



Analysis and Experimental Research on Data Characteristics of BDS Positioning Error

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Abstract. Aiming at the problem that the integrity evaluation of satellite navigation and its augmentation system needs to analyze the characteristics of positioning error data, this paper proposes to use the time series analysis method to analyze the characteristics of the positioning error data of BDS single-point positioning and differential positioning from multiple angles of self-correlation, extremity and thick tail, which can provide the basis for deducing the integrity risk value by establishing an accurate positioning error distribution model. The result shows that the self-correlation of BDS differential positioning error is significantly lower than that of single-point positioning error. And the error of the two positioning methods in vertical and horizontal directions shows the characteristic of thick tail. Compared with the normal distribution, the single-point vertical positioning error has the most serious thick tail, reaching 420.3% and the differential horizontal positioning error has the lightest thick tail, only 22.9%. This indicates that the differential positioning method has better positioning performance and integrity.

Keywords: Satellite navigation · Positioning error · Data characteristic · Integrity evaluation

1 Introduction

With the sustained growth of global air transport business, civil aviation requires higher and higher positioning performance and integrity of satellite navigation system. Due to the influence of satellite clock error, ephemeris error, ionospheric delay, tropospheric delay and multipath effect, the positioning error of satellite navigation system may exceed the threshold set by International Civil Aviation Organization (ICAO). And the probability that the system can not detect the transfinite events which lead to the misleading information of the aircraft operation is called integrity risk [1]. Nowadays, with the rapid development of BeiDou Satellite Navigation System (BDS) and its augmentation system, the integrity evaluation of the system has been paid more and more attention. It is an effective evaluation method by establishing the distribution model of positioning error to derive integrity risk value. The data characteristics of positioning error are the important basis for choosing a reasonable distribution model. At present, most domestic literatures focus on the temporal and spatial characteristics of

BDS positioning error in the whole country. Few articles analyze the characteristics of BDS positioning error from the positioning domain. Paper [2] uses the continuous observation data of the domestic regional monitoring network to show that the positioning error is relatively stable in time scale and uneven in space scale. Paper [3] analyzes the main factors affecting the dynamic positioning error of BDS and evaluates the positioning performance of the system. The research methods of the above literature have reference significance for analyzing the data characteristics of BDS positioning error.

In this paper, the static single-point positioning error and differential positioning error of BDS are taken as the sample data. And the time series analysis method is used to study the positioning error characteristics of the two positioning methods from the angles of self-correlation, extremity and thick tail. The thick tail of positioning error is qualitatively and quantitatively verified based on the distribution statistics and kurtosis coefficients, respectively. The analysis results can provide a basis to establish an accurate error distribution model in the integrity evaluation of BDS.

2 The Error Sources of BDS Positioning

The origin of BDS positioning error can be divided into three sources as shown in Fig. 1.

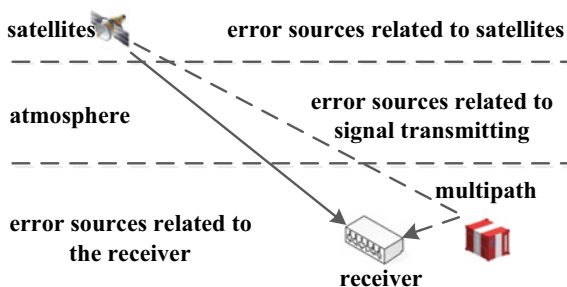


Fig. 1. Error sources of BDS positioning

- (1) Error related to the satellites: These errors mainly include satellite clock error and ephemeris error which are caused by the inability of ground monitoring station to make absolutely accurate measurements and predictions of satellite orbit and frequency drift of the satellite clock [4, 5]. The ephemeris error varies very slowly with time and has strong spatial and temporal correlation.
- (2) Error related to the signal transmitting: Signal transmitting from the satellite to the receiver needs to pass through the atmosphere. The influence of atmosphere on signal transmitting is atmospheric delay which is usually divided into ionospheric delay and tropospheric delay with high spatial correlation [6].

- (3) Error related to the receivers: Receivers may be subject to varying degrees of multipath effects and electromagnetic interference. This part of error also includes the noise of the receiver itself. Multipath error does not show normal distribution but sinusoidal motion with a period of several minutes as the satellite moves [7].

