(1) Assuming that the long side of the military training airspace is parallel to the air route, military aircraft and civil aviation aircraft fly at the same altitude.
(2) The interval between military training airspace and air route is 10 km.
(3) The positions of military aircraft and civil aircraft are independent of each other, and each uses independent navigation facilities.
(4) Considering that in military flight training, most aircraft are turning when approaching the boundary of airspace, this paper calculates the collision probability of military and civil aviation aircraft by taking the high slope circling training as an example.

3.2 Event Collision Model for Military and Civil Aviation

A rectangular collision box is defined with military training aircraft A as the center. The length, width, and height of the collision box are λ_{hx}, λ_{hy}, λ_{hz}:

$$\begin{cases} \lambda_{hx} = \lambda_{mx} + \lambda_{jx} \\ \lambda_{hy} = \lambda_{my} + \lambda_{jy} \\ \lambda_{hz} = \lambda_{mz} + \lambda_{jz} \end{cases} \tag{2}$$

where λ_{jx}, λ_{jy}, λ_{jz} and λ_{mx}, λ_{my}, λ_{mz} are the fuselage length, wingspan, and fuselage of military aircraft and civil aircraft, respectively, and consider the civil aircraft as a point B. When point B is in contact with the collision box A, It can be considered that two aircraft have collided.

Unlike the traditional Event model, military aircraft and each passenger airliner can be considered as pairs of aircraft with potential collision risks. Therefore, the Event model can be rewritten as

$$\begin{aligned} N_{\mathrm{ay}} = {} & \mathrm{GERh}\frac{2E(S)}{2L}\left(\lambda_{hx} + \frac{U_{\mathrm{s}}\lambda_{hy}}{V_{\mathrm{s}}}\right)P_{\mathrm{z}}(0)\left(1 + \frac{W_{\mathrm{s}}\lambda_{hy}}{V_{\mathrm{s}}\lambda_{hz}}\right)\beta \\ & + \mathrm{GERh}\frac{2E(O)}{2L}\left(\lambda_{hx} + \frac{U_{\mathrm{o}}\lambda_{hy}}{V_{\mathrm{o}}}\right)P_{\mathrm{z}}(0)\left(1 + \frac{W_{\mathrm{o}}\lambda_{hy}}{V_{\mathrm{o}}\lambda_{hz}}\right)\beta \end{aligned} \tag{3}$$

Among them, β is the probability of military aircraft training in the military training airspace within 1 h. $2E(S)$ and $2E(O)$, respectively, indicate how many pairs of military and civil aircraft are in the same direction and in the opposite direction in the air route of length $2L$. U_S, V_S, W_S, and U_O, V_O, W_O are relative speeds in the longitudinal, lateral, and vertical directions when aircraft A and aircraft B fly in the same direction and in the opposite direction. The relative speeds of the two aircrafts in the lateral and vertical directions are the same whether they are in the same direction or in the reverse direction, so $V_S = V_O$ and $W_S = W_O$.

In the traditional Event model, GERh is defined as the frequency at which the collision box passes through the separation sheet, replacing the GERh with the statistical frequency of the civil aircraft's lateral interval of zero in one hour. However, there is no corresponding statistical data on the assessment of the risk of collision between military aircraft and civil aircraft. This paper estimates the GEHR value of military and civil aircraft based on the lateral position deviation probability model of the civil aircraft [4] and the lateral position deviation probability model of the military aircraft.

4 Military and Civil Aircraft Lateral Overlap Probability

4.1 Coordinate System

The track of flight training was plotted on an assumed XY coordinate system with the center of the track circle as origin. The X-axis is perpendicular to the training airspace boundary, and the positive half of the X-axis points to the side of the route. According to the relevant provisions, the safety interval between the training airspace and the air route is 10 km. Assume that the width of the air route is 20 km. At this time, the equation of the center line of the air route is $x = R_0 + 20$ km.

4.2 Lateral Position Deviation of Civil Aviation Aircraft

In general, the yaw of the aircraft on the route is mainly caused by the navigation error, and its probability density function follows the double exponential distribution with zero expectation [5]. which is

$$f_{\mathrm{norm}_y}(y_1) = \frac{1}{2\sigma_y} \exp\left(-\left|\frac{R_0 + 20 - y^1}{\sigma_y}\right|\right) \tag{4}$$

where σ_y is the parameter corresponding to the yaw error probability density function, and this parameter can be determined by the RNP value -n. According to the definition of RNP, n refers to 95% of the time, the aircraft is flying within the range of n nautical miles on both sides of the route centerline. According to different RNP values, the parameter σ_y under the corresponding navigation condition can be calculated. According to the calculation of Ref. [5], when the RNP value is 4, σ_y is 1.33.