The above error can be classified into deviation and noise according to the characteristics of magnitude and speed of changing. The deviation varies slowly in a certain period of time. For example, the magnitude of ionospheric delay generally remains unchanged in a few seconds or even minutes. The noise changes quickly and is difficult to measure. But its mean, variance, correlation function and other statistical indicators in a certain period of time can be calculated.

3 Characteristic Analysis and Experimental Study of BDS Positioning Error

The data characteristics of positioning error are the guarantee of choosing a reasonable distribution model and improving the accuracy of BDS integrity risk evaluation. BDS has two positioning methods: Single-point positioning and differential positioning. The error of single-point positioning can reach to tens of meters without any compensation which is difficult to meet the positioning performance requirements of many occasions. Differential positioning uses the spatial and temporal correlation of satellite clock error, ephemeris error and atmospheric delay error to eliminate the measurement error of user stations and improve the positioning performance by calculating the error correction of reference stations with known positions. Formulas (1) and (2) are pseudo-range observer expressions $\rho_u^{(i)}$ and $\rho_{uc}^{(i)}$ for single-point positioning and differential positioning, respectively.

$$\rho_u^{(i)} = R_u^{(i)} + c(\Delta t_u - t) + g_u^{(i)} + I_u^{(i)} + T_u^{(i)} + \varepsilon_u^{(i)} \quad (1)$$

$$\rho_{uc}^{(i)} = R_u^{(i)} + c\Delta t_{ur} + g_{ur}^{(i)} + I_{ur}^{(i)} + T_{ur}^{(i)} + \varepsilon_{ur}^{(i)} \quad (2)$$

Among them, superscript i represents satellite number, subscript u, r represent user receiver and reference receiver, respectively. $R_u^{(i)}$ is the real distance from satellite i to the user receiver. $\Delta t_u, t, g_u^{(i)}, I_u^{(i)}, T_u^{(i)}, \varepsilon_u^{(i)}$ represent clock error of the user receiver, satellite clock error, ephemeris error, ionospheric error, tropospheric error, noise, respectively, $\rho_{uc}^{(i)}, g_{ur}^{(i)}, I_{ur}^{(i)}, T_{ur}^{(i)}, \varepsilon_{ur}^{(i)}$ represent the corresponding correction residues and satellite clock error can be eliminated completely. $g_{ur}^{(i)}, I_{ur}^{(i)}, T_{ur}^{(i)}$ are nearly to zero under short baseline. c is the speed of light. Differential positioning could eliminate the amount of common error.

Because the ionospheric delay error, tropospheric delay error and multipath effect error show non-zero mean and asymmetric non-Gaussian characteristics, the superposition of the error makes the positioning error projected to the position domain shows non-Gaussian characteristics.

3.1 Experimental Environment and Positioning Data Processing

The purpose of the experiment is to collect the sample data of single-point positioning and differential positioning. The experimental equipment includes four sets of BDS reference receivers and antennas, one data processor, one set of very high-frequency data broadcast (VDB) transmitter and antenna, one set of BDS test receiver and antenna and one test computer. Some equipment is shown in Fig. 2.



Fig. 2. Some experimental equipment

Four reference receiver antennas are erected at four calibrated corners on the top floor, respectively, and the test receiver antenna is placed at a calibrated measuring point. Data processor uses BDS data of four reference receivers to calculate differential correction. Test computer realizes single-point positioning and differential positioning by applying correction in positioning calculation. The schematic diagram of the whole experimental device is shown in Fig. 3. The actual position coordinates of the reference receiver antennas and the test receiver antenna are shown in Table 1.

The calculating period of the test computer is 1 s and 20,000 sample data expressed in WGS-84 rectangular coordinate system are collected according to the two methods of single-point positioning and differential positioning, respectively. Positioning error $(\Delta x, \Delta y, \Delta z)$ which includes the horizontal positioning error Δd and the height positioning error Δh is the difference between the calculated value and the true value of the measured point. The positioning error vector in station-origin coordinate system $(\Delta e, \Delta n, \Delta u)$ can be obtained by using the transformation Formula (3):

$$\begin{bmatrix} \Delta e \\ \Delta n \\ \Delta u \end{bmatrix} = \begin{bmatrix} -\sin \lambda & \cos \lambda & 0 \\ \sin \phi \cos \lambda & -\sin \phi \sin \lambda & \cos \phi \\ \cos \phi \cos \lambda & \cos \phi \sin \lambda & \sin \phi \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} \quad (3)$$

$\Delta e, \Delta n, \Delta u$ are eastward error, northward error, upward error, respectively. λ, ϕ is the longitude and latitude of the measured point. Thus, we can deduce the horizontal

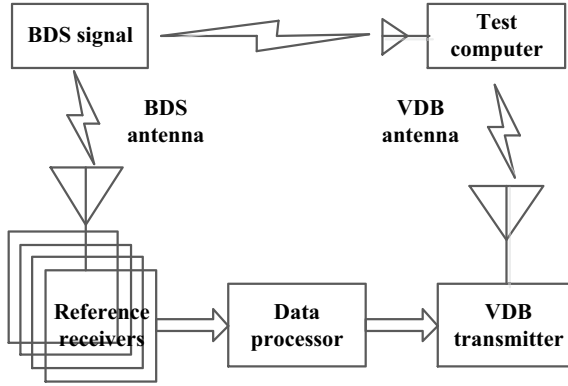


Fig. 3. Schematic diagram of the whole experimental device

Table 1. Position coordinates of the reference receiver antennas and test receiver antenna

Coordinates	x (m)	y (m)	z (m)
Reference point 1	-2,613,360.05	4,740,275.02	3,362,287.98
Reference point 2	-2,613,353.52	4,740,285.43	3,362,275.35
Reference point 3	-2,613,327.20	4,740,297.81	3,362,282.69
Reference point 4	-2,613,329.96	4,740,288.58	3,362,300.42
Measured point	-2,613,328.61	4,740,296.76	3,362,284.98

positioning error (Formula 4) and the height positioning error (Formula 5) from the positioning error vector:

$$\Delta d = \sqrt{\Delta e^2 + \Delta n^2} \tag{4}$$

$$\Delta h = \Delta u \tag{5}$$

Due to the space limit of the paper, only the vertical positioning error of the two positioning methods is shown in Figs. 4 and 5.

3.2 The Self-correlation of Positioning Error

Because there are some correlations among the slow variable error such as ionospheric delay error, tropospheric delay error and multipath effect error included in the source of positioning error, there will be some self-correlation in the positioning error. Before modeling the positioning error distribution in BDS integrity evaluation, it is necessary to determine that the error samples approximately meet the requirements of independent and identical distribution. The positioning error of satellite navigation system can be regarded as a set of time series. The stationarity (approximate independence) of data can be judged by calculating the self-correlation function of the error sequence with

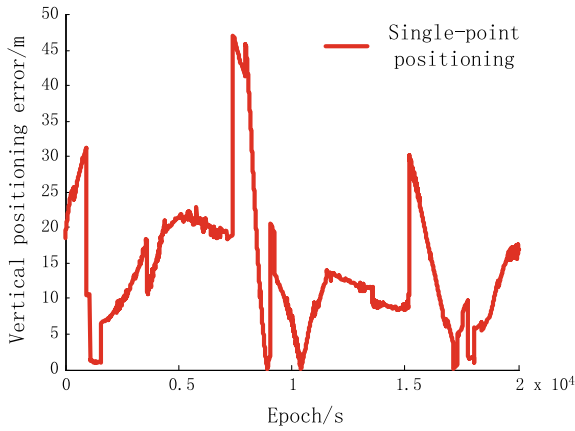


Fig. 4. Single-point positioning error sequence

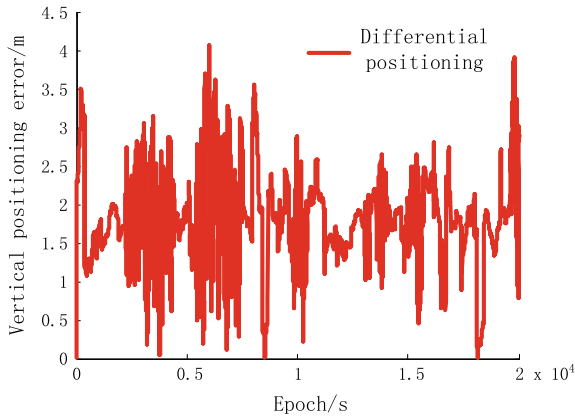


Fig. 5. Differential positioning error sequence

time series analysis method. Formula (6) is used to calculate the self-correlation function.