4.3 Probability Model of Lateral Position Deviation of Military Aircraft

Military aircraft are often in maneuvers during training flights. And there is no statistical data on the lateral position deviation of military aircraft. Therefore, the lateral positional deviation of military aircraft in the training airspace can only be studied through mathematical models. According to the characteristics of circling flight, its lateral deviation is affected by the turning radius, the center of the turning track, and the wind.

4.3.1 Turning Radius Probability Density Function

Pilot's motion error in military training flight follow Gaussian distribution

$$\gamma(M_{\text{last}}) = \frac{1}{\sqrt{2\pi}\sigma_{\text{p}}} \exp\left(-\frac{(M_{\text{last}} - M_{\text{shlould}})^2}{2\sigma_{\text{p}}^2}\right) \tag{5}$$

where M_{shlould} is the expected motion, M_{error} is the actual motion, and σ_{p} is the standard deviation. In the process of circling training, the factors affecting the lateral deviation of the aircraft are mainly the speed of the aircraft, the heading, the turning gradient γ, the crosswind, the position of the starting point of the circling, and the navigation accuracy. As shown in Fig. 2, it is assumed that the aircraft starts to fly in the clockwise direction from the S point, and the desired circling path is as shown by the solid line, and the circling path under the influence of the action error and the omnidirectional wind is, respectively, indicated by two broken lines.

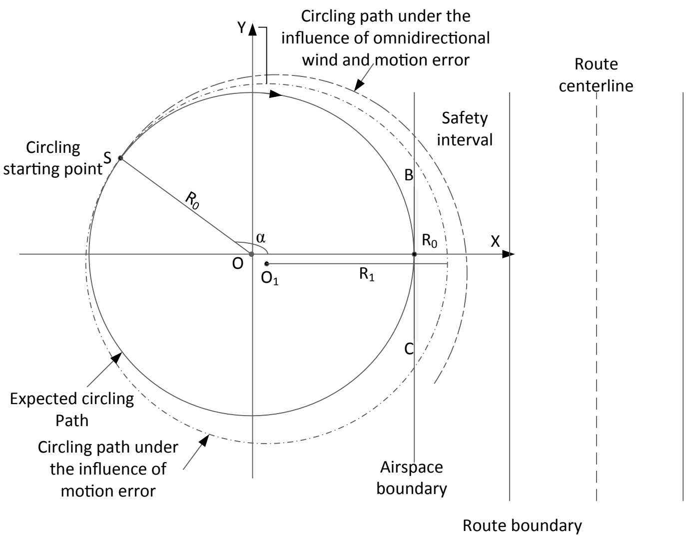

Fig. 2. The lateral spacing of the high slope circling training airspace and the route

Suppose the expected flight speed is $V = 900$ km/h, the actual speed is V', the expected turning slope is $\gamma = 45°$, and the actual turning slope is γ', both of which are affected by the pilot's motion error. Then, the turning radius under the influence of speed error and turning slope error is:

$$R_1 = \frac{(V')^2}{g\tan(\gamma')} \tag{6}$$

The actual speed and the actual turning slope obey the following probability distribution.

$$\begin{aligned} f_\gamma(\gamma') &= \frac{1}{\sqrt{2\pi}\sigma_\gamma}\exp\left(-\frac{(45-\gamma')^2}{2\sigma_\gamma^2}\right) \\ f_V(V') &= \frac{1}{\sqrt{2\pi}\sigma_V}\exp\left(-\frac{(250-V')^2}{2\sigma_V^2}\right) \end{aligned} \tag{7}$$

4.3.2 Center of the Circling Path

Suppose the aircraft starts to turn from the $S(x_s, y_s)$ point in the heading θ, then the actual center O_1 (x_1, y_1) is

$$\begin{cases} x_1 = x_s + R_1\cos\theta \\ y_1 = y_s - R_1\sin\theta \end{cases} \tag{8}$$

It is assumed that the heading error is affected by navigation accuracy, airborne navigation equipment error, and pilots' operation error. All of them obey the Gauss distribution in angle. Therefore, the joint probability distribution function of heading error also follows the Gauss distribution.

$$f_\theta(\theta') = \frac{1}{\sqrt{2\pi}\sigma_\theta}\exp\left(-\frac{(\vartheta-\theta')^2}{2\sigma_\theta^2}\right) \tag{10}$$

According to the conservative values given in ICAO 8168 document, the standard deviation is theta = 2.6°, where theta is the heading angle in the ideal state and the expression is:

$$\vartheta = \begin{cases} \pi - \arctan\left(\frac{y_S}{x_S}\right) & x_S > 0, y_s \neq 0 \\ 2\pi - \arctan\left(\frac{y_S}{x_S}\right) & x_S < 0, y_s \neq 0 \\ \frac{\pi}{2} & x_S = 0, y_S = R_0 \\ -\frac{\pi}{2} & x_S = 0, y_S = -R_0 \\ \pi & x_S = R_0, y_S = 0 \\ 0 & x_S = -R_0, y_s = 0 \end{cases} \tag{11}$$

4.3.3 Omnidirectional Wind

The plane is inevitably affected by the wind. However, a certain wind direction cannot be specified when estimating the track. Therefore, ICAO defines a wind direction that is the most unfavorable to the aircraft—omnidirectional wind. The omnidirectional wind direction is always perpendicular to the current heading of the aircraft and points in the opposite direction of the turn. Assuming that the omnidirectional wind speed is w, the real-time turning radius can be expressed as

$$R_1' = R_1 + \frac{w}{\alpha} \tag{12}$$

where α is the number of angles that the aircraft turns when it turns to a heading parallel to the airspace boundary. At this time, the military aircraft is closest to the civil aircraft. Therefore, the minimum lateral distance for defining a military and civil aircraft is

$$\Delta L = y' - x_1 + R_1' \tag{13}$$

where y' is the abscissa of the actual position of the civil aircraft.

5 Simulation of Military and Civil Aviation Lateral Deviation

There are nonlinear terms in the lateral deviation expression of military and civil aircraft, so it is difficult to obtain an analytical expression of the probability density function of the lateral spacing. Assume that each military airport has 24 fighters, each of which requires 200 h of flight training per year. There are five training airspaces in the airport area of responsibility. For each training flight, the duration of the military aircraft's activity in the airspace is half of the total flight duration. Then, the average frequency of training flights in each hourly airspace is

$$\beta = \frac{24 \times 200}{365 \times 24} \times \frac{1}{2} \times \frac{1}{5} = 0.054 \tag{14}$$

Assume that the starting point of the hover training flight is $S_2\left(-\frac{\sqrt{2}}{2}R_0, \frac{\sqrt{2}}{2}R_0\right)$. Other parameters involved in the simulation are as follows in Table 1.

Among the above parameters, the military civil aircraft size uses the public data of the F16 and A380 passenger aircraft [4]. The vertical safety interval uses a safety interval of 10 km from our radar. The Monte Carlo method is used to select the random number corresponding to the probability density function of speed and slope, and the simulation of the lateral deviation of military aircraft and civil aviation aircraft with n = 1,000,000 times is carried out. The results are shown in Figs. 3 and 4.

It is assumed that the training airspace boundary is tangent to the circling path, that is, no interval margin is left in the training airspace. The lateral deviation value of the civil aircraft obtained from the simulation is subtracted from the lateral deviation of the military aircraft. Calculate the number of simulations where the difference is less than

Table 1. Lateral collision probability simulation parameter

Parameters	Value	Parameters	Value
E (S)	0.61	λ_{mz} (m)	24.1
E (O)	0.01	λ_{jx} (m)	15.6
V (m/s)	250	λ_{jy} (m)	9.45
γ (°)	45	λ_{jz} (m)	5.09
σ_V	15	$U_s(m/s)$	3
σ_γ	2	$U_o(m/s)$	497
$P_z(0)$	0.5	$V_s$or $V_o(m/s)$	5.23
L (km)	10	W_s or W_o (m/s)	0.58
λ_{mx} (m)	72.8	β	0.054
λ_{my} (m)	79.8		

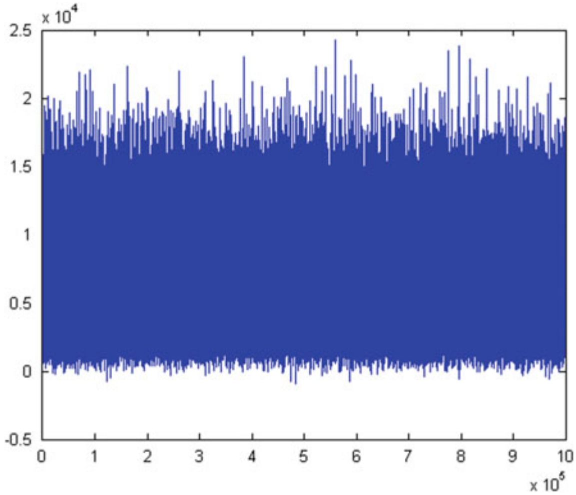

Fig. 3. Military aircraft lateral deviation

zero. From this, the frequency of the lateral separation loss of the military and civil aviation aircraft is GERh = 8.38×10^{-05}.