$$\rho_k = \frac{\sum_{i=1}^{n-k} (x_i - \bar{x})(x_{i+k} - \bar{x})}{\sum_{i=1}^n (x_i - \bar{x})^2} \quad (6)$$

Among them, k is the order of delay and n is the number of samples. If the value of self-correlation function is 1 at zeros and close to 0 at other positions, it indicates that the sequence is likely to be stationary and the distribution model of error sequence can be established directly. If the value of self-correlation function is obviously not zero at non-zeros, it indicates that the sequence is very likely to be non-stationary. This requires increasing the sampling interval to reduce the correlation before establishing

the distribution model. Since the characteristics of horizontal and vertical positioning errors are identical, this paper only calculates the self-correlation function of vertical positioning error of single-point positioning and differential positioning, as shown in Figs. 6 and 7.

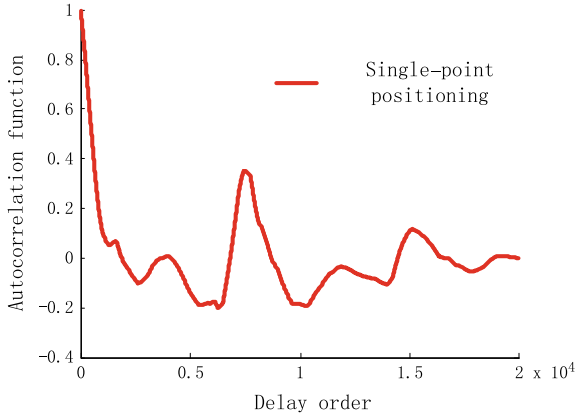


Fig. 6. Self-correlation function of single-point positioning error sequence

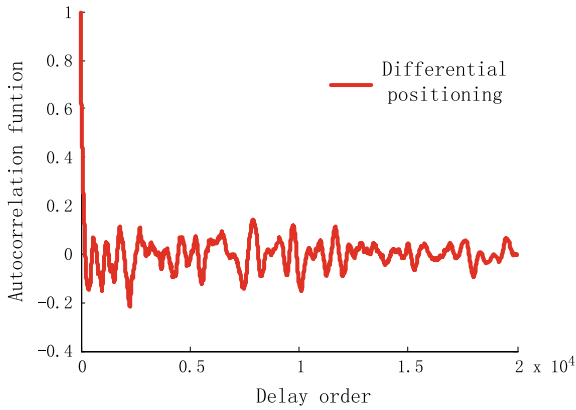


Fig. 7. Self-correlation function of differential positioning error sequence

It can be seen qualitatively from Figs. 6 and 7 that the self-correlation of the single-point positioning error sequence is high and the samples are not independent which makes it difficult to meet the requirements of modeling. However, the self-correlation of the differential positioning error sequence is low and the samples are nearly independent. So the distribution model can be directly established for integrity evaluation.

3.3 The Extremity of Positioning Error

According to the RTCA DO-253 document standard [8], the integrity risk value of satellite navigation ground-based augmentation system (GBAS) is 10^{-8} – 10^{-7} . If the frequency estimation is used to calculate the integrity risk, the required sample number should be 10^8 at least. Even if the sampling frequency is 1 Hz, the acquisition time will reach several years. So we can see that the event of positioning error exceeding the limit is extreme and it is difficult to appear in short-term observation experiments. Therefore, the integrity evaluation of the system needs to deduce the integrity risk value by establishing the distribution model of positioning error.

3.4 The Thick Tail of Positioning Error

The integrity risk evaluation of satellite navigation system mainly focuses on the characteristics of the distribution tail of positioning error as the occurrence of larger error. Therefore, whether the positioning error has a thick tail is the key to choose the distribution model. Compared with the normal distribution, if the tail descent rate of a given distribution is slower than that of the normal distribution, it can be judged that the distribution is thick-tailed. The hist function of MATLAB software can be used to make intuitive statistics on the distribution of vertical positioning error in Sect. 3.1 and the result is shown in Figs. 8 and 9.