Considering the two cases of the same direction and reverse flight of military and civil aircraft, respectively, it is assumed that the aircraft with the same direction and reverse flight within 2L distance has $E(O)$ = 0.61, $E(S)$ = 0.01 pairs. Put GERh and the above parameters into Eq. 3. It is calculated that the collision probability of the same direction flight N_{ay_s} is 2.58×10^{-08}, the collision probability of reverse flight N_{ay_r} is 2.60×10^{-08}, and the total collision probability N_{ay_t} is 5.18×10^{-08}. Take eight different circling starting points and calculate their collision probability, respectively (Table 2).

From $S1$ to $S5$, that is, the starting point of the circling clockwise from the negative X-axis direction to the positive X-axis direction, the collision probability is gradually

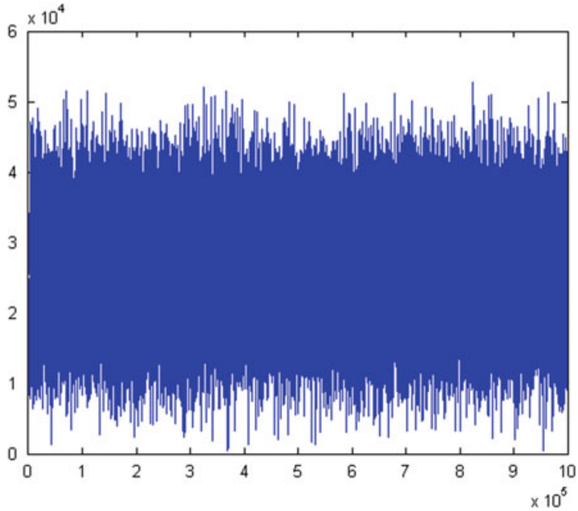

Fig. 4. Civil aircraft lateral deviation

Table 2. Military and civil aircraft collision probability without interval margin

Starting points	GERh	N_{ay_s} (Times/hour)	N_{ay_r} (Times/hour)	N_{ay_t} (Times/hour)
$S_1(-R_0, 0)$	9.56×10^{-05}	2.94×10^{-08}	2.96×10^{-08}	5.90×10^{-08}
$S_2\left(-\frac{\sqrt{2}}{2}R_0, \frac{\sqrt{2}}{2}R_0\right)$	8.38×10^{-05}	2.58×10^{-08}	2.60×10^{-08}	5.18×10^{-08}
$S_3(0, R_0)$	7.30×10^{-05}	2.25×10^{-08}	2.26×10^{-08}	4.51×10^{-08}
$S_4\left(\frac{\sqrt{2}}{2}R_0, \frac{\sqrt{2}}{2}R_0\right)$	6.34×10^{-05}	1.94×10^{-08}	1.95×10^{-08}	3.90×10^{-08}
$S_5(R_0, 0)$	5.45×10^{-05}	1.68×10^{-08}	1.69×10^{-08}	3.37×10^{-08}
$S_6\left(\frac{\sqrt{2}}{2}R_0, -\frac{\sqrt{2}}{2}R_0\right)$	1.05×10^{-04}	3.22×10^{-08}	3.24×10^{-08}	6.45×10^{-08}
$S_7(0, -R_0)$	1.25×10^{-04}	3.83×10^{-08}	3.86×10^{-08}	7.69×10^{-08}
$S_8\left(-\frac{\sqrt{2}}{2}R_0, -\frac{\sqrt{2}}{2}R_0\right)$	1.14×10^{-04}	3.51×10^{-08}	3.53×10^{-08}	7.05×10^{-08}

reduced. This is mainly because the military aircraft moves clockwise, the time to reach one side of the route is relatively short, and it is less affected by the omnidirectional wind. The deviation is mainly due to the pilot's operational error, and if the starting point of the turn is closer to the route, the less likely the military aircraft is to deviate from the training airspace due to pilot operational errors;

From *S*5 to *S*8, that is, clockwise from the positive *X*-axis direction to the negative *X*-axis direction, the collision probability gradually increases. The probability of collision is mainly affected by the pilot's operational error. But each point is affected by the omnidirectional wind. For example, points *S*3 and *S*7 at both positive and negative sides of the *Y*-axis are affected by the pilot's operational error. However, the path from the $S7$ point to the side of the route is longer, and it is also affected by the omnidirectional wind, so the probability of collision at the S7 point is high.