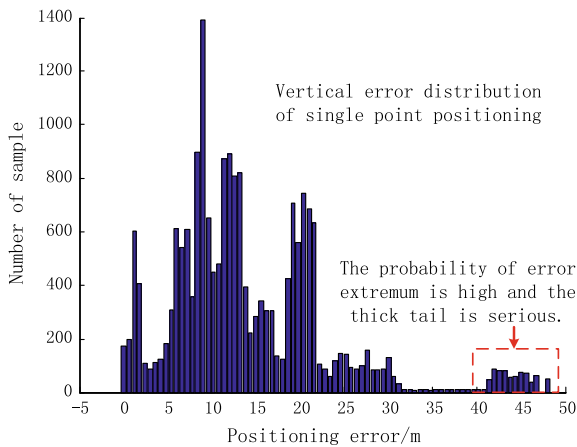


Fig. 8. Vertical error distribution of single-point positioning

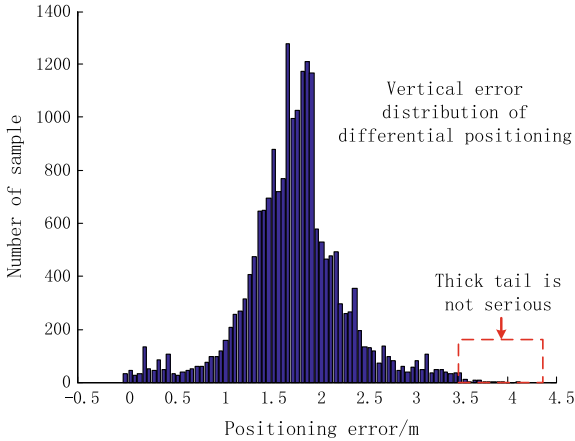


Fig. 9. Vertical error distribution of differential positioning

From Figs. 8 and 9, it can be qualitatively concluded that the thick tail of single-point positioning vertical error is serious and error exceeding event is easy to occur. While the tail of differential positioning vertical error is very light and the system has low integrity risk. Thick-tailed feature of the sample distribution can also be quantitatively judged by the kurtosis as shown in Formula (7):

$$\beta = \frac{\sum_{i=1}^n (x_i - \bar{x})^4}{(n - 1)s^2} \tag{7}$$

β is the kurtosis of sample. x , n are the number of sample. \bar{x} , s is the mean and variance, respectively. Kurtosis is a dimensionless variable and linear transformation does not change the kurtosis value of the variable. Kurtosis is used to characterize the degree of dispersion of the variable. The thicker the tail distribution is, the larger the kurtosis is. The kurtosis of the normal distribution is 3. For a given variables, if the kurtosis is greater than 3, the distribution of the variable is thick-tailed. $(\beta - 3)/3$ can be used to characterize the degree of thick tail of the variable distribution relative to the normal distribution. The result of calculating the kurtosis and the degree of thick tail of the experimental data in Sect. 3.1 is shown in Table 2.

Table 2. Kurtosis of positioning error

Error type	Kurtosis	Thick tail or not	The degree of thick tail (%)
Single-point vertical error	13.6107	Yes	420.3
Single-point horizontal error	3.9181	Yes	30.6
Differential vertical error	4.8195	Yes	60.7
Differential horizontal error	3.6868	Yes	22.9

As shown in Table 2, the positioning error of the two positioning methods has the characteristics of thick tail. Compared with the normal distribution, the single-point vertical positioning error has the highest thick tail, reaching 420.3% and the differential horizontal positioning error has the lowest thick tail, only 22.9%. It indicates that the integrity risk of single-point positioning is higher than that of differential positioning.

4 Conclusion

Establishing an accurate positioning error distribution model is the key to BDS integrity evaluation and the characteristics of positioning error are the basis for selecting the distribution model. Based on the analysis of positioning error sources, this paper studies the self-correlation, extremity and thick tail of positioning error with the measured data of BDS single-point positioning and differential positioning by using time series analysis method. The experimental results show that the self-correlation of single-point positioning error is high and it is difficult to directly establish the evaluation model. And the distribution tail is heavy which is prone to error exceeding the limit and increases the integrity risk of the system. Because of eliminating most of the common error, differential positioning error has low self-correlation and is easy to model. And evaluate the integrity of the system, and the distribution tail is light, which reduces the integrity risk of the system.

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