For the same circling starting point, when circling to the nearest position of the route, the collision probability of the military flight with the civil aviation aircraft in the same direction is lower than the collision probability of the reverse flight. The total collision probability of other circling starting points is larger than 5×10^{-9} except for all start points. Therefore, if the boundary of the circling path is regarded as the boundary of the airspace, there is a certain risk of collision. Therefore, we calculate the collision probability under various interval margin. When the ideal path and the airspace boundary have a safety margin of 7 km, the collision risk is shown in Table 3.

Table 3. Collision probability at 7 km safety margin

Starting points	GERh	N_{ay_s} (Times/hour)	N_{ay_r} (Times/hour)	N_{ay_t} (Times/hour)
$S_1(-R_0, 0)$	4.80×10^{-06}	1.48×10^{-09}	1.49×10^{-09}	2.96×10^{-09}
$S_2\left(-\frac{\sqrt{2}}{2}R_0, \frac{\sqrt{2}}{2}R_0\right)$	5.50×10^{-06}	1.69×10^{-09}	1.70×10^{-09}	3.40×10^{-09}
$S_3(0, R_0)$	5.30×10^{-06}	1.63×10^{-09}	1.64×10^{-09}	3.27×10^{-09}
$S_4\left(\frac{\sqrt{2}}{2}R_0, \frac{\sqrt{2}}{2}R_0\right)$	4.70×10^{-06}	1.44×10^{-09}	1.45×10^{-09}	2.89×10^{-09}
$S_5(R_0, 0)$	3.60×10^{-06}	1.11×10^{-09}	1.12×10^{-09}	2.22×10^{-09}
$S_6\left(\frac{\sqrt{2}}{2}R_0, -\frac{\sqrt{2}}{2}R_0\right)$	5.10×10^{-06}	1.57×10^{-09}	1.58×10^{-09}	3.15×10^{-09}
$S_7(0, -R_0)$	5.70×10^{-06}	1.75×10^{-09}	1.77×10^{-09}	3.52×10^{-09}
$S_8\left(-\frac{\sqrt{2}}{2}R_0, -\frac{\sqrt{2}}{2}R_0\right)$	5.40×10^{-06}	1.66×10^{-09}	1.67×10^{-09}	3.33×10^{-09}

6 Summary

This paper studies the collision risk assessment of military and civil aircraft. The Event model has been improved to assess the probability of collision between a military training aircraft and a civil aircraft. Taking the circling training flight as an example, the simulation study shows that the collision risk of the military and civil aircraft is related to the starting circling position and the circling direction of the military aircraft. When there is no interval margin in the training airspace, the collision risk of military and civil aircraft is larger than the standard requirement. According to the simulation results, the collision risk of military and civil aircraft can be reduced in the following ways without improving the navigation accuracy and the pilot's ability.

1. In order to reduce the risk of collision, the starting point of the circling training should be set to be close to the side of the civil air route and circling in the opposite direction of the route.
2. The heading of the circling flight shall be determined according to the heading of the civil aircraft at the level. When the military aircraft is closest to the air route, its heading should be the same as that of the civil aviation aircraft.

3. According to the most demanding conditions, the collision risk can be met when the ideal circling path maintains a 7 km margin with the airspace boundary close to the air route. In the other directions, since it is not adjacent to the air route, the interval margin does not need to be too large. According to the calculation of the interval margin of 7 km in the approaching route and 2 km in other directions, the circling training airspace size can be set to 22 km × 17 km.

References

1. Reich PG (1966) Analysis of long range air traffic systems: separation standards. J Inst Navig 19:331–347
2. Brooker P (2003) Lateral collision risk in air traffic systems: a "Post-Reich" event model. J Navig 56:399–409
3. Meng X, Zhang P et al (2010) Aircraft collision risk assessment at intersecting air routes. J Beijing Univ Aeronaut Astronaut 36:1–5 (in Chinese)
4. Cao X, Zhang Z (2015) Cross route collision risk assessment based on improved Event model. J Civ Aviat Univ China 33:1–4 (in Chinese)
5. Zhang X, Zhu D, Yang C (2014) Safety assessment of RNP4 Chengdu-Lasa parallel routes. Sci Technol Eng 14:307–310 (in Chinese